“Territorial Games with Sidepayments, both Voluntary and Mandatory”

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Abstract

This paper examines the affect of sidepayments, both voluntary and mandatory, on the core solution in the class of collective decision problems that can be classified as territorial games. In a territorial game, a winner's power may be limited, so that it can enact some but not all things. Consequently, its complement (a "limited" loser) has power to preclude those things a "limited" winner cannot enact. The power to preclude an outcome gives an agent a "local" veto over that particular outcome. For this paper, each individual agent's preferences over outcomes are assumed to be strictly increasing over that portion of his own wealth available for sidepayments, given any fixed decision about the other aspects of the outcome.

The results indicate that an outcome involving sidepayments belongs to the core solution only if every individual agent receiving compensation has (1) a local veto over that outcome, (2) a right to a minimum mandatory sidepayment at that outcome, or (3) a veto over the distribution of a mandatory sidepayment for which it is a potential recipient at the outcome. Moreover, when considering only voluntary sidepayments, if no individual has a local veto, then an outcome belongs to the core of the game only if it also belongs to the core of the game without sidepayments.

The paper investigates also the effect of substituting mandatory compensation (a damage rule) for a local veto with the right to negotiate voluntary sidepayments (a property rule). Such a rule modification at a single "location" tends to extract wealth from the previously propertied, regardless of the "location" of the actually chosen outcome.
Territorial games without sidepayments are described and explored in Slotnick (1995). The distinguishing feature of a territorial game is that a winning coalition might not be able to do everything, but only some things. Naturally, when a winner's power to enact is limited, its complement (although a loser) manifests a corresponding power to prevent. This power of prevention is a "localized" power to veto those outcomes beyond the grasp of the limited winner. Those outcomes which a coalition can preclude (whether as a winner, blocker or loser) are its territory. This paper continues that exploration by examining the effects on equilibrium (as embodied in the core solution) when allowing the possibility of voluntary and mandatory sidepayments in a territorial game.

This paper employs the methodology of Laing and Slotnick's (1994) investigation of simple games with voluntary sidepayments, deriving parallel results for the larger class of territorial games. The methodology is extended to incorporate results for mandatory sidepayments.

The local veto of a territorial game appears to embody many of the exclusionary aspects of private property rights. Without the provision for sidepayments, however, a market for such property rights can not enter into the model. When sidepayments are allowed, a person may trade her property right for money — in other words, she may join a winning coalition in exchange for a sidepayment.

Such a property rule that allows voluntary sidepayments can be compared to and contrasted with a damage rule that guarantees a mandatory sidepayment in lieu of the right to preclude (see Calabresi and Melamed, 1972; cf. Coase, 1960). A local rule change, from property rule to damage rule, will affect wealth throughout the system. Depending on the decision context, either rule could create greater stability than the other.

Section 1 describes territorial games with sidepayments and the decision context in which they will be investigated. Section 2 examines preference assumptions and domination. Section 3 presents the main results of the paper. Section 4 explores the effect of substituting mandatory compensation for a local veto right.

1. The decision context.

The section describes an outcome space designed to study sidepayments, extends the "collegial" concepts from simple to territorial games and "adjusts" territorial games to accommodate sidepayments.
1.1. Outcomes.

In this paper, outcomes are represented in a way that facilitates comparing games with and without sidepayments. Let $X$ denote the set of available decision alternatives without regard to sidepayments. $X$ contains at least two outcomes, one of which is a default outcome (denoted as $z$). Let $M$ denote the set of positive real valued vectors $m = (m_1, \ldots, m_N)$. Essentially, $m$ is a decision about the distribution of wealth (represented by a set of private goods) across the $n$ agents, where $m_i$ denotes the net monetary value of the wealth of agent $i \in N$. More particularly, the vector $m \in M$ denotes the final distribution of that portion of material wealth that has been segregated from $X$ and available for sidepayments.

Now we can represent an outcome in a territorial game with sidepayments by the pair $(x, m)$, with "alternative" $x \in X$ and wealth distribution $m \in M$. When no sidepayments are made at location $x$, we say that each agent $i$ keeps his endowment ("endowed wealth"), denoted $e_i(x)$, and that $m = (e_1(x), \ldots, e_N(x)) = e(x)$. More succinctly, $e_i(x)$ denotes the net endowed wealth of agent $i \in N$ in the set of segregated private goods if $x$ is chosen. An individual's endowed wealth is finite and may vary with $x$; decisions about $x$ can affect the endowed wealth of the various agents. The results of this paper apply whether or not endowed wealth is constant.

1.2. Winning, blocking and losing in territorial games.

In the territorial game (without sidepayments) over $X$, every coalition is winning, blocking or losing. A winning coalition can choose (i.e. enact) the outcome of the game from its effective set. A blocking coalition can effect (i.e. enforce) only the default outcome. A losing coalition can enforce no outcome. However a losing coalition automatically precludes any outcomes for which its complement (a winning coalition) is not effective. The territorial games discussed in this paper are assumed to be essential, superadditive, decisive and proper.

A simple game is said to be collegial if there exists a set of players (the collegium) who belong to every winning coalition. In parallel fashion, for a territorial game, the local collegium at some $x \in X$ is denoted $S^*(x)$ and defined to be that subset of agents who belong to every winning coalition that can enact $x$. A game is locally collegial at $x$ if and only if $S^*(x)$ is not empty; otherwise it is locally noncollegial. In other words, a game is locally collegial at $x$ if some individual acting unilaterally can prevent $x$ from being enacted. In this sense every member of the local collegium is said to have a veto over $x$. This does not guarantee that he or she can enforce the default outcome. In fact, an individual constitutes a solo blocking coalition only if that individual is a member of the "universal collegium," i.e. if she is in the local collegium for all $x \in X$. 
1.3. Adjustments to allow sidepayments.

We define winning, blocking and losing coalitions of territorial games with sidepayments in terms of their ability to enact and preclude alternatives \( x \) in the set \( X \). In addition, a coalition also can redistribute its own total wealth\(^6\) (in \( m \)) via voluntary sidepayments. We assume that, for any given \( x \in X \), there are no transaction costs in the transfer of sidepayments. Thus any coalition, if it chooses, can enact any alternative in its effective set and redistribute its own wealth among its own members regardless of any actions of agents not in the coalition. Also, if it wishes, a coalition may offer some of its wealth to other agents not in the coalition.

Sidepayments need not be voluntary. Suppose that a winning coalition \( S \) is given the power to enact \( x \) but only if it pays to a set of agents \( S' \) an exogenously determined fee, denoted \( f(x:S\rightarrow S') \geq 0 \). In other words, \( f(x:S\rightarrow S') \) is the required payment made by coalition \( S \) to coalition \( S' \) at \( x \). Such a fee, to enact or "use" alternative \( x \), is defined to be a mandatory sidepayment. This mandatory sidepayment is determined exogenously to the collective choice over \( X \). For example, many proposed rules for locating a radioactive waste disposal facility would require that the facility's developer pay a fee to the affected locality.

Mandatory sidepayments in this paper are coupled with the ability of agents to voluntarily transfer their assets — including those assets received via the mandatory sidepayments. Let \( f_j(x:S\rightarrow S') \geq 0 \) be the payment that agent \( j \in S' \) receives under some distribution of \( f(x:S\rightarrow S') \). Then, a winning coalition \( S \) can enact any \( x \) in its effective set and distribute its endowment so that its total wealth is no greater than (1) its total endowment, plus (2) that portion of the mandated sidepayments paid to its members less (3) mandated sidepayments it must pay to others. Consequently, a winning coalition, \( S \), would be able to enact \( x \) only if (1) \( x \) is in its effective set, and (2) it has (or gets) enough wealth to make the sidepayments that must be made to agents outside \( S \).

The mandated sidepayment \( f(x:S\rightarrow S') \) may be a fair (or "market") value as determined by a referee or other "objective" observer: for example, a jury's monetary evaluation of actual physical injuries. Alternatively, the mandated sidepayment may attempt to approximate preferences: for example, a jury's award for "pain and suffering." Such an award \( f(x:S\rightarrow S') \) may focus entirely on damages to \( S' \) or may depend on the motives or character of the agent \( S \) who inflicted the harm. At the other extreme, \( f(x:S\rightarrow S') \) may be a flat fee or otherwise have no relation to the identity of \( S \), the preferences of \( S' \) or the unique characteristics of \( x \).

A mandatory sidepayment will be defined as path independent when a set of agents \( S' \) receives a "usage" fee at \( x \) regardless of which winning coalition enacts \( x \): \( f(x\rightarrow S') = f(x:S\rightarrow S') = k > 0 \) for all winning \( S \). Under the assumption of path independence, the rights and entitlements of a set of agents do not vary with its "opposition." As a consequence of this, we can identify all such sets of agents receiving fees at \( x \), without regard to which coalitions actually form at \( x \). Path independence of every mandatory sidepayment is assumed throughout this paper.
1.4. Distribution of a mandatory sidepayment within a coalition.

For purposes of this paper, we need only ask two questions concerning the distribution of a coalition's path independent usage fee $f(x;\to S)$.

First, does the allocation mechanism entitle any agent to a minimum payment? Define $f^*_j(x;\to S)$ to be the minimum sidepayment that must be allocated from $f(x;\to S)$ to agent $j \in S$.

Second, does the method of distributing $f(x;\to S)$ grant any agent in $S$ a veto power over the distribution at $x$? That is, is the allocation mechanism locally collegial? Define $S^*[f(x;\to S)]$ to be the set of agents with veto power over the distribution of $f(x;\to S)$. If no sidepayment is mandated to the set of agents $S$ at $x$, we say $f(x;\to S) = 0$, and define $S^*[f(x;\to S)]$ to be empty. Notice $S^*[f(x;\to S)]$ may contain agents outside $S$.

Of course, an agent could enjoy both a veto and a minimum entitlement.

1.5. Games of confiscation: endogenously determined taxes and subsidies.

In contrast to mandatory compensation, consider confiscation. Suppose that a winning coalition can determine not only $x$, but also the final allocation $m$ for all agents. In other words, a winning coalition can establish taxes and subsidies. Under these conditions, any winning coalition could enact any alternative $x$ in its effective set and also expropriate other agents' wealth for the coalition's private benefit. In fact, under the assumptions about preferences used in this paper, we can show that for games of confiscation (1) an outcome $(x, m)$ is in the core only if the game is locally collegial at $x$, and (2) the core solution permits the local collegium to acquire all the confiscatable wealth available at $x$. As a corollary, for games of confiscation, if there is no local collegium then the core is empty.

To the extent that a local veto represents private property exclusionary rights, these results imply that when a government has the authority to tax all, (1) wealth will gravitate towards those holding property and (2) if the government operates under "majority" rule, private property is necessary to prevent public chaos.

2. Preferences and dominance.

The preferences of each agent are represented by an asymmetric strict preference relation over the outcomes $(x, m) \in X \times M$. Each agent's preferences over wealth ($M$) are assumed to be monotonically increasing.
in his own wealth ($m_i$) and preferentially independent of $X$. In other words, at a given location $x$, he prefers one outcome $(x,m)$ to another $(x,m')$ if and only if his wealth in the first ($m_i$) is greater than his wealth in the second ($m_i'$). Furthermore, each agent $i \in N$ is assumed to regard (for every fixed $x \in X$) the outcomes $(x,m)$ and $(x,m')$ as being equivalent if $m_i = m_i'$. A coalition strictly prefers one outcome $(x,m)$ to another $(x',m')$ if and only if every member of the coalition strictly prefers the first to the second.

We say that an outcome $(x,m)$ is dominated via coalition $S$ if $S$ can choose a strategy such that, whatever agents not in $S$ decide to do, some outcome will be reached that $S$ strictly prefers to $(x,m)$.

In particular, $(x,m)$ is dominated via a blocking coalition if and only if it can enact default ($z$) and distribute its wealth among its members such that every distribution of the wealth of the other agents (not in the blocking coalition) leads to an outcome $(z,m')$ that the blocking coalition strictly prefers to $(x,m)$.

Moreover, any outcome $(x,m)$ is dominated via a winning coalition if and only if (A) it can (1) enact some $x'$ in its effective set, (2) choose a strategy with respect to distribution of usage fees at $x'$ and (3) choose a strategy for distributing its allocatable wealth among its members, such that (B) every distribution of the allocatable wealth of the other agents (not in the winning coalition) leads to an outcome $(x',m')$ that the winning coalition strictly prefers to $(x,m)$.

Any outcome $(x,m)$ is dominated via a losing coalition if it can (1) choose a strategy with respect to distribution of usage fees and (2) choose a strategy for distributing its allocatable wealth among its members, such that every choice by its complement (a winning coalition) of any $x'$ in that winning coalition’s effective set and every distribution of the allocatable wealth of that winning coalition at $x'$ leads to an outcome that the losing coalition strictly prefers to $(x,m)$.

Now we can define the set of all feasible outcomes that are not dominated via any coalition to be the core of the game.

3. The core in territorial games with sidepayments.

The results in this section establish necessary conditions on the transfer of sidepayments if the outcome is to belong to the core. 7

**Lemma 1. (Conservation of wealth).** In any territorial game with sidepayments, any outcome which throws away wealth will be dominated.
In other words, by Lemma 1, if the sum of the agents’ wealth is less than the sum of their endowments, the outcome will be dominated. At the same time, to be feasible, the total wealth available at an outcome can not be greater than the sum of the individual endowments.

Lemma 2. (Accumulation of wealth – How much?). In any territorial game with sidepayments, an outcome \((x, m)\) will be dominated if at that outcome some agent who can not veto \(x\) has accumulated more than the sum of

(1) her endowment,

(2) her minimum mandated sidepayments and

(3) the mandated sidepayments of which she is to receive a portion and over the distribution of which she has a veto.\(^8\)

Lemma 2 begins to identify “effectual” agents who (at equilibrium) can accumulate wealth beyond their endowments. An agent will be deemed effectual at \(x\) if he or she (1) has a local veto, or (2) is entitled to receive a minimum distribution from a mandated sidepayment, or (3) belongs to a set of agents designated to receive a mandated sidepayment (and has a veto over the distribution of that sidepayment). Denote the agent as ineffectual at \(x\) if he or she meets none of these three criteria.

Lemma 3. (Accumulation of wealth – Who?). In any territorial game with sidepayments, an outcome is dominated if at that outcome any ineffectual agent has accumulated wealth beyond that agent’s endowment.

In other words, only effectual agents can accumulate additional wealth. Obviously, one agent can accumulate wealth only at the expense of another. Theorem 1 follows immediately.

Theorem 1. (Transfer of wealth). In a territorial game, an outcome \((x, m)\) that involves any transfer of wealth through sidepayments belongs to the core only if at least one of these conditions obtains:

(1) The game is locally collegial at \(x\) (i.e. someone has a veto at \(x\)).

(2) A sidepayment is mandated at \(x\) and its distribution rule requires minimum payments to at least one individual agent.

(3) A sidepayment is mandated at \(x\) to a set of agents \(S\) and its distribution is determined by a collegium, at least one of whom is in \(S\).

Furthermore, any agent receiving a sidepayment in the core must be effectual.
As with simple games, when only voluntary sidepayments are permitted in territorial game, any feasible outcome \((x,e(x))\) in which each agent keeps his own endowment belongs to the core, only if the outcome \(x\) also is a core solution to the associated game without sidepayments.\(^9\)

**Theorem 2.** If no one has a veto at \(x\), then the outcome \((x,m)\) belongs to the core only if (1) each agent keeps his endowment \((m=e(x))\) and (2) this outcome is also in the core of the associated game without sidepayments.

The ability of an individual or group to forego (exchange) his or their local veto for a voluntary sidepayment enables a "market" to develop for such preclusive veto rights. Theorem 2 implies that there is "no market" for such preclusive rights when they are able to be exercised only in concert with others – because the market will only clear at zero. A partnership, for example, can be bought for nothing (or practically nothing) if a majority of the partners has the right to sell the entire partnership’s assets and to deny any particular partner a share of the proceeds. By Theorem 1(a), there can be a market for private property (tertiary) rights only to the extent that at least one individual agent has a veto with respect to the property (territory).

For simple games these Theorems imply that there is no market for (noncollegial) votes (Laing and Slotznick, 1994). In territorial games when a city (or other collectivity) is given a local veto, no outcome in the core will transfer any wealth to an individual voter of that city. The only transfers might be to the city administration (or individual administrators) which can veto the sale of the right.

By Lemma 3, even a mandated sidepayment to a collectivity will only channel funds to the agency administrating the distribution of the funds – unless individuals are specifically entitled to a minimum distribution. Given these results, it is not surprising that much of the funds in any government subsidy or entitlement program go towards administration and that graft appears endemic. Indeed it is amazing that any of the funds reach the intended recipients.

In the international arena, these results suggest that (1) foreign aid will be given by one country only to an agent (or other country) who can veto the first country’s foreign policy objectives, (2) only marginal amounts of truly humanitarian aid will be given and (3) the foreign aid that is given will not reach the recipient country’s general population, but rather individuals or bureaucracies (including local militias and their warlords) with local veto power. Consider disaster relief in Somalia or Bosnia-Herzegovina. Additionally, these results imply that the more democratic the government, the less foreign aid it will get.

Further delineation requires an additional preference assumption: that the strict preference ordering of each agent over the feasible outcomes is transitive.
Theorem 3. Outcome \((x,m)\) belongs to the core of the game with sidepayments only if

1. All of the following coalitions are effectual:
   a. Every blocking coalition that strictly prefers the status quo \(z\) to \(x\) (without sidepayments).
   b. Every limited winning coalition \(S\) that dominates \(x\) by an outcome \(x'\) (without sidepayments).
   c. Every limited losing coalition that dominates \(x\).
2. All of the following receive sidepayments:
   a. Some effectual member of any winning coalition \(S\) that dominates \(x\) via some \(x'\) (without sidepayments).
   b. Every member of the local collegium at \(x\) who strictly prefers every alternative outside his territory to \(x\) (without sidepayments).
   c. No ineffectual agent.

Part 2(b) of Theorem 3 has particular implications for NIMBY ("not in my backyard") preferences. These preferences describe a limited loser who likes everything outside his territory better than anything inside. They are particularly pervasive in situations involving public goods, such as choosing a hazardous waste disposal site. The "backyard" of a NIMBY preferred person will not be in the core of the territorial game without sidepayments (Slotnick, 1995a). By Theorem 3, outcomes in that "backyard" may become part of the core in the game with sidepayments, but only if the NIMBY preferred owner of the "backyard" is paid off.

4. Property rights versus damage rules.

The foregoing results suggest that for certain purposes, the ability to receive sidepayments for a local veto (a property right or property rule) is similar to a local entitlement to distribution (or damage rule). Let us consider briefly what happens when the property right of a "limited" losing coalition \(S\) at a particular \(x\) is exchanged for a path independent damage (or compensation) rule that is independent of the winning coalition enacting \(x\). Both the limited loser and its complement, a "limited" winner, are affected.

4.1. The effect on the limited loser.

Limited loser \(S\) might (but need not) dominate fewer alternatives.
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Under a property rule, suppose limited loser S can veto x, so it can veto all wealth distributions (m) that could be associated with x (i.e. at x, S can veto every \((x,m)\)). Under a damage rule, S can still "veto" those wealth distributions at x that transfer to S less than the mandated sidepayment (i.e. at x, S can veto any \((x,m)\) for which \(\Sigma_{i \in S} m_i < eg(x) + f(x \rightarrow S)\)). Obviously, in changing from a property rule to a damage rule, there are fewer distributions at x which loser S can veto. Hence there are fewer outcomes in its territory and fewer outcomes at x that S could dominate (depending of course on S's preferences).

Less obviously, there are fewer outcomes outside x that might be dominated by limited loser S. Consider first: despite the rule change at x, S may still be able to veto outcomes at other locations (say \(x'\)). Suppose that S did not like the outcomes at \(x'\), and would have vetoed them when S also had a local veto at \(x\). But suppose further that S prefers these outcomes (at \(x'\)) to the outcomes (at \(x\)) that S can no longer veto under a damage rule. By definition, these outcomes at \(x'\) are no longer dominated via limited loser S.

4.2. The effect on a limited winner.

The complement of limited loser S (limited winner NS) might (but need not) dominate more outcomes.

Obviously, after changing from a property rule to a damage rule at x, limited winner NS will not pay S more than \(f(x \rightarrow S)\). Consider the outcomes at x with wealth distributions that require a greater payment to S. If such outcomes had not been dominated before by an outcome at another location (not x), they will be dominated for NS by an outcome at x that gives S exactly \(f(x \rightarrow S)\).

Less obviously, there are more outcomes outside x that might be dominated by limited winner NS. First, consider the outcomes at x that winner NS can enact after (but not before) the rule change. If NS prefers these outcomes (at \(x\)) to some outcomes at another location (say \(x'\)) then the outcomes (at \(x'\)) are now dominated via NS. (Notice that outcomes at \(x'\) might have been dominated prior to the rule change by outcomes not at x.)

4.3. The combined effects.

As we see, the rule change at x has two effects. Loser S might dominate fewer alternatives. Its complement, winner NS, might dominate more alternatives. The net result of these two effects of a rule change.
at x is that for various alternatives in the core, NS will insist on a greater share of the wealth and S will be willing (or be forced) to accept less, both at x and elsewhere.

Importantly, a core can not be guaranteed under either the property rule or the damage rule. In fact, the rule change could increase or decrease the size of the core. A change from a property rule to a damage rule could create a core where none existed, or destroy an existing core. In other words, regardless of the wealth effect noted above, the rule change at x can be either stabilizing or destabilizing for the entire system.

The most surprising effect of a rule change at x is the possible increase in the number of core alternatives outside x. This effect is primarily confined to NIMBY situations where agents prefer everything outside their territory to anything inside the territory. For example, suppose a state has refused to permit any hazardous waste dumps. Suppose further that a new federal law gives a developer the right to put a hazardous waste dump at certain sites which happen to be near major cities in the state (although the developer must pay a usage fee), but permits the state to veto other sites. It seems fairly obvious that in many cases, the state and developer would find a mutually advantageous site (and payment) remote from any major city.

6. Conclusion.

This essay explored the core solution in territorial games with voluntary or mandatory sidepayments, in which each agent, given any particular alternative in the underlying territorial game, wants to maximize his wealth. At a core solution, a sidepayment can be made only to an agent with (1) a local veto, (2) a mandated minimum sidepayment or (3) a veto over the distribution of a mandatory sidepayment for which he is a potential recipient. Some such sidepayment must be made if the core of the game with voluntary sidepayments is to be larger than the core of the underlying territorial game without sidepayments.

Changing a local veto (property right) to a mandatory sidepayment (right to compensation) may increase the core, decrease it, alter it or leave it unchanged. The tendency however, is to transfer wealth from the previously propertied to other agents throughout the economy.

In sum, neither sidepayments in general, nor confiscating property rights (even if replaced with compensation) are panaceas for unstable, defaulting or otherwise problematic decision contexts.
Lemma 2 (repeated): In any territorial game with sidepayments, the feasible outcome \((x,m)\) in the set of feasible outcomes \(Y \subseteq X \times M\) is dominated if there exists any agent \(j \in N\) who has no local veto and for whom
\[
m_j > \phi(x) + \sum_{S \subseteq j} \text{s.t. } j \notin S^* \left[ \sum_{j \notin S^*} f^*_j(x \rightarrow S) + \sum_{j \in S^*} f^*_j(x \rightarrow S) \right] f(x \rightarrow S').
\]

Proof: Choose any \((x,m)\) from the set of feasible alternatives \(Y \subseteq X \times M\) and any \(j \in NS^*(x)\) such that
\[
\delta_j = m_j - \phi(x) - \sum_{S \subseteq j} \text{s.t. } j \notin S^* \left[ \sum_{j \notin S^*} f^*_j(x \rightarrow S) \right] f(x \rightarrow S') - \sum_{S \subseteq j} \text{s.t. } j \notin S^* \left[ \sum_{j \in S^*} f^*_j(x \rightarrow S) \right] f^*_j(x \rightarrow S) > 0.
\]

Since \(j\) is not in \(S^*(x)\), there must be some winning coalition \(S\) that can enact \(x\) which contains \(S^*(x)\) but not \(j\). Then \(N_j\) contains \(S^*(x)\) and thus, by superadditivity, is winning and can enact \(x\).

The wealth that \(N_j\) may distribute among its members is its endowment \(\phi(N_j)\) less any mandatory sidepayments it must make to \(j\). \(N_j\) must pay \(f^*_j(x \rightarrow S)\) to \(j\), for every mandatory sidepayment that must be made to some set of agents \(S\) that contains \(j\), for which \(j\) is entitled to a mandatory minimum: \(\sum_{S \subseteq j} f^*_j(x \rightarrow S)\).

Now consider a mandatory sidepayment that must be made to some set of agents \(S'\) that contains \(j\), and for which \(j\) has a veto over its distribution. Because of agent \(j\)'s veto, \(S' \backslash j\) cannot control the distribution of \(f(x \rightarrow S')\) and all or a portion of this usage fee might be made to \(j\). Indeed, \(N_j\) might have to pay \(j\) the entire usage fee whenever \(j\) has a veto over a distribution of a mandatory sidepayment: \(\sum_{S \subseteq j} f^*_j(x \rightarrow S)\).

When \(S'\) is mandated a sidepayment, and some \(j \in S'\) receives that entire sidepayment \(f(x \rightarrow S')\), then \(f(x \rightarrow S')\) includes any minimum sidepayment to which \(j\) is entitled. Consequently, the maximum sidepayments which \(N_j\) must pay to \(j\) are: \(\sum_{S \subseteq j} f^*_j(x \rightarrow S') + \sum_{S \subseteq j} \text{s.t. } j \notin S^* \left[ \sum_{j \notin S^*} f^*_j(x \rightarrow S) \right] f^*_j(x \rightarrow S)\).

But then \(N_j\) can distribute among its members at least:
\[
\phi(N) - \sum_{S \subseteq j} f(x \rightarrow S') - \sum_{S \subseteq j} \text{s.t. } j \notin S^* \left[ \sum_{j \notin S^*} f^*_j(x \rightarrow S) \right] f^*_j(x \rightarrow S).
\]

Then \(N_j\) has a strategy leading to an outcome \((x,m')\) such that \(m_j = m_j' + \delta_j(n-1)\) for every \(i \in N_j\) and at the same time refuse any offered sidepayments from \(j\). Thus, \((x,m)\) is dominated via \(N_j\).


Endnotes

1 A more mathematically detailed discussion of these issues can be found in Slotnick (1995b). The idea was first presented in a preliminary fashion at the annual meetings of the Public Choice Society held in San Francisco, California on March 18-20, 1988. Comments are welcome and can be sent to P.O. Box 23, Mt. Gretna, PA 17064.

2 A simple game is a type of territorial game in which every winning coalition can enact any outcome. See Slotnick (1995a, 1995b).

3 Much of the "law and economics" literature is concerned with choosing between damage and property rules. The framework of territorial games with voluntary and mandatory sidepayments allows us to mathematically examine how equilibria change with the vesting of various rights and entitlements.

4 The effective set of a winning coalition contains at least two outcomes, one of which is default, and may but need not include all of X. The effective set of a blocking coalition is z (i.e. default or the status quo). The effective set of a losing coalition is empty.

5 For a more detailed discussion see Slotnick (1995a, 1995b).

6 The total wealth of coalition S, denoted es(x), is the sum of the endowments of all members of the coalition.

7 These results have been proven for the special case of simple games with voluntary sidepayments by Laing and Slotnick (1994). (In that paper, wealth was represented by the letter "t", as in t, ti, and T, rather than the m, m, and M of this paper.) Although the wording in Laing and Slotnick (1994) does not apply to territorial games, most of their proofs are sufficiently general to establish the corresponding results presented here, for territorial games with voluntary (but not mandatory) sidepayments. One need only substitute S*(x) for S* and use qualifying words such as "local" or "locally" to modify the words "collegial", "noncollegial" and "collegium". The corresponding results for mandatory sidepayments follow straightforwardly from Lemma 2 of this paper, proven in the Appendix.

8 To prevent double counting for parts (2) and (3), if the agent has both a veto over the distribution of and a minimum entitlement with respect to some particular usage fee, then only the veto is counted.

9 For every territorial game which permits voluntary but not mandatory sidepayments, we can associate a territorial game without sidepayments. Both games have the same winners, blockers and losers. Both have the same set of X alternatives. For the associated game without sidepayments, however, at each x ∈ X the only feasible wealth distribution vector is m = e(x).

10 Because the loser has a veto (i.e. a territory) it can preclude certain outcomes, thus "limiting" its losses. Of course some losers have no veto (and no territory) so are not "limited" (Slotnick, 1995a). The complement of a limited loser is a called "limited" winner, because there are limits on its power.

11 In other words, we remove x from the territory of S but now specify f(x, y) = k > 0.