“Local Vetoes and Territorial Games”

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Local vetoes and territorial games

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Abstract

Territorial games are characterized by the interaction of the power to enact and the power to prevent. This broad class of games (which includes simple games) provides particularly appropriate ways to model public choices in which individuals also have civil rights.

What distinguishes a winning coalition in a territorial game, is that a winner might not be able to do everything -- but only some things. Although the complement of a winning coalition is a losing coalition, to the extent that the winning coalition's power is limited, the complement of that winning coalition has become invested with the power to constrain the outcome of the game. The outcomes which such a limited losing coalition (or indeed any coalition) can prevent (or veto) constitute its territory.

This terminology recognizes that the power to exclude is one of the primary aspects of private property and individual rights. Importantly, in many of the most commonly examined preference situations in the collective choice literature (e.g. convex preference sets or a sufficiently unattractive status quo), the introduction of the exclusionary aspect of property rights can only add to the core, and in this sense, can only increase stability. On the other hand, for games with NIMBY preferences ("not in my back yard") most often associated with public goods, the introduction of property rights can also be destabilizing.
Local vetoes and territorial games

The limited "localized" veto powers arising from property or "territorial" rights frequently prevent a choice situation from being modeled as a simple game— even if the underlying collective choice mechanism is a simple game such as majority rule. The purpose of this paper is to provide a conceptual framework for analyzing a broad class of such games, in which some "losers" have limited blocking power and correspondingly some "winners" have limits on their power: such games that are almost — but not quite — simple.

The games examined in this paper are "almost" simple for a number of reasons. First, each winning coalition has the power to choose an alternative and enact it as the outcome of the game. Second, no losing coalition has the power to enact any alternative. Third, every blocking coalition can only ensure that the default outcome (for example, a status quo) occurs.

Yet these games are "not quite" simple for two reasons mentioned above. First, a winning coalition may be restricted to choosing from only part of the universe of alternatives. Second, a losing coalition may have power to exclude (or bar) some alternatives, thus constraining the outcome of the game. Moreover, in the context of these games, the first implies the second — that is — when a winner’s power "to enact" is limited, the winner’s complement has gained the power "to prevent" or "to veto" those outcomes beyond the winner’s reach.

The outcomes forbidden to a limited winner constitute the "territory" of its complement, a limited loser. Indeed the territory of any coalition consists of those alternatives which it (by its sole action) can prevent from becoming the outcome of the game. This interaction, between the power to enact and the power to prevent, characterizes a territorial game.

1. An illustrative example.

Consider a residential subdivision constructed around a commonly owned area, as shown in Figure 1. The homeowners are discussing whether to build a swimming pool for their common use. Rocky ground limits the possible locations to the large oval area labeled Y (that includes its boundary). The default outcome (z) or "no pool" also is feasible, but cannot be shown in the figure. Suppose that a homeowners’ association collects monthly membership fees and administers various easements which affect use of the common area, to wit:

a. the association can use common funds to make capital improvements (such as a swimming pool) if a simple majority of homeowners approves:
b. common law property rights ensure that no construction can occur on an individual’s lot (including its boundary) without that individual’s consent;

c. the easements do not allow construction on the common area closer than fifty feet to an individual’s lot without that individual’s consent, unless a super majority of the homeowners approves (i.e. 4 out of 5);

d. construction in the rest of the common area, requires only approval by a simple majority.

Insert Figure 1 about here.

In the site selection game over \( X = Y \cup z \), each player is identified by the number of lot he or she owns. Player 2 is a “universal” loser who can enact no outcome and prevent no outcome. She has “no” enactment (or “effective”) set (denoted \( E_2 \)) and “no” territory (denoted \( T_2 \)). Both her effective set and her territory are empty. (The same conclusions hold for players 3, 4 and 5 operating unilaterally.) Player 1 also is a loser in that he can enact no outcome; however, by rule b, he can veto any site on his lot. Consequently, player 1 is a “limited” loser whose territory is the vertically shaded area (including that area’s entire boundary). His complement, coalition \((2,3,4,5)\), is a “limited” winner which can enact only the outcomes in the cross-hatched area and the horizontally shaded area. Coalition \((1,2,3)\), on the other hand, is a “universal” winner which can enact any feasible alternative, including retaining the default outcome of “no pool.”

Another “limited” loser is coalition \((1,2)\) with a territory consisting of both the vertically shaded area (by rule b) and the cross-hatched area (by rule c). Its complement \((3,4,5)\) is a “limited” winner that can enact only (1) default, or (2) those outcomes in the horizontally shaded area (including that area’s entire boundary). Notice that no individual has a veto over the cross-hatched area.

This game has no blocking coalitions. A complete listing of winners and losers as well as their territories and effective sets can be found in Table 1.

Insert Table 1 about here.

From our homeowners example, it is obvious that territorial games are appropriate models for a variety of spatial games, particularly ones involving property rights. But this class of games includes decision problems that are much more complex.
Table 1: Coalition Characterization

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<tr>
<th>Universal Winners</th>
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Effective sets.

The effective set (Es) of each universal winner consists of the entire set of feasible alternatives. The effective set of each limited loser or universal loser consists of the empty set. The effective set of coalition (2,3,4,5) consists of the union of the horizontally shaded area, the cross-hatched area, and default. The effective set of coalition (2,3,4), (2,3,5), (2,4,5) or (3,4,5) consists of the union of the horizontally shaded area and default.

Territories.

The territory (Ts) of each universal or limited winner consists of the entire set of feasible alternatives. The territory of each universal loser consists of the empty set. The territory of coalition (1) consists of the vertically shaded area. The territory of coalition (1,2), (1,3), (1,4) or (1,5) is the union of the vertically shaded area and the cross-hatched area.
Consider the game of locating a nuclear power plant in some municipality. An individual voter with property rights may prevent anyone from putting a power plant on his property. Yet the power company (which has a vote in the game of site selection) may already own the site where it wants to put the plant. Nonetheless, a municipality (representing a majority of its voters) may have the power to veto any site within its boundaries. Indeed, until recently, regulations of the U.S. Nuclear Regulatory Commission required that a nuclear power plant could not be constructed until emergency evacuation procedures had been developed and approved by the county in which the plant was to be located. By failing to approve the evacuation procedures, a county could (and frequently did) preserve the status quo (no power plant site). In such a federated system, seemingly innocuous and incidental regulations may drastically affect powers to preclude and enact, and this may be modeled as a territorial game.

Just as importantly, territorial games need not model only site selection and spatial contexts. In hiring decisions, for example, a specialist may have a veto over job candidates who are in her specialty, but no veto over generalists or candidates in other specialties. Consider also the power of a committee in a legislature to prevent any vote (by the legislature as a whole) on a measure within its purview.

2. Dominated outcomes.

Many game-theoretic models of choice rely upon some notion of domination. In a territorial game (as in a simple game), for every coalition that is not limited losing, one alternative dominates another (for a particular coalition) if and only if the first is strictly preferred to the second by every member of the coalition and the coalition has the power to enact the first.

For a limited losing coalition the situation is more complex. A limited losing coalition is not effective for any individual alternative, but can ensure that the outcome lies outside its territory. Hence, for alternative y to be dominated via limited losing coalition S, y must belong to the territory of S and S must strictly prefer everything outside its territory to y.

Insert Figure 2 about here.

Consider the homeowners example, redrawn in Figure 2 to show some of player 1's indifference contours. Let h₁ be player 1's house. Suppose that player 1 abhors swimming and believes that communal splashing will destroy the tranquility of his backyard. He prefers any swimming pool to be as far from his house as possible. For him, h₁ is the worst possible outcome. (He is indifferent between
outcomes equally far away). For player 1, location x is worse than any outcome outside of his lot and player 1 can veto x as a site. Hence location x is dominated via the coalition consisting of player 1.

Player 1 does not like location y either, but it is outside his territory. Consequently, y cannot be dominated via coalition (1). Location w is not much better. Although player 1 can veto w, his veto can not preclude y (which is worse than w). Hence w is not dominated via coalition (1).

3. Undominated outcomes.

Outcomes are usually considered stable if they are not dominated. In a territorial game (as in a simple game), the alternatives not dominated by a universal winner consist of its Pareto optimal set. The alternatives not dominated by a blocking coalition are those alternatives that are not strictly inferior to the status quo for any member of the blocking coalition. And of course, nothing is dominated by a universal loser.

For a limited winner in a territorial game, however, the undominated alternatives include not only the coalition’s Pareto optimal set, but also non-Pareto-optimal outcomes which are inferior to (and only inferior to) alternatives for which the coalition is not effective.

For a limited losing coalition, the analysis is more complex. First, a limited losing coalition can not dominate anything outside its territory. Second, when a limited losing coalition’s territory is “lower preference bounded,” (see below) the limited losing coalition can not dominate anything inside or outside its territory.

4. Lower preference bounding.

A coalition’s territory is lower preference bounded if an only if there is some alternative y outside the territory that is sufficiently unattractive to the coalition, in the sense that for every alternative x in its territory some member of the coalition finds y at least as bad as x. If T_S is lower preference bounded for one or more members of S, then T_S is lower preference bounded for S.

When the default outcome (such as a status quo) is sufficiently unattractive (but not necessarily everyone’s worst outcome), then neither blocking nor limited losing coalitions can dominate anything. With a zero-valued or unattractive default, both blockers and losers can be “ignored.”

Of additional interest is when individual preference sets are weakly convex in a convex, compact alternative space. 8 In two particular circumstances, lower preference bounding is pervasive: first, when a limited loser’s territory is “strictly in the interior” of the alternative space and second, when individual
preferences are a (decreasing) function of distance from bliss and a limited loser’s territory is “near” bliss for one of its members but “small” in relation to the alternative space.

Much economic and game-theoretic literature assumes either weakly convex preference sets or a zero-valued default, thereby fostering lower preference bounding.

5. The dominion.

The **dominion** of the game is the set of all feasible outcomes that are dominated via no limited losing coalition.

Of course, an outcome that does not belong to the territory of any limited loser is necessarily in the dominion. Notice that everything is in the dominion when the territories of all limited losing coalitions are lower preference bounded.

The homeowners example is redrawn in Figure 3. Each player wants the swimming pool as far as possible from his house. However their preferences for default differ. For player 1, the “no pool” outcome is the unique best alternative. Player 2 is indifferent between default and \( \lambda \) (the alternative in \( E(2,3,4) \) closest to player 2). Players 3, 4 and 5 definitely want a pool (just far away), so for them, “no pool” is the unique worst outcome. Consequently, for any limited losing coalition that includes 3, 4 or 5, its territory is lower preference bounded.

In the example, no outcome is dominated by any limited losing coalition that includes 3, 4 or 5. Limited loser (1) only dominates those outcomes as close to his home as (or closer to his home than) \( \alpha \). Limited loser (1,2) only dominates those outcomes both strictly closer to 1’s home than \( \delta \) and strictly closer to 2’s home than \( \gamma \) and \( \lambda \). The dominion is the cross-hatched area plus default.

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Insert Figure 3 about here.

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6. The heartland.

The **heartland** of the game is defined to be the set of all feasible outcomes that are not dominated via any (universal or limited) winning coalition.

Consider a particular simple game and a particular territorial game, played over the same outcomes, with identical individual preference configurations. We say the two games are associated if and only if the winners, blockers and losers are the same for both games. (A winner in the simple game
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Territorial games

can be either a universal or limited winner in the associated territorial game.) Notice that a territorial game is associated with exactly one simple game, but a simple game may be associated with more than one territorial game.

Obviously (but importantly) the heartland of a territorial game contains the heartland of the associated simple game.

Consider the territorial game shown in Figure 3. The heartland of that territorial game consists of part of the boundary of the feasible set \([\alpha, \lambda] \), whereas the heartland of the associated simple game is a \(\alpha^{12} \).

7. The core.

The core of the game is the set of all feasible outcomes that are not dominated via any coalition. For the homeowners example (Figure 3), the core of the territorial game consists of two disjoint portions of the boundary of the feasible set, \((\alpha, \gamma] \cup [5, \lambda] \). The core of the associated simple game is the single point \(\alpha\).

Notice that \(\alpha\) is not contained in the core of the territorial game, even though it is in the heartland of the territorial game. This divergence between equilibria in the two associated games occurs because of the influence of limited losers.

The property of lower preference bounding identifies situations in which such total divergence of equilibria can not develop. The core of a territorial game in which the territory of every limited losing coalition is lower preference bounded contains the core of the associated simple game.

In a territorial game, if default is sufficiently unattractive, then the core equals the heartland.

8. The core and NIMBY preferences.

Strictly speaking, NIMBY requires that everything in a coalition's territory is worse than everything outside the territory — that is — all of a coalition's "worse" outcomes are in its territory. Domination by limited losers will occur. By modifying territories in Figure 3, one can easily demonstrate that with NIMBY preferences the existence of territories can either increase stability, alter stability or decrease stability when compared to the associated simple game.

Consider, however, the limiting case in which each non-default alternative is in the territory of at least one individual who has NIMBY preferences. It is straightforward to show that the core equals the default alternative.

In simple games with concave preference sets ("the farther from my house the better"), Laing and Slotznick (1990) found that stability would be increased if individuals found all alternatives beyond some indifference contour to be equivalent. Such a preference horizon recalls the admonition: "out of sight, out of mind." If enough feasible alternatives are beyond this horizon (this set of alternatives being the "frontier") then a core can be guaranteed. Essentially the frontier becomes a dumping ground for undesirables.

A similar phenomenon can arise in a concave territorial game with respect to the individual player's territories. Let the territorial frontier be the region between a coalition's preference horizon and its territorial boundary. Define the no man's land to be the set of alternatives outside every individual's territory. Suppose that every individual has a territory and concave preferences with a preference horizon within his territory.13 If these territories do not overlap (except on their boundaries), then the domain will consist of no man's land (including the common ground), the individuals' territorial boundaries and the adjacent territorial frontiers. Because of the assumed universal indifference beyond territorial boundaries, these outcomes are also not dominated with respect to winning and blocking coalitions — and are included in the core.

In light of the foregoing, we note that the Three Mile Island nuclear power plant is located in the middle of a river at the corner of three counties. Consider also the power plant at Four Corners, which serves Los Angeles but is located at the corner of New Mexico, Arizona, Utah and Colorado.

10. Conclusion.

This paper has confronted a variety of frequently encountered collective decision problems that can not be modeled in terms of simple games, and presented a tractable way to classify and represent these situations. When these problems differ from simple games it is because winning coalitions may be less than omnipotent — their complementary losers gaining an implicit power to constrain and veto. This interaction between the power to enact and the power to prevent characterizes these territorial contexts. Despite many unresolved issues, territorial games seem the most appropriate model for exploring NIMBY contexts, such as the siting of obnoxious facilities through collective decision making, in those situations where individuals enjoy certain property rights.


Slotnick, B. (1995b). Territorial games with sidepayments, both voluntary and mandatory. (working paper)
A more mathematically detailed discussion of these issues (including appropriate proofs) can be found in Slotnick (1995a). The ideas were first presented in a preliminary fashion at the annual meetings of the Public Choice Society, held in Tucson, Arizona on March 27-29, 1987. As usual the prods and prickles of Jim Laing have been invaluable. The comments of Ken Koford were also helpful. Additional comments are welcome and should be sent to P.O. Box 23, Mt. Gretna, PA 17064.

The territory of a winner (whether or not limited) includes all alternatives. (Notice that a winner, by choosing to enact a particular outcome, precludes every other outcome, and that every winner can choose between at least two outcomes.) The territory of a blocking coalition is everything but default. The territory of a "universal" loser (a loser whose complement can enact anything) is the empty set.

This and the other games discussed in this paper are assumed to be proper, decisive, essential and superadditive. Properness ensures that when a set of disjoint coalitions forms and each chooses its strategy, at most one outcome can occur. Decisiveness ensures that when a set of disjoint coalitions forms and each chooses its strategy, at least one outcome occurs. The essential property requires that there is at least one winning coalition, and at least one outcome other than default.

Using this notation, a territorial game is proper if and only if for every partition $S$ of the set of players and for all $S, S' \in S$, both $E_S \cap T_{S'} = \emptyset$ and $T_S \cap E_{S'} = \emptyset$. A territorial game is superadditive if and only if given $S \subseteq S'$, both $E_S \subseteq E_{S'}$ and $T_S \subseteq T_{S'}$.

Eminent domain and other issues concerning sidepayments in territorial games are addressed in Slotnick (1995b). Laws of eminent domain permit a governmental (or quasi-governmental) body to "take" property rights, but require that the taker must pay market value for the rights. Eminent domain, in effect exchanges the veto power of a person's property right for a mandated sidepayment. This paper focuses on the more elementary issues concerning territorial games without sidepayments.

The author wishes to thank Ken Koford for this example.

This paper employs a dominance relation (using strict preferences), which emphasizes choosing a strategy, rather than choosing a single outcome.

The preference relation $R$ is assumed to be reflexive, complete, transitive and continuous.

One can view these alternatives that are not in any limited loser's territory (i.e. no one has "property" rights over these alternatives) as the "commons ground" or the "commons."

Player 1 is indifferent between $a$ and the point on the boundary between his lot (territory) and the common area that is closest to his home. The indifference curve is shown as a dashed line. Alternative $a$ is a point on the boundary of $Y$ that intersects that indifference curve.

Player 1 is indifferent between $b$ (a point on the boundary of $Y$) and the point on the boundary of $E(3,4,5)$ that is closest to his home. Player 2 is indifferent between $y$ and $\lambda$ (both on the boundary of $Y$). $\lambda$ is the point on the boundary of $E(3,4,5)$ that is closest to her home.

By construction, $\alpha$ is the point on the boundary of $Y$ farthest from the home of player 4.
The common ground is a subset of the no man’s land. In addition, if every alternative that is in any limited loser’s territory is in some individual’s territory, then the common ground is identical to the no man’s land. The commons, because it is no man’s land and no one can be excluded can be expected to become a repository for facilities and outcomes with negative externalities. Compare the fabled “tragedy of the commons.”