In many important decisions, people are uncertain or ambiguous concerning the magnitude of the probabilities of events that can affect outcomes. The classic theory of decision-making argues that people's decisions should not be affected by whether knowledge of a probability is precise or ambiguous. This chapter presents a descriptive model of how people cope with ambiguous probabilities in decision-making. The model predicts that ambiguity matters. 'Decision weights' associated with ambiguous probabilities are assumed to be reached via an anchoring-and-adjustment process in which people anchor on an estimate of the probability and then adjust this as a result of mentally simulating alternative values of the probability. The mental simulation process is affected by both the amount of ambiguity and whether outcomes are large or small gains and/or losses. One important factor that determines people's attitudes towards ambiguity is the nature of the role they assume in making decisions. This interaction between roles and attitudes towards ambiguity is explored in a series of three experiments which test and validate explicit predictions of the ambiguity model. The experiments concern the purchase and sale of insurance, a legal decision-making situation in which plaintiffs and defendants must decide whether to go to trial or settle out of court, and the purchase and sale of industrial equipment involving warranties and discounts. Finally, recognizing that ambiguity concerning probabilities is but one source of lack of knowledge in decision-making, a conceptual framework is suggested to guide future work on this topic.

The theory of decision-making under uncertainty (von Neumann and Morgenstern, 1947; Savage, 1954) has proven to be enormously useful for both helping people make better decisions and understanding how decisions are made. There are many interesting 'how to' applications in the literature (see, e.g., Keeney and Raiffa, 1976) and descriptive
implications of the theory have been used extensively in fields such as finance, economics, and public policy. Despite these successes, both the theory's prescriptive and descriptive adequacy have been repeatedly questioned (see, e.g., Einhorn and Hogarth, 1981; Slovic, Fischhoff, and Lichtenstein, 1988). At the origin of these questions are several paradoxical choice situations in which what seem to be reasonable answers turn out to violate the prescriptions of the theory. These paradoxes have been extensively studied and have inspired many prescriptive and descriptive variations of the theory of decision-making under uncertainty (for overviews see Machina, 1987; Weber and Camerer, 1987; Fishburn, 1988).

The starting point of this chapter is one of these paradoxes, first presented by Daniel Ellsberg (1961). To illustrate briefly the essence of Ellsberg's paradox, consider two situations in which you stand to win $1,000 if you correctly guess the outcome of a flip of a coin, i.e., heads or tails. You can only observe the toss of a coin in one of the two situations and it is your task to decide in which of the two situations you would rather guess the outcome. In the first situation you know that the coin is fair, i.e., there are equal chances of observing heads and tails. In the second situation, however, you do not know whether the coin is fair, i.e., the chances of observing, say, heads could be greater, less than, or the same as observing tails. What would you do? Elect to play in the first situation, the second, or would you be indifferent between the two?

When faced with this choice, most people opt for the first situation where they know they have an even chance of winning $1,000. The fact that the odds of winning in the second situation are unknown makes it less attractive. When comparing the two situations, the first has only one source of uncertainty (i.e., whether heads or tails will appear on the toss of a coin), and this can be precisely quantified (the probability of heads is 0.50 as is that of tails). On the other hand, the second situation is characterized by two sources of uncertainty, the first as to whether heads or tails will appear, the second concerning the probability of heads or tails. Ellsberg referred to this uncertainty about the level of one's uncertainty as ambiguity.

Many readers may legitimately ask why preferring the situation in which the probability of winning $1,000 is known precisely is paradoxical vis-à-vis the theory of decision-making under uncertainty. To see this, ask yourself whether you would pick heads or tails when faced with the opportunity of winning $1,000 if the coin displayed your choice. Most people are indifferent between heads or tails whether or not they know the coin to be fair. Suppose, however, that the game is redefined so that the prize depends on observing heads. In this case, most people would prefer to play with a fair coin where there is no ambiguity. Similarly, if the game were redefined so that the prize depended on tails, most people would still prefer the fair coin. However, since heads and tails are mutually exclusive, how can one rationally defend preferring the coin for which the odds are known?

Whereas Ellsberg's paradox is typically demonstrated using artificial games of chance, the prevalence of ambiguity—and thus the importance of his challenge to the theory of decision-making under uncertainty—should not be underestimated. Indeed, as stated by Ellsberg (1961):

Ambiguity is a subjective variable, but it should be possible to identify 'objectively' some situations likely to present high ambiguity, by noting situations where available information is scanty or obviously unreliable or highly conflicting; or where expressed confidence in estimates tends to be low. Thus, as compared with the effects of familiar production decisions or well-known random processes (like coin-flipping or roulette), the results of Research and Development, or the performance of a new President, or the tactics of an unfamiliar opponent are all likely to appear ambiguous. (pp. 660–61)

In this chapter, we present a descriptive model that shows how people react to ambiguity. Implications of this model are then discussed and illustrated by three different kinds of experiments in which people exhibit different attitudes towards ambiguity. Sometimes they avoid it (as in the example above), sometimes they seek it, and sometimes they appear indifferent to its presence. In some cases, the same person may exhibit all three attitudes when faced with different levels of probability for the same decision-making task. In other situations, a person's role determines his or her attitude towards ambiguity. Finally, recognizing that there are different sources of ambiguity even in simple decision-making tasks, we suggest a framework for an agenda of future research on the effects of ambiguity.

**THE AMBIGUITY MODEL**

The theory of decision-making under uncertainty assumes that people evaluate the attractiveness of a given alternative by weighting the utility of the outcome by the probability of obtaining it. The basic premise of the ambiguity model is that people do not use probabilities to weight outcomes in ambiguous situations. Instead, probabilities are replaced by subjective weights that do not necessarily have the mathematical properties of probabilities.

The main psychological assumption underlying the model is that the subjective weights given to ambiguous probabilities are the end result of a mental anchoring-and-adjustment process (cf. Tversky and Kahneman, 1974; Einhorn and Hogarth, 1985). People are assumed to anchor on a particular estimate of the probability and then adjust this by imagining, via a mental simulation process, other values that the probability could
take. To illustrate, consider a situation in which you are concerned about the chances of an accident occurring in a new industrial facility. A study conducted by technical experts assesses the risk as $P = 0.001$, but they have doubts about the precision of this estimate. In the process assumed here, it is postulated that you would first anchor on a given value of probability (e.g., the 0.001 provided by the experts) and then imagine or 'try out' other values the probability could take, both below and above the anchor. Depending on the circumstances (see below), you would not necessarily accord equal weight in imagination to possible values of the probabilities on both sides of the anchor. For instance, in the present example values above the anchor may well weigh more heavily in imagination than those below (the occurrence of accidents might be salient). The resulting weight given to the ambiguous probability is taken to reflect both the initial anchor and the net effect of the mental simulation process and can be written:

$$S(p_A) = p_A + (k_A - k_s)$$

(11.1)

where $p_A$ is the anchor, $k_A$ represents the values and weight accorded in the mental simulation to values of $p$ greater than the anchor, and $k_s$ corresponds to the weighted values below the anchor.

To make these notions operational, one needs to specify (1) how the anchor, $p_A$, is established, (2) what effects the amount of mental simulation (i.e., the ranges of alternative probability values considered), and (3) what determines the sign or direction of the adjustment process.

1. In ambiguous circumstances, some initial value of the probability is assumed to be typically available to the decision-maker. This may be a figure based on historical data, provided by experts (as in the example above), or selected from memory.

2. If the decision-maker has sufficient knowledge to assign a unique value to the probability there would be little or no mental simulation (however, see, Hogarth and Einhorn, 1990). When the probability is ambiguous, one would expect considerable simulation, the extent of which is assumed to be positively related to the amount of perceived ambiguity.

3. The sign of the adjustment process is determined by the person's attitude towards ambiguity. This could reflect personal dispositions towards optimism or pessimism, but we argue that it is largely dependent on situational variables such as the sign or size of outcomes or whether the context of the situation induces caution (as when considering insurance) or playfulness (as when gambling).

The manner in which imagination affects the anchor value, $p_A$, in the ambiguity model can be shown by depicting the judgemental compromise that results from the anchoring and adjustment process as a function of the anchor probability. This is illustrated in the three panels of figure 11.1.

In interpreting the panels of figure 11.1, recall that two forces cause the final judgement to deviate from the anchor. These are the amount of perceived ambiguity and the person's attitude towards ambiguity in the circumstances. The former determines the amount of mental simulation and thus the extent to which the ambiguity function deviates from the diagonal (45°) line; the more the perceived ambiguity, the greater the deviation. The latter determines the direction of the adjustment and thus the point at which the ambiguity function crosses the diagonal.

Consider first the extreme anchors of $p_A = 0$ and $p_A = 1$. In both cases, the adjustment can only be in one direction, up for $p_A = 0$ and down for $p_A = 1$, thereby illustrating the fact that the location of $p_A$ places constraints on the ranges of values that can be imagined above and below the anchor. Thus $S(p_A) > p_A$ when $p_A = 0$ and $S(p_A) < p_A$ when $p_A = 1$. In general, $S(p_A)$ will overweight small probabilities and underweight large ones; what changes from situation to situation is the point at which the ambiguity function crosses over the diagonal (45°) line, i.e., where over weighting changes to under weighting. In figure 11.1a, values of the probability below the anchor are weighted in imagination more heavily than those above, and the cross-over point lies below $p_A = 0.5$. In figure 11.1b, values above the anchor are weighted more heavily than those below. Here the cross-over point lies above 0.5. In figure 11.1c, values above and below the anchor are weighted equally such that the crossover occurs at 0.5.

To summarize, the ambiguity function shows overweighting of small
anchor values but underweighting of larger ones. The point at which the function changes from over- to underweighting depends on the person’s attitude towards ambiguity. For example, assuming that people are generally cautious in the face of risk (or engage in something akin to ‘defensive pessimism’, Noreen and Cantor, 1986), the ambiguity function would resemble that shown in figure 11.1a if the decision-maker is concerned with the possibility of obtaining a positive outcome. On the other hand, when faced with the possibility of a loss (e.g., when assessing the risk of a new technology), the function would be better represented by figure 11.1b. This is because caution induces greater concern for possible values of the probability lying below rather than above the anchor in the case of potential gains, whereas the contrary holds for losses. We also argue that the location of the cross-over point will be affected by the degree of caution engendered by the situation. Thus, when facing the ambiguous chance of gaining a very large sum of money, the cross-over point will be closer to \( p_A = 0 \) than in a case where a small sum is involved. Similarly, when faced with a large potential loss, the cross-over point will be closer to \( p_A = 1 \) than in a situation involving a small loss.

**Implications**

An important implication of the ambiguity model is that, relative to anchor probabilities, it does not predict that people will always avoid situations characterized by ambiguous probabilities. Indeed, in some cases people will prefer ambiguity, specifically when faced with either a small probability of a gain (as in figure 11.1a) or a large probability of a loss (as in figure 11.1b). In addition, there will be cases where people are relatively insensitive to the effects of ambiguity, i.e., when the anchor probability is in the region of the cross-over point.

We now describe three experiments in which we exploit the model’s implications concerning attitudes towards ambiguity. The experiments are set in three different settings. The first concerns the effects of ambiguity on the purchase and sale of insurance; the second involves a legal decision-making situation; and the third deals with the purchase and sale of industrial equipment. The intention of the experiments is to explore how ambiguity affects competitive situations by having differential impacts on opposing parties. In particular, does asymmetry in the manner in which ambiguity affects the two sides of a decision or transaction confer competitive advantage on one of the parties?

<table>
<thead>
<tr>
<th></th>
<th>CONSUMERS</th>
<th>firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-ambiguous</td>
<td>Ambiguous</td>
</tr>
<tr>
<td></td>
<td>Well-known processes (1)</td>
<td>Typical situation (2)</td>
</tr>
<tr>
<td></td>
<td>New technologies - inside information for consumers (3)</td>
<td>New technologies - processes poorly understood (4)</td>
</tr>
</tbody>
</table>


**Experiment 1**

**Rationale** In traditional economic analysis of insurance markets (based on the theory of decision-making under uncertainty), two variables are relevant in the pricing of insurance. These are the probability of a potential loss occurring (e.g., the probability of an automobile accident), and the amount of the loss (e.g., the magnitude of the damage). It is further assumed that insurance is bought and sold because buyers (consumers) are more risk averse than sellers (insurance firms) reflecting different levels of wealth (your insurance company is wealthier than you are!). However, we shall argue that the purchase and sale of insurance can also be affected by ambiguity in ways that are not accounted for by the standard theory.

To simplify the analysis, imagine that consumers and firms either are or are not ambiguous about the probability relevant to an insurance contract. These possibilities can be represented in the form of a 2 \( \times \) 2 matrix as shown in figure 11.2 together with examples illustrating each cell. Next assume that both firms and consumers have the same anchor probabilities, \( p_A \), but that firms exhibit greater caution in their attitude towards ambiguity than consumers. The rationale for the latter assumption is that there is an important asymmetry in attitude between accepting and transferring a risk. Specifically, compared to the person transferring risk, one would expect the person accepting risk (e.g., an insurer) to give more weight in imagination to possible values of the probability of loss that are greater than the anchor value (see also Thaler, 1980; Hershey, Kneuthure...
and Schoemaker, 1982). This asymmetry can be translated within the ambiguity model by showing different ambiguity functions for consumers and firms as illustrated in figure 11.3. Note from the figure that the ambiguity function for firms lies uniformly above that for consumers thereby indicating a more cautious attitude in the presence of ambiguity. When neither consumers nor firms are ambiguous, their respective ambiguity functions coincide with the diagonal (45°) line.

Several testable predictions are suggested by figure 11.3. First, consider situations where consumers are ambiguous. For low probabilities more weight is given to values above the anchor than those below; for high values of probability, the opposite is likely to be true. Comparing what consumers would be willing to pay in ambiguous as opposed to non-ambiguous situations, one would thus expect willingness to pay larger premiums under conditions of ambiguity for low probability events (i.e., to avoid ambiguity) but less willingness to buy insurance for high probability of loss events (i.e., exhibiting preference for ambiguity). Second, firms would want to charge higher premiums for ambiguity across most of the probability range. However, aversion to ambiguity would be expected to decrease as probabilities increase. Third, and as a corollary to the preceding, firms are generally expected to be more averse to ambiguity than consumers. We now turn to experimental tests of these predictions.

Subjects There were two groups of subjects: 101 professional actuaries and 116 MBA students. The professional actuaries were members of the Casualty Actuarial Society and formed a subset of the members residing in North America who responded to different questionnaires as part of a mail survey conducted in 1986. (In total, 489 of 1,165 persons or 42 percent of the membership provided usable responses in this survey. Mean length of experience as actuaries reported by the respondents was 13.8 years.) The MBA students responded to questionnaires handed out in a class on decision-making at the University of Chicago, Graduate School of Business. All these students had taken prior work in economics and statistics and were ‘sophisticated’ about the relevant issues, although clearly not as knowledgeable about insurance as the actuaries.

Stimuli and design The experimental stimulus involved a scenario in which the owner of a small business with net assets of $110,000 seeks to insure against a $100,000 loss that could result from claims concerning a defective product. Subjects assigned the role of consumers were told to imagine that they were the owner of the business. Subjects assigned the role of firms were asked to imagine that they headed a department in a large insurance company and were authorized to set premiums for the level of risk involved. The question was worded to indicate a single risk.

Ambiguity was manipulated by factors involving how well the manufacturing process was understood, whether the reliability of the machines used in the process was known, and the state of the manufacturing records. In both ambiguous and non-ambiguous cases a specific probability level was stated (e.g., 0.01). However, a comment was also added as to whether one could ‘feel confident’ (non-ambiguous case) or ‘experience considerable uncertainty’ (ambiguous case) concerning the estimate. Uniformity of perceptions of ambiguity was controlled by describing the situations by the same words in both the consumer and firm versions.

Four variables were manipulated in the study. These were role (consumer or firm), ambiguity (ambiguous or non-ambiguous version of the stimulus), probability of loss (p = 0.01, 0.33, 0.65, 0.90), and type of respondent (actuaries or MBA students). Subjects were assigned the role of either consumer or firm. Consumers stated the maximum premiums they would be prepared to pay, whereas firms were asked to state the minimum premiums they would be prepared to charge. Each subject responded to both the ambiguous and non-ambiguous versions of the stimuli that related to his or her role but at only one probability level (i.e.,
responses at the different probability levels were made by different subjects. For the actuaries, the two versions (ambiguous and non-ambiguous) of the stimulus for this experiment were the first and last of several questions they were asked to answer. Each question appeared on a different page of the questionnaire and the order of the ambiguous and non-ambiguous versions was randomized across subjects. For the MBA subjects, the stimuli were included among a series of problems related to decision-making, each on a different page of an experimental booklet in which the ambiguous and non-ambiguous versions were also physically separated by several items. Subjects were instructed to work systematically through the booklet at their own pace without looking back at previous responses.

In summary, the design of the experiment involved four factors: three involved comparisons between subjects (i.e., role of consumer or firm, probability level, and type of respondent), and one within subjects (i.e., ambiguous versus non-ambiguous scenarios). There were 217 subjects; 10 actuaries and 116 MBA students.

Results Table 11.1 summarizes the results of the experiment by showing the median prices in all experimental conditions. Medians are shown rather than means because several distributions within cells were quite skewed. Results conformed with predictions of the ambiguity model. Consumers were averse to ambiguity (as measured by willingness to pay higher prices) for low probability events; however, as the probability level increased, the attitude towards ambiguity changed from aversion to preference (compare columns 1 versus 2 for the actuaries, and 3 versus 4 for the MBA students). For firms, there is also aversion to ambiguity for low probability of loss events and the level of the aversion decreases as probabilities increase (compare columns 5 versus 6 for the actuaries, and 7 versus 8 for the students). Firms, however, never prefer ambiguity.

Second, there are differences between prices firms are willing to charge and how much consumers are willing to pay for given probability and loss levels. This is important because it suggests how ambiguity affects the case with which transactions can be made in insurance markets. Consider the case where, for low probability of loss events, consumers are ambiguous but firms are not (e.g., automobile insurance where firms have statistical data on accidents and thefts which individuals lack. This is the prototypical case in which much insurance is sold – see the upper left cell in figure 11.2). Comparing the entries at the $p = 0.01$ level for columns 1 versus 6 (for the actuaries) and 3 versus 8 (for the MBA students), it is clear that consumers are prepared to pay much more than firms require. However, when firms also become ambiguous (compare columns 5 versus 1 and 7 versus 3), it is not clear that it will be easy for consumers to find firms willing to supply insurance at prices they are willing to pay.

A third interesting comparison can be made between the responses of the actuaries and MBA students. On the one hand, the responses of both groups are qualitatively similar in the patterns of their reactions towards ambiguity. On the other hand, the actuaries' prices are generally higher than those of the students when ambiguity is held constant (compare columns 1 versus 3, 2 versus 4, 5 versus 7, and 6 versus 8). The reason for this result is unclear but suggests that actuaries have a greater appreciation of the risks underlying insurance contracts than MBA students and are therefore willing both to pay and charge more. Discussion with actuaries and their comments on the questionnaires revealed that they specifically considered ambiguity in the determination of premiums, taking the price over the level for non-ambiguous probabilities.

Table 11.1. Experiment 1: Median prices ($) of firms and consumers

<table>
<thead>
<tr>
<th>Probability of loss (n)</th>
<th>$</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Actuaries</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Ambiguous</td>
<td>5,000</td>
<td>1,500</td>
</tr>
<tr>
<td>(2) Non-ambiguous</td>
<td>46,000</td>
<td>40,000</td>
</tr>
<tr>
<td>(3) Ambiguous</td>
<td>5,000</td>
<td>1,500</td>
</tr>
<tr>
<td>(4) Non-ambiguous</td>
<td>50,000</td>
<td>40,000</td>
</tr>
<tr>
<td><strong>MBA students</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) Ambiguous</td>
<td>40,000</td>
<td>60,000</td>
</tr>
<tr>
<td>(6) Non-ambiguous</td>
<td>80,000</td>
<td>80,000</td>
</tr>
<tr>
<td>(7) Ambiguous</td>
<td>40,000</td>
<td>60,000</td>
</tr>
<tr>
<td>(8) Non-ambiguous</td>
<td>80,000</td>
<td>80,000</td>
</tr>
</tbody>
</table>

Note: (n) indicates number of subjects in experimental condition.
These observations are supported by statistical tests involving an appropriate analysis of a variance model with three between-subject factors (probability level, role, and type of subject), the within-subject factor of ambiguity, and the different possible between- and within-factor interactions. This analysis shows significant main effects for probability level ($p < 0.0001$), role (i.e., firm or consumer, $p < 0.0001$), type of subject ($p < 0.01$), and ambiguity ($p < 0.02$). Moreover, there are significant interactions with respect to probability level $\times$ role ($p < 0.0003$), ambiguity $\times$ probability level ($p < 0.0001$), ambiguity $\times$ role ($p < 0.0001$), and ambiguity $\times$ type of respondent ($p < 0.001$).

**Experiment 2**

**Rationale** Imagine a case of civil litigation where both plaintiff and defendant must decide whether to accept an out-of-court settlement or risk going to court. For the plaintiff, this decision is naturally framed as either accepting a sure sum (the settlement) or going to court with the possibility of gaining a larger sum or losing all. For the defendant, it is the reverse: either lose money for sure (the settlement) or go to court with the chance of losing either more or nothing. To continue the example, imagine that the two parties agree on both the probability that the plaintiff will win the case and the amount that each is prepared to pay the other to settle out of court. Assume further that this amount is equal to the expected value at stake in the court case. Ignoring consideration of legal costs, what actions do different choice theories predict would be taken by plaintiff and defendant?

The theory of decision-making under uncertainty predicts that, provided the plaintiff and the defendant are risk-averse and agree on the probability of the outcome of the case, they will both prefer to settle out of court (Gould, 1973). This contrasts with the predictions of prospect theory (Kahneman and Tversky, 1979), the leading alternative descriptive theory of decision-making under risk. These are that the plaintiff will take the risk-averse action (i.e., settle out of court) provided the probability of winning the case is not very small; the defendant, however, will take the risky action (i.e., go to court). And indeed, the prospect theory predictions have been upheld in experimental tests of this legal decision-making situation (Hogarth, 1987, chapter 5).

However, what happens if probabilities are ambiguous? First, note that neither the standard theory nor prospect theory make specific predictions concerning the effects of ambiguity. The ambiguity model makes the following predictions: (1) For plaintiffs, when probabilities of winning the case are moderate or large, ambiguity implies underweighting the anchor probabilities (see figure 11.1a) thereby encouraging the parties to settle out of court. In other words, under ambiguity plaintiffs will be more likely to choose the riskless option (i.e., settle out of court) than when probabilities are not ambiguous. (2) For defendants, the predictions are more complex. For high probability of loss events, ambiguous probabilities are underweighted relative to their anchors (see figure 11.1b) such that defendants would be expected to continue to take the risky option (go to court). Indeed, for high probability of loss events the model predicts greater risk seeking under ambiguity when probabilities are ambiguous as opposed to non-ambiguous. However, in the presence of ambiguity, the tendency to take the risky alternative will be reduced, relative to the non-ambiguous case, as the probability of losing the case decreases. This prediction follows from the implication that, for losses, there is over-weighting of anchor probabilities when these are small or moderate (see figure 11.1b). To summarize, defendants are predicted to exhibit risk-seeking behaviour at high probability of loss levels irrespective of ambiguity. At moderate probability levels, however, defendants with ambiguous information about probabilities will exhibit more risk-averse behaviour (i.e., settle out of court) than those with precise probability estimates. The following experiment was designed to test these predictions.

**Subjects** Subjects were 80 MBA students at the University of Chicago taking a course in decision-making (these were not the same students as those in Experiment 1). As assignments given in the first and second weeks of the course, students were required to complete two questionnaires which contained several decision-making problems that were to be debriefed and discussed later in the course.

**Task and method** Subjects were allocated at random to four experimental conditions that were created by crossing two kinds of role (plaintiff or defendant) by two types of probabilistic information (ambiguous or non-ambiguous). In addition, two levels of probability were varied as a within-subject factor by setting the probability of the plaintiff winning the trial at 0.80 in the first questionnaire, and at 0.50 in the second which was completed one week later.

The stimulus consisted of a short scenario which stated: whether the subject was the plaintiff or defendant; the amount at stake in the case (subjects were asked to imagine that this was $20,000 of their own money); an estimate by the party's lawyer of the probability that the case would be won by the plaintiff (see below); and knowledge supplied by each party's lawyer that the opposing party would settle for a given sum (minimum for the plaintiff, maximum for the defendant). This sum was $16,000 in the 0.80 probability condition, and $10,000 in the 0.50
condition. The scenario made no mention of the reasons underlying the litigation and subjects were instructed to ignore legal costs. Subjects were required to decide between accepting the out-of-court settlement (i.e., $16,000 or $10,000) or to risk going to court.

Ambiguity was manipulated in the scenarios by stating, in the ambiguous case, that in response to a query about the chances of winning or losing the case, the lawyer gave a best guess 'after some hesitation' and that 'given the nature of the case, he feels very uneasy about providing you with such a figure'. In contrast, the non-ambiguous version simply stated 'your lawyer believes there is a... chance that...'.

Results Table 11.2 summarizes results of the experiment by reporting the percentages of subjects choosing to settle out of court in each experimental condition. Recall first that, for plaintiffs, the model predicts that ambiguity will act as a force towards risk aversion. For defendants, on the other hand, ambiguity is only expected to induce risk-averse behaviour at low or moderate probability levels, e.g., at 0.50 but not at 0.80. Results show that, at the 0.80 probability level, the vast majority of plaintiffs chose to settle out of court, whereas most defendants took the risky option of going to court. Moreover, there are no differences in responses due to ambiguity, 89% per cent versus 61% per cent for the plaintiffs, and 25% per cent versus 10% per cent for the defendants. However, note that, in the non-ambiguous condition, since all plaintiffs chose the riskless option and most defendants the risky option, choices could not be sensitive to effects of ambiguity.

At the 0.50 probability level, the pattern of responses for the plaintiffs is almost identical to that at 0.80 with, again, no effects for ambiguity. However this is not the case for defendants where the difference between the percentages of subjects wishing to settle out of court in the non-ambiguous and ambiguous conditions (2 vs 57) is significant ($x^2 = 3.63, df = 1$). In addition, whereas there is no statistically significant difference between responses of the same subjects in the non-ambiguous condition at the 0.80 and 0.50 probability levels (25% per cent versus 6% per cent), the difference between responses at these two probability levels in the ambiguous condition (10% per cent versus 57% per cent) is significant (Cochran's test, $Q = 10.00, df = 1, p = .0016$). These results clearly support the predictions of the ambiguous model (for further details, see Hogarth, 1989).

Experiment 3

Rationale The preceding experiment examined situations involving two parties (defendants and plaintiffs) facing risky situations involving only

<table>
<thead>
<tr>
<th>Probability level</th>
<th>0.80</th>
<th>0.50</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plaintiffs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-ambiguous</td>
<td>89</td>
<td>89</td>
<td>(19)</td>
</tr>
<tr>
<td>Ambiguous</td>
<td>96</td>
<td>83</td>
<td>(24)</td>
</tr>
<tr>
<td>Defendants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-ambiguous</td>
<td>25</td>
<td>6</td>
<td>(16)</td>
</tr>
<tr>
<td>Ambiguous</td>
<td>10</td>
<td>57</td>
<td>(21)</td>
</tr>
</tbody>
</table>

Note: (n) indicates number of subjects in experimental condition.


losses or gains. It is therefore also instructive to consider what happens when two parties face risky situations involving both possible losses and gains. Of particular interest are situations where the structure of a transaction is such that ambiguity has differential effects on the evaluations made by the two parties. Specifically, if ambiguity has little effect on the evaluation made by one party, but does affect the other, competitive advantages can accrue to one of the parties.

To explore this possibility, consider situations where two parties are on opposing sides of transactions that can be thought of as involving risky choices of the following type:

Party A has: a large probability of a modest gain, and a small probability of a large loss.

Party B has: a large probability of a modest loss, and a small probability of a large gain.

To be specific, describe A’s situation as involving a 0.9 chance of winning $2,000 accompanied by a 0.1 chance of losing $8,000, and B’s situation as a 0.9 chance of losing $2,000 accompanied by a 0.1 chance of winning $8,000.

To evaluate the effects of ambiguity on the situations faced by A and B, recall that figure 11.1a represents a typical ambiguity function for gains, whereas figure 11.1b depicts one associated with losses. This implies that, for party A, an ambiguous 0.9 chance of gaining $2,000 will be evaluated as less attractive than if the 0.9 chance were not ambiguous (see figure 11.1a); in addition, an ambiguous 0.1 chance of losing $8,000 will be evaluated as more aversive than a 0.1 chance that is not ambiguous (see
was made operational by two versions of the experimental stimuli where the probabilities of the relevant events were given in either ambiguous or non-ambiguous form. Subjects were allocated at random to the four cells of the 2 x 2 design.

The scenario used in this experiment involved the purchase and sale of industrial equipment valued at about $100,000. Buyers had the opportunity of obtaining the equipment from one of four suppliers (Alpha and Beta) who differed in respect of their terms of sale. Alpha's price included a warranty against a specific type of breakdown. Beta did not offer a warranty but was willing to sell at a discount relative to Alpha. The problem was structured so that the buyer was asked to consider Beta's offer as involving a potential gain of $2,000 (the discount) against a potential loss of $8,000, where the latter was the difference between the $10,000 cost of repairing the breakdown (should it occur) and the $2,000 discount.

In the seller version of the questionnaire, subjects were told that, although their usual policy was to sell machinery with warranties against specific breakdowns, a customer had requested to forgo the warranty for a $2,000 discount. This was described as 'a one-shot deal and would have no repercussions on the rest of your business.' The net effect of the deal was described as 'if you sell the machine with a discount, you are facing a potential loss of $2,000 if no breakdown occurs during the warranty period (i.e., the amount of the discount). However, you also stand to gain $8,000 if a breakdown occurs (i.e., you would save repair costs of $10,000 but allow a discount of $2,000).'

Ambiguity was manipulated in the same manner in both the buyer and seller versions of the scenarios. In the ambiguous case, the machinery being sold was described as being 'based on new design principles' and that, although there was a 'best estimate' of the probability of a breakdown within the warranty period, you experience considerable uncertainty about this estimate. In the non-ambiguous version, subjects were told that 'extensive records existed concerning the machine's breakdown record and that you can confidently estimate the probability of a breakdown occurring within the warranty period'. The anchor probability of the breakdown occurring within the warranty period was given as 0.1 (for both buyers and sellers).

Subjects made two responses to the scenario. Buyers were required to choose between Alpha (i.e., buy with warranty but no discount) or Beta (no warranty but discount). In addition, they were asked to state 'the minimum discount they would be prepared to accept to buy the equipment without the warranty'. Sellers were asked whether they would sell the machinery 'at a discount of $2,000 but with no warranty' or 'without the discount but with the warranty'. Their second question was 'What is
Table 11.3: Preferences and discounts of buyers and sellers

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Ambiguous %</th>
<th>Non-ambiguous %</th>
<th>Ambiguous %</th>
<th>Non-ambiguous %</th>
</tr>
</thead>
<tbody>
<tr>
<td>For discount warranty</td>
<td>36</td>
<td>68</td>
<td>51</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>n</td>
<td>10</td>
<td>10</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Stated discounts</td>
<td>$</td>
<td>$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>Minimum for buyers, maximum for sellers</td>
<td>$3,000</td>
<td>$2,000</td>
<td>$1,986</td>
<td>$1,580</td>
</tr>
<tr>
<td>Medians</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>4,049</td>
<td>3,011</td>
<td>1,986</td>
<td>1,580</td>
</tr>
</tbody>
</table>


the maximum discount you would be prepared to grant if you were to sell the machinery without the warranty?

Results Following the rationale given above, recall that we predicted that, whereas the buyer's decision should be sensitive to ambiguity, this would not be the case for the seller. Table 11.3 presents the results of the experiment in terms of (a) responses concerning preferences for discounts versus warranties and (b) minimum (for buyers) and maximum (for sellers) amounts that the parties would accept (for buyers) or grant (for sellers) in lieu of a warranty.

Consider first the data in respect of preferences for discounts versus warranties. For buyers, whereas 64 per cent of subjects chose the warranty in the ambiguous condition, the corresponding figure was 32 per cent in the non-ambiguous condition ($\chi^2 = 6.35, df = 1, p < 0.02$). In other words, choices made by buyers were consistent with avoiding ambiguity. For sellers, however, whereas 47 per cent chose the warranty in the ambiguous condition, this figure is 61 per cent in the non-ambiguous condition and the difference is not statistically significant ($\chi^2 = 1.28, df = 1$). To summarize, buyers (party A) were sensitive to ambiguity in this situation as predicted whereas sellers (party B) were not.

Results of the choice data are supported by estimates of minimum (for buyers) and maximum (for sellers) discounts. Median and mean discounts stated by buyers in the ambiguous condition exceed those in the non-

ambiguous condition, $5,000 versus $2,000 and $4,049 versus $3,011, respectively. The difference between medians is statistically significant ($p < 0.05, \text{one-tailed Mann-Whitney test}$) although the difference between means is not ($t = 1.39, p = 0.086, \text{one-tailed test}$). Differences between the mean and median discounts of sellers in the ambiguous and non-ambiguous conditions are both small and statistically insignificant.

DISCUSSION

We first comment on the three experiments reported above and then suggest a framework for an agenda of future research on the effects of ambiguity in decision-making under uncertainty.

The experiments illustrate different aspects of the ways in which ambiguity and role affect decision-making under uncertainty. In Experiment 1, we demonstrated the effects of ambiguity on the purchase and sale of insurance. Although the prices stated by both firms (i.e., sellers) and consumers were sensitive to ambiguity, the two parties differed in their reactions to ambiguity. Firms were generally more averse to ambiguity than consumers. In particular, whereas aversion to ambiguity decreased with increases in the probability of a loss for both firms and consumers, firms never reached the point where they would accept to insure an ambiguous risk for a price less than a non-ambiguous risk with an equivalent anchor probability. Consumers, on the other hand, did show preferences for ambiguity in that they were not prepared to pay as much to insure an ambiguous as opposed to non-ambiguous risk when the probability of loss was high.

In addition to illustrating the conditions under which people are averse to or prefer ambiguity, these results help to illuminate some aspects of insurance markets that have puzzled scholars working within the tradition of the classic model of decision-making under uncertainty. One of these puzzles concerns why so many travellers purchase 'flight' insurance at airports even though prices greatly exceed the rates of readily available life insurance (Eisner and Strotz, 1961). There may be a number of reasons why consumers are interested in purchasing such insurance. Whereas the chance of an accident during a particular flight is small, it is ambiguous. Furthermore, when accidents do occur, media coverage is typically extensive such that it is easy to imagine scenarios under which accidents can happen. In short, given an event with an ambiguous probability and the ease with which disastrous scenarios can be imagined, the ambiguity model clearly predicts large upward adjustments of the implicit decision weights associated with the decision to buy flight insurance.

The results of Experiment 1 also shed light on the conditions under
which insurance can or cannot be easily acquired by consumers. When both firms and consumers are ambiguous, consumers will have difficulty in finding firms offering insurance at prices they are prepared to pay. On the other hand, for low probability events where consumers are ambiguous, but firms are not, there will be an active market for coverage. Consider, for example, home (theft and fire) and life insurance. For most of us, the probabilities of losses in these domains are experienced as being both low and ambiguous. For insurance companies, on the other hand, the probabilities of these events can be estimated precisely from statistics and there is no ambiguity.

In Experiment 2, we examined how ambiguity affects risk taking in the context of a legal decision-making situation. Our starting point was the asymmetry in risk attitudes predicted by Kahneman and Tversky's (1979) descriptive theory of decision-making under risk. In short, this theory predicts that people tend to be risk averse when faced with risky situations involving gains but risk seeking in the face of potential losses. Translated to the legal scenario examined in the experiment, this means that, whereas plaintiffs (who face potential gains) will have a strong tendency to seek settlements out of court, defendants (who face possible losses) will prefer to take their chances by going to court. However, this analysis ignores the effects of ambiguity and does not square with the empirical observation that the vast majority of civil suits in the USA are in fact settled out of court thereby implying that this option is preferred by both plaintiffs and defendants (cf. Gould, 1973).

In our analysis, we predicted that, for moderate anchor probabilities (about 0.5), the behavior of defendants would be more cautious in the presence of ambiguity thereby counteracting the tendency to go to court. And indeed, this was what was observed and suggests that, in the presence of ambiguity, parties to civil litigation will typically prefer to settle out of court. Moreover, since most people lack experience with civil litigation and are, perforce, ambiguous about the chances of winning their cases, the ambiguity model provides a rationale for why most cases are in fact settled out of court. We note, parenthetically, that understanding the effects of ambiguity in situations such as these could be important in different types of negotiations. For example, building on Kahneman and Tversky's (1979) work, it has been suggested that in bargaining situations one might be able to exploit asymmetries in risk attitudes towards losses and gains by being able to 'second-guess' the decisions of an opponent (see, e.g., Bazar- man, 1983; Hogarth, 1987, chapter 5). Thus in a situation such as our court scenario, a plaintiff could construct his or her bargaining strategy assuming that the defendant would tend to be risk seeking. However, in the presence of ambiguity, this assumption could well be erroneous.

Experiments 1 and 2 illustrated how ambiguity affects certain kinds of decisions and showed that, whereas people tend to avoid ambiguity, there are also situations in which ambiguity is preferred. Experiment 3 demonstrated a further phenomenon. Two parties can be on opposite sides of a transaction that is affected by ambiguity. However, ambiguity only affects the way one but not both parties evaluate the situation. Although the particular task examined in Experiment 3 involved the purchase and sale of industrial equipment, the paradigm of two parties being on opposite sides of a transaction where both could stand to gain or lose (as opposed to only gain or lose) has more general application. One area is the market for protective services where people can hedge risks by different means. Consider, for example, trading in financial instruments, e.g., stocks, bonds, commodities, futures, options, and portfolio insurance. As shown in Experiment 3, if people put themselves into a situation where they have a large probability of gaining a small amount and a small probability of losing a large amount, ambiguity will impact negatively on their position. However, by taking a position that involves a large probability of losing a small amount accompanied by a small probability of gaining a large amount, the subjective evaluation of the position will not be affected by ambiguity. It would be interesting to ascertain empirically what kinds of professional traders tend to structure deals for themselves that are more similar to the first or second type of situation, and whether this leads to strategic advantages in buying and selling. For example, do individuals differ in the types of trades they undertake for different types of institutions?

In summary, we have demonstrated that by incorporating the effects of ambiguity into more traditional models of decision-making under uncertainty we can model and predict many interesting real-world phenomena. In particular, we have shown that attitudes towards ambiguity are largely determined by characteristics of situations people face and/or the roles they are asked to assume and that people do not necessarily exhibit only aversion to ambiguity. However, from a descriptive viewpoint, people face other ambiguities aside from probabilities in decision-making. To illustrate, consider the following scenario:

You receive a telephone call from the department store where you purchased a refrigerator two years ago. The caller informs you that the warranty on your refrigerator has expired and asks whether you wish to renew it at an annual rate of $50. The only further information supplied is the minimum cost of labor for someone to visit your home should your refrigerator malfunction ($35). The warranty would cover the costs of both labor and parts. You are quite vague, even ignorant, about the chances of your refrigerator breaking down; you also find it difficult to estimate the costs of possible repairs.

As this scenario illustrates, even in a relatively simple situation involving the purchase of a warranty, decision-makers can experience ambiguity,
Table 11.4. Situations arising from different levels of knowledge concerning probabilities and outcomes

<table>
<thead>
<tr>
<th>Knowledge about outcomes</th>
<th>Known</th>
<th>Ambiguous</th>
<th>Ignorant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Known</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Ambiguous</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Ignorant</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

not only with respect to probabilities, but also with respect to the amounts at stake or outcomes. Moreover, if we define ambiguity as resulting from lack of knowledge of probabilities and outcomes, then it should be clear that there can be a continuum extending from complete knowledge to complete ignorance, for both probabilities and outcomes.

Dividing these continua for probabilities and outcomes into three sections representing ‘knowledge’, ‘ambiguity’, and ‘ignorance’, therefore, we can define a $3 \times 3$ matrix of situations involving different levels of partial knowledge as illustrated in Table 11.4.

Table 11.4 contains nine cells or types of situations for relatively simple kinds of decision tasks. Descriptive work had really only considered two cells, number 1 where both probabilities and outcomes are assumed to be known (as in the classic model of decision-making under uncertainty), and number 2 where, although outcomes are assumed known, knowledge about probabilities is ambiguous (the case considered in this chapter). However, as illustrated in the refrigerator scenario considered above, there must also be many real-life situations that involve the other seven cells of Table 11.4.

Building on the work described in this chapter, therefore, Table 11.4 suggests a rich agenda for research on understanding the effects of ambiguity and ignorance on decision-making. Contrast, for example, decisions in cell 9 (‘ignorant’ – ‘ignorant’) with decisions in cells 1 or 2. How does one take risky decisions when lacking even rough estimates of probabilities or outcomes (consider again the refrigerator scenario, above)? What models might best describe behaviour in these kinds of situations? What are the effects of ambiguity concerning outcomes over and above ambiguity with respect to probabilities? Does role (e.g., buyer versus seller) affect ambiguity concerning outcomes in the same manner as ambiguity concerning probabilities? Does ambiguity regarding probabilities of losses help explain the limited interest by insurers in marketing policies such as pollution insurance, earthquake coverage, and political risk protection? These are fascinating and open questions. They are also the kinds of questions we shall be addressing in future research.

NOTE

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REFERENCES


