“Incumbent Reputations and Ideological Campaign Contributions in Spatial Competition”

91-01-01

Daniel Ingberman
Incumbent Reputations and Ideological Campaign Contributions in Spatial Competition

By DANIEL E. INGBERMAN*

January, 1991

Abstract: This paper develops an equilibrium model of incumbent-challenger spatial competition with campaign contributions. In the model contributions serve to provide information about the attributes of the candidates to the risk-averse electorate. Incumbents are distinguished from challengers by voters' perceptions -- without contributions, an incumbent who is located at his "reputation" is a less risky lottery than any challenger, but this risk-advantage deteriorates as the incumbent's position deviates from his reputation -- as well as by their strategic positions: the incumbent moves first and the challenger moves second. Each candidate takes into account the contributions that both candidates receive in (Cournot-Nash) contributor equilibrium. It is shown that for any pair of positions, if the sequential contribution process has non-refundable contributions, then for every pair of candidate positions, contributor equilibrium exists and is unique, and incumbent-challenger equilibrium exists and is unique. Furthermore, the equilibrium effects of contributions on the candidates' positions and probabilities of winning depend on the relative strengths of the effects of contributions and reputation.

*Assistant Professor of Public Policy and Management, The Wharton School of The University of Pennsylvania, Philadelphia PA 19107-6372. For their comments on this essay and earlier working papers, thanks are due to: David Austen-Smith, Randall Calvert, Dennis Eppe, Gerald Faulhaber, Howard Kranewitter, Krishna Ladha, Allan Meitze, Peter Ordeshook, Thomas Palfrey, Keith Poole, Thomas Romer, Howard Rosenhal, and Dennis Yao, and seminar participants at Caltech, Carnegie Mellon University, Columbia University, the Hoover Institution, and the University of Pennsylvania. Support from the Wharton Junior Faculty Research Fund is also gratefully acknowledged. The standard caveat applies.

Order review © "Formal Theories of Politics II" in J. Math Model.
1. Introduction

Nearly two decades of rapid growth in campaign expenditures in the U.S., especially at the national level, has provoked a plethora of comment from a wide variety of sources.\(^1\) Despite this attention, many normative and policy issues remain open questions, including the possible effects of contribution on the competitive balance in elections, and the extent to which campaign contributions present an undue economic influence on political resource allocation. No less important is the need to provide coherent theoretical explanations for reported empirical anomalies and regularities, including the interactions between incumbency and contributions.\(^2\)

This essay develops a framework designed to disentangle some of these issues using an equilibrium unidimensional spatial model of two candidate political competition constructed from primitive assumptions concerning the beliefs, perceptions, and motives of the players -- voters, candidates, and contributors -- as well as basic postulates delineating their strategic interrelationships. In this sense the analysis is closely related to Austen-Smith's (1987) work. In contrast to Austen-Smith, however, this essay diverges from the standard spatial assumptions of candidate anonymity and symmetry in order to capture empirically observed differences between incumbent and challenging candidates. Building on Bernhardt and Ingberman (1985) and Ingberman (1986), (1988), (1989) in this model incumbent and challenging candidates are distinguished by their strategic relationship (the incumbent is first mover) as well as by voters' (asymmetric) perceptions of them. In particular, the Downsian-flavored assumptions of incumbency and reputation effects are used as variables in the analysis, which yields a compelling equilibrium and associated comparative statics.

Viewed from standard theoretical perspectives, many of the results of the model are surprising. For example, denote the equilibrium positions of incumbent and challenger as \(L\) and \(R\), where \(L\) lies to the left of \(R\) (as shown below, in this model candidates choose strictly distinct positions in equilibrium). Depending on "initial conditions" (e.g., the location of the incumbent's reputation and the relative strengths of the effects of incumbency, reputation, and contributions) and the form of contributor direct payoffs as a function of candidates' positions, a *ceteris paribus* increase in contributions to \(L\) for all pairs of platforms could, in equilibrium, i) push both candidates' positions to the left and decrease \(L\)'s probability of winning; ii) pull both candidates' positions to the right and decrease \(L\)'s probability of winning; iii) "polarize" the candidates (pull \(L\)'s position to the left and \(R\)'s to the right) and increase \(L\)'s probability of winning; or iv) reduce "polarization" (push \(L\)'s position to the right and \(R\)'s to the left) and increase \(L\)'s probability of winning. Qualitatively different varieties of result emerge when contributors' direct payoffs are independent of candidate positions.
Structure of the Analysis: Following the "reputation" model of spatial competition (Bernhardt and Ingberman 1985, Ingberman 1986, 1988, 1989), this model diverges from most of spatial theory by distinguishing the incumbent from the challenger and assuming that voters are fundamentally uncertain about the post-election outcomes associated with the two candidates. The candidates' strategic relationship is a further divergence from standard spatial models of political competition. Consistent with the notion that voters' perceptions of the incumbent are based (in part) on his performance in office, the incumbent is assumed to be first-mover (Stackelberg leader) vis-a-vis the challenger: the incumbent irrevocably chooses a position, taking into account the subsequent optimal response of the challenger. Abstracting from contributions for the moment, think of voters inferring the incumbent's position from his actions in office. But the voters have not had the opportunity to observe the challenger in the same fashion. Depending on how they evaluate the incumbent's performance, this asymmetry between the candidates through voters' perceptions can hurt or help the incumbent. Some of the assumed features distinguishing incumbents from challengers therefore concern voters' perceptions, and are summarized as "incumbency" and "reputation" effects; these features are treated as variables in the analysis. When both incumbency and reputation effects are imposed, the heuristic (absent contributions) is as follows. Voters associate the incumbent with a "reputation" based on his past performance, and when his current position is identical to his reputation, they perceive him to represent a less risky lottery than any challenger. But this *ceteris paribus* risk-advantage deteriorates as the incumbent's current position diverges from voters' common perceptions of his reputation.

In the model, the electoral role of contributions is to finance candidates' costs of providing information about themselves to the risk-averse electorate; candidates value contributions only to the extent that they help to win elections (by reducing the risk voters associate with a candidate). Many -- if not all -- if the costly activities in political campaigns fit this information-provision interpretation: in addition to political advertising, the costing activities of "campaigning" include: travelling to give speeches, canvassing door-to-door, printing and disseminating literature, and supporting the staff necessary to coordinate these activities.

A specification of contributors' incentives and the strategic hierarchy between the candidates and contributors completes the model. As motivation, note that between 25-30% of the growing sums expended in U.S. congressional campaigns are provided by several thousand active PAC's who represent many diverse industry, labor, religious and ideological groups, among others. Of similar interest from a modelling perspective is the "oral tradition" that PAC's seek "access" (for example, the opportunity to attempt to convince a politician that acting in the PAC's interest is compatible with the legislator's interest). For these reasons, contributors are assumed to act "competitively" in the sense that they take the outcomes associated with the candidates and the actions of all other contributors as given when they
make their contributions, and seek to influence the probability that their favored candidates win, rather than to influence the actions they might take in office. Unlike voters, contributors treat the candidates as riskless (or symmetrically risky, independent of contributors’ expenditures.) Within this framework the degree of correlation between the outcomes contributors value and voters values can be taken as a variable in the analysis ("ideological" vs. "non-ideological" contribution). Finally, the candidates play Stackelberg against the contributors (i.e. each candidate takes into account the contributions that both candidates will receive in equilibrium).

Organization: The remainder of this essay proceeds as follows. Section 2 introduces the model, the strategic hierarchy, and defines equilibrium. Section 3 provides sufficient conditions for existence of equilibrium. Section 4 develops in detail a parameterized model "ideological contributions" that illustrates the relationship between contributions and reputation in contributor and candidate equilibrium.

Section 5 concludes. Unless noted otherwise, proofs are contained in the appendix.

2. The Model

There are three types of players in this model: voters, contributors and candidates. Each has preferences and expectations. Their payoffs are determined through interaction with other players in the context of particular strategic hierarchies and the induced mechanisms linking candidate positions, contributions, and electoral probabilities.

2.1. Voters

2.1.1. Preferences and Participation

(A1) (Ideal Points) Let \( \theta \) denote the "ideal point" of a voter of type \( \theta \) in \( R^1 \); there is a continuum of \( \theta \)'s, each of zero measure.

Within this unidimensional framework, voters treat candidates as lotteries over post-election outcomes that are completely described by mean and variance (or risk) components. Moreover, for any level of risk, voter preferences are symmetric about their ideal points:

(A2) (Expected Utility)
(i) (Symmetric Mean-Risk Expected Utility) \( U(\mu, \theta, \nu) = \text{expected utility of voter } \theta \text{ for the lottery } (\mu, \nu) \); \( \mu \) denotes the mean of the lottery, and \( \nu \) its riskiness\(^3\). \( U(\cdot) \) is twice continuously differentiable everywhere, with \( U_1(\cdot), U_2(\cdot) < 0 \) (subscripts denote partial derivatives with respect to each argument).
(ii) (Shapes and Relative Orientations of Indifference Curves) \( U_{11} < 0, U_{22} \leq 0, U_{12} \geq 0. \)

(A3) (Participation and Voting)
(i) Let \( p(\theta) \) denote the (exogenous) probability that a voter of type \( \theta \) participates in the election;
i.e. \( p(\theta) \) depends only on \( \theta \).

(ii) Any \( \theta \) who actually participates votes for the candidate who offers the highest expected utility. In the case of indifference \( \theta \) tosses a fair coin to determine his vote.

(iii) Let \( \theta(m) \) denote candidates' perceptions (at the time of their location decision) of the twice continuously differentiable density function over the location of the median at the time of the election (induced by \( p(\theta) \)) and \( \phi(m) \) the continuous (cumulative) distribution function associated with \( \theta(m) \).

Let \( m \) denote the expected median value of \( \theta \).

(A3) has several interpretations. First, Palfrey and Rosenthal (1985) show that with strategic voter abstention, a "large" electorate, and fixed outcomes in a binary election, no voter with a positive cost of voting participates in equilibrium. Hence (A3) (i) and (ii) might reflect the distribution of negative voting costs as a function of \( \theta \). In that case, \( \theta(m) \) (as defined in (A3) (iii)) is the density over the median at the time of the election, induced by individual voters' participation probabilities \( p(\theta) \).

Alternatively, voters' tastes may be stochastic. Under this second interpretation (A3) (iii) summarizes candidates' beliefs when they choose their positions concerning the identity of the median voter at the time of the election.

2.1.2. Voters' Beliefs

By contrast with standard models of spatial competition, this model distinguishes between incumbent and challenging candidates; let \( I, C \) = mean post-election outcomes (platforms, positions) that all voters associate with the incumbent and challenger, respectively. In particular, voters perceive the incumbent candidate to have a "reputation" that colors the lottery they associate with him; let \( IP = \) voters' point perception of the incumbent's reputation. Here the term "reputation" denotes the electorate's point perception of the past record of the incumbent; in this sense one could view the term "record" as a synonym for "reputation". One goal of this analysis is to disentangle the effects of incumbent "reputation" (as defined in (A4) below) from the effects of contributions on equilibrium candidate positions, probabilities of winning, and campaign contributions. As such, think of (A4) as the base case in an analysis in which the "reputation" effect is a variables.

Let \( D_I(I,C) \) and \( D_C(I,C) \) denote the contributions received by the incumbent and the challenger, respectively, as a function of candidate positions, and let \( v_I([I-IP],D_I(I,C)), v_C(D_C(I,C)) = \{v_I(\cdot), v_C(\cdot)\} = \) riskiness associated with the incumbent and challenger, respectively, as a function of the incumbent's reputation, and, through contributions, both candidates' positions.

(A4) The functions \( v_I(\cdot) \) and \( v_C(\cdot) \) are twice continuously differentiable in both arguments, and display the following properties:

1. (Reputation Effect) For any fixed \( D_I \) and \( D_C \), movement away from his reputation increases the incumbent's risk at an increasing rate: \( \frac{\partial v_I(\cdot)}{\partial |I-IP|} > 0, \frac{\partial^2 v_I(\cdot)}{\partial |I-IP|^2} > 0 \).
(2) (Incumbency Effect) For any level of contributions, an incumbent located at his reputation is less risky than the challenger: \( \forall D_I, D_C \geq 0, v_I(I=IP,D_I) < v_C(D_C) \).

(3) (Contribution Effect) (i) Contributions decrease the riskiness associated with a candidate at a decreasing rate: for \( J=I,C \),
\[
\frac{\partial v_J(\cdot)}{\partial D_J} < 0, \quad \frac{\partial^2 v_J(\cdot)}{\partial D_J^2} > 0.
\]
(ii) Holding contributions constant, the challenger's risk is independent of \( C \): \( \forall C \),
\[
\frac{\partial^2 v_J(\cdot)}{\partial D_J^2} = 0.
\]
(iii) A candidate's riskiness is unaffected by the contributions received by his opponent:
\[
\frac{\partial v_I}{\partial D_C} = \frac{\partial v_C}{\partial D_I} = 0.
\]

Contributors act only after the candidates choose their positions (see below), so (A4)(3) (i) posits that contributions directly affect only the riskiness of the lotteries associated with the candidates \( (v_C(\cdot), v_I(\cdot)) \), but not the means \( (C,I) \). This seems to be justified empirically: political advertising tends to emphasize individual-specific characteristics such as "competence" and "credibility" rather than the particular outcomes a candidate seeks to bring about. Furthermore, (A4)(3) allows a candidate to use his contributions only to reduce his own riskiness; that is, "negative advertising" (increasing the riskiness of one's opponent) is excluded from this analysis.

The remainder of assumption (A4) is identical to the basic reputation model of Bernardt and Ingberman (1985), Ingberman (1986), and Ingberman (1989) and is motivated extensively there. Although one can construct a variety of scenarios in which assumptions like (A4) are justified, a natural way to think about incumbency and reputation effects is to assume that at least some electorally salient candidate attributes (e.g., "competence", "judgement", "true motivations", etc.) are not directly observable to voters. To the extent that these unobservable candidate characteristics can be indirectly inferred from candidate behavior, the incumbent, because the voters observe him performing in office, will have a risk-advantage over the challenger. This is the incumbency effect. However, if changes in the incumbent's position confound voters' estimation of these salient but unobservable idiosyncratic incumbent attributes, then the incumbent's risk-advantage over the challenger will increasingly deteriorate as the incumbent's position deviates from voters' notions of his past positions. This is the reputation effect.

2.2. Candidates

The incumbent and challenger are solely motivated to win election -- they have no preferences over policy, nor do they value contributions except as a vehicle to gain office. Candidates have perfect
information about the environment, save for the actual identity of the median voter at the time of the election.

2.3. Contributors

(A5) (Contributor Payoffs and Beliefs)

(1) (a) (Additively Separable Direct Contributor Payoffs) Let $A(K, S_A) = A(K) - \alpha(S_A)$, and $B(K, S_B) = B(K) - \beta(S_B)$ denote the direct payoffs of A and B if candidate K wins, K = I, C, and contributions are given by $S_A$ and $S_B$ respectively. $A(K, S_A)$ and $B(K, S_B)$ do not depend on candidate risks $\nu_I(\cdot)$ and $\nu_C(\cdot)$. (b) The costs of contributions $\alpha(S_A)$ and $\beta(S_B)$ are (weakly) convex, and $A(K, S_A)$ and $B(K, S_B)$ are twice continuously differentiable in all arguments.

(2) (Spatially-Motivated Contributions) Contributors' direct evaluations of the candidates depend on candidates' positions $\{I, C\}$. (a) Contributors are indifferent between candidates with identical positions, have strict preferences between the candidates as long as $I \neq C$. (b) Contributors are "polarized" in the sense that if there are two candidates with platforms $\{L, R\}$ such that $L < R$, then A always prefers L and B always prefers R.

(A5) (2) limits the analysis to the case of strongly correlated voter and contributor preferences (contributor A always prefers the candidate whose position lies on the left, and contributor B always prefers the candidate on the right). This assumption will be relaxed in section 3. (A5) (1) (a) says that contributors' information is "better" than voters' in the sense that any risk contributors do associate with the outcomes represented by the candidates is independent of candidates' positions, their incumbent-challenger status, or the contributions any candidate has received.

2.4. Play of the Game

2.4.1. Electoral Probabilities Let $\hat{\theta}(I, C, \nu_I(\cdot), \nu_C(\cdot)) = \text{ideal point of an indifferent voter (type) when candidates are associated with lotteries } \{I, \nu_I(\cdot)\}, \{C, \nu_C(\cdot)\}$.

LEMMA 1: (1) For any two lotteries $\{J, \nu_J\}, \{K, \nu_K\}$, if $J \neq K$ and there exists a $\hat{\theta} = \hat{\theta}(J, K, \nu_J, \nu_K)$ that solves $U(\hat{\theta}) = \nu_K$, then $\hat{\theta}$ is unique. (2) For any two lotteries, $\{J, \nu_J\}, \{K, \nu_K\}$ such that $\nu_K \neq \nu_J$, the first partial derivatives of the function $\hat{\theta}(I, K, \nu_J, \nu_K)$ with respect to each of its arguments exist and are continuous.


By Lemma 1(1), whenever $I \neq C$, the unique indifferent voter $\hat{\theta}(I, C, \nu_I, \nu_C)$ divides the set of voters into two disjoint regions: to the left of $\hat{\theta}$ all voters prefer one candidate, while to the right of $\hat{\theta}$ all voters prefer the other. The $\theta$ who is the median in the set of $\theta$'s who actually participate in the election is therefore "pivotal" in the sense that the candidate s/he prefers in equilibrium will win the election. This makes electoral probabilities easy to compute. For instance, if $I < \hat{\theta}(\cdot) < C$, then $Q(\cdot) = \Phi(\hat{\theta}(\cdot))$ (= probability that the realized median voter at the time of the election lies to the left of $\hat{\theta}$).
2.4.2. Strategic Hierarchy

The timing of play is as follows. First the incumbent selects a position. Under the reputation effect, voters' perception of the incumbent's position \( I \) and the relationship between his contributions and voters' perception of the risk associated with him, \( \nu_I(\cdot) \) depend on his reputation or record (IR) and his actions and pronouncements in office. After I and the function \( \nu_I(\cdot) \) are determined, the challenger selects his position, C. Then contributors make their contributions, according to the subgame described below. Finally, the median ideal point is realized according to the density of \( \phi(m) \), the election is held, and the winner (the candidate preferred by the realized median \( \theta \)) is declared.

The timing of the model suggests the following strategic hierarchy.

(A6) (Extensive Form)

1. (Non-refundable Cournot Contributions) The contributors move last, taking into account the effects of their contributions on electoral probabilities, holding as fixed actions of other contributors and the outcomes associated with the candidates. In particular, contributors interact in the style of Cournot: at any point in time, contributors believe that the other contributor's expectations will not vary with their own. Contributors continue (simultaneously) in this way until neither contributor wants to increase their expenditures. However, once an expenditure is made, it cannot be taken back.

2. (Challenger Location) The challenger takes I as given and selects C to minimize \( Q(\cdot) \), taking into account the contributions that both candidates will receive as a function of his location. That is, the challenger plays Nash against the incumbent but Stackelberg against the contributors.

3. (Incumbent Location) The incumbent is first-mover and Stackelberg leader: he chooses his position to maximize his probability of winning, taking into account the subsequent optimal reaction of the challenger and the contributors, and the behavior of voters.

Assumption (A6) (1) posits a particular and important dynamic for contributor interaction: contributors repeatedly and simultaneously have the opportunity to contribute, until neither wants to increase its expenditures --- i.e., once contributions have been made, they cannot be retrieved from the recipient candidate. In addition, contributors are myopic --- they believe that the other contributor's actions are independent of its own. As a result, the equilibrium result of this contributor interaction
equilibrium need not be subgame perfect: a contributor might regret its earlier expenditures, given the other contributor's equilibrium strategy. In this sense, the contributor is somewhat more plausibly interpreted as a model of many-contributor, rather than two-contributor, competition.

2.4.3. Contributor Equilibrium; Definition

For any \((I, C)\), let \( Q(S_{AI}, S_{AC} | S_B) \) denote the incumbent's probability of winning as a function of the contributions received by both candidates from contributor A, given the vector of B's contributions. The contributions tatonnement consists of repeated stages of simultaneous contribution. Under the assumption that contributions are non-refundable, at each stage \( t \), each contributor's
expenditure is the maximum of its expenditure at the last stage and the optimal expenditure, given the other contributor's expenditure at the previous stage. Thus, in the sequential contributions game, at any time $t$, contributor A's problem is to

$$\max_{S_{AI}(t), S_{AC}(t)} A(I, S_{AI}(t), S_{AC}(t), S_B(t-1)) + A(C, S_{AI}(t), S_{AC}(t)) [1 - Q(S_{AI}(t), S_{AC}(t), S_B(t-1))]$$

(1)

subject to $S_{AI}(t) \geq S_{AI}(t-1), S_{AC}(t) \geq S_{AC}(t-1)$

That is, as postulated in assumption (A6), at any point in time A can only increase his expenditures; expenditures that have already been made cannot be taken back. Let $S^*_A(t | I, C, S_B(t-1)) = \{S_{AI}(t | I, C, S_B(t-1)), S_{AC}(t | I, C, S_B(t-1))\}$ denote the solution to this problem, provided it is well-defined. B's problem, and its solution, are defined analogously.

**Definition:** For any $(I, C)$, an equilibrium in the contributor subgame (a contributor equilibrium) is a pair $(S^*_A, S^*_B)$ that simultaneously solves contributors' problems (1).

Provided that for every $t$, contributor responses (solutions to problems (1)) are well-defined, $D_I(I, C) = S^*_A + S^*_B$ and $D_C(I, C) = S^*_A + S^*_B$ (note that the potential indirect dependence of contributions on $\nu_I$ and $\nu_C$ is suppressed for notational simplicity). However, this definition is actually overly general, since (A5) implies that contributors, if they contribute at all, give only to the spatially closest candidate:

**Lemma 2:** Under (A5), in any contribution equilibrium each contributor contributes to (at most) one candidate: neither A or B contributes when $I = C$, and when $I \neq C$, each contributor contributes to the spatially closest candidate.

**Proof:** Obvious from construction. By contributing to the farther candidate, a contributor increases the probability that the outcome he likes least will, in fact, occur.

Thus, if $I < C$, then A contributes to the incumbent and B contributes to the challenger.

2.4.4. *Incumbent-Challenger Equilibrium with Contributions*

For any l, the challenger's problem is to:

$$\min_{C} Q[I, C, \nu_I[D_I(I, C)], \nu_C[D_C(I, C)]]$$

(2)

treating I as given, but taking into account the contributions that both candidates will receive as a function of his position. Provided it is well-defined, let $C^*(I) = C^*(I, \nu_I[D_I(I, C)], \nu_C[D_C(I, C)])$ be the
solution to (2). Thus $C^*(I)$ is the challenger's optimal choice of $C$ when the incumbent is located at $I$ and the candidates receive contributions $D_I(I,C)$ and $D_C(I,C)$ as a function of $(I,C)$.

The incumbent chooses $I$, taking into account the challenger's and contributors' optimal responses, to

$$\max_I Q(I,C^*(I),\nu_I([D_I(I,C^*(I)]),\nu_C(D_C(I,C^*(I)]))$$

(3)

Let $I^* = I^*[C^*(I),\nu_I([IP-I]),\nu_C(D_C(I,C^*(I)])]$ denote the solution to (3), provided it is well-defined. Thus, when the candidates receive contributions as functions of their positions $D_I(I,C)$ and $D_C(I,C)$, $I^*$ is the incumbent's optimal choice of $I$, taking into account the optimal reactions of the challenger and the contributors.

**Definition:** An incumbent-challenger equilibrium with contributions is a pair of lotteries $((C^*, \nu_C^*), (I^*, \nu_I^*))$ and associated electoral probabilities $Q^* = Q[I^*, C^*, \nu_I^*, \nu_C^*], \nu_I^*, \nu_C^*$, where $C^*$ and $I^*$ solve problems (2) and (3) and $\nu_C^*$ and $\nu_I^*$ are the implied values of $\nu_C$ and $\nu_I$, taking into account the contributor equilibrium: $C^* = C^*(I^*) = C^*[I^*, \nu_C(D_C(I^*, C^*)), \nu_I(D_I(I^*, C^*))]; I^* = I^*[C^*(I), \nu_I([IP-I], D_I(I^*, C^*), I^*, C^*)]; \nu_C^* = \nu_C(D_C(I^*, C^*));$ and $\nu_I^* = \nu_I([IP-I^*, D_I(I^*, C^*)]$. Because the incumbent is first-mover, and constrained by his reputation, the incumbent need not always be able to obtain a positive equilibrium probability of winning. In that case, of course, the model places no restrictions on at least the position of the incumbent, and many of the results obtained may not then hold. In order to avoid this somewhat paradoxical possibility, assume

(A7) $Q(IP) > 0$.

3. Existence of Equilibrium

For a variety of reasons, equilibria need not exist in this type of model without further assumptions. This section attempts to clarify some of these issues.

3.1. Existence of Contributor Equilibrium

The existence of contributor equilibrium is obviously necessary to the existence of equilibrium candidate locations in this model. Because candidate decisions depend on the anticipated responses of the contributors, uniqueness of contributor equilibrium is important as well.

**Lemma 3:** Suppose assumption (A6) (1) does not hold, so that contributions are refundable. Suppose also that for all $(I,C)$, each contributor's problem (equations (1)) is concave in its own expenditures, and convex in the contributions received by the challenger. Then

(a) Contributor equilibrium exists for every $(I,C)$.
(b) If (i) for any lottery \( \{\mu, \nu\} \), voter expected utilities \( U(\cdot) \) are additively separable in \( \theta, \mu \) and \( \nu \), and (ii) for all \( \{I, C, D_L, D_C\} \), \( \frac{\partial^2 \Phi}{\partial \theta^2} = 0 \), then under assumption (A5) contributor equilibrium exists. This equilibrium is the unique, globally stable outcome of every contribution tatonnement.

With non-refundable contributions, however, standard (continuity and curvature) conditions such as those in Lemma 3.b are not enough to ensure the uniqueness of contributor equilibrium. An example illustrates. Suppose each contributor’s optimal contribution to the closest candidate is always decreasing in the contributions received by the farthest candidate. Then, because the contributors are myopic, any Nash pair with refundable contributions will be an equilibrium to some contribution tatonnement with non-refundable contributions. Call such a Nash pair \( \{D^*_A, D^*_B\} \). Since each contributor’s optimal expenditures are (assumed to be) decreasing in the other contributor’s expenditures, for any \( \epsilon > 0 \) and any \( \delta > 0 \), \( \{D^*_A + \epsilon, D^*_B + \delta\} \) must also be a Nash equilibrium with non-refundable contributions. Hence the contributions equilibrium in this model is not generally unique, although, for any fixed \( \{I, C\} \), since contributors’ reaction functions are single-valued, the particular contributions tatonnement that leads from \( (0,0) \) to \( \{D^*_A, D^*_B\} \) is unique.

Moreover, although contributors’ choice sets are compact and their objective functions continuous, the quasiconcavity of the contributors’ objective function, which is sufficient to establish existence of contributor equilibrium when contributions are refundable, need not be satisfied.\(^{11}\) However, with non-refundable contributions, there are many non-quasiconcave parametric specifications that produce reasonable equilibria (especially under the many-contributors interpretation of the model). One such specification (that nevertheless satisfies the hypotheses of Lemma 3(b)) whenever electoral probabilities are strictly between zero and one is

\( (A8) \) (Quadratic utility and uniform medians) (1) Voter \( \theta \)'s utility over sure outcomes is quadratic; i.e., for any \( \theta \) and any riskless outcome \( x \), \( u(\{x, \theta\}) = -(x-\theta)^2 \). (2) \( \varphi(m) \) is uniform over the interval \([1, r]\).

**Lemma 4:** (Existence and Uniqueness in the Contributor Subgame) Under assumptions (A1)-(A8), for all \( \{I, C\} \),

1. Contributor equilibrium exists.
2. Given the preferences of contributors and voters, there exists a compact interval \([l, r_+]\), where \( l < l \) and \( r_+ > r \), and contributions \( D^*_A \) and \( D^*_B \) that do not depend on \( I \) and \( C \), such that, for all \( \hat{\theta}(0,0) \in [l, r_+] \), \( Q^*(D^*_A, D^*_B) = Q^*(D^*_A, D^*_B) \). For all \( \hat{\theta}(0,0) \not\in [l, r_+] \), \( \{D^*_A, D^*_B\} = (0,0) \). Thus, equilibrium electoral probabilities are unique.

It is important to see that under (A8), even though \( Q(\cdot) \) has the appropriate curvature and continuity properties whenever electoral probabilities are strictly positive, contributors' objective functions (equations (1)) are not everywhere concave, and hence Lemma 3 does not apply. In fact, without the assumption of nonrefundable contributions, this special case reflects a tendency of non-
existence of (Cournot), contributions equilibria whenever the distribution over the median is bounded and the indifferent voter without contributions, \( \hat{q}(0,0) \), is close to an edge of the support of the distribution.\(^\text{12}\) Non-refundability of contributions leads to existence in these cases, but the equilibrium is not subgame perfect: when non-refundability is required for existence of contributions equilibrium, the constraint that contributions are non-refundable must be binding.\(^\text{13}\)

**Corollary 4.1:** If contributions are refundable, then under assumption (A8), there exist candidate positions, and positive electoral probabilities without contributions, so that no Nash contribution equilibrium exists.

To see the intuition behind Corollary 4.1, suppose the location of the indifferent voter without contributions (\( \hat{\theta}(0,0) \)) is chosen such that if A contributes nothing, a small expenditure by contributor B ensures the victory of the rightmost candidate, R (i.e., \( D_B > \hat{D}^{-1}_B(0) \)). At any point in the contributions game, contributor B never wants to spend more than the minimum needed to ensure candidate R's victory, but, as noted in the proof of Lemma 4, contributor A has an incentive to contribute up to \( D_A \) as long as the location of the indifferent voter, given B's contributions, lies "close enough" to the support of the median (i.e., A contributes up to \( D_A \) as long as \( \hat{\theta}(0,D_B) \geq L \)). However, when B's expenditures get large enough (and in particular, at \( D_B = \hat{D}_B \)), A optimally contributes nothing, since the indifferent voter when A spends nothing is so far off the bounded support of the median that the increase in the A's favored candidate's probability of winning does not justify the expenditure of \( D_A \). Thus, because contributions are refundable, there can be no Nash equilibrium in contributions for this choice of \( \hat{\theta}(0,0) \) when voters' utility is quadratic and the median voter is distributed uniformly over a bounded interval.

3.2. Existence of Incumbent-Challenger Equilibrium with Contributions

3.2.1. The Challenger's Optimal Location Decision

The challenger is free to locate C either to the right of I to the left of I, or equal to I. Provided they exist, define

\[
C^L(I) = \arg\min_{C < I} Q(I, C, D(I, I), D_C(I, I)),
\]

\[
C^R(I) = \arg\min_{C > I} Q(I, C, D(I, I), D_C(I, I)),
\]

(4)

\[
Q^L(I) = Q(I, C^L(I)), \text{ and } Q^R(I) = Q(I, C^R(I))
\]
If the challenger sets $C = I$, then contributors are indifferent between the candidates and do not contribute. Thus, for any $I$ such that the incumbent represents a strictly less risky lottery than the challenger when there are no contributions (i.e., $v_I(|I-IP|,0) < v_C(0)$),

$$Q(I) = Q[I,C^*(I^*)] = \min\{Q^L(I), Q^R(I)\}. \quad (5)$$

Clearly, the incumbent never chooses a position $I$ where he is more risky than the challenger without contributions, since the challenger could then win with probability one by choosing the same position as the incumbent, and thereby representing a lottery with the same mean but a lower risk than the incumbent. Hence, when $I$ is such that $v_I(|I-IP|,0) = v_C(0)$,

$$Q(I) = Q[I,C^*(I^*)] = \min\{5, Q^L(I), Q^R(I)\}. \quad (6)$$

3.2.2. The Incumbent's Optimal Location Decision

The incumbent's maximization of $Q(I)$ is thus constrained by the risks that would be associated with the candidates if there were no contributions. Let $1^0L < IP$ solve $v_I(IP-1^0L,0) = v_C(0)$, and $1^0R > IP$ solve $v_I(1^0R-IP,0) = v_C(0)$. As long as the incumbent can choose some positions that yield a positive probability of winning, the incumbents optimal position $I^*$ must always lie between $1^0L$ and $1^0R$; however, $Q(I)$ is typically discontinuous at $1^0R$ and $1^0L$. Similarly, under some specifications of contributor preferences, at some positions $I$ the challenger could locate $C$ arbitrarily close to $I$ and be associated at a lower risk than the incumbent, and in that way win with probability one. Clearly the incumbent never chooses any such location.\textsuperscript{14}

3.2.3. Existence of Incumbent-Challenger Equilibrium with Contributions

Due to the sequential nature of candidate location decisions, incumbent-challenger equilibrium exists under a wide set of circumstances. One particularly useful parameterization is

(A9) $(\epsilon_m$ distinctness) There exists a smallest distance, $\epsilon_m > 0$, between any non-identical positions.

Assumption (A9) is similar to the imposition of an $\epsilon_m$ wide grid over the set of possible candidate locations. The assumption means that discontinuities in electoral probabilities ($Q(I)$) that arise due to voter or contributor behavior (e.g., at $1^0L$ and $1^0R$) do not lead to the non-existence of well-defined candidate choices.\textsuperscript{15} And if, for every $I$, there exists a well-defined challenger reaction, given the subsequent contributor equilibrium, then the incumbent's probability of winning is a well-defined function of his location, and under (A9), his optimal location exists. This establishes,

**Lemma 5:** Assume (A8) or any other sufficient condition for existence and uniqueness of contributor equilibrium for all $\{C,I\}$. Then, for any $\epsilon_m > 0$, (A9) is a sufficient condition for the existence of incumbent-challenger equilibrium with contributions.
4. Equilibrium Contributions, Candidate Positions, and Electoral Probabilities

This section characterizes the incumbent-challenger equilibrium with contributions. A more tightly parameterized environment is explored with particular attention to the effects of contributions on equilibrium incumbent and challenger positions and electoral probabilities. Variables in the analysis include the strength of the reputation effect and the dependence of contributions on incumbent-challenger status as well as ideology. When contributions do depend on ideology, the electoral effects of contributions depend primarily on the equilibrium interaction between (1) the incumbent's need to "guard his flank" and move toward increased contributions, and (2) the reputation effect, which, if all else equal, leads the incumbent to locate (weakly) closer to his reputation than otherwise, and hence can lead him to move in the opposite direction from increased contributions.

4.1. "Ideological Contributions"

For expository simplicity, the term "ideological contributions" will be used below to describe the parametric environment that includes voters having quadratic utility, the median at the time of the election being distributed uniformly, and risk-neutral contributors.

(A10) Assumption (A8) (quadratic voter utility and uniformly distributed median), augmented with risk-neutral contributors: Let \( A(K, S_A) = a \cdot K - \alpha \cdot S_A \) (\( a < 0, \alpha > 0 \)), and \( B(K, S_B) = b \cdot K - \beta \cdot S_A \) (\( b > 0, \beta > 0 \)).

**Lemma 6:** Under (A10),

1. Contributions equilibrium exists and is unique for all \( \{I, C\} \).
2. For all \( \hat{\theta}(0, 0) \in [L, R_+] \), both candidates receive positive contributions, and for all \( C < I \), \( \{D^*_C, D^*_I\} = \{D^*_A(\nu_C(\cdot)), D^*_B(\nu_I(\cdot))\} \), and for all \( C > I \), \( \{D^*_C, D^*_I\} = \{D^*_B(\nu_C(\cdot)), D^*_A(\nu_I(\cdot))\} \), where \( D^*_A(\nu_C(\cdot)), D^*_B(\nu_I(\cdot)) \) and \( D^*_A(\nu_I(\cdot)) \) are as defined in the proof of Lemma 4.

In other words, in equilibrium with linear contributor payoffs, quadratic voter utility and a uniformly distributed median, each contributor always contributes to the closest candidate, and contributions do not depend on the positions of the candidates, as long as the candidates are distinct. (Of course, since \( \nu_I(\cdot) \) can depend on the incumbent's position through the reputation effect, the incumbent's equilibrium contributions typically depend on \( I \).) Therefore, the incumbent's probability of winning \( Q(I, C) \) is a well-defined (and for \( C \neq I \), smooth) function of candidate positions for \( \hat{\theta}(0, 0) \in [L, R_+] \).

Lemma 6 illustrates an important facet of contributor equilibrium: contributions do not vary as candidate positions vary, as long as the left-right orientation of the candidates and everything else is held fixed. However, since the challenger and incumbent are distinguished according to their risks, which is
affected both by contributions and the incumbent's choice of position. Thus, because the marginal effects of contributions might differ for the incumbent and the challenger, contributions probably depend on the relative left-right orientation of the candidates. In particular, if the indifferent voter without contributions lies close enough to the support of the median, and $I < C$, then by Lemma 6, in equilibrium contributor $A$ spends $D_A^*(\nu_I)$ in favor of the incumbent and contributor $B$ spends $D_B^*(\nu_C)$ in favor of the challenger, while if $C < I$, then contributor $A$ spends $D_A^*(\nu_C)$ in favor of the challenger, while $B$ spends $D_B^*(\nu_I)$ in favor of the incumbent.

For notational simplicity, for candidate $J = I, C$ and contributor $X = A, B$, let $D_X^*(\nu_I) = D_X^*(I)$. The next result shows that under this specification of ideological contributions, the challenger’s reaction function and hence, the incumbent’s probability of winning as a function of $I$, are well-defined. In particular, the challenger always locates its position equal to the ideal point of the indifferent voter.

**Lemma 7:** Under (A10), for all $I$ such that $\nu_I(\cdot) < \nu_C(\cdot)$

1. $C^*(I) = \hat{\theta}(C, I, D_C^*, D_I^*)$; the challenger's optimal position is equal to the ideal point of the voter who is indifferent between the challenger and the incumbent.

2. $Q(I, \nu_I([I, IP]), C^*(I), D_C^*, D_I^*) = \min(Q^L(I), Q^R(I))$, where $Q^L(I) = Q(I, \nu_I([I, IP]), C^*(I), D_C^*, D_I^*)$ and $Q^R(I) = Q(I, \nu_I([I, IP]), C^*(I), D_C^*, D_I^*)$ where, as in Lemma 6, $\{D_C^*, D_I^*\} = \{D_A^*(C), D_B^*(I)\}$ when $C < I$, and when $C > I$, $\{D_C^*, D_I^*\} = \{D_A^*(I), D_B^*(C)\}$.

To complete the specification of the challenger's reaction function, observe that since $\nu_I(\cdot)$ and $\nu_C(\cdot)$ are continuous in their arguments, and $D_A^*(\cdot)$ and $D_B^*(\cdot)$ are constants, $Q(I)$ is well-defined and smooth as long as $\nu_I < \nu_C$. However because the incumbent is constrained to be first-mover, his probability of winning falls to zero once $\nu_I > \nu_C$; if the challenger can locate either on top or arbitrarily close to a more risky incumbent, the challenger will win with probability one. Thus, one disadvantage to the incumbent of being first-mover is that $Q(\cdot)$ must be discontinuous at any $I$ such that $\nu_I([I, IP], D_A^*(I)) = \nu_C(D_B^*(C))$, $\nu_I([I, IP], D_B^*(I)) = \nu_C(D_B^*(C)$ or $\nu_I([I, IP], 0) = \nu_C(0)$. In other words, the incumbent's equilibrium location can be constrained by the challenger's optimal responses when there are no contributions, and when the equilibrium left-right orientation of the candidates is reversed.

In order to more fully characterize the incumbent's location decision, assume that $C^*(IP) > IP$; this is a minor assumption, since the incumbent located at his reputation has an assumed risk-advantage over the challenger. Observe that if there were no reputation effect in the model, the incumbent's assumed a priori risk-advantage would be fixed for all pairs of candidate positions having the same left-right orientation. Hence, from the proof of Lemma 7 (equation (15)) we can see that $Q^R(I)$ and $Q^L(I)$ would be upward- and downward-sloping linear functions of $I$ with exactly one crossing point. With the
reputation effect, however, and the assumption that $C^*(IP) > IP$, we can infer that $Q^L(I) < Q^C(I)$. Provided the following incumbent positions (to the right of IP) exist, let $\tilde{I}$, $I^0$, and $I^L$, respectively, be implicitly defined by

$$Q^R \left[ \tilde{I}, C^R(\tilde{I}, D^A(\tilde{I}), D^B(C)) \right] = Q^L \left[ I, C^L(I, D^A(I), D^B(C)) \right]$$

(7)

$$\nu_I(\tilde{I}, |I^0, IP|, \beta) = \nu_C(0), \text{ and}$$

(8)

$$\nu_I(I^L, |I^L-IP|, D^B(I)) = \nu_C(D^A(C))$$

(9)

and define

$$I^R = \arg\max_{I \geq IP} Q^R(I).$$

(10)

Then

**Lemma 8:** For $\varepsilon_m$ sufficiently small, under (A10), (1) If $C^*(IP) > IP$, then $I^* = \min\{I^0, I^R, I^L, \tilde{I}\}$. (2) $C^*(I^*) > I^*$. The bottom line of Lemma 8 is that the candidates always take distinct positions in equilibrium; part (2) also says however, that the incumbent never locates so that the challenger has an incentive to locate on the "wrong" side of the incumbent. Perhaps more importantly, Lemmas 6, 7 and 8 provide a computational basis for comparative statics. These results are explored in the next section.

4.2. The Reputation Effect and the Electoral Impact of Ideological Contributions

4.2.1. Candidate Positions and Electoral Probabilities without the Reputation Effect

Suppose there is no reputation effect -- that is, modify assumption (A4) so that, holding contributions fixed, the incumbent's risk does not depend on his position or reputation ($\nu_I$ is independent of $I$ and IP). Then, as long as candidate electorate risks and contributor expenditures are such that the incumbent is less risky than the challenger in either relative left-right configuration of candidate positions, the incumbent's equilibrium position always moves in the direction of increased contributions. More precisely,
LEMMA 9: Suppose there is no reputation effect, but the incumbency effect holds, so that 
\nu_C(D^*_A(C)) > \nu_I(D^*_B(I)) and \nu_C(D^*_B(C)) > \nu_I(D^*_A(I)). Then in incumbent-challenger 
equilibrium with contributions,

1. \( I^* = 1 \)
2. \( \frac{\partial I^*}{\partial D_A} < 0, \frac{\partial I^*}{\partial D_B} > 0. \)

Figure 1: Comparative Statics of an Increase in \( D^*_A(\cdot) \). Without the Reputation Effect

As shown in Figure 1, with "ideological contributions" (assumption (A10)) but without the 
reputation effect, \( Q^R(I) \) and \( Q^L(I) \) are linear functions of \( I \), and \( R^R, I^0 \) and \( I^L \) are not defined when 
there is no reputation. As a result, \( I^* = 1(D^*_A(\cdot), D^*_B(\cdot)) \), and the incumbent's position is solely 
determined by the contributions that both candidates receive as a result of the incumbent's location and 
the challenger's subsequent optimal response; in other words, the incumbent's optimal location drives the 
challenger to indifference between \( C^R(I) \) and \( C^L(I) \). Thus, as \( D^*_A(\cdot) \) increases, all else equal, \( Q^R(I) \)
increases (shown as the change between QR(1) and QL(2) in the figure) while QL(I) decreases (shown as the change from QL(1) to QL(2) in the figure). Hence, the incumbent moves to the left (11 to 12 in the figure) in order to minimize the benefit that the challenger could obtain from the increase, if positions were to remain fixed.

Now consider the effects of the increase in $D_A(t)$ on the incumbent's probability of winning. Observe that when there is no reputation effect, in equilibrium the challenger is indifferent between locating on the incumbent's right (at $CR(I^*)$) or on the incumbent's left (at $CL(I^*)$). Thus, in equilibrium each candidate gains or loses the same amount from a given increase in either contributor's contributions, whether or not that candidate is actually the recipient of those contributions in equilibrium. Indeed, under weak conditions, the incumbent always loses from an increase in either $D_A(t)$ or $D_B(t)$:

**Lemma 10**: Suppose that for $J = C, I$, $\partial \nu_J/\partial D_J$ is increasing in $\nu_J(0)$, for any $D_J$. Then if there is no reputation effect, the incumbent's equilibrium probability of winning $Q(I^*)$ is decreasing in the contributions received by either candidate.

In summary: When the incumbency effect holds but there is no reputation effect, then the incumbent always locates at $\bar{I}$; this drives the second-moving challenger to indifference between $CR(I)$ and $CL(I)$. As a result, the incumbent's equilibrium position always moves in the direction of increased contributions, whether or not the incumbent actually receives the increased contributions. By always locating at $\bar{I}(D_A(\cdot), D_B(\cdot))$, the incumbent is "guarding his flank" and minimizing the gain the challenger could obtain as recipient of the increased contributions. Nevertheless, because in equilibrium the challenger's probability of winning always increases as either candidate's contributions increase, if the challenger wins in equilibrium with probability less than one-half, then (since the challenger's optimal response to the less risky incumbent is to locate so that his position equals the ideal point of the indifferent voter) the challenger's position always becomes more moderate (closer expected median) as either candidate's contributions increase, while the incumbent's position always moves in the direction of increased expenditures. If, in equilibrium, the incumbent is the recipient of the increased contributions, then the distance between the candidates (which is purely a function of the incumbent's risk-advantage) increases, while if the challenger is the recipient of the increased contributions, the distance between the candidates decreases.

### 4.2.2. Candidate Positions and Electoral Probabilities under the Reputation Effect

Without the reputation effect, the incumbent's location decision and its comparative statics were determined by the effects of contributions alone, and as a result, $I^* = \bar{I}$ when there is no reputation effect. By contrast, with the reputation effect, the constraints imposed by reputation are binding if $\bar{I} \neq I^*$. And,
whenever the incumbent's location decision is constrained by his reputation, comparative statics deviate markedly from their pattern without the reputation effect.

![Diagram](image)

Figure 2: An Increase in $D_A^\hat{}(\cdot)$ with the reputation effect

To see this, suppose as before that $C^\ast(IP) > IP$. With the reputation effect, $Q^R(I)$ and $Q^L(I)$ are strictly concave and $Q^R(I)$ is single-peaked, as shown in Figure 2. In the no-reputation case, the incumbent locates so that the challenger is indifferent between $C^R(I^\ast)$ and $C^L(I^\ast)$; by contrast, with the reputation effect, there will exist IPs so that the incumbent has a strict preference to locate at $I^R$, and the challenger strictly prefers $C^R(I^\ast)$ to $C^L(I^\ast)$. For example, as shown in Figure 2, initially $D_A^\hat{}(\cdot)$ and $D_B^\hat{}(\cdot)$ are such that $I^\ast = I^R (= I_1$ in the figure). Then $D_A^\hat{}$ increases, and the incumbent's new equilibrium position is such that $I^\ast = \bar{I}$ (shown as $I_2$ in the figure). (Since $I^R$ does not depend on $\phi(m)$, but $\bar{I}$ does, for any IP, $D_A^\hat{}(\cdot)$, and $D_B^\hat{}(\cdot)$, one can choose $l$ and $r$ to produce this situation.)

Thus there will be an ideological component that determines the comparative statics of the incumbent's probability of winning with respect to contributions: "ideological incumbents", for whom the constraints of reputation are binding (i.e., $I^R = I^\ast$) will tend to gain from the contributions they obtain in
equilibrium, while "moderate incumbents" who are constrained more by their challengers' abilities to outflank them than they are by their reputations (i.e., \( \bar{I} = I^* \)) will tend to be made worse off by the contributions that either candidate receives in equilibrium. To see this, suppose for simplicity \( I^R, \bar{I} < I^L, \bar{I} \). Then, when the effects of contributions dominate the effects of reputation, \( \bar{I} < I^R \), but when the constraints of reputation dominate, \( I^R < \bar{I} \). The key to comparative statics with the reputation effect, then, are the effects of changes in \( D_A^* \) and \( D_B^* \) on the locations of \( I^R \) and \( \bar{I} \).\(^{19}\) Observe that since \( \bar{I} \) is decreasing with \( D_A^* \) (increasing with \( D_B^* \)) but \( I^R \) is increasing in \( D_A^* \), as \( D_A^* \) increases (\( D_B^* \) decreases), it becomes more likely that the effect of contributions predominate (i.e., eventually should be the case that \( \bar{I} < I^R \) as \( D_A^* \) continues to increase). Thus, when \( \bar{I} > I^R \), \( I^R \) and \( Q^* \) are increasing in \( D_A^* \), while if \( \bar{I} < I^R \), then \( Q^* \) is decreasing in \( D_A^* \). In other words, as long as the challenger prefers to locate on the right of \( I^R \), the incumbent is made better off by an increase in \( D_A^* \), and both candidates move away from the direction of increased contributions. However, once \( D_A^* \) increases so much that the challenger prefers to \( C_I^L(I^R) \) to \( C_I^R(I^R) \), then the incumbent is made worse off by increases in \( D_A^* \), and the incumbent moves toward the direction of increased contributions.\(^{20}\)

In summary: By contrast to the case when the incumbent is unconstrained by a reputation, with the reputation effect, the incumbent's equilibrium probability of winning can increase with increased contributions. In particular, the incumbent likes increases in contributions (\( D_A^* \) or \( D_B^* \)) as long as he is the equilibrium recipient of those increased contributions (i.e., when the effects of reputation are important relative to the effects of contributions), and the incumbent dislikes any contributions that the challenger receives (i.e., when the effects of contributions are strong relative to reputation and \( I^* = \bar{I} \)). When the effects of reputation predominate (\( I^* = I^R \)), the incumbent and challenger both always move away from the direction of increased contributions, and when contributions are important relative to reputation, (\( I^* = \bar{I} \)), the incumbent always moves away from increased contributions, and the challenger moves closer to the incumbent.

4.3. Extensions

4.3.1. Contributors' Risk Attitudes

Ingberman (1986), (1988)\(^{21}\) shows that in ideological contribution environments with non-linear contributor payoffs, \( Q^L(I) \) and \( Q^R(I) \), and the candidates' problems in general, become increasingly less well-behaved as contributor direct payoff functions become increasingly more non-linear. Each candidate's contributions will tend to depend explicitly on both candidates' positions. As can be seen from contributors' optimization problems (equations (14)), when direct payoffs over candidate positions concave (risk-aversion) and linear in contributions, each contributor's expenditure in favor of the closest candidate increases as either candidate's position moves away from the contributor, while if contributors'
direct payoffs over candidate positions are strictly convex (risk-loving), contributions to the spatially closest candidate increase as either candidate's position moves closer to the contributor.

In each case, a number of additional assumptions are required to characterize incumbent-challenger equilibrium. To see this, pick any position for the incumbent at which \( v_I(|I-IP|,0) < v_C(0) \), and consider the challenger's optimal response. If contributors' direct payoffs over candidate positions are strictly concave, then as \( C \) approaches \( I \), the challenger's contributions increase, and the incumbent's decrease. Alternatively, if contributors' payoffs are convex, then as \( C \) approaches \( I \), the incumbent's contributions increase and the challenger's decrease. Thus, candidates' optimal locations need not be well-defined. Moreover, when they are well-defined, the shapes of \( Q^R(I) \) and \( Q^L(I) \), and the comparative statics of positions and probabilities, can vary from the basic case of ideological contributions with risk-neutral contributors.

4.3.2. Non-Ideological Contributions

Suppose that contributors' direct valuations of the candidates depend only on incumbent-challenger status, and not on either candidate's position. Beyond redefining the function \( v_I(\cdot) \) and \( v_C(\cdot) \), non-ideological contributions is a special case of ideological contributions. The fundamental difference between the two cases is that since non-ideological contributions are determined by candidate identities rather than their positions, the incumbent's equilibrium probability of winning is always increasing in the contributions he receives, and decreasing in the contributions received by his opponent. Since the challenger's equilibrium position equals the ideal point of the indifferent voter, the challenger's position moves farther from the expected median as the incumbent's probability of winning increases (at least when \( Q(I) > .5 \)).

The density governing the location of the median at the time of the election (\( \phi(m) \)) is assumed to be symmetric about the expected median, which means that with non-ideological contributions, \( \bar{I} = \bar{m} \), the expected value of the median voter. That is, with \( C^*(IP) > IP \), the incumbent's equilibrium position lies closer to the median than his reputation, but the incumbent never locates to the right of the expected median ideal point, since that would give the challenger a strict preference to locate on the incumbent's left. Since \( I^R \) is increasing in the incumbent's contributions and decreasing with the challenger's, for any initial reputation \( IP \), as the incumbent's contributions \( D_I \) increase his position moves towards, but never past, the expected median; the reverse holds for increases in \( D_C \). As in the ideological contributions case, the incumbent's equilibrium location depends on the relative strengths of the effects of reputation and contributions. Unlike the ideological contributions case, however, increased contributions always leads the incumbent to locate (weakly) closer to the expected median than he would without the increased contribution.
5. Summary and Conclusions

This paper developed a sequential model of incumbent-challenger spatial competition with contributions. The basic assumption of the model is that voters have more and better information about the incumbent, all else equal; this idea is implemented in the posited incumbency and reputation effects. In the model, contributions finance campaign advertising, so that the electoral role of contributions is to provide information to the electorate.

Under the assumptions made, contributor equilibrium, and incumbent-challenger equilibrium with contributions, exists. Indeed, the paper demonstrates the necessity of non-refundable contributions (or a similar requirement) to guarantee the existence of contributor equilibrium everywhere under commonly used assumptions (quadratic voter utility, uniform distribution over the median voter at the time of the election, and linear contributor payoffs). Under the tightly parameterized framework, the comparative statics of ideological and non-ideological contributions are compared, as a function of the strength of the reputation effect. In particular, non-ideological contributions always increase the recipient candidate's equilibrium probability of winning, but with ideological contributions, when the incumbent is unconstrained by his reputation, increased contributions to either candidate reduce the incumbent's equilibrium reelection probability. When the incumbent is constrained by his reputation, then depending on the relative strengths of the effects of contributions and reputation, the incumbent's equilibrium probability of winning is non-monotonic in his contributions: for at least some initial conditions the incumbent's probability of winning is increasing in his contributions up to some critical level, and then decreasing in the contributions he receives in equilibrium beyond that level.
Appendix

Proof of Lemma 3: (a) By assumption, contributor payoffs are continuous in all variables. Contributor strategy sets (possible levels of expenditures) are compact, since no contributor ever plans to spend more than the utility difference between the two candidates, undiscounted by any electoral probability. Now standard arguments\(^{23}\) show that the simultaneous Cournot tatonnement without the no-takeback assumption (A6) yields at least one Nash equilibrium at which each contributor is maximizing its expected payoffs, given the response of the other. This equilibrium must also be an equilibrium in the sequential Cournot tatonnement when contributions are, in fact, refundable.

(b) Contributors' direct payoffs \(A(\cdot)\) and \(B(\cdot)\) are additively separable in contributions and political outcomes, by assumption (A5)(1). Suppose for simplicity that \(I < C\). Hence \(S_A = D_I\) and \(S_B = D_C\). Therefore, a sufficient condition for the global stability of the contribution tatonnement with refundable contributions\(^{24}\) is

\[
\frac{\partial^2 Q}{\partial D_t^2} < 0, \quad \frac{\partial^2 Q}{\partial D_C^2} > 0.
\]

\[
2\cdot \left| \frac{\partial^2 Q}{\partial D_t \partial D_C} \right| = \left| \frac{\partial^2 Q}{\partial D_C^2} \right| \quad \text{and} \quad 2\cdot \left| \frac{\partial^2 Q}{\partial D_I \partial D_C} \right| < \left| \frac{\partial^2 Q}{\partial D_I^2} \right| \tag{11}
\]

Focussing on contributor \(A\), differentiation yields

\[
\frac{\partial^2 Q}{\partial D_I^2} = \frac{\partial Q}{\partial \theta} \frac{\partial^2 \nu_I}{\partial \nu_I \partial \theta} \frac{\partial^2 \nu_I}{\partial \nu_I \partial D_I^2} + \frac{\partial Q}{\partial \theta} \frac{\partial^2 \nu_I}{\partial \nu_I \partial \theta^2} \left( \frac{\partial \nu_I}{\partial D_I} \right)^2 + \frac{\partial Q}{\partial \theta} \frac{\partial^2 \nu_I}{\partial \theta^2} \left( \frac{\partial \nu_I}{\partial D_I} \right)^2 \tag{12}
\]

and

\[
\frac{\partial^2 Q}{\partial D_I \partial D_C} = \left[ \frac{\partial \nu_I}{\partial D_I} \right] \left[ \left( \frac{\partial^2 Q}{\partial \theta^2} \right) \left( \frac{\partial \theta}{\partial \nu_I} \right) \left( \frac{\partial \nu_C}{\partial D_C} \right) + \left( \frac{\partial Q}{\partial \theta} \right) \left( \frac{\partial^2 \theta}{\partial \theta \partial \nu_I} \right) \left( \frac{\partial \nu_C}{\partial D_C} \right) \right].
\]

Because a candidate's contributions affects only its own riskiness, and voter expected utilities are additively separable in \(\nu\) and \(\mu\), \(\left( \frac{\partial \theta}{\partial \nu_I \partial \nu_C} \right) = 0\). By assumption, \(\frac{\partial^2 \Phi}{\partial \theta^2} = 0\), which implies that \(\frac{\partial^2 Q}{\partial \theta^2} = 0\).

Hence \(\frac{\partial^2 Q}{\partial D_I \partial D_C} = 0\), and \(\frac{\partial^2 Q}{\partial D_I^2} = \frac{\partial Q}{\partial \theta} \frac{\partial^2 \nu_I}{\partial \theta^2} \frac{\partial^2 \nu_I}{\partial \nu_I \partial D_I^2}\), which by assumption, is not zero. Therefore, the conditions given in equation (10) are satisfied, which completes the proof.

Proof of Lemma 4: For notational simplicity, suppose that \(I < C\); this does not affect the analysis in any way. Define \(D_A^1(\cdot)\) implicitly by \(\hat{\theta}(D_A^1(\cdot), D_B^1(\cdot, \nu)) = r\). Because \(\phi(m)\) is bounded, \(\nu_T(\cdot)\) and \(\nu_C(\cdot)\) are continuous in contributions, and \(U(\cdot)\) is continuous in \(\nu\), there exists a smallest \(\hat{\theta}(0, \cdot) = l < 1\) at which \(A \) is indifferent between contributing or not. Analogously define \(r_L > r\) as the maximum value of \(\hat{\theta}(\cdot, 0)\) for which \(B\) optimally responds with \(D_B^1\). Solving the contributors' problems (1), using (A8), shows that at any stage \(t\), contributor's optimal expenditures at that stage are given by
\[ D_A^*(t) = \begin{cases} \max(D_A^*(t), D_A) \\ \text{if } \hat{\theta}^*[0,D_B^*(t-1)] \leq [l,r] \text{ and } \hat{\theta}[D_A^*, D_B^*(t-1)] \leq r \\ \max(D_A^*(t-1), D_A(t)) \end{cases} \]

\[ D_B^*(t) = \begin{cases} \max(D_B^*(t), D_B) \\ \text{if } \hat{\theta}[D_A^*, D_B^*(t-1)] > r \\ \text{else} \\ \max(D_B^*(t-1), D_B(t)) \end{cases} \]

where \( D_A \) and \( D_B \) are interior solutions to contributors' problems (equations (2)) when \( \hat{\theta}(\cdot) \) lies within the interval \([l,r]\). That is, with quadratic voter utility,

\[ \hat{\theta}(I, C, \nu_I, \nu_C) = \frac{I + C}{2} + \frac{\nu_C - \nu_I}{2(C-I)} \text{ and } \]

\[ D_A^* = \arg\min \left\{ \frac{A(I) - A(C)}{2(C-I)(r-I)} \cdot (\nu_I(S_A)) \cdot (\alpha(S_A)) \right\} \]

\[ D_B^* = \arg\min \left\{ \frac{B(C) - B(I)}{2(C-I)(r-I)} \cdot (\nu_C(S_B)) \cdot (\beta(S_B)) \right\} \]

Observe that at any \( t \), for contributor \( X = A, B \), \( D_X^*(t) \leq D_X^* \) and \( D_X^* \) is independent of \( D_Y^* \), for \( Y = A, B \). \( X \neq Y \). Thus, for \( \hat{\theta}(0,0) \in [l,r] \), \( D_X^*(t) \) is single-valued, and for \( \hat{\theta}(0,0) \notin [l,r] \), \( Q^* = Q(0,0) \).

Because \( D_A^* \) does not depend on \( D_B^* \) or \( \hat{\theta}^*(\cdot, D_B) \), \( D_A^* = D_A^* \) for all \( \hat{\theta}^*(0, D_B), \theta^*(D_A, D_B) \in [l,r] \). It now remains to show that \( Q^* = Q(D_A^*, D_B^*) \) for all \( \hat{\theta}(0,0) \in [l,r] \). There are two cases. (Case 1) \( Q^* \in (0,1) \). Then \( \{D_A^*, D_B^*\} = \{D_A^*, D_B^*\} \), and \( Q^* = Q(D_A^*, D_B^*) \); for any \( \hat{\theta}(\cdot) \in (l,r) \) and \( D_X(t) < D_X^* \) means that contributor \( X \) is not optimizing, a contradiction. wants to increase its expenditure, and no contributor ever wants to spend more than \( D_X^* \). (Case 2) \( Q^* \notin (0,1) \). Let \( W \) denote the contributor whose favored candidate wins with probability one and \( N \) denote the contributor whose favored candidate loses with probability one. Because \( D_W^*(t) \leq D_W^*(t) \), \( D_N^* = D_N^* \). Thus, since \( W \)'s candidate wins in equilibrium with probability one, \( Q(D_N^*, D_W^*(t)) = Q(D_N^*, D_W^*) \), \( Q^*(D_A^*, D_B^*) = Q^*(D_A^*, D_B^*) \).

This completes the proof. \( \blacksquare \)
PROOF OF COROLLARY 4.1: The contributions game is as before, except that contributions are refundable. Therefore, contributor reaction functions are given by

\[
D_A^*(t) = \begin{cases} 
D_A^* & \text{if } \hat{\theta}[0,D_B^*(t-1)] \in [l, r] \text{ and } \hat{\theta}[D_A^*,D_B^*(t-1)] \leq r \\
D_A^+(t) & \text{if } \hat{\theta}[D_A^*,D_B^*(t-1)] > r \\
0 & \text{otherwise}
\end{cases}
\]

\[
D_B^*(t) = \begin{cases} 
D_B & \text{if } \hat{\theta}[D_A^*(t-1),0] \in [l, r_+] \text{ and } \hat{\theta}[D_B^*,D_A^*(t-1)] \geq l \\
D_B^+(t) & \text{if } \hat{\theta}[D_B^*,D_A^*(t-1)] < l \\
0 & \text{otherwise}
\end{cases}
\]

Choose \{(I,C), \{\upsilon_I(0),\upsilon_C(0)\}\} and \(\hat{\theta}(0,0) \in [l, r]\) to make \(\hat{\theta}(0,0) \cdot l\) small enough so that i) \(D_B^{-1}(0) < D_B^*\), ii) \(\hat{\theta}(0,D_B) < l\), and iii) \(\hat{\theta}(D_A^*,0) < r\). From contributors' reaction functions, and from iii),

(1) \(D_A^*(D_B) = D_A^*\) whenever \(D_B = D_B^*\), and \(D_A^*(D_B) = 0\), and (2) \(D_B^*(D_A) = D_B^{-1}(0)\) when \(D_A = 0\), and

\(D_B^*(D_A^*) = D_B^*\). Combining (1) and (2) demonstrates the non-existence of contributor equilibrium for the chosen initial conditions and completes the proof.

PROOF OF LEMMA 5: (1) is implied by Lemma 4. (2) follows from substitution of contributors' payoffs as specified in (A10) in equation (14) in the proof of Lemma 4.

PROOF OF LEMMA 7: Whenever choosing among \(C \neq I\), the challenger maximizes or minimizes the location of the indifferent voter as a function of \(C\). Since, holding the relative left-right orientation of the candidates fixed, contributions are fixed at \(\{D_A^*(\cdot), D_B^*(\cdot)\}\), it must be true that \(C^*(I) = \hat{\theta}(I, C^*(I))\); if the challenger's position is not equal to the position of the indifferent voter, the challenger could locate closer to the indifferent voter and increase his probability of winning. (This can be seen directly by differentiating equation (13)). This establishes part (1) of the lemma, which is obviously true when \(C = I\).

Direct calculation shows that

\[
Q^R_I(I) = \frac{I + \sqrt{\nu_C(D_B^*(C)) \cdot \upsilon_I([I-IP], D_A^*(I))}}{r \cdot l}
\]

\[
Q^L_I(I) = 1 - \frac{I - \sqrt{\nu_C(D_A^*(C)) \cdot \upsilon_I([I-IP], D_B^*(I))}}{r \cdot l}
\]

which establishes (2) and completes the proof.

PROOF OF LEMMA 8: When \(C = C^*(IP)\), \(Q(IP, C^*(IP)) < Q(IP)\), so for any \(\epsilon > 0\), \(Q(IP, C) < Q(IP)\). Thus \(I^* > IP\). A similar argument shows that \(I^* \leq I\), i.e., \(Q(I) > Q(I)\) for \(I > I\).
Observe that $I^* \leq I^0$, otherwise $C^*(I) = I$ and $Q(I) = 0$. Similarly, $I^* \leq I^L$, otherwise for some $\varepsilon_m$,

$Q(C = I^{L^-\varepsilon_m}, I^{L-\varepsilon_m}) < Q(I^0)$. Now assume $I^R < I < I^0, I^L$, so that for all $I \in (I^R, I^L)$, $Q^R(I) < Q^L(I)$ and $Q(I) = Q^R(I)$. By definition, $Q^R(I^R) > Q^R(I)$ for all $I \geq I^R$. A similar argument applies when $I^R < I < I^0$, which establishes that $I^* = \min\{I^0, I^R, I^L, I\}$ and proves (1).

Differentiating equation (15) under (A4), shows that $\nu_I(I^R, I^L, I) < \nu_C(D^A_B(C))$. Now if $I^R = \min\{I^0, I^R, I^L, I\}$, then by construction $C^*(I^*) = C^R(I^*) > I^*$. Suppose instead then that $I^R \neq \min\{I^0, I^R, I^L, I\}$. Since $\nu_I(\cdot)$ is increasing in $I$-IP, for $I = I^L$ or $I = I$ we know $C^*(I^*) = C^R(I^*) > I$. Now suppose $I^0 = \min\{I^0, I^R, I^L, I\}$. Clearly, unless $s = I^0/(r-I)$, both candidates cannot simultaneously choose to locate at $I^0$. Hence if $I^0 = \min\{I^0, I^R, I^L, I\}$, either $I^* = I^0$ and $C = C^R(I^0)$ or $I^* = I^0 - \varepsilon_m$ and $C = C^R(I^0 - \varepsilon_m)$, which shows that $C^*(I) > I$ and completes the proof.\[\Box\]

**Proof of Lemma 9:** Without the reputation effect, $\nu_C(D^A_B(C), \nu_I(D^A_B(I), D^B_B(C)), \text{ and } \nu_I(D^A_B(I))$ are independent of all distinct $(C, I)$ pairs. Hence, $\partial Q^R(I)/\partial I = -\partial Q^L(I)/\partial I = 1$, $I^0$ and $I^L$ are undefined, and $I^R$ solves $\hat{\nu}(I^R, C^*(I^R), D^A_B(I), D^B_B(C)) = \alpha$. Thus, by Lemma 8, (1) follows. To establish (2), suppose that $a$ and $\alpha$ change so that $\hat{D}^A_B$ increases for all $\hat{\nu}(0, 0) \in [I, r_\alpha]$. This increases $Q^R(I)$ and decreases $Q^L(I)$ for every $I$, so $\partial \hat{\nu}_I/\partial D^A_B < 0$, and (2) follows.\[\Box\]

**Proof of Lemma 10:** $\frac{\partial Q^R(I)}{\partial I} = \frac{\partial Q^L(I)}{\partial I} = 1$ is shown directly above. Hence $Q(I)$ is decreasing in both contributors' expenditures if the increase in, say, $D^A_B$, would cause the challenger's variance (if he were the recipient) to fall more than the incumbent's variance (if he were the recipient). The hypothesis of the lemma is equivalent to a statement that for any two candidates $J$ and $K$, $\nu_C > \nu_I$ implies that $\frac{\partial \nu_I}{\partial D^D_J} > \frac{\partial \nu_K}{\partial D^D_K}$. Since the challenger is ex ante more risky than the incumbent, at any level of spending $D^D$,

$\frac{\partial \nu_C}{\partial D^D} > \frac{\partial \nu_I}{\partial D^D}$. Hence, for contributor $X = A, B$, from equation (14) (and Lemma 6) we can see that $D^X_A(I) < D^X_B(C)$. Hence the challenger will obtain a larger decrease in his variance from the increase in $D^X_B(C)$ than the incumbent will obtain from $D^X_A(I)$, so that the increase in $D^X(\cdot)$ must cause $Q^*(\cdot) = Q(I)$ to fall.\[\Box\]
Footnotes


2. For example, incumbent candidates in the U.S. Congress win almost all the time (generally, about 95+% and receive nearly all the PAC contributions (usually 85-90%). Empirical and theoretical papers that seek to understand the connection between contributions, ideology, and political behavior often take Stigler, (1971), as motivation, include Chappell (1981), Goughlin (1986), Jacobsen (1980), (1983), (1985), Kait and Zupan (1984), Kau and Rubin (1982), Peltzman (1985), Poole and Romer, (1985), and Poole and Rosenthal (1986).

3. Here 'risk' is used in the sense of Rothschild and Stiglitz (1970). Once can show that under the variance interpretation of risk, only quadratic voter utility satisfies these restrictions; see, eg., Chamberlain (1984). The seminal papers in modelling candidates as lotteries include Shepsle, (1972), and McKelvey, (1980). Bernhardt and Inghberman, (1985), Austen-Smith (1987), and Inghberman, (1989) all use the assumption of risk-averse voters in models of spatial competition.

4. Observe that in a model without campaign contributions, assumption (A3) is not required; see, for example, Aranson, Hinich, and Ordeshook, (1974), Glazer, Grofman, and Owen, (1989), or Inghberman (1986). However, in this model a more risky candidate loses with probability one by taking the same position as a less risky candidate. Hence, unlike the probabilistic voting environment of Coughlin and Nitzan, (1981)a,b, Coughlin, (1983), (1986), Enelow and Hinich (1981), (1982), (1984), or the probabilistic participation environment of Hinich, Ledyard, and Ordeshook (1972), Ledyard, (1984), here candidate uncertainty regarding the location of the median at the time of the election does not affect the existence or non-existence of candidate equilibrium.

5. Although convenient, (A4)(3) (iii) does not crucially affect the qualitative nature of the results of the basic reputation model (in fact, for any I, in contributor equilibrium both $\psi_1(\cdot)$ and $\psi_C(\cdot)$ can depend on the challenger's choice of C).

6. There is a large body of empirical evidence which either directly or indirectly supports this assumption; see in particular Ferejohn (1977), who examines a variety of alternative hypotheses regarding the source of the incumbency advantage in elections. Regarding the empirical and theoretical aspects of the incumbency advantage in U.S. --which is far from a new phenomenon -- congressional and other elections, see Cover (1977), Cover and Mayhew (1977), Eubank and Gow (1983), Mayhew (1974), Jacobson (1983), Mayhew (1974), Popkin, Gorman, Phillips, and Smith (1976).

7. This interpretation is similar to the "predictive dimension" notion of Enelow and Hinich (1981), (1982), (1984), and Enelow, Hinich and Mendell (1986). The major difference is the idea that estimation of non-social as well as spatial candidate characteristics might depend on candidate reputations. Alternatively, think of the reputation effects as a "credibility effect": voters may believe that candidates differ in their inherent preferences over the possible tradeoffs between keeping old promises and maintaining popular platforms. Or, because each individual voter has a small impact on the outcome of the election, voters may find it rational not to expend the resources necessary to inform themselves about the candidates' performances or the structure of the game. In that case, "rules of thumb" resembling the reputation effect may be rational forecasting techniques, taking into account the
marginal benefits and costs of acquiring and processing information. On this interpretation, see Fiorina, (1981).

8. For analysis of candidate competition in which candidates have other goals beyond winning, see Wittman (1977), (1983) and Calvert (1983). For a model of campaign contributions constructed on this basis, see Edelman (1983), and for models of politically determined macroeconomic policy, see Alesina ()..

9. As in "Cournot tatonnement," the path that leads Cournot-type duopolists to a Nash equilibrium. See, for example, Moulin, (1986), ch. 6, pp. 125-144.

10. See Ingberman, (1990) for a model of subgame perfect contributions.

11. Intuitively, the marginal benefit of contributions on electoral probabilities equals the product of the partial derivatives of $Q(\cdot)$ with respect to $\theta^*(\cdot)$, and $\theta^*(\cdot)$ with respect to expenditures. If the cumulative distribution function governing the identity of the pivotal median voter at the time of the election ($\Phi(m)$) has sufficient curvature, then one contributor's objective function may not be everywhere quasiconcave. This can also occur when the support of $\phi(m)$ is bounded, so the marginal benefits of contributions could be zero (or small) until contributions reach a minimum floor. Alternatively, for certain specifications of voters' utility over sure outcomes ($u(\cdot)$), the function $\theta^*(\cdot)$ need not possess the desired curvature. In either case, contributor equilibrium may not exist, or if it does, equilibrium contributions could vary discontinuously with candidate positions.

12. To see this note that since $\phi(m)$ is bounded, for some initial locations of the indifferent voter without contributions, $\theta(0,0)$, it can be the case that given the contributions of the other contributor, a contributor has to spend a strictly positive amount before the indifferent voter lies within the interval $[L,F]$. Since these expenditures incur marginal costs, but produce no marginal benefits until $\theta^*(D)$ lies within $[L,F]$, this contributor's objective function is not quasiconcave for these parameter choices. As a result, the contributor's reaction function, if it exists, need not be continuous, and equilibrium cannot be guaranteed when contributions are refundable.

13. The basic problem when contributions are refundable emerges when (1) one candidate wins with high enough probability ex ante (i.e., without contributions) so that, holding the contributions of the other contributor fixed, the initially advantaged contributor can set the location of the indifferent voter equal to the edge of the bounded support of the distribution of the median, $\phi(m)$, with a trivial expenditure, and (2) at least one contributor obtains net negative benefits at the contributions tatonnement equilibrium that would result under the assumption of nonrefundable contributions. When contributions are refundable, the advantaged contributor never wants to spend more than the minimum needed to set the probability of his favored candidate's election equal to one. But the disadvantaged contributor, since he holds the other's expenditures fixed, cannot be maximizing his net expected benefits (equation (1)) when the indifferent voter is located at the edge of the support of $\phi(m)$ and the disadvantaged contributor is spending zero (by assumption A5). Hence, whenever one candidate wins with high probability in equilibrium, the "losing contributor" must be spending an amount that that equates marginal benefits and marginal costs of the last dollar contributed -- i.e., in equilibrium, neither contributor can gain net expected benefits by unilaterally increasing his expenditure. But since the losing contributor is therefore spending a positive amount for a small probability that his favored candidate wins, there is clearly no guarantee that a contribution equilibrium always exists when contributions are refundable.

14. Also, the incumbent's probability of winning is probably discontinuous at that point, as long as equilibrium contributions are continuous functions of candidate positions. The reason is that when the less risky challenger can locate arbitrarily close to the more risky incumbent, the indifferent voter
becomes arbitrarily far from both positions, while the indifferent voter between equally risky candidates lies exactly midway between the candidates' positions.

15. For example, suppose that a candidate's probability of winning increases monotonically as its position approaches some location $X$, but the candidate's probability of winning is discontinuous at $X$. Without assumption (A8) non-existence of equilibrium might occur because as the challenger (incumbent) wants to locate arbitrarily close to point $X$, but not equal to it.

16. However, if $\bar{I} < I_0, I^R, I^L$, then the challenger is indifferent between locating on the incumbent's left or right.

17. Observe that if this did not hold, then the incumbent, as first-mover, would lose with probability one.

18. Figure 3 was produced by letting varying the parameters of contributor $A$'s utility function (a and $\alpha$) for the case where $\nu = \frac{\rho}{m + D}$ for $I = I, C$, and $\rho_{I} = |I - I|$. Many different parameter combinations that satisfy the assumptions of the model produce figures of the general form shown. One such combination of parameter values is $I = 0, r = 10, I = 0, m_{I} = 2, m_{C} = 1, \rho_{C} = 20$, and $\rho_{I}(0) = 1$.

19. Observe that $I^0$ does not depend on contributions, and the comparative statics of $I^L$ move in the same direction of the comparative statics of $\bar{I}$. Consider, therefore, changes in $a, b, \alpha$ or $\beta$ that increase (decrease) $D_A(\cdot)$ or $D_B(\cdot)$. Direct calculation, using equation (11) under assumption (A9), shows that $\bar{I}$ is increasing in $D_B(\cdot)$ and decreasing in $D_A(\cdot)$, while $I^R$ is increasing in $D_A(\cdot)$ and decreasing in $D_B(\cdot)$. Using the result of Lemma 6 $[C^*(I) = \theta^*(I, C^*(I)), \nu(I, C(\cdot))]$, and so direct calculation shows that $C^R(\cdot)$ and $Q^R(\cdot)$ are increasing in $D_A(\cdot)$ (decreasing in $D_B(\cdot)$), while the reverse is true for $C^L(\cdot)$ and $Q^L(\cdot)$.

20. Another way to see this is to consider the polar case of the infinite reputation effect in which the incumbent cannot deviate from IP without losing with probability one. Suppose initially that $C^*(I) > IP$. Then as long as $C^*(I) > IP$, increases in $D_A(\cdot)$ (which comes to the incumbent in equilibrium) increase the incumbent's probability of winning, while increases in $D_B(\cdot)$ decrease the incumbent's probability of winning. Once $D_A(\cdot)$ increases to the point that $C^*(IP) < IP$, it must then be the case that $Q^*$ decreases with $D_A$. (Note that if the incumbent were unconstrained by his reputation, contributions would always make him worse off, as above, but he would do better initially if he could always locate at $I$ without reputational costs.)

21. These campaign contributions models do not impose non-refundable contributions, and hence only apply to those equilibria in this model in which the non-refundability constraint is non-binding, a strict subset of the cases treated in this paper. However, subject to restrictions on initial conditions ($\theta^*(0,0)$, and the parameters of the contributor payoffs), the equilibria of the models are identical.

22. Thus, all else equal, an incumbent with $IP = \bar{m}$ gains less from a given increase in his contributions than an incumbent with $IP = m$.

Bibliography


