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A NONCOOPERATIVE MODEL OF BARGAINING IN SIMPLE SPATIAL GAMES

by

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In a seminal paper, Reinhard Selten (1981) presents a noncooperative bargaining model for a class of characteristic function games and investigates within this context a concept of stable demand vectors proposed by Wulf Albers (1975). In the following pages I follow the lead established by Selten to demonstrate that the noncooperative approach extends naturally to cooperative games without side-payments in which a point in a Euclidean set of alternatives is to be selected in accordance with a simple collective decision rule (Laing, Nakabayashi, and Slotznick, 1983). Within this context, I examine Albers' concept of stable demand vectors and a closely related concept from the theory of cooperative games called the competitive solution (McKelvey, Ordeshook, and Winer, 1978) to determine whether such cooperative solutions can be implemented noncooperatively via stationary equilibrium strategies for bargaining in simple spatial games without side-payments. The results identify some interesting contrasts to Selten's analysis that arise at "the knife's edge" in this class of games.*

*In applying the noncooperative approach to this class of problems, I have benefitted from discussions with Wulf Albers and Taekwon Kim. I am grateful especially to Reinhard Selten for counseling me at critical stages in the development of these results, for creating the general approach to the noncooperative analysis of cooperative games that guides this work, and for inviting me to join the research group on "Game Equilibrium Models," Zentrum fuer interdisziplinaere Forschung, Universitaet Bielefeld, Federal Republic of Germany, during the period January 15 through June 30, 1988, so that I could devote full time to this research. Also, I wish to express appreciation for the support provided me by the National Science Foundation (Grant SES87-09476) and by the Department of Decision Sciences, the Wharton School, and Research Fund of the University of Pennsylvania.
1 THE GAME

A Collective Decision Problem

This paper investigates nondictatorial simple collective decision problems without side-payments in which the negotiations among players are governed by rules specified by the process mechanism described in the next section.

In a simple collective decision problem one element \( x \) must be chosen from an arbitrary set \( X \) of decision alternatives in accordance with a simple collective decision rule (Laing, Nakabayashi, and Slotznick, 1983). In this paper it is assumed that that \( X \) is a compact and convex subset of finite-dimensional Euclidean space. Such a rule may be defined by specifying for each subset \( S \subseteq N \) of players the subset \( E_S \subseteq X \) of alternatives that \( S \), acting as a coalition, is empowered to enact as the decision outcome. The rule is simple if the possible coalitions of players may be partitioned into the three sets of (W) winning, (L) losing, and (B) blocking coalitions such that \( S \in W \) iff \( E_S = X \), \( S \in L \) iff \( E_S = \emptyset \), and \( S \in B \) iff \( E_S = z \), where \( z \in X \) denotes the default option representing the status quo. The rule is superadditive if every superset of a winning coalition is a winning coalition (and every subset of a losing coalition is a losing coalition). Such a rule is essential if \( N \in W \) & \( X \neq \emptyset \), and proper if for every partition \( P \) of \( N \) into coalitions and for every distinct \( S \) and \( S' \) in \( P \), \( E_S \cap E_{S'} = \emptyset \) or \( E_S \setminus E_{S'} = z \). The noncooperative bargaining model specified below applies to collective decision problems in which the rule is simple, superadditive, essential, and proper. I shall assume also that the rule is nondictatorial, in that every winning coalition contains more than one player. For the special case in which the \( n \) player’s roles under the rules are symmetric, the decision is governed by \( m/n \) majority rule, where \( m \) is any integer such that \( n/2 < m \leq n \) and \( S \) is a winning coalition iff \( S \) contains at least \( m \) members.

A Noncooperative Representation of Negotiations

This subsection characterizes informally, then specifies in detail, a recursive game form for conducting negotiations in such cooperative games. The procedure is in the spirit of that specified by Selten(1981).
An overview. Some introductory comments may be used to characterize the basic style of the game form specified below. The game begins with a chance move by Nature as to which player will first have the "initiative": an opportunity to initiate a proposal. At any step in the process only the player who has the floor may send a message. The player who has the floor occupies just one of two positions — initiator or responder. In the initiator position, player \( i \in N \setminus i \) can choose either (1) to pass the initiative to any other player \( j \in N \setminus i \) or pass to a chance selection of initiator by Nature, or (2) to make a proposal \( (x, S) \) in which \( i \in S \) offers an alternative \( x \in X \) that \( S \) can enact, and specifies the order in which players in \( S \setminus i \) are queued for responding to the proposal. In the responder position that follows a proposal \( (x, S) \) from \( i \), player \( r \in S \setminus i \) (as responder) can choose to either (1) accept the proposal, or (2) reject (thus cancelling) the proposal and assume the initiator position. If in round \( t \) all responders agree to the proposal \( (x, S) \), then the game ends with this as the final outcome. This procedure incorporates a parameter \( t \) that can be set to limit the maximum number of rounds in the negotiation process, such that if no final decision is enacted by the end of round number \( t \), then the default outcome \( z \) becomes the final result. Adopt the convention of saying that a new round (or trial) in the negotiation process begins when any player assumes the initiator position.

A Bargaining Mechanism. In the following procedure, the symbol "M:" identifies an action by the governing mechanism, whereas "P:" represents a choice opportunity for the player who has the floor to choose one of the options proceeded by "\( \cdot \)."

**A.0 Initialize Parameters (START)**

**M:** Specify the set \( N \) of \( n \) players, the set \( X \) of decision alternatives, the default option \( z \in X \), and the outcomes in \( X \) that each coalition \( S \subseteq N \) can enact, denoted by \( E_S \), if one of its members has the initiative, thus defining the admissible proposals. Set the parameters \( t = 0 \) and \( t - \) the maximum number of rounds available for negotiations.

**A.1 Chance move**

**M:** Select \( i \in N \) at random and set \textit{initiator}=\( i \).
A.2 Initiation of proposal

M: Set $t = t+1$. If $t > t$, then set $(x^*, S^*, t) = (z, \emptyset, t)$ and go to A.4 (default outcome). Otherwise, set $S^* = \emptyset$ and open the floor to the initiator, $i$.

P: The initiator $i$ must choose just one of the following options:
   - **pass** [optional: to $j \in N \setminus i$].
     
     [M: If $j$ is specified, set initiator $= j$ and go to A.2. Otherwise, go to A.1.]

     * propose $x \in E_{i\cup R}$ to $R \subseteq N \setminus i$ (ordering $R$).
     
     [M: Set $S^* = S^* \cup i$ and $x^* = x$.]

A.3 Response to proposal.

M: If $R = \emptyset$ go to A.4. Otherwise, designate the first player in $R$ as responder, $j$, and open the floor to $j$.

P: The responder, $j$, must choose just one of the following options:
   - **reject** $(x, S)$ [and assume the initiator position].
     
     [M: Designate $j$ as the initiator, $i$. Go to A.2.]

   - **accept** $(x, S)$.
     
     [M: Set $S^* = S^* \cup j$, $R = R \setminus j$. Go to A.3.]

A.4 Final Decision

M: Announce final decision $= (x^*, S^*, t)$. STOP.

**Effectiveness in the bargaining procedure:** Seizing the opportunity. Note that this specification of the bargaining mechanism uses the symbol $E_{i}$ rather than the symbol $E'_{i}$ that was employed earlier to define a simple collective decision problem. This nuance in notation is introduced to deal with a complication that arises in implementing simple collective decision rules via this procedure. Under the procedure it is possible for any set $S$ of two or more players to filibuster (monopolize the floor) if some member $i$ of $S$ becomes the initiator. For example, suppose $i$ passes to $j$, $j$ passes to $i$, $i$ to $j$, and so on. Equivalently, suppose $i$ makes a proposal that includes $j$ as first responder, $j$ rejects and makes a proposal listing $i$ as the first responder, and so on. Each exercise of the initiative in either sequence begins a new round. By these tactics, $i$ and $j$ could continue the game indefinitely or through the maximum number of rounds permitted by the rules, and thus ensure the continuation of the status quo. Moreover, if the rules limit the game to at most $t$ rounds, then $i$ and $j$ can exchange the floor in
this way until one of them attains the initiative for the ultimatum subgame to be played in round $t$: "Take it or leave it."

Therefore, care must be taken in defining coalition effectiveness in the bargaining game. In this context, I shall follow an approach similar to that employed by Elaine Bennett (1988) for another class of games by interpreting effectiveness of coalition $S$ as the ability to seize the opportunity to enact an outcome if a member of $S$ attains the initiator position. Precisely, for any nonempty subset $S \subseteq N$ of players, let $E_S \subseteq X$ constitute the alternatives that, under the rules, $S$ can enact as the outcome of the game if some member $i$ of $S$ attains the initiative and makes a proposal to $S \setminus i$. Because of the filibuster possibilities, if $S$ contains at least two members, then $E_S = E_S^1 \cup \cdots \cup E_S^r$; otherwise, $E_S = E_S^i$. A coalition containing at least two players is effective for the status quo in the bargaining game, and thus is not a losing coalition in the bargaining game, even though it is "losing" in the simple collective decision game. Note also that if $S$ belongs to the set of winning($W$) or blocking($B$) coalitions, as specified in the simple collective decision game, then $S$ has the identical effectiveness in the associated bargaining game: $E_S = E_S^i$.

In the sequel, I shall assume that (1) the rules place no limit on the number of rounds, hence there is no ultimatum subgame, and (2) the status quo is unattractive, thus no one has incentive for indefinite play. At equilibrium, under these conditions, there is no filibuster.

**Payoffs**

Payoffs are defined over endpoints of the game, where each such outcome may be denoted as $(x, S, t)$, indicating that the alternative $x$ is enacted in round $t$ by coalition $S$ as the final decision, thus ending the game. Let the ordered set $T = \{0, 1, 2, \ldots\}$ index the rounds in the negotiation process. Assume that the preferences of each player $i \in N$ over the possible outcomes are represented by the continuous real-valued payoff function $h_i : X \times T \to R$. Also assume that payoffs are neither discounted nor subject to transaction costs, such that the payoff to any player $i$ for an outcome in which the alternative $x \in X$ is enacted by any coalition $C \subseteq N$ in any round $t \in T$ is given by $h_i(x, t) = u_i(x)$, and that the function $u_i : X \to R$ is strictly quasiconcave. Consequently,
no indifference contour associated with \( u_i \) in \( X \) is "thick" or "flat."

The payoff function \( u_i \) has some useful properties. Denote an upper contour set in \( X \) associated with \( u_i \) in either of the following two ways:

\[
R_i(x) = \{ y \in X \mid u_i(y) \geq u_i(x) \}, \text{ given any fixed } x \in X; \\
R_i(q_i) = \{ y \in X \mid u_i(y) \geq q_i \}, \text{ given any fixed real number, } q_i \in \mathbb{R}.
\]

The interior of \( R_i(x) \) [respectively, \( R_i(q_i) \)] relative to \( X \) constitutes the strict preference set, denoted by \( P_i(x) \) [respectively, \( P_i(q_i) \)]. The above assumptions concerning \( X \) and \( u_i(\cdot) \) are sufficient to guarantee that, for every \( x \in X \), \( R_i(x) \) is nonempty, compact, and strictly convex. [Any set \( V \) is said to be strictly convex if, for all \( x \in V \), \( y \in V \setminus x \) and any \( \lambda \) such that \( 0 < \lambda < 1 \), \( w = [\lambda x + (1-\lambda)y] \) lies in the interior of \( V \).] Moreover, \( u_i(\cdot) \) attains a maximum at a unique element of \( X \), denoted as the bliss point \( \beta_i \), and also attains a minimum in \( X \). Therefore, without loss of generality, the function \( u_i \) may be scaled such that, for every player \( i \) and every alternative \( x \in X \), \( 0 \leq u_i(x) \leq u_i(\beta_i) \). Thus, each player's payoffs are bounded.

Preference sets in \( X \) for any nonempty subset \( S \) of players associated with the \( u_i \)'s may be defined as follows:

\[
R_S(x) = \cap_{i \in S} R_i(x), \text{ given any fixed } x \in X; \\
R_S(q) = \cap_{i \in S} R_i(q_i), \text{ given any fixed vector of } n \text{ real numbers, } q_i \in \mathbb{R}^n.
\]

The interior of these sets constitute the strict preference sets \( P_S(x) \) and \( P_S(q) \), respectively. Then the (weak) Pareto optimal set of alternatives in \( X \) for any nonempty subset \( S \) of players may be defined simply as \( \beta_S = \{ y \in X \mid P_S(y) = \emptyset \} \).

The assumptions imply that, for every nonempty subset \( S \) of players and \( x \in X \), \( R_S(x) \) is nonempty, compact, and strictly convex, whereas, for every vector \( q \) of \( n \) real numbers, \( R_S(q) \) is compact and strictly convex (but can be empty). These properties facilitate the analysis. For example, they provide one justification for limiting attention to behavioral strategies that have finite carrier.

\textit{N.B.} It is assumed throughout this paper that the default outcome, \( z \), is unattractive, in that for every player \( i \) there is a winning coalition \( C \) containing \( i \) and an alternative \( x \in E_C = X \) such that every member of \( C \) strictly
prefers \( x \) to \( z \). Since \( z \) is interpreted to represent an unattractive status quo that remains in effect unless a different alternative is enacted, each player belongs to some winning coalition that seeks a decision in a finite number of rounds.

An Abstract Representation of the Bargaining Game

This game may be represented as a special instance of a general type of recursive game of perfect information. Such a game may be specified by the expression \( G = (N, Y, y^0, O, P, A, \pi, \tau, h) \), where

\( N = \{1, \ldots, i, \ldots, n\} \) is a finite set of players.
\( Y \) is a set of nodes.
\( O \subseteq Y \) is a set of decision outcomes representing alternative end points of the game.
\( Y \setminus O \) is a set of nodes representing choice opportunities by Nature or any player.
\( y^0 \in Y \setminus O \) is a pronouced node indicating the first move of the game.
\( P = (P_0, P_1, \ldots, P_n) \) is a partition of \( Y \setminus O \) such that \( P_0 \) denotes the moves assigned to Nature and \( (\forall i \in N) \ P_i = P_1 \cup P_2 \), the union of positions in which player \( i \) is (1) initiator and (2) responder, respectively.
\( A \) is a choice function that assigns to every \( y \in Y \setminus O \) a nonempty set \( A(y) \subseteq Y \) of alternatives.
\( \pi \) is a probability function such that \( \pi[A(y)] \) governs any move \( y \in P_0 \).
\( \tau: Y \setminus T = \{0, 1, 2, \ldots\} \) is an indexing of the rounds such that \( \tau(y^0) = 0 \) and move \( y \in Y \) occurs in round \( t = \tau(y) \).
\( h: O \rightarrow \mathbb{R}^n \) is a function that assigns a payoff vector to every outcome \( \sigma \in O \).

If the rules place no limit on the maximum length of the game, then the types of nodes in \( Y \setminus O \) and the choices available at each such node are as follows.

\( y \in P_0 \) denotes a chance move by Nature in accordance with the simple lottery \( \pi[A(y)] \), where \( A(y) = \{p_i | i \in N, p_i \in P_1\} \) is the set of initiator positions and for every \( p_i \in A(y) \), \( \pi(p_i) > 0 \).
\( p_i \in P_1 \) denotes a node at which player \( i \in N \) has the initiative to make a proposal.

The choice set \( A(p_i) \) at any initiator node \( p_i \) consists of the following,
mutually exclusive options.

yeP₀: pass (to random selection of initiator).

pj∈P[j]: pass the initiative to any specified player j∈N\i.

rj∈P[j]: propose (x,C), where C⊆N is an ordered set of players that
includes i, x∈C, and j is the first player in C\i.

rj=(x,C,S,j)∈P[j] denotes a responder position of player j in relation to a
proposal to enact the outcome (x,C). S includes the initiator of this
proposal and any responder who has accepted it, and j is the first
player in C\S.

The choice set A(rj) at any responder node rj consists of the following
options.

pj∈P'[j]: reject the proposal and assume the initiator position.

rk∈(x,C,S∪j,k)∈P'[k], if S∪j≠C, where k is the first player in C\(S∪j):
accept the proposal, so that the floor then passes to the next
responder, k.

(x,C)∈O if S∪j=C: accept the proposal and end the game.

An endpoint of the game may be denoted (x,C,t)∈O, indicating that x has
been enacted in round t by coalition C; in this case each player i∈N
realizes the payoff h_i(x,t). Given that payoffs are assumed to be
independent of both the coalition enacting the outcome and the round, for
convenience I shall represent the outcome (x,C,t), in an abuse of
notation, variously as (x,t), (x,C), or x, depending on the context.

All details defining the game are assumed to be common knowledge.

2 NONCOOPERATIVE SOLUTION THEORY

This section summarizes the general noncooperative solution theory for
such recursive bargaining games as presented by Selten (1981, §3).

Behavioral Strategies

Local strategies. Define a local strategy by at any node y∈Y\O as a
probability distribution over A(y) with finite carrier. (For parsimony, this
also applies to the moves of Nature, so that (∀y∈P₀) b_y=π[A(y)].) To avoid
measure-theoretic complications, the analysis is confined to local strategies
that assign positive probability to a finite number of options. This domain restriction might seem problematic when y represents an initiator position, since, in selecting a proposal, the initiator must choose a point in the convex, real-valued set X of decision alternatives. However, I shall prove in Section 3 that the initiator has only a small number of optimal proposals, given that he must satisfy constraints associated with the payoff expectations of responders to his proposal.

Global strategies. A global strategy, denoted by b, is a function that assigns to every $y \in Y \setminus \mathcal{O}$ a local strategy, $b_y$.

Stationary strategies. The following analysis focuses exclusively on strategies that are, in the following sense, stationary. Any behavioral strategy $b$ is said to be stationary if, for every player $i \in \mathcal{N}$, $b_y = b_{y'}$ for every pair of moves $y$ and $y'$ in $P_i$ such that either (1) $y$ and $y'$ are initiator positions of player $i$, or (2) $y$ and $y'$ are responder positions of player $i$ such that $y = y' = (x, C, S, i)$. Any stationary behavioral strategy has certain advantages. First, it is parsimonious, in that the behavior it prescribes for any round $t$ in the game does not depend on the history of play through round $t-1$. Second, any stationary strategy is subgame consistent, in that it prescribes the same behavior in any two subgames that maintain the players' identities and in all respects are essentially equivalent up to positive linear transformations of the payoffs.

Subgame consistency. To define subgame consistency more precisely, let $G' = (N', Y', y_0', C', P', A', \pi', \tau', h')$ and $G'' = (N'', Y'', y_0'', C'', P'', A'', \pi'', \tau'', h'')$ denote any two subgames of the game $G$. Call the subgame $G''$ isomorphic to $G'$ if $N' = N'' := \mathcal{N}$ and there exists a bijective homomorphism $f: G' \rightarrow G''$ such that, for every $y' \in Y'$ and its image $f(y') = y'' \in Y''$, all four of these conditions obtain: (1) for every $k \in (0, 1, \ldots, n)$, $y' \in P_k$ iff $y'' \in P_k'$, (2) $A''(y'') = A'(y')$, (3) $y' \in P_0'$ implies $\pi''[A''(y'')]' = \pi'[A'(y')]$, and (4) for every player $i \in \mathcal{N}$, outcome $y' \in O'$ and image $y'' = f(y') \in O''$, $h_i''(y'')$ is a positive linear transformation of $h_i(y')$. Then define a global strategy $b$ for game $G$ to be subgame consistent if it possesses the following property: for every pair of subgames $G'$ and $G''$ of $G$, if $G''$ is isomorphic to $G'$ then, for each decision node $y' \in Y' \setminus (O' \cup P_0')$ of $G'$ and image $y'' = f(y') \in Y''$ of $G''$, $b_y'' = b_{y'}$. 

9
Expected Payoffs

In computations of expected payoffs, the expression \((y,t)\) indicates that node \(y \in Y\) occurs in round \(t\) of the game. Let \(p[(x,t') \in O \mid (y,t),b,t]\) denote the conditional probability that the game ends with outcome \((x,t')\), \(x \in X\), given that (1) the node \((y,t)\) has been reached, (2) the global strategy \(b\) governs behavior in all subsequent play, and (3) the game ends in or before round \(t\).

If the game is constrained by the rules to last at most a finite number \(t\) of rounds, then, given that each local strategy has finite carrier, only a finite number of outcomes have positive probability, and the expected payoff to any player \(j \in N\) at any node \((y,t)\) is given by the expression
\[
h_j(b \mid (y,t)) = \Sigma_{t' \in \{t, t+1\}} \Sigma_{x \in X} \Sigma_{o \in O} p[(x,t') \in O \mid (y,t),b,t] h_j(x,t'),
\]
where round \(t+1\) is included to account for the selection of the status quo outcome \(z\) by default.

On the other hand, if the rules place no constraint on the maximum length of the game, then \(h_j(b \mid (y,t)) =
\[
p[\omega \mid (y,t),b] u_i(z) + \lim_{t \to \infty} \Sigma_{t' \in \{t, t+1\}} \Sigma_{x \in X} \Sigma_{o \in O} p[(x,t') \in O \mid (y,t),b,t] h_j(x,t'),
\]
where \(p[\omega \mid (y,t),b]\) denotes the probability that play will continue forever, given that \((y,t)\) has been reached and \(b\) governs behavior. Since each local strategy \(b\) has finite carrier, only a finite number of outcomes in \(X\) have positive probability. Moreover, for all \(x \in X\) and \(t' \in \{t, t+1\}\), \(h_j(x,t') \geq 0\). Therefore, the sum across \(x\) and \(t'\) is nondecreasing in \(t\). Also, the sum is bounded above by \(u_j(b_j)\). Therefore, the limit exists and, thus, \(h_j(b \mid (y,t))\) is well defined.

The assumptions (1) that \(X\) is convex, (2) that, for any player \(j \in N\), \(x \in X\) and round \(t\), \(h_j(x,t) - u_j(x)\) and (3) that \(u_j(\cdot)\) is strictly quasiconcave, together imply that the expected payoff function is strictly risk averse: if two distinct outcomes \(x\) and \(y\) in \(X\) yield the same payoff to \(j\), then \(j\) strictly prefers the "sure thing" outcome \(w = \lambda x + (1-\lambda)y\) to the lottery that yields \(x\) with probability \(\lambda\) and otherwise yields \(y\).
Stationary Equilibrium

For any player \( j \in N \) and any move \( y \in P_j \), define a local strategy \( b_y \) to be optimal at \( y \) if, for every \( w \in A(y) \), \( b_y(w) > 0 \) implies that there is no \( v \in A(y) \) such that \( h_j(b_y | v) > h_j(b_y | w) \). A global strategy \( b \) is defined to be a stationary equilibrium if, to every node \( y \in Y \setminus O \), \( b \) assigns a local strategy \( b_y \) that is optimal at \( y \). A global strategy \( c_j \) is said to be a deviation by player \( j \) from \( b \) if the local strategies assigned by \( c_j \) and \( b \) differ only at one or more nodes assigned to player \( j \). If \( b \) is a stationary equilibrium of the game, then, for every \( j \in N \) and \( y \in P_j \) and for every deviation \( c_j \) of \( j \) from \( b \), \( h_j(b | y) \geq h_j(c_j | y) \) [Selten, 1981: Thm. 1]. Also, since a stationary equilibrium is optimal at each decision node, it is subgame perfect (Selten, 1975).

3 STATIONARY EQUILIBRIUM POINTS IN THE SIMPLE SPATIAL BARGAINING GAME

This section investigates stationary equilibrium strategies in simple collective decision games on a Euclidean set \( X \) of decision alternatives in which the process of negotiations is governed by the recursive bargaining procedure defined above. N.B. It is assumed throughout that the rules impose no limit on the length of the game and that the status quo is unattractive.

Preliminary Results

For any stationary global strategy \( b \) and every player \( j \in N \), define the quota of player \( j \) associated with \( b \) as the expected payoff to \( j \) of assuming the initiative, given that behavior is governed by \( b \): \( q_j(b) = h_j(b | p_j \in P_j) \). Then let \( q(b) = (q_1(b), \ldots, q_j(b), \ldots, q_n(b)) \) denote the quota vector associated with \( b \).

Lemma 1 (lower bound on responder’s payoff). If \( b \) is any stationary equilibrium with associated quota vector \( q(b) \), then, for every player \( j \in N \) and responder position \( r_j \in P_j^2 \), \( h_j(b | r_j) \geq q_j(b) \).

Proof. Suppose there exists some \( j \in N \) and \( r_j \in P_j^2 \) such that \( h_j(b | r_j) < q_j(b) \). Then \( b \) is not optimal at \( r_j \), since \( j \) has an option \( p_j \in A(r_j) \cap P_j \) for which \( h_j(b | r_j) < h_j(b | p_j) = q_j(b) \). Hence, \( b \) is not a stationary equilibrium. //
Lemma 2 (upper bound on equilibrium payoffs). Let b denote any stationary equilibrium with associated quota vector q(b). Then, for every player j \in N and every node y \in Y \setminus (UP j) such that y \in P^0 is a chance move assigned to Nature or y \in P^1_k is an initiator position assigned to any other player k \in N \setminus j, the expected payoff to j at y cannot exceed his quota: h_j(b | y) \leq q_j(b).

Proof. If j has the initiative at position p_j \in P_j, then j can pass to y \in P^1_j for any i \in N \setminus j or to y \in P^0_0 y^0 (Nature). Since b is optimal at p_j, for any such y it must be true that h_j(b | y) \leq h_j(b | p_j) = q_j(b). Also, j's expected payoff at the first move of the game, y^0 \epsilon P^1_0, is a convex combination of his expected payoffs across the initiator positions that may be selected at y^0. Therefore, h_j(b | y^0) \leq q_j(b). //

Note that Lemma 2 applies to the game's starting point. Therefore, for any stationary equilibrium point, each player's expected payoff for participating in the game as a whole does not exceed his quota. The lemma implies also that there is a first-mover advantage: at equilibrium, the initiator's expected payoff equals his quota.

Lemma 3 (guaranteed acceptances). Let b be any stationary equilibrium with associated quota vector q(b). Then any proposal (x, C) by any initiator i \in N will be accepted with certainty if, for every responder j \in C \setminus i, u_j(x) > q_j(b).

Proof. Consider any proposal (x, C) and any responder position r_j \in P^2_j such that everyone in S=C \setminus j has initiated or accepted the proposal. Thus, j is the last responder in C. Then, since b is optimal at r_j, j must accept the proposal, thereby ending the game. Therefore, by induction on responder positions, every player in C \setminus i must accept the proposal. //

Lemma 4 (limited play). If b is a stationary equilibrium and default is unattractive, then the probability of infinitely long play equals zero.

Proof (Selten, personal communication). Suppose that there exists a node y \in Y in the game such that p(\epsilon | y, b) \neq 0. Then, in accordance with b, there must exist an infinite sequence s of rounds that is reached with positive probability from y such that s contains only a finite number of rounds in
which the game ends with probability greater than zero (but less than one, by assumption). Since the number of such rounds is finite, there must be a last such round, say \( t \), in which the game can end in accordance with \( b \). Now consider the subgame in the infinite sequence that begins with round \( t+1 \) and suppose that player \( i \) has the initiative. If the play continues into round \( t+2 \) in accordance with \( b \), then each player \( j \in N \) expects a payoff equal to \( u_j(z) \). By assumption, default is unattractive, so, by definition, there exists a winning coalition \( C \) containing \( i \) and an alternative \( x \in E_C \) such that, for every player \( j \in C \), \( u_j(x) > u_j(z) \). Thus, player \( i \) can propose the outcome \((x, C)\) and, by Lemma 3, every responder must accept it, thereby ending the game. Therefore, \( b \) is not a stationary equilibrium. //

**Lemma 5 (quota-feasible outcomes).** Let \( b \) be a stationary equilibrium with associated quota vector \( q(b) \) and outcomes \( O(b) \). Then,

1. For every player \( i \in N \), there exists an outcome \((x, C) \in O(b)\) such that \( u_i(x) > q_i(b) \), and

2. For every outcome \((x, C) \in O(b)\) such that \( C \neq \emptyset \), and for every member \( i \) of \( C \), \( u_i(x) > q_i(b) \).

**Proof.** Part (1) follows immediately from the fact that \( q_i(b) \) is a probability-weighted average of \( i \)'s payoffs for outcomes in \( O(b) \).

The following proof of (2) is a restatement of one given by Selten (1981: see the relevant portion of his proof of Lemma 3). Let \( s \) be any sequence of positions in \( Y \) that ends with the outcome \((x, C) \in O(b)\) and consider any player \( i \in C \). There must be a position \( y \in P_i^L \) in \( s \) such that after \( y \) there is no initiator position \( p_k \) in \( s \): either \( y \in P_i^L \), so that \( i \) proposes the outcome \((x, C)\), or \( y \in P_i^P \), so that \( i \) is a responder to the proposal. In either case, Lemma 1 and the definition of \( q_i(b) \) together imply \( q_i(b) \leq h_i(b|y_i) \). Let \( \mathbb{D} \) denote the conditional probability, given that position \( y_i \) is reached and \( b \) governs behavior, that no responder after \( y_i \) in \( s \) rejects the proposal if \( i \) initiates or accepts it at \( y_i \). Note that \( \mathbb{D} > 0 \), since \((x, C) \in O(b|y_i)\). Thus the proposal is rejected by one of these responders with probability \( 1-\mathbb{D} \); in this event, Lemma 2 implies that player \( i \)'s payoff cannot exceed \( q_i(b) \). Hence, \( q_i(b) \leq h_i(b|y_i) \leq \mathbb{D} u_i(x) + (1-\mathbb{D}) q_i(b) \). Given \( \mathbb{D} > 0 \), this implies \( q_i(b) \leq u_i(x) \). Since \((x, C) \in O(b)\) and \( i \in C \) were selected arbitrarily, it follows that, for each \((x, C) \in O(b)\) and \( i \in C \), \( u_i(x) \geq q_i(b) \). //
Lemma 6 (efficient agreements). Let b denote any stationary equilibrium with associated quota vector q(b). For any winning coalition C and alternative x ∈ X, the outcome (x, C) is assigned positive probability by b only if x is Pareto optimal for C.

Proof. Suppose that, in accordance with b, (1) player i initiates at position p_1 ∈ P_i a proposal (x, C) such that C ∈ W and x is not Pareto optimal for C, and (2) every responder j ∈ C \ i agrees, thus ending the game. In this event, Lemma 1 implies that, for every j ∈ C \ i, u_j(x) ≥ q_j(b). Then, by the definitions of Pareto optimality and winning coalition, there exists an alternative x' ∈ X that C could enact that every member of C strictly prefers to x. Consider a deviation c_i from b such that i proposes (x', C). Then, for every responder j ∈ C \ i to i's proposal, u_j(x') > q_j(b); hence, Lemma 3 implies that the outcome (x', C) will occur with certainty if i proposes it. But then h_i(c_i | p_1) > h_i(b | p_1), so b is not optimal at p_1. This contradicts the assumption that b is a stationary equilibrium. //

Stationary Equilibria Based on Extreme Proposals and Accommodating Responses

One type of stationary equilibrium is based on proposals that are, in the sense to be defined, extreme. Let b be any stationary strategy with associated quota vector q(b). For any player i ∈ N and any initiator position p_1 ∈ P_i, let E_i(q) = \{ C ∈ W : i ∈ C, E_i \cap \{ C \cap i(q) \neq \emptyset \} \} denote the effective coalitions for which i might generate an acceptable proposal. These are the coalitions to which i belongs that can enact an outcome such that the payoff to everyone C \ i is a least as good as his quota. For any coalition C ∈ E_i(q), let x^*_C denote any decision alternative in X such that

\[ u_i(x^*_C) = \max_{x \in E_i \cap \{ C \cap i(q) \}} u_i(x). \]

Next, define

\[ E_i^*(q) = \{ C ∈ E_i(q) : u_i(x^*_C) = \max_{C' \in E_i(q)} u_i(x^*_{C'}) \}, \]

\[ A_i^*(q) = \{ r_j ∈ P_j : j ∈ N \setminus i, r_j = (x, C, S, j), C ∈ E_i^*(q), x = x^*_C \}, \]

where, by definition, A_i^*(q) constitutes the set of extreme (maximal) proposals by the initiator, i, given the quota vector q(b).
Lemma 7 (existence, finiteness, and uniqueness of extreme proposals). Let $b$ denote any stationary equilibrium with associated quota vector $q(b)$. Then, for every player $i \in N$ and initiator position $p_i \in P_i^1$, the set of extreme proposals is nonempty, finite, and unique.

Proof. Consider any player $i \in N$ and any initiator position, $p_i \in P_i^1$, assigned to $i$. By Lemma 2, if $i$ passes or makes a proposal that is rejected, then his expected payoff cannot exceed $q_i(b)$, whereas, by Lemma 5(2), if he proposes an outcome in $O(b)$ that is accepted by all responders, then his expected payoff is no less than $q_i(b)$. By Lemma 1 and the local optimality of any stationary equilibrium strategy, a proposal $(x, C)$ by $i$ will be accepted only if, for every responder $j \in C \setminus i$, $u_j(x) \geq q_j(b)$. Lemma 5 implies that, at equilibrium, such a proposal exists. Hence, the set of effective coalitions for which $i$ has a feasible proposal, $E_i(q) = \{C \in WUB \mid i \in C, \quad Ec \cap Rc \setminus i(q) \neq \emptyset\}$, is not empty.

For any $C \in E_i(q)$, consider any decision alternative $x^*_C$ in $X$ such that

$$u_i(x^*_C) = \max_{x \in C \cap Rc \setminus i(q)} u_i(x).$$

If $C \notin E_i(q)$, then $Ec \cap Rc \setminus i(q) = \emptyset$, so $x^*_C = z$ is the default outcome. Now suppose $C \in E_i(q)$, so that $E_i = X$, hence $Ec \cap Rc \setminus i(q) = Rc \setminus i(q)$. Then, since $u_i$ is continuous and $Rc \setminus i(q)$ is nonempty and compact, $u_i$ attains a maximum in $Rc \setminus i(q)$ at some point, $x^*_C$. Moreover, $u_i(\cdot)$ is strictly quasiconcave and $Rc \setminus i(q)$ is convex, hence $x^*_C$ is the unique maximal element of $Rc \setminus i(q)$. Thus for each $C \in E_i(q)$ there is a unique point $x^*_C$ such that the proposal $(x^*_C, C)$ maximizes $u_i$. By definition, $E_i^*(q)$ contains each coalition $C$ to which $i$ belongs such that the associated point $x^*_C$ maximizes $u_i(x^*_C)$ across $C \in E_i(q)$. Since a unique point $x^*_C$ is associated with each coalition $C$ in $E_i^*(q)$, and there is a finite number of permutations of $C \setminus i$ and of coalitions in $E_i^*(q)$, then, at equilibrium, the set $A_i^*(q)$ of extreme proposals by the initiator $i$ is nonempty, finite, and unique. 

The following result identifies one important type of equilibrium.
Theorem 1 (EA equilibria: extreme proposals and accommodating responses). Let b be a stationary strategy with associated quota vector q(b) such that b has the property:

(A) For every player r∈N and responder position y′∈(x,C,S,r)∈P²,

\[ b_y(p_r \in P_r) = \begin{cases} 0 & \text{if } (\forall k \in S \setminus C) u_k(x) \leq q_k(b). \end{cases} \]

Then b is a stationary equilibrium if and only if b has the property:

(E) For every player i∈N, initiative y∈P², and option y′∈A_i(y),

\[ b(y') > 0 \text{ implies } y' \in A_i^x(q), \]

where A_i^x(q) is the set of extreme proposals, given q.

Proof. Sufficiency. Let b denote any stationary strategy that satisfies property A. Suppose that b does not possess property E. Then there exists some i∈N, y∈P², and y′∈A_i(y) such that b_y(y′)>0 but y′ is not an element of A_i^x(q). There are two possibilities. First, A_i^x(q)≠∅; hence b is not optimal at y. Second, A_i^x(q)=∅; hence Lemma 7 applies. Thus, in either case, b is not a stationary equilibrium.

Necessity. Let b denote any stationary strategy that satisfies A, and assume that b also exhibits property E. Select arbitrarily any player i∈N, option y∈P², and alternative y′∈A_i(y) such that b_y(y′)>0. By property E, y′=(x_i^*,C,S,k)∈A_i^x(q|y). Consider any responder r∈C\{i} to the proposal. Either r is the last responder in C or, in accordance with property A, every responder in C\{S∪i} will accept the proposal. In either case, by definition of the extreme proposals set, r expects a payoff of at least q_r(b) if he conforms to the strategy b and thus accepts the proposal. If he rejects the proposal, then, by definition, r’s expected payoff equals q_r(b). Therefore, r has no incentive to deviate from b. Since r was chosen arbitrarily, this conclusion applies to every responder to the proposal, if it is offered. Therefore, the initiator i expects the proposal y′ to be accepted, if he offers it at y. But this proposal was selected from the set A_i^x(q|y) of proposals that maximize i’s expected payoff at y, given that all responders are governed by b. Hence, i has no incentive to deviate from b. Therefore, b is a stationary equilibrium. //

Hereinafter, the term EA strategy refers to any stationary strategy that satisfies both properties E and A.
Corollary 1 (EA quota points). Let b denote any EA strategy, with associated quota vector q(b), that exhibits properties E and A. Then b is a stationary equilibrium such that, for every player i ∈ N, initiative y ∈ P_i, and proposal (x, C, S, r) ∈ A_i(y),

\[ b_y(x, C, S, r) > 0 \implies u_i(x) = q_i(b). \]

Proof. This follows immediately from Theorem 1, Lemma 7, and the definitions of quota and extreme proposals set. //

Path Independence, Instrumentality, and Expeditiousness

Among other things, EA strategies exhibit a pure form of path independence. The local strategy induced by a global stationary strategy b at any initiator position is totally independent of the path through which this node is reached. On the other hand, the local strategy induced by b at a responder position (x, C, S, r) can depend on S, hence on the history of prior play in the current round. The independence concept I have in mind rules out this possibility. Let me first characterize this concept informally by saying that a strategy b is path independent in the bargaining game if \( b_y = b_{y'} \) for every player j ∈ N and for every pair of positions y and y' assigned to j such that the subgames beginning at y and y' are essentially equivalent, differing only in the path by which these subgames are reached. In the context of the games at issue in this paper, the independence concept may be specified precisely as follows. Define any global strategy b to be path independent if it is stationary and for every player r ∈ N and every pair of responder positions y - (x, C, S, r) and y' - (x', C', S', r) assigned to r, \([x - x']\) and \([S - C - S']\) implies \( b_y = b_{y'} \). A path independent strategy is history-free at every choice position, whereas a stationary strategy need be history independent only at initiator positions, given the way in which responder positions are represented. Thus, path independence is a natural extension fully in the spirit of stationarity. In particular, for any game G as defined earlier, let G' denote an associated game that differs from G only in that the generic descriptions of each responder position (x, C, S, r) and outcome (x, C) in G are replaced, respectively, by (x, C\S, r) and (x) in G'. Then any global strategy b in game G is path independent if and only if b induces an associated strategy b' in G' that is stationary (and subgame consistent).
EA strategies have some appealing features. First, every EA strategy is path independent. Second, any EA strategy is instrumental, in the sense that it is motivated solely to achieve valued outcomes, and focuses exclusively on that goal, ignoring any details about the process that are not essential to reaching this ultimate objective. Third, any EA equilibrium strategy is expeditious, in the sense that, for every subgame that begins with an initiator position, it induces a proposal at that position that is accepted, thus ending the game. Thus, at an EA equilibrium, the game ends in the first round.

On the other hand, this type of equilibrium is based on a quite accommodating pure strategy: each responder accepts a proposal with certainty even in the knife-edge case in which he is indifferent as to whether he accepts. Corollary 1 indicates that, at an EA equilibrium, the initiator also operates at the knife’s edge, proposing via a pure or completely mixed local strategy a point that yields exactly his quota.

Noncooperative Implementation of Semistable Demands

Demands in spatial games. Albers’ (1975) concept of semistable demand vectors for characteristic function games may be extended to this class of simple spatial games without sidepayments as follows (Albers, personal communication). Any demand vector \( d \in \mathbb{R}^N \) is semistable if it satisfies two conditions:

1. Everyone is feasible. For every player \( i \in N \), there exists a coalition \( C \subseteq N \) and alternative \( x \in E \cap R_C(d) \) such that \( u_i(x) \geq d_i \).
2. There is no slack. For every coalition \( C \subseteq N \), \( E \cap R_C(d) = \emptyset \).

Quotas and demands. I now begin to investigate the relationship between quotas and semistable demands. Lemma 5(1) has established that any quota vector \( q(b) \) associated with a stationary equilibrium for these games satisfies condition (1): at equilibrium, everyone is feasible. By the following lemma, the quota vector associated with a stationary equilibrium also contains no slack, as required by condition (2): no coalition is effective for any outcome in \( X \) that strictly exceeds all its members’ quotas.
Lemma 8 (no slack). \textit{Let }b\textit{ be any stationary equilibrium with the associated quota vector }q(b). \textit{Then, for every coalition }C \subseteq N, E_C \cap P_C(q) = \emptyset.\textit{ }

\textbf{Proof.} Suppose that }b\textit{ is a stationary global strategy with associated quota vector }q(b)\textit{ but there exists a coalition }C \subseteq N\textit{ and decision alternative }x \in E_C \cap P_C(q)\textit{. Consider any member }i\textit{ of }C\textit{ and position }y \in P_i\textit{ such that }i\textit{ has the initiative or, by rejecting any proposal, can assume it at position }p_i \in P_i\textit{. Then there exists an outcome }\langle x, C \rangle\textit{ that }i\textit{ can propose at }p_i\textit{ such that, for every }j \in C, u_j(x) > q_j(b). \textit{If }i\textit{ proposes the outcome }\langle x, C \rangle, \textit{it will be accepted with certainty, by Lemma 3, and }i\textit{ expects to achieve the payoff }u_i(x) > q_i(b) = h_i(b|p_i). \textit{Therefore }b\textit{ is not optimal at }y. //

Thus, the quota vector associated with any stationary equilibrium constitutes a demand vector that is semistable.

\textbf{On implementing semistable demands.} The next theorem identifies an interesting contrast for simple spatial games with the results obtained by Selten (1981) for one-stage characteristic function games. Let me say that any global behavioral strategy }b\textit{ with associated quota vector }q(b)\textit{ implements the demand vector }d\textit{ if }b\textit{ is a stationary equilibrium and }d=q(b). \textit{Selten}(1981, Theorem 2) has shown for one-stage characteristic function games not only that the quota vector associated with any stationary equilibrium is a semistable demand vector, but also that every semistable demand vector can be implemented by a path independent, pure stationary equilibrium that exhibits properties E and A. \textit{Therefore, the second part of the following result identifies a distinctive feature of simple spatial games.}

\textbf{Theorem 2 (semistable demands and property A).} Every stationary equilibrium }b\textit{ implements a semistable demand vector, }d=q(b), \textit{whether or not }b\textit{ satisfies property A. However, there exist games in which not all semistable demand vectors are implemented by a stationary equilibrium that exhibits property A.}

\textbf{Proof.} The first part of the theorem follows immediately from Lemmas 5(1) and 8. The second part is demonstrated below via an example. //

[ Figures 1, 2, and 3 about here.]
Figure 1: A semistable demand vector for the House
Figure 2: Another semistable demand vector for the House
Figure 3: Stable demands and competitive solution points for the House
A Simple Majority Decision Problem: "The House"

Figure 1 depicts a five-person, simple majority rule decision problem (with an empty core) that has been dubbed "The House." As in all examples to be examined in this paper, a point is to be selected from \( X \subset \mathbb{R}^2 \). Each player \( i \in N \) seeks to minimize the Euclidean distance of the decision point in \( X \) from his bliss point, \( \beta_i \), and is indifferent among points that are equidistant from \( \beta_i \). Thus, the arc in \( X \) of any circle centered on \( \beta_i \) is an indifference class of player \( i \). Assume in the following examples that the default outcome is unattractive and thus may be ignored.

Semistable demands for the House. Consider the demands shown as arcs in Figure 1. Only the two decision points, labelled \( x_1 \) and \( x_3 \) in this figure, satisfy the demands of a simple majority and thus are feasible. Each player's demand is satisfied by at least one of these points, and there is no slack. Therefore, this demand vector \( d \) is semistable. Now consider a stationary strategy \( b \) by which any player who has the initiative proposes a feasible point that satisfies his demand, and each responder accepts with certainty any proposal that satisfies his and every subsequent responder's demand. No one has incentive to deviate from this strategy unilaterally, so it is an equilibrium. Moreover, given \( b \), each initiator expects to achieve a payoff equal to his demand, so \( q(b) = d \). Hence, this is an EA equilibrium that implements the demand vector. The same conclusion applies to the semistable demand vector plotted in Figure 2.

Stable demands for the House. A quite different conclusion applies to the situation depicted by Figure 3. The arcs plotted in this figure assume special importance in the literature, since they both form the basis for a cooperative solution of this game called the competitive solution (McKelvey, Ordeshook, and Winer, 1978; also see Laing and Olmsted, 1978, and Forman and Laing, 1982) and constitute a demand vector that not only is semistable but also possesses certain other appealing properties such that it has been called stable (Albers, 1987; Bennett and Winer, 1983).

I now investigate whether this stable demand vector can be implemented via a stationary equilibrium strategy for the recursive bargaining game. For this
analysis it is useful to adopt the following nonstandard notation. Given any 
player \(i \in N\), let \(i^m\) (respectively, \(i^\omega\-m\)) denote the player whose bliss point 
is the \(m\)th one clockwise (resp., counterclockwise) in Figure 3 from the bliss 
point of player \(i\): e.g., \(5^\omega-1\) and \(1^\omega-2-4\). Note that, in Figure 3, the 
competitive solution points have been indexed such that \(x_i\) denotes player \(i\)’s 
favorite competitive solution point: \(u_i(x_i) > d_i\).

A path dependent strategy: "Don’t be greedy." Reinhard Selten and, 
individually, Harrison Wagner (1987:20-21) have suggested a class of 
estatoric equilibria that implement the stable demand vector in "The House." 
By any strategy in this class, the initiator, \(i\), proposes a competitive 
solution outcome \((x, C) \in ((x_i^\omega_1, i^\omega_1, i^\omega_2), (x_i^\omega_1, i^\omega_1, i^\omega_1, \ 1^\omega-1, \ 1^\omega-2), (x_i^\omega_1, i^\omega-2, i^\omega-1))\) such that \(u_i(x) = d_i\). Each responder, \(j\), 
rejects any proposal of an outcome \((x, C)\) by \(i\) if \(x \neq x_i\), but, if \(x = x_i\), then \(j\) accepts iff 
\(h_j(b|\text{accept}) \geq d_j\).

Strategies of this type might seem natural in normal social affairs, since 
we often refuse to cooperate with someone who proposes his best outcome. Such 
an equilibrium could be selected on the basis a "standard of behavior," to 
borrow vonNeumann and Morgenstern’s (1953: 41f) term, that devolves from the 
broader social environment surrounding the game.

On the other hand, such strategies have their weaknesses in this context. 
Within a game theoretic framework, at least, it seems to me that equilibrium 
should arise endogenously from within the game, rather than being imposed on 
the game exogenously by importing a "standard of behavior" from the social 
environment. (For an extensive analysis of this issue, see Forman and Laing, 
1982.) A more direct approach would be to develop such a strategy as a 
solution to a game in which preferences depend on various aspects of plays of 
the game. But I have assumed that each player’s preferences are instrumental, 
in the sense that the player’s payoffs across the alternative outcomes \(((x, C))\) 
that could be enacted in round \(t\) depend solely on \(x\), regardless of which 
coalition \(C\) enacts alternative \(x\) or who proposed this outcome. Thus, it seems 
appropriate to consider only global strategies that focus exclusively on this 
objective. For this reason, the "Don’t be greedy" strategy seems not quite 
 germane to the present context. It is not path independent, instrumental, or
expeditious. Each responder’s acceptance decision depends not only on whether the expected payoffs meet each responder’s demand (cf. "price") but also on who is making the proposal. By this strategy, player 3, as the last responder, accepts an offer to enact the competitive solution point $x_2$ if this is proposed by player 1, but rejects the proposal if, instead, it is initiated by player 2.

On path independent implementation of stable demands. Limiting attention now exclusively to strategies that are path independent and instrumental, I conclude the proof of Theorem 2 by demonstrating that no stationary EA equilibrium implements the stable demand vector given in Figure 3.

Proof of Theorem 2 (conclusion). Given the demands plotted for the game in Figure 3, suppose that the stationary strategy $b$ satisfies property $A$, prescribing that each responder accept with certainty any proposal that satisfies every responder’s demand. Without loss of generality, suppose that player 1 is the initiator at position $p_1$. If $b$ is to be optimal at $p_1$, then it must satisfy property $E$ by instructing 1 to propose the outcome $(x_1, 125)$ or $(x_1, 152)$. If proposed, this outcome will result with certainty. Hence, $h_1(b^1|p_1) = q_1(b) = u_1(x_1) = 1 > d_1$, therefore $q(b^1) 
eq d$. A similar conclusion will be reached for a second game, the five-person Star, that is analyzed later in this paper. //

No stationary equilibrium strategy $b$ that assigns such an accommodating pure strategy to each responder position can implement this demand vector for the House, since there exists an outcome that would be accepted if proposed by player 1 for which his payoff strictly exceeds his demand. This observation is generalized in the following useful result.

Theorem 3 (semistable demands implementable with accommodating responses). Let $b$ denote any stationary global strategy exhibiting property $A$ such that the associated quota vector $q(b)$ constitutes a semistable demand vector, $d$. Then $b$ is an equilibrium if and only if, for every option $x \in X$ and player $i \in N$, $x \in \bigcup_{C \subset N} E_i \cap R_i(d)$ implies $u_i(x) \leq d_i$.  

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Proof. Sufficiency. Choose any player \( i \in N \) and suppose that there exists a coalition \( C \subseteq N \) and alternative \( x^* \in E_C \cap R_C(d) \) such that \( u_i(x^*) > d_i = q_i(b) \). The effectiveness relation is superadditive. Therefore, at any initiator position \( p_i \in P_i \), \( i \) could make the extreme proposal \( (x^*, I \cup C, I, r) \) and, by property A, it would be accepted by all responders. In this case, \( i \)'s expected payoff equals \( u_i(x^*) > q_i(b) \). Hence, \( b \) is not an equilibrium.

Necessity. Suppose that for every player \( i \) and each alternative \( x \) that is feasible, given the demands, \( u_i(x) \leq d_i \). Since \( d \) is a semistable demand vector, by definition there exists for each player \( i \) some alternative \( x^* \) and coalition \( C \) such that \( x^* \in E_C \cap R_C \) and \( u_i(x^*) \geq d_i \). Hence \( u_i(x^*) - d_i \). Consider any initiative \( y \in P_i \) of player \( i \). Since \( q_i(b) \) is a probability weighted average of \( i \)'s payoffs in \( O(b \mid y) \) then for all \( x' \in O(b \mid y) \), \( u_i(x') = q_i(b) \). Hence, \( i \) has no option that would yield him a greater payoff, and thus no incentive to deviate from \( b \) at \( y \). By property A, every responder achieves at least his quota in any proposal that he accepts, in accordance with \( b \), and thus has no incentive to deviate from \( b \). Therefore, \( b \) is an equilibrium. //

Examples of EA-implementable demands

Theorem 3 provides a simple criterion for determining whether a demand vector can be implemented via an EA equilibrium. The following examples illustrate this point.

A game with a core solution. The five-person simple majority game depicted in Figure 4 satisfies Plotz's(1967) conditions and thus has a unique core point at \( x^* - \beta_2 \). Only this point is feasible, given the stable demand vector \( d \) plotted in the figure, and, thus, Theorem 3 implies that these demands can be implemented in the bargaining game via an EA stationary equilibrium.

Three games with an empty core. Figure 5 provides the basis for the following two examples. First, consider the three-person, simple majority game involving only players 1, 2, and 4 and the (pair-wise tangent) demands for these players, as plotted in the figure. This stable demand vector supports

[Figures 4, 5, and 6 about here.]
Figure 4: Stable demands in a game with a core solution
Figure 5: Stable demands and competitive solution points in a Two-Insiders game
Figure 6: Stable demands and competitive solution points in a modified Two-Insiders game
the competitive solution of this three-person game and, by Theorem 3, can be implemented via an EA equilibrium strategy since none of the three feasible outcomes exceeds anyone's demand.

Second, let us add players 3 and 5 with bliss points at \( \beta_3 \) and \( \beta_5 \), respectively, and demands as plotted in Figure 5, thus defining the five-person, simple majority "two-insiders" game (Laing and Olmsted, 1978). Then the vector of demands plotted as arcs in the figure is stable and provides the basis for the competitive solution of this game. [If anyone increased his demand in this game, then the resulting demand vector would not be stable in the sense of Albers(1987) or Bennett and Winer(1983).] Only the three competitive solution points — \( x_{12} \), \( x_{14} \) and \( x_{24} \) — are feasible, given these demands. Yet, for players 3 and 5, there exists a solution point that exceeds the player's demand, hence, by Theorem 3, this demand vector cannot be implemented in the bargaining game via an EA stationary equilibrium.

Finally, suppose that we modify the last game slightly by moving the bliss points of players 3 and 5 to the positions shown in Figure 6. Then, if the demands of these two players are as plotted, the resulting demand vector again defines the same competitive solution points as before, and is stable. In this knife's edge case, each player is indifferent among the competitive solution points that satisfy his demand. Therefore, by Theorem 3, this stable demand vector equals the quota vector associated with an EA equilibrium.

The Knife's Edge

These conclusions depend crucially on the strategies employed by any responders who is indifferent between accepting a proposal and, instead, rejecting it to initiate his own proposal. The next result demonstrates that, in any outcome under a stationary equilibrium that is enacted by a winning coalition, at least one responder must face this situation.

**Lemma 9 (the knife's edge).** Let \( b \) denote any stationary equilibrium strategy with associated quota vector \( q(b) \) and outcomes \( O(b) \). For every winning coalition \( C \) and for every proposal \( (x, C, S, j) \) by any initiator \( i \in N \) to enact an outcome \( (x, C) \in O(b) \), there exists some responder \( k \in C \setminus i \) such that \( u_k(x) = q_k(b) \).
Proof. Suppose that, in accordance with a stationary equilibrium \(b\), some player \(i\) initiates the proposal \((x,C,S,j)\) to enact an outcome \((x,C)\in O(b)\) by a winning coalition, \(C\). Assume also, contrary to the lemma's assertion, that for every \(k\in C\setminus i\), \(u_k(x)\neq q_k(b)\). Then, by Lemma 1, \(x\in P_{C\setminus i}(q)\). There are two possibilities, depending on whether \(x=\beta_i\).

First, suppose that \(x\neq\beta_i\). Then, since \(X\) and \(P_{C\setminus i}(q)\) are convex and the latter is open, there exists some \(\lambda\in (0,1)\) such that \(w = [\lambda x+(1-\lambda)\beta_i] \in P_{C\setminus i}(q)\). Then \(u_i(w) > u_i(x)\), since \(u_i\) is strictly quasiconcave, and, by Lemma 3, \(u_i(x) > q_i(b)\). Therefore, \(u_i(w) > q_i(b)\). But then \(P_{C}(q) \neq \emptyset\); hence, \(q_i(b)\) contains slack, in contradiction of Lemma 8.

Second, consider the remaining possibility: \(x=\beta_i\). Select any responder \(k\in C\setminus i\) to the proposal by \(i\) to enact the outcome \((\beta_i,C)\) and suppose that it is \(k\)'s move to respond. Then \(k\) can reject the proposal, take the initiative, and propose the same outcome, \((\beta_i,C)\), listing \(i\) as the first responder in \(C\setminus k\). By Lemma 3, \(i\) is guaranteed that every responder in \(C\setminus (i\cup k)\) will accept the proposal if \(i\) does. Thus, at this position, \(h_i(b\|r_i) = u_i(\beta_i)\). Since \(b\) is optimal at \(r_i\) and \(\beta_i\) is the unique point in \(X\) that maximizes \(u_i\), \(k\) can be certain that, for every \((x',C')\in O(b\|r_i)\), \(x' = \beta_i\). Therefore, \(h_k(b\|p_k) = q_k(b) \geq u_k(\beta_i) = u_k(x)\), contradicting the assumption that \(q_k(b) < u_k(x)\). //

The next example, unlike "The House," is perfectly symmetric and thus facilitates more complete investigations of strategies at the knife's edge.

Implementing Stable Demands in a Symmetric Simple Majority Problem: The Star

EFS strategies. Given the complete symmetry of the five-person simple \((3/3's)\) majority problem depicted in Figure 7, I shall limit attention to stationary strategies that not only are path independent, but also are symmetric. In particular, I shall assume in the analyses of "The Star" that (1) at each chance move, Nature selects each player as initiator with probability \(1/5\), (2) all players adopt identical strategies, (3) every initiator's strategy is uniformly and completely mixed over the proposals in the support set of \(b\) at each of his initiator positions, (4) only simple

[Figure 7 about here.]
Figure 7: Stable demands and competitive solution points in a symmetric game: The Star
(three-person) majorities are considered, and (5) the response strategies satisfy the following properties, where \( j \) and \( k \), respectively, denote the first and second responder listed in a proposal, and \( \mathbf{q}(b) \) is the quota vector associated with the stationary strategy \( b \):

1. **(F)** \( j \) rejects with probability \( \begin{cases} 0 \\ 1 \end{cases} \) if \( h_j(b|\text{accept}) \begin{cases} < \\ > \end{cases} q_j(b) \), where \( f \geq 0 \).
2. **(S)** \( k \) rejects with probability \( \begin{cases} 0 \\ 1 \end{cases} \) if \( h_k(b|\text{accept}) \begin{cases} < \\ > \end{cases} q_k(b) \), where \( s \geq 0 \).

Since only three-person coalitions are under consideration, allow me to adopt the following convention for identifying a strategy's properties. Let the term EPS strategy refer to a stationary strategy that induces a local strategy for the initiator, first responder, and second responder that satisfies, respectively, property \( F \), \( F \), and \( S \). Also, \( A \) will be inserted in place of \( F \) or \( S \) to denote the special case in which the corresponding responder accepts with certainty any proposal for which the expected payoff equals his quota (\( f = 0 \) or \( s = 0 \), respectively). By this convention, for example, EAS describes a stationary strategy under which the initiator makes extreme proposals, and, in the first and second responder's strategies, \( f = 0 \) and \( s = 0 \). EAA means that the global strategy satisfies properties \( F \) and \( A \).

**Stable demands and competitive solution points in the Star.** The vector of demands plotted in Figure 7 is stable (in sense of Albers, 1987, and Bennett and Winer, 1983) and supports the competitive solution points of this game (McKelvey, Ordezhook, and Winer, 1978). Moreover, this is the unique demand vector \( d \) for this game that not only is semistable, but also is symmetric, in the sense that [given the way payoffs are scaled below] for every pair of players \( i \) and \( j \), \( d_i = d_j \).

**Scaling the payoffs.** Exactly five decision alternatives satisfy these demands of some simple majority in this game, and each of these points is denoted by \( x_i \), indicating that player \( i \) strictly prefers this point to the other four that are consistent with these demands. Assume that the utility function over the decision alternatives \( X \) of each player \( i \in N \) is linear in distance from his bliss point and, without loss of generality, scale this function (and the demand vector) such that \( u_i(x_i) = 1 \), \( u_i(x_i@1) = u_i(x_i@2) = q \), and \( u_i(x_i@2) = u_i(x_i@2) = 0 \), where the subscripts follow the nonstandard notation adopted earlier, and \( q = 0.545^+ \). The analysis will concern proposals that one
of these five outcomes be adopted as the majority-rule decision.

Stationary strategies in the Star. The following results, as indicated by an asterisk, are proved for a special case: the simple majority rule Star with an unattractive status quo. They delimit the path independent and symmetric strategies that constitute stationary equilibria for this game.

Corollary 2* (quota demands). For the Star under simple majority rule, let q(b) be the quota vector associated with any stationary strategy b that is both path independent and symmetric, and let d denote the unique vector of demands that is both symmetric and semistable, as plotted in Figure 7. Then b is a stationary equilibrium only if q(b)=d.

Proof. By the first part of Theorem 2, every stationary equilibrium b implements a demand vector d=q(b) that is semistable. Since b is symmetric and path independent, then q(b) is symmetric. But d is the unique semistable and symmetric demand vector for the Star. Therefore, the symmetric and path independent strategy b is a stationary equilibrium for the Star only if q(b)=d. //

Lemma 10* (initiator’s strict advantage). In the Star under simple majority rule, given that default is unattractive, let b denote any stationary strategy equilibrium that is both symmetric and path independent. Then — for every player iεN and every initiative p_i^x_p_i^y of i, and for every position y such that either yεP^0 or yεP^j for some jεN\i — h_i(b|p_i) > h_i(b|y).

Proof. Since b is an equilibrium and default is unattractive, we may ignore the possibility of infinite play, by Lemma 4. Without loss of generality, given all the symmetry, consider the situation of player 1. Let b_k|p_i denote the probability that alternative x_k is enacted as the outcome, given that the game has reached the initiator position p_i assigned to player i and that b governs behavior. Then, by symmetry and the way payoff functions are scaled,

\[ h_1(b|p_i) = b_{11} + 2qb_{21} = q \]

Therefore,

\[ b_{11} = q(1-2b_{21}) \]
If player 2 has the initiative, then the expected payoff to 1 is
\[
h_1(b|p_2) = b_{12} + q(b_{22} + b_{52})
\]
By symmetry, this may be rewritten
\[
h_1(b|p_2) = b_{21} + q(b_{11} + b_{31})
= b_{21} + q[b_{11} + .5(1-b_{11} - 2b_{21})]
\]
Substituting by [1] in this expression and combining terms yields
\[
h_1(b|p_2) = b_{21}[1-q(1+q)] + q(1-q)/2
\]
Therefore
\[
[2] \quad h_1(b|p) < q \iff b_{21} < q(1-q)/2[1-q(1+q)] \approx .789^+
\]
By symmetry, \( b_{51} = b_{21} \), and \( \Sigma k b_k = 1 \). Hence \( b_{21} \leq 1/2 \) and the strict inequality in [2] obtains. Therefore, \( h_1(b|p_2) \) [hence, by symmetry, \( h_1(b|p_3) \)] is strictly less than 1's quota.
The proof for \( p_3 \) (hence \( p_4 \)) is similar:
\[
h_1(b|p_3) = b_{13} + q(b_{23} + b_{53}) = b_{31} + q(b_{21} + b_{31})
= -b_{21} + .5(1+q)(1-b_{11})
\]
Using [1] to substitute for \( b_{11} \) in this expression and simplifying yields
\[
h_1(b|p_3) = -b_{21}[1-q(1+q)] + (1+q^2)/2
\]
Therefore,
\[
h_1(b|p_3) < q \iff -b_{21}[1-q(1+q)] + (1+q^2)/2 < q
\]
Simplifying yields
\[
[3] \quad h_1(b|p_3) < q \iff b_{21} > [1-q(2+q)]/[1-q(1+q)]
\]
The numerator of the fraction in [3] is negative, while the denominator is strictly positive; thus, since \( b_{21} \geq 0 \), \( h_1(b|p_3) < q \).
Finally, at any move assigned \( y \in \mathbb{P} \) assigned to Nature, there is positive probability that some \( j \neq 1 \) will be selected as initiator, and \( h_1(b|p_j) < q \) for all \( j \in N \backslash 1 \). Therefore, \( h_1(b|y) < q \). \(/\)

Lemma II* (never propose the worst). For the Star under simple majority rule, let \( b \) denote any stationary strategy that is both symmetric and path independent. For any player \( i \in N \) and any initiator position \( p_i \in P_i \), if \( b \) assigns positive probability at \( p_i \) to any proposal by \( i \) to enact either of the alternatives \( x_i \in \omega_2 \) or \( x_i \in \omega_{-1} \), then \( b \) is not an equilibrium.

Proof. Suppose that at \( p_i \) player \( i \) proposes the outcome \( (x, c) \), where \( x \in (x_i \in \omega_2, x_i \in \omega_{-1}) \) and \( u_i(x) < q \). Then, lest Lemma 5(2) be contradicted, some
responder \( j \in C \setminus i \) must reject the proposal, hence, by Lemma 10*, the expected payoff to \( i \) for making the proposal is less than \( q \). Therefore, \( b \) is not optimal at \( p_1 \).

**Extreme proposals.** The next result identifies a stationary equilibrium instructing each initiator to propose his favorite competitive solution point.

Proposition 1* (EPA equilibrium in the Star). In the simple majority rule Star with unattractive status quo, let \( d \) be the stable demand vector that defines the competitive solution and, for each player \( i \in N \), let \( x_i \) denote player \( i \)'s favorite competitive solution point. Assume that \( b \) is a global stationary strategy with associated quota vector \( q(b) \cdot d \) such that \( b \) (1) induces local response strategies that satisfy properties \( F \) and \( S \), and (2) instructs each initiator, \( i \in N \), to propose each of the outcomes \((x_i, i, i^{\oplus 1}, i^{\oplus -1})\) and \((x_i, i, i^{\oplus -1}, i^{\oplus 1})\) with probability \( 1/2 \). Then \( b \) is a stationary equilibrium if and only if \( s = 0 \) and \( f = f^* \), where \( f^* \in (0,1) \) is unique (\( f^* \approx 0.8297 \)).

**Proof. Necessity.** Given any probability \( f \geq 0 \), assume that \( s > 0 \). Without loss of generality, suppose that \( b \) is an equilibrium that implements \( d \) and assume that, in accordance with \( b \), player 1 exercises the initiative at \( p_1 \) by proposing \((x_1, C)\). Since \( x_1 \neq z \), \( C \neq W \). If \( C \cap \{3,4\} \neq \emptyset \) then the proposal would be rejected, hence, by Lemma 10*, \( b \) would not be an equilibrium. Without loss of generality, given the symmetry, suppose now that 1 proposes the outcome \((x_1, 152)\). Note that \( u_2(x_1) = q \), so 2 is indifferent to accepting this proposal, thus ending the game, or rejecting it to take the initiative. Therefore, in accordance with \( b \), 2 rejects the proposal with probability \( s \), so the expected payoff to player 5 for accepting the proposal is

\[
h_5(b|\text{accept}) = (1-s)u_5(x_1) + s h_5(b|p_2) = (1-s)q + s h_5(b|p_2)
\]

If \( s > 0 \), then the right hand side is strictly less than \( q \), by Lemma 10*. Consequently, 5 will reject the proposal and assume the initiative; hence, again by Lemma 10*, the proposal is not optimal for player 1 at \( p_1 \), contradicting the assumption that \( b \) is a stationary equilibrium. Therefore, \( b \) is a stationary equilibrium only if \( s = 0 \). Moreover, given \( s = 0 \) and \( u_1(x_1) > d_1 \), \( f = 0 \) implies that \( b \) is not an equilibrium, by Theorem 3. Therefore, if \( b \) is an equilibrium then \( s = 0 \) and \( f > 0 \).
Now assume \( s=0 \) and \( f>0 \). Given the way payoffs are scaled and that the game and global strategy \( b \) are both symmetric, then the expected payoff to 1 if he has the initiative is

\[
h_1(b|P_1) = b_{11} + q(b_{21} + b_{31}) = b_{11} + 2qb_{21}
\]

Hence, since \( h_1(b|P_1) - d_1 = q \),

\[4\] \( b_{11} + 2qb_{21} = q \)

In accordance with the Markov process induced by \( b \),

\[5\] \( b_{11} = \frac{1}{2} f(b_{12} + b_{15}) + (1-f) \), and

\[6\] \( b_{21} = \frac{1}{2} f(b_{22} + b_{25}) \)

Since \( b \) is symmetric, we may rewrite \( [5] \) as

\[5'\] \( b_{11} = fb_{21} + (1-f) \)

and \( [6] \) as

\[
\begin{align*}
b_{21} &= \frac{1}{2} f(b_{11} + b_{31}), \text{ hence} \\
b_{21} &= \frac{1}{2} f[b_{11} + 0.5(1-b_{11} - 2b_{21})]
\end{align*}
\]

Rearranging terms in the last equation yields

\[6'\] \( b_{11} = \frac{2b_{21}(2+f)}{f} - 1 \)

which, given \( f>0 \), is well defined.

Solving \( [5'] \) and \( [6'] \) simultaneously,

\[
f b_{21} + (1-f) = \frac{2b_{21}(2+f)}{f} - 1
\]

hence

\[7\] \( b_{21} = \frac{f(2-f)}{[4+f(2-f)]} \)

where the denominator is greater than zero, given that \( f \) is a probability.

Substituting this expression for \( b_{21} \) in \( [5'] \) yields

\[8\] \( b_{11} = \frac{[f^2(2-f)/[4+f(2-f)]) + (1-f)}{f} \)


\[9\] \( -(1+q)f^2 - 2(1-q)f + 4(1-q) = 0 \)

Given that \( f \) is a probability, \( [9] \) has a unique solution:

\[ f^* = 0.8297161245 \]

Therefore, \( b \) is a stationary equilibrium in the Star only if \( s=0 \) and \( f=f^* \).

**Sufficiency.** Now suppose that behavior is governed by \( b \) and that \( s=0 \) and \( f=f^* \). Without loss of generality, consider again the proposal \((x_1, 152)\). The above calculations have established that, by making this proposal, 1 expects to attain his quota. No other proposal can yield him a greater payoff. Therefore 1 has no incentive to deviate from \( b \). Given the opportunity 2 will accept the proposal, and both 5 and 2 will attain their quotas. Therefore 2 has no incentive to deviate from \( b \). On the other
hand, if $5$ rejects and assumes the initiative, then, again, $5$ expects to achieve only his quota. Thus no player in this coalition has incentive to deviate from $b$ and these players were chosen arbitrarily. Therefore, $b$ is a stationary equilibrium. //

Thus, the proceeding results have begun to delimit the symmetric, path independent, stationary strategies that implement the stable demand vector for the Star. Lemma 11* eliminates strategies that induce any initiator to propose a competitive solution point that falls short of his demand. Proposition 1* then identifies a very special, knife-edge case ($s=0$ and $f=f^*$) in which the stable demand vector for the Star is implemented by a strategy that induces the initiator to propose the competitive solution point that exceeds his demand. I next investigate whether the stable demand vector for this game can be implemented via strategies that induce each initiator to propose only outcomes that exactly yield his demand.

**Modest proposals.** In general, define the following property that any stationary strategy $b$ might possess:

(Q) For every player $i \in N$, initiative $y \in P^i_x$, and option $y' \in A_i(y) \ of\ i\ at\ y$, $b_y(y') > 0$ implies that $y'$ is a proposal of some outcome $(x,C)$ satisfying $u_i(x) = q_i(b)$. Thus, $b$ satisfies property Q if it induces a local strategy at every initiator position such that positive probability is assigned at that position only to proposals that, if accepted, would yield the initiator his quota exactly; such a proposal is said to be **modest**.

**Proposition 2***( QFS implementation of stable demands in the Star).** For the Star under simple majority rule with an unattractive status quo, let $d$ denote the stable demand vector and let $b$ denote any symmetric, stationary global strategy that seeks only a bare majority and exhibits properties Q, F, and S. Then $b$ implements the stable demand vector $d$ if and only if, for every proposal that is induced by $b$ to enact an outcome $(x,i,j,k)$, either

(a) $f>0, s=0, u_j(x)<d_j$ and $u_k(x)<d_k$, or

(b) $f=0, s>0, u_j(x)>d_j$ and $u_k(x)>d_k$,

where $j$ and $k$, respectively, are first and second responders to the proposal.
Proof. Suppose \( b \) is a stationary and symmetric QFS equilibrium that implements the stable demand vector \( d \). For any \( i \in N \), consider any proposal by \( i \) to enact an outcome \((x, ijk)\), in accordance with \( b \). By Theorem 2 and the definition of a competitive solution, \( x \) is a competitive solution point that satisfies both responders' demands. Since \( b \) is a stationary, symmetric equilibrium that satisfies \( Q \) and implements \( d \), then the outcome \( x \) exceeds one responder's demand and satisfies exactly the demand of the other. By Theorem 3, at least one responder \( r \in \{j,k\} \), if indifferent, must reject the proposal with strictly positive probability. However, if the proposal is rejected with positive probability, then, by Lemma 10*, the initiator's expected payoff does not satisfy his demand. But this contradicts the assumption that \( b \) is a stationary equilibrium that implements the stable demand vector. Therefore, the indifferent responder must accept with certainty, while the responder who, if indifferent, would reject the proposal with positive probability, must be offered an outcome that exceeds his demand. //

Therefore, if \( b \) is a symmetric QFS strategy that implements the stable demand vector for the Star, then \( b \) must be a QAS or QFA stationary equilibrium, as characterized by Proposition 2*. The next two results deal with these possibilities, thus completing the characterization of symmetric stationary equilibria that implement the stable demand vector for this game.

Proposition 3* (QFA implementation of stable demands in the Star). For the Star, let \( b \) denote a QFS stationary global strategy such that (1) the associated quota vector \( q(b) \) equals the stable demand vector \( d \), (2) \( b \) induces local behavioral response strategies such that \( f>0 \), \( s=0 \), and (3) for every player \( i \in N \) and initiative \( p_i^1 \), \( b \) instructs \( i \) to propose one of the two competitive solution outcomes \((x_1, 1, i^{1*}, i^{2*})\) or \((x_1, 1, i^{1*}, i^{2*})\), each with probability \( \frac{1}{2} \). Then \( b \) is a stationary equilibrium if and only if \( f \geq 2(1-q) \) \([\approx 0.909^+]\).

Proof. Without loss of generality, given the symmetries, suppose that, at initiative \( p_1^1 \), player 1 proposes the outcome \((x_2, 123)\). As second responder, 3 would accept this proposal with certainty since \( s=0 \) and \( u_3(x_3)=q \). Then 2 also must accept with certainty, since \( u_2 \) accepts \((x_2, 123)\).
123)\r\n\r\nIn this situation, neither responder has incentive to deviate from \( b \). Therefore, the expected payoff to 1 for making this proposal is \( u_1(x_2) = q \).

To determine whether 1 has reason to deviate from \( b \) at \( p_1 \) note that only the outcome \( x_1 \) could increase 1's payoff, given the others' demands. Again without loss of generality, given symmetry, suppose that \( c_1 \) is a global strategy in which player 1 deviates unilaterally from \( b \) at position \( p_1 \) by proposing the outcome \( (x_1, 152) \). Since \( s = 0 \) and \( u_2(x_1) = q \), player 2 would accept this proposal with certainty. Therefore, \( u_2[\text{accept} (x_1, 152)] = u_2(x_1) = q \), hence 2 is indifferent and, in accordance with \( b \), rejects the proposal with probability \( f > 0 \). If 2 rejects this proposal to take the initiative, then, in accordance with \( b \), 5 proposes \( (x_1, 512) \) with probability \( 1/2 \) and otherwise proposes \( (x_4, 543) \). Player 1 has no incentive to reject the first proposal since 2 would accept it, thus giving 1 his favorite competitive solution point. The second proposal also would be accepted with certainty, in accordance with \( b \). Therefore, the expected payoff to 1 for deviating from \( b \) at \( p_1 \) by proposing \( (x_1, 152) \) is
\[
h_1(c_1 | p_1) = (1-f)u_1(x_1) + (f/2)[u_1(x_1) + u_1(x_4)] = 1 - (f/2).
\]
Therefore 1 has no incentive to deviate unilaterally from \( b \), hence \( b \) is a stationary equilibrium, iff
\[
h_1(c_1 | p_1) = 1 - (f/2) \leq h_1(b | p_1) = q
\]
or, equivalently, \( f \geq 2(1-q) \) \[ \approx 0.909^+ \]. //

Thus, the QFA strategy \( b \) implements the stable demands in the Star if and only if it satisfies a rather severe restriction.

The next result indicates that the stable demands in this game can be implemented under considerably less restrictive conditions.

Proposition 4* (QAS implementation of stable demands in the Star). For the Star, let \( b \) denote a QFS stationary global strategy such that (1) the associated quota vector \( q(b) \) equals the stable demand vector, \( d \), (2) \( b \) induces local behavioral response strategies such that \( f = 0 \) and \( s > 0 \), and (3) for every player \( i \in N \) and initiative \( p \in P_1 \), \( b \) instructs \( i \) to propose one of the two competitive solution outcomes \( (x_1, 0, 0, 1) \) or \( (x_0, 0, 1, 1) \) or \( (x_1, 0, 0, 1) \), each with probability \( 1/2 \). Then \( b \) is a stationary equilibrium.
Proof. Without loss of generality, given the symmetries, suppose that 1 has the initiative at \( p_1 \) and proposes the outcome \((x_2, 132)\). Since \( u_2(x_2) = 1 > q \), 2 has no incentive to deviate from \( b \) as second responder to this proposal, and would accept with certainty. Consequently, at 3's move as first responder to this proposal, \( u_3(\text{accept } (x_2, 132)) = u_3(x_2) - q \). Hence, 3 has no incentive to deviate unilaterally from \( b \) and would accept the proposal with certainty. Therefore, \( u_1(\text{propose } (x_2, 132)) = q \). Only the outcome \( x_1 \) would yield 1 a greater payoff. Without loss of generality, given symmetry, suppose 1 deviates from \( b \) at \( p_1 \) by proposing the outcome \((x_1, 152)\). Since \( u_2(x_1) = q \), 2 would reject this proposal with probability \( \varepsilon > 0 \), in accordance with \( b \). If 2 rejected this proposal to assume the initiative, then he would propose \((x_1, 251)\) with probability \( 1/2 \) and, otherwise, \((x_3, 243)\). Either of these proposals would be accepted with certainty. As a result, by accepting 1's proposal as first responder, player 5 expects to achieve

\[
u_5(\text{accept } (x_1, 152)) = (1 - \varepsilon) u_5(x_1) + 0.5 \varepsilon [u_5(x_1) + u_5(x_4)] = (1 - 0.5 \varepsilon) q < q.
\]

Therefore, 5 would reject this proposal, then propose \((x_1, 521)\) with probability \( 1/2 \) and, otherwise, \((x_4, 534)\) and either proposal would be accepted. Consequently, in deviating from \( b \) at \( p_1 \) by proposing \((x_1, 152)\), 1 expects a payoff of \( 0.5 [u_1(x_1) + u_1(x_4)] \) = \( 1/2 < q \). Therefore, neither the initiator nor any responder has incentive to deviate from \( b \). //

In sum, the last three propositions indicate three types of stationary equilibria that implement the stable demand vector in the Star. However, the unique EFA equilibrium of Proposition 2* based on extreme proposals requires that a very stringent condition be satisfied by the responders' strategies. In this sense, the equilibria based on modest proposals — the OFA and QAS strategies, respectively, of Propositions 3* and 4* — provide a more direct way of implementing the stable demand vector in this game. For this reason I shall now investigate whether analogous strategies can be constructed to implement the stable demands for the House.

**QFS Implementations of Stable Demands in the House**

In constructing symmetric QFS strategies for the House, I shall impose only those symmetries present in this game. It is apparent by inspection of
Figure 8, which reproduces for convenience Figure 3, that players 1 and 5 are in symmetric roles, as are players 2 and 4. The linear payoff functions across the competitive solution points will be normalized as in Table 1.

<table>
<thead>
<tr>
<th>COMPETITIVE SOLUTION POINTS</th>
<th>x_1</th>
<th>x_2</th>
<th>x_3</th>
<th>x_4</th>
<th>x_5</th>
</tr>
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<td>1</td>
<td>q_1</td>
<td>0</td>
<td>w_1</td>
<td>q_1</td>
</tr>
<tr>
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<td>q_2</td>
<td>1</td>
<td>q_2</td>
<td>w_2</td>
<td>0</td>
</tr>
<tr>
<td>3:</td>
<td>0</td>
<td>q_3</td>
<td>1</td>
<td>q_3</td>
<td>0</td>
</tr>
<tr>
<td>4:</td>
<td>0</td>
<td>w_4 = w_2</td>
<td>q_4 = q_2</td>
<td>1</td>
<td>q_4 = q_2</td>
</tr>
<tr>
<td>5:</td>
<td>q_5 - q_1</td>
<td>w_5 = w_1</td>
<td>0</td>
<td>q_5 - q_1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: q_1 \approx .535^*, w_1 \approx .137^*, q_2 \approx .669^*, w_2 \approx .193^+, q_3 \approx .503^*.

Table 1. Normalized Linear Payoff Functions for the House

I shall consider QFA strategies for the House that preserve the symmetries among players and seek to enact competitive solution outcomes via three-person majorities.

**QFA strategies for the House.** Given a proposal of any outcome (x,ijk), assume that the local response strategies are: the second responder k \in N rejects the proposal with probability \pi = 0 if he is indifferent, and the first responder j \in N rejects with probability f_k > 0 if he is indifferent, where, by symmetry, f_1 = f_5 and f_2 = f_4. Also stipulate that the symmetric QFA global strategies at issue induce local strategies at each initiative to propose competitive solution outcomes as specified in Table 2.

<table>
<thead>
<tr>
<th>INITIATOR:</th>
<th>PROPOSES</th>
<th>WITH</th>
<th>OTHERWISE PROPOSES</th>
<th>OUTCOME:</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUTCOME:</td>
<td>PROBABILITY:</td>
<td></td>
<td>OUTCOME:</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(x_2,123)</td>
<td>m_1</td>
<td>(x_5,154)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(x_3,234)</td>
<td>m_2</td>
<td>(x_1,215)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(x_2,321)</td>
<td>1/2</td>
<td>(x_4,345)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(x_3,432)</td>
<td>m_4 = m_2</td>
<td>(x_5,451)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(x_4,543)</td>
<td>m_5 = m_1</td>
<td>(x_1,512)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Modest Proposals Induced by a Symmetric QFA Strategy to Enact Three-Person Competitive Solution Outcomes in the House

Given these specifications, the proof given in the appendix constructs the following result.

[ Figure 8 about here. ]
Figure 8: Stable demands and competitive solution points for the House
Proposition 5* (QFA implementation of stable demands in the House). Let $b$ denote any such symmetric QFA global strategy for the House with associated quotas $q(b)=d$, the stable demand vector. Then $b$ is a stationary equilibrium if and only if the following conditions all obtain:

1. $1 \geq f_1 \geq (1-q_2) + \frac{(1-q_1)/(1-w_1)}{1-q_2}$ \quad [$= .870^+]$
2. $1 \geq f_2 \geq 2-(q_1 + q_3)$ \quad [$= .963^-]$ 
3. $1 \geq f_3 \geq 2(1-q_2)/(1-w_2)$ \quad [$= .821^-$]
4. $1 - \frac{(1-q_2)/f_1}{f_1} \geq m_1 \geq \frac{(1-q_1)}{f_1(1-w_1)}$
5. $1 - \frac{(1-q_3)/f_2}{f_2} \geq m_2 \geq \frac{(1-q_1)}{f_2}$

**Proof.** See appendix.

If the responders' strategies satisfy the first three of these conditions, then the last two inequalities imply, to three places, that $m_1 \in (.539, .668)$ and $m_2 \in (.465, .503)$. Thus, at such a QFA equilibrium, the first responders reject with high probability any proposal to which they are indifferent, while, except for player 3, each initiator's strategy tends to be biased in favor of proposing a specific competitive solution point that yields him his demand: e.g., player 1 leans in favor of proposing $x_2$, whereas player 2, if he leans at all, tends to favor $x_1$.

**QAS strategies for the House.** Now assume that, given a proposal of any outcome $(x,ijk)$, the local response strategies are, instead, as follows: the first responder $j \in N$ rejects the proposal with probability $f=0$ if he is indifferent, and the second responder $k \in N$ rejects with probability $s \geq 0$ if he is indifferent, where, by symmetry, $s_1=s_5$ and $s_2=s_4$. Also stipulate that the symmetric QAS global strategies at issue induce modest proposals at each initiative in accordance with the local strategies specified in Table 3.

<table>
<thead>
<tr>
<th>INITIATOR</th>
<th>PROPOSES WITH</th>
<th>OTHERWISE PROPOSES</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUTCOME:</td>
<td>PROBABILITY:</td>
<td>OUTCOME:</td>
</tr>
<tr>
<td>1</td>
<td>$(x_2,123)$</td>
<td>$m_1$</td>
</tr>
<tr>
<td>2</td>
<td>$(x_3,243)$</td>
<td>$m_2$</td>
</tr>
<tr>
<td>3</td>
<td>$(x_2,312)$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>4</td>
<td>$(x_3,423)$</td>
<td>$m_4=m_2$</td>
</tr>
<tr>
<td>5</td>
<td>$(x_4,534)$</td>
<td>$m_5=m_1$</td>
</tr>
</tbody>
</table>

**Table 3.** Modest Proposals Induced by a Symmetric QAS Strategy to Enact Three-Person Competitive Solution Outcomes in the House
Given these specifications, the following result is proved in the appendix.

Proposition 6* (QAS implementation of stable demands in the House). Let $b$ denote any such symmetric QAS global strategy for the House with associated quotas $q(b)=c$, the stable demand vector. Then $b$ is a stationary equilibrium if and only if both of the following conditions obtain:
\[
q_2 \geq m_1 \geq (1-q_1)/(1-w_1) \quad \text{and} \\
q_3 \geq m_2 \geq (1-q_1)
\]
[where $q_2 \approx .669^*$, $(1-q_1)/(1-w_1) \approx .539^*$, $q_3 \approx .503^*$, and $(1-q_1) \approx .465^*$].

Proof. See the appendix.

Note that the two conditions on initiator strategies identified by Proposition 6* are identical to those of Proposition 5* if $f_1=f_2=1$. Moreover, both types of equilibria induce initiator strategies that are biased in the same way: player 1 clearly leans towards proposing that $x_2$ be enacted, whereas player 2, if he is biased at all, tends to lean towards proposing $x_1$. These equilibria also have implications about the probability with which each outcome is reached. For example, if at any chance node $y$ the initiator is selected from a completely mixed uniform distribution and if, in accordance with the QAS equilibrium strategy $b$ of Proposition 6*, $m_1=.6$ and $m_2=.5$, then $b(x_1|y)=b(x_5|y)=.18$, $b(x_2|y)=b(x_4|y)=.22$, and $b(x_3|y)=.20$. Perhaps most importantly, both types of QFS equilibria provide a rather natural means of implementing the stable demand vector and competitive solution in this game.

4 CONCLUSIONS

This paper has demonstrated that the noncooperative approach used by Selten to analyze bargaining in characteristic function games extends naturally to simple spatial games without sidepayments. The results indicate that the cooperative solution concepts of Albers' stable demand vector and McKelvey, Ordeshook, and Winer's competitive solution can be implemented in a direct way via stationary equilibrium strategies that govern each player's negotiations in the bargaining problem represented as a noncooperative game in extensive form. These noncooperative equilibria add precision to the characterization of not only the cooperative solution but also the process through which this
solution is reached in rational play. For example, the equilibrium strategies of Propositions 5* and 6* not only specify what choices would be made at each decision node in the game tree; they also make precise statements about the probabilities with which various outcomes of the game will be reached. These features illustrate the important advantages to be gained by creating an integrated theory of both cooperative and noncooperative games. In this way one may hope to rest cooperative solution theory on first principles pertaining to the strategy used by each player to reach a cooperative agreement that maximizes his payoff, rather than on arbitrary dicta about "reasonable" properties that the cooperative solution should satisfy, or on informal appeals to intuition about the process through which the cooperative solution is reached. One also can hope that in time it will be possible to derive the cooperative solution by solving the noncooperative bargaining game, but this may require advances in the theory about how rational players select among alternative noncooperative equilibria. At the very least, noncooperative equilibrium analysis can be used to determine whether or not a cooperative solution concept is consistent with a detailed analysis of the bargaining process. It also provides a crisp benchmark for identifying ways in which cognitively limited human beings systematically deviate in their bargaining from perfectly rational play.

APPENDIX

Proof of Proposition 5*. To avoid unnecessary repetition, note that, by conforming to b, each responder will accept any proposal that is consistent with b, thereby attaining a payoff that satisfies his demand (quota). A responder has incentive to deviate unilaterally from b if and only if he expects to achieve a greater payoff by rejecting the proposal and then, as initiator, making a proposal that deviates from b. Therefore we need only consider unilateral deviations from b that might tempt the initiator, under the assumption that no responder (qua responder) has incentive to deviate from b. Each such deviation will be examined and necessary and sufficient conditions constructed so that the initiator has no incentive to deviate in this way from b. Without loss of generality, since b is stationary and symmetric, we consider one initiative each for players 1, 2, and 3.
**Initiator-1.** Player 1 has two deviations from b at his initiative p1 that might tempt him.

First, suppose that 1 proposes \((x_1, 125)\). Since 5 would accept, 2 is indifferent and, in accordance with b, rejects the proposal with probability \(f_2 > 0\). Given the initiative, 2 would propose \((x_3, 234)\) with probability \(m_2\) and, otherwise, \((x_1, 215)\); 2's proposal would be accepted. Therefore the expected payoff to 1 for making this proposal is

\[
h_1[\text{propose } (x_1, 125)] = (1-f_2)u_1(x_1) + f_2[m_2u_1(x_3)+(1-m_2)u_1(x_1)]
\]

\[= (1-f_2)+f_2(1-m_2) = 1-f_2 m_2\]

Then \(h_1[\text{propose } (x_1, 125)] \leq q_1\), hence 1 has no incentive to deviate from b in this way, if and only if \(f_2 m_2 \geq (1-q_1)\).

Second, assume that 1 proposes \((x_1, 152)\). Since 2 would accept and \(u_5(x_1) = q_5\), 5 would reject 1's proposal with probability \(f_5 - f_1 > 0\). Given the initiative, 5 would propose \((x_4, 543)\) with probability \(m_5 - m_1\) and, otherwise, \((x_1, 512)\); either proposal would be accepted. Hence

\[
h_1[\text{propose } (x_1, 152)] = (1-f_1)u_1(x_1) + f_1[m_1 u_1(x_4)+(1-m_1)u_1(x_1)]
\]

\[= (1-f_1)+f_1(m_1 w_1+(1-m_1)) = 1-f_1 m_1 (1-w_1)\]

Then \(h_1[\text{propose } (x_1, 125)] \leq q_1\) if and only if \(f_1 m_1 \geq (1-q_1)/(1-w_1)\).

Therefore, 1 has no incentive to deviate in either way from b at his initiative \(p_1\) if and only if both \(f_1 m_1 \geq (1-q_1)/(1-w_1)\) and \(f_2 m_2 \geq (1-q_1)\). By symmetry, an analogous conclusion applies to any initiative of player 5.

**Initiator-2.** There are two deviations that might tempt 2 at \(p_2 \leq P_1\).

First, suppose 2 deviates from b by proposing \((x_2, 213)\). In accordance with b, the second responder would accept, yielding the first responder, 1, his quota. Hence 1 would reject the proposal with probability \(f_1 > 0\) and, given the initiative, would propose \((x_2, 123)\) with probability \(m_1\) and, otherwise, \((x_2, 154)\); 1's proposal would be accepted. Hence

\[
h_2[\text{propose } (x_2, 213)] = (1-f_1)u_2(x_2) + f_1[m_1 u_2(x_2)+(1-m_1)u_2(x_5)]
\]

\[= (1-f_1)+f_1 m_1 = 1-f_1 (1-m_1)\]

Consequently, \(h_2[\text{propose } (x_2, 213)] \leq q_2\) if and only if \(f_1 (1-m_1) \geq (1-q_2)\).

Second, assume that 2 proposes \((x_2, 231)\). Since 1 would accept, yielding 3 his quota, 3 would reject this proposal with probability \(f_3 > 0\). Given the initiative, 3 would propose \((x_2, 321)\) or \((x_4, 345)\), each with probability \(1/2\), and his proposal would be accepted. Hence

\[
h_2[\text{propose } (x_2, 231)] = (1-f_3)u_2(x_2) + f_3/2[u_2(x_2)+u_2(x_4)]
\]
\[(1-f_3)+(f_3/2)(1+w_2) = 1-(f_3/2)(1-w_2)\]

Then \(h_2[\text{proposes } (x_2, 231)] \leq q_2 \) if and only if \( f_3 \geq 2(1-q_2)/(1-w_2) \).

Therefore, 2 has no incentive to deviate from \( b \) at \( p_2 \) if and only if both \( f_1(1-m_1) \geq (1-q_2) \) and \( f_3 \geq 2(1-q_2)/(1-w_2) \). By symmetry, the analogous conclusion obtains when 4 has the initiative.

**Initiator=3.** Although 3 also has two tempting deviations from \( b \) at his initiative, they are symmetric. Therefore, without loss of generality, assume that 3 deviates from \( b \) by proposing \((x_3, 324)\). Since 4 would accept this proposal and 2 is indifferent, 2 would reject the proposal with probability \( f_2 > 0 \). Given the initiative, 2 would propose \((x_3, 324)\) with probability \( m_2 \), \((x_1, 215)\) otherwise, and the proposal would be accepted. Hence

\[h_3[\text{proposes } (x_3, 324)] = (1-f_2)u_3(x_3) + f_2[m_2u_3(x_3)+(1-m_2)u_3(x_1)]\]

\[-(1-f_2)+f_2m_2 = 1-f_2(1-m_2)\]

Then \(h_3[\text{proposes } (x_3, 324)] \leq q_3 \) if and only if \( f_2(1-m_2) \geq (1-q_3) \).

**Initiator=\( i \in N \).** Therefore, it follows from the above results for the various initiators that \( b \) is a stationary equilibrium if and only if the following system of inequalities is satisfied:

\[(5.1) \quad f_1m_1 \geq (1-q_1)/(1-v_1) \quad \text{and} \quad f_1(1-m_1) \geq (1-q_1) \]

\[(5.2) \quad f_2m_2 \geq (1-q_2) \quad \text{and} \quad f_2(1-m_2) \geq (1-q_2) \]

\[(5.3) \quad f_3 \geq 2(1-q_2)/(1-w_2) \]

Conditions (5.1) and (5.2) may be replaced by their respective solutions:

\[(5.1') \quad f_1 \geq (1-q_2) + [(1-q_1)/(1-v_1)] \quad \text{and} \quad 1-[(1-q_2)/f_1] \geq m_1 \geq (1-q_1)/f_1(1-v_1) \]

\[(5.2') \quad f_2 \geq 2/(q_1+q_3) \quad \text{and} \quad 1-[(1-q_3)/f_2] \geq m_2 \geq (1-q_1)/f_2. \]

**Proof of Proposition 6*.** As in the proof of Proposition 5*, only deviations from \( b \) by initiators 1, 2 and 3 need be analyzed to construct conditions that are necessary and sufficient for equilibrium.
Initiator=1. Player 1's only temptation for a deviation from b is to propose that his favorite competitive solution point, x₁, be enacted. Given the others' demands, 1 has only two such deviant proposals.

First, suppose 1 deviates from b by proposing (x₁,125). As second responder, player 5 would be indifferent between accepting this proposal or rejecting it to take the initiative. Hence, in accordance with b, 5 would reject the proposal with probability s₅ = s₁ > 0 and, as initiator, propose instead the outcome (x₄, 534) with probability m₅ = m₁ and, otherwise, propose (x₁, 521). Either proposal would be accepted. Therefore, by accepting 1's proposal as first responder, player 2 expects the payoff

\[ h₂[\text{accept } (x₁, 125)] = (1-s₂)u₂(x₁) + s₂ [m₁u₂(x₄) + (1-m₁)u₂(x₁)] \]
\[ = (1-s₂)q₂ + s₂ [m₁w₂ + (1-m₁)q₂] \]

Consequently, since s₂ > 0 and q₂ > w₂ > 0, \( h₂[\text{accept } (x₁, 125)] \) < q₁, hence 2 rejects this proposal, iff m₁ > 0. Given m₁ > 0, 2 would reject this proposal, then propose (x₃, 243) with probability m₂ and, otherwise, (x₁, 251). Either of these proposals would be accepted. Thus, at 1's initiative,

\[ h₁[\text{propose } (x₁, 125)] = m₂u₁(x₃) + (1-m₂)u₁(x₁) = 1-m₂ \]

Therefore, 1 has no incentive to deviate from b in this manner if and only if both m₁ > 0 and 1-m₂ ≤ q₁.

Second, suppose that 1 deviates at his initiative p₁ by proposing (x₁, 152). Player 2 would reject this proposal with probability q₂ > 0 and, given the initiative, propose (x₃, 243) with probability m₂ and, otherwise, (x₁, 251). Either of 2's counterproposals would be accepted. Knowing this, by accepting 1's proposal as first responder, 5 expects the payoff

\[ h₅[\text{accept } (x₁, 152)] = (1-s₂)u₅(x₁) + s₂ [m₂u₅(x₃) + (1-m₂)u₅(x₁)] \]
\[ = (1-s₂)q₅ + s₂ (1-m₂)q₅ = (1-s₂ m₂)q₅ \]

Thus, since s₂ > 0, player 5, in accordance with b, would reject this offer if and only if m₂ > 0. Given the initiative, 5 would then propose (x₄, 534) with probability m₅ = m₁ and, otherwise, (x₁, 521). Either proposal would be accepted. Consequently, the expected payoff to 1 for deviating from b in this way is

\[ h₁[\text{propose } (x₁, 152)] = m₁u₁(x₄) + (1-m₁)u₁(x₁) = m₁w₁ + (1-m₁)w₁ \]
\[ = 1-m₁ (1-w₁) \]
\[ ≤ q₁ \text{ iff } m₁ ≥ (1-q₁)/(1-w₁). \]

Therefore, 1 has no incentive to make this proposal iff m₁ satisfies the last condition.
Combining these results, when player 1 exercises the initiative, neither
any responder nor 1 has incentive to deviate from b if and only if the
following conditions both obtain: \( m_1 \geq (1-q_1)/(1-w_1) \) and \( m_2 \geq (1-q_1) \). By
symmetry, these same conditions together are necessary and sufficient for b
to be an equilibrium at any initiator position assigned to player 5.

Initiator=2. Player 2 has two deviant proposals that might be tempting
when he has the initiative.

First, suppose 2 deviates from b by proposing \((x_2, 213)\). Player 3 would
reject this proposal with probability \( s_3 > 0 \) and, given the initiative, would
propose \((x_2, 312)\) with probability \( 1/2 \) and, otherwise, \((x_4, 354)\). Either
proposal would be accepted. Therefore, as first responder to 2’s proposal, player 1 expects

\[
h_1[\text{accept } (x_2, 213)] = (1-s_3)u_1(x_2) + (s_3/2)[u_1(x_2) + u_1(x_4)]
\]

\[
= (1-s_3)q_1 + (s_3/2)(q_1+w_1)
\]

\[< q_1 \text{ since } s_3 > 0 \text{ and } w_1 < q_1.
\]

Therefore 1 would reject 2’s proposal and, instead, propose \((x_2, 132)\) with
probability \( m_1 \) and, otherwise, \((x_5, 145)\); 1’s proposal would be accepted. Hence,

\[
h_2[\text{propose } (x_2, 213)] = m_1u_2(x_2) + (1-m_1)u_2(x_3) = m_1
\]

and thus 2 has no incentive to deviate in this way if and only if \( m_1 \leq q_2 \).
Assume hereinafter that \( m_1 \) does not exceed this upper bound.

Second, suppose 2 deviates from b at \( p_2 \) by proposing \((x_2, 231)\). Player 1
would reject this proposal with probability \( s_1 > 0 \) and, given the initiative,
would propose \((x_2, 132)\) with probability \( m_1 \) and, otherwise \((x_5, 145)\). Either
proposal would be accepted. Therefore,

\[
h_3[\text{accept } (x_2, 231)] = (1-s_1)u_3(x_2) + s_1[m_1u_3(x_2) + (1-m_1)u_3(x_5)]
\]

\[
= (1-s_1)q_3 + s_1m_1q_3
\]

\[< q_3 \text{ since } s_1 > 0 \text{ and } m_1 \leq q_2 < 1.
\]

Therefore 3 will reject 2’s offer and propose \((x_2, 312)\) or \((x_4, 354)\), each
with probability \( 1/2 \). Either proposal would be accepted. Hence,

\[
h_2[\text{propose } (x_2, 231)] = [u_2(x_2) + u_2(x_4)]/2 = (1+w_2)/2 = 0.596 < q_2
\]

and 2 has disincentive to deviate from b by proposing \((x_2, 213)\).

Thus, 2 has no incentive to deviate from b at \( p_2 \) in any way iff \( m_1 \leq q_2 \).
By symmetry, the analogous conclusion applies at any initiator position
assigned to player 4.
Initiator = 3. As initiator, player 3 also may be tempted by two deviations from b, but these cases are symmetric. Therefore, without loss of generality, suppose that player 3 deviates by proposing \((x_3, 324)\). Player 4 would reject this proposal with probability \(s_4 = s_2\) and, given the initiative, would propose \((x_3, 423)\) with probability \(m_4 = m_2\), \((x_5, 415)\) otherwise, and either proposal would be accepted. Hence,

\[
h_2[\text{accept } (x_3, 324)] = (1-s_2)u_2(x_3) + s_2[m_2u_2(x_3) + (1-m_2)u_2(x_3)]
\]

\[
= [1-s_2(1-m_2)]q_2
\]

\[
< q_2 \text{ iff } m_2 < 1, \text{ given that } 0 < s_2.
\]

Thus, provided that \(m_2 < 1\), player 2 would reject 3’s proposal and, given the initiative, propose \((x_3, 243)\) with probability \(m_2\) and, otherwise, \((x_1, 251)\) and either proposal would be accepted. Consequently,

\[
h_3[\text{propose } (x_3, 324)] = m_2u_2(x_3) + (1-m_2)u_3(x_1) = m_2.
\]

Therefore player 3 has no incentive to deviate from b when he has the initiative iff \(m_2 \leq q_3\).

Initiator = i \in N. Taken together, the results obtained when each player enjoys the initiative imply that the symmetric QAS stationary strategy for the House as specified is an equilibrium if and only if it always induces completely mixed local initiator strategies that satisfy both \((1-q_1)/(1-w_1) \leq m_1 \leq q_2\) and \((1-q_1) \leq m_2 \leq q_3\). //

REFERENCES


