Optimal Bidding and Contracting Strategies in Supply Chains for Non-storable Goods

DJ Wu
Department of Management
Bennett S. LeBow College of Business
Drexel University
Philadelphia, PA 19104, U.S.A.
Tel: 215 895 2121
Email: wudj@drexel.edu

Paul R. Kleindorfer
Department of Operations and Information Management
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104, U.S.A.
Tel: 215 898 5830
Email: kleindor@grace.wharton.upenn.edu

Jin E. Zhang
Department of Economics and Finance
City University of Hong Kong
83 Tat Chee Avenue
Kowloon, Hong Kong
Tel: (852) 2788 7315
Email: efzhang@cityu.edu.hk

Version: June 1999

Keywords: Bidding; Contracting; Supply Chain Management; Non-storable Goods
Abstract

This paper models the interaction of long-term contracting and spot market transactions between Firms and Retailers for non-storable commodities. The problem is of particular interest in supply chains concerned with capital intensive production processes in which reservation of capacity and JIT delivery are important elements of operations. The basic framework allows buyers (called “Retailers”) to purchase from suppliers (called “Firms”) a commodity that is essentially non-storable. Firms and Retailers may either contract for delivery in advance (the “contracting” option) or they may sell and buy some or all of their output/input in a spot market. Contract pricing involves both a reservation fee per unit of capacity and an execution fee per unit of output if capacity is called. The key question addressed is the structure of the optimal portfolios of contracting and spot market transactions for these Firms and Retailers, and the pricing thereof in market equilibrium. The model is especially suitable for analyzing electric power, hotel rooms and airline seats and other dated services, but applies as well to contracting for JIT performance in capital intensive supply chains. The paper extends our earlier theoretical results for the Single-Firm case to the Multi-Firm case.
1 Introduction

The evolution of supply chain management has seen an increasing emphasis on long-term relationships. However, it has been recognized that single-sourcing in these relationships can result in opportunism and small-number bargaining problems ex post. This has given rise to a concern with multiple-source contracting that allows suppliers and buyers to obtain the benefits of competition. A problem that must be solved in this regard is the optimal structuring of the portfolio of contracts for either buyers or sellers when facing uncertainty and heterogeneous supplier costs. The resulting contracting problem is complicated because it entails the so-called two-good problem. The first good is the capability of producing, i.e., capacity itself. The second is the actual output produced. In long-term contracting, especially in capital intensive industries, both of these goods are the focus of contracts. A reservation fee per unit of capacity is typically required, given the opportunity costs a supplier faces in tying up scarce capacity through a commitment to a particular buyer. In addition, there is a cost per unit of output produced, the latter reflecting the out-of-pocket costs of actual production.

This paper is concerned with the two-good contracting problem, in which there is an additional feature present, the existence of a spot market for the good in question. This spot market can be thought of as a back-stop technology, output from which can be purchased, typically at a premium, at short notice. Suppliers (called here “Firms”) can also reserve some of their capacity to sell output into the spot market, perhaps at some risk that they may find no customers (called here “Retailers”) for their output. Competition from other Firms in the contract market and the risks of not finding Retailers for output in the spot market, as well as price volatility risks in the spot market, all lead to a delicate balancing act for participants in this market. This paper provides a general framework for analyzing the market equilibrium and optimal portfolio strategies for Firms and Retailers in such a market. We focus our attention here on capacity and production decisions, and not on inventory decisions. This would correspond either to cases in which the commodity in question is non-storable, such as electric power or dated services like car rentals and airline
seats (Belobaba, 1989; Brunelle and McGill, 1993; Gallego and van Ryzin, 1994, 1997), or cases in which JIT delivery is essential and Firms and Retailers intend to have continuous delivers with near-zero inventories in the pipeline. Another example of growing importance is E-commerce, in which the internet is used to link Firms and Retailers together, with purchases possible either through contracting markets or through auction-based spot markets, mediated through internet compatible data methods for conveying product and production information (e.g., Laudon and Laudon, 1999).

A good example of the type of market to which our analysis is applicable is electric power. In this case, in the newly restructured electricity market (e.g., Chao and Huntington, 1998), producing Firms (Generators) and Retailers (Load Serving Entities) can sign long-term bilateral contracts to cover the Retailer's needs, or Firms and Retailers can interact "on the day" in a spot market. How much of their respective capacity and demand Firms and Retailers should or will contract for in the bilateral contracting market, and how much they will leave open for spot transactions, is a fundamental question. Interestingly, this question has not been previously solved.

To be sure, the problem of commodity pricing, including futures contracts, has been the subject of study for some time in the finance and economics literature, e.g., Newbery and Stiglitz (1981), Kawai (1983), Chambers and Bailey (1996), Deaton and Lazoque (1996), Schwartz (1997) and Dana (1998). The differences between these studies and the problem of interest here are several folds. First, we consider Firms with heterogeneous production costs; second, we consider uncertain access to the spot market by Firms; third and most importantly we consider the two-good problem in which both capacity and output are bid into the contracting market. It should be noted, however, that the standard commodities contracting problem has solved the problem in continuous time, whereas the model treated below considers only a two-period version of the problem. We return to this point and other generalizations to the current framework in the conclusions to the paper. Finally, contracting has been the subject of considerable interest in both economics and management science. The economics work has been recently summarized
in the Buckley and Michie (1996) book of readings and the management science work in the edited volume by Tayur, Ganesan and Magazine (1999). This latter work addresses a whole range of contracting issues in supply chain management. However, this work does not address the interaction of quantity and pricing commitments, with spot market backstops, which are the essential ingredients of the real problems motivating this research.

In the problem we model (which might be thought of as the “month ahead” market), Firms and Retailers interact through an electronic bulletin board, posting bids and offers until agreement has been reached. Capacity not committed through this contracting market is assumed to be offered on the spot market, but may go unused because of the risk of not finding customers or transmission capacity at the last minute. Retailers face another type of risk for demand not contracted for in the bilaterals market, namely price volatility in the spot market. Such price volatility can be quite severe as seen in the midwest (ECAR - East Center Area Reliability Coordination Agreement - region) electric power market in the United States in the summer of 1998 where prices as high as $8,000/mWh materialized in the spot market (where “normal” prices would be in the $20-$50 range/mWh).\(^1\) Such experiences as this have caused Load Serving Entities (our Retailers) in the electric power market to pay close attention to the proper balance in their supply portfolio between long-term contracting and spot purchases.

The nature of the results obtained is roughly this. We assume there are \(I\) Firms and \(J\) Retailers. Firm \(i\) has knowledge on its own cost structure \((b_i, \beta_i)\), where \(b_i\) is the unit variable cost of production and \(\beta_i\) is the unit cost of capacity. The price distribution of the spot market and Retailers’ demand functions are assumed to be common knowledge. Firm \(i\) bids the following information on the electronic bulletin board, \([s_i, g_i, L_i], \; i = 1, \ldots, I\), where \(s_i\) is the subscription fee per unit of reserved capacity, \(g_i\) is the contract price per unit of output actually demanded if the contract is exercised, and \(L_i\) is the capacity Firm \(i\) bids into the contract market. Retailer \(j\) decides how much to contract from each Firm, signing contracts of the form \(Q_{ij}\) with each Firm \(i\).

\(^1\)Other countries like the U.K. have also experienced large peaks in spot market prices. It should be noted that different countries use different contracting institutions for the long-term market; these important institutional details will not be the subject of this paper.
Given a common knowledge distribution of the spot price $P_s$, Firms and Retailers adjust their bids and offers until equilibrium is reached. We show that when Firms properly anticipate demands to their bids, $[s_i, g_i, L_i], \ i = 1, \ldots, I$, (referred to as the von Stackelberg assumption), then a dominant strategy for their contract execution price is to truthfully reveal the Firms’ variable cost, i.e., $g_i = b_i$, with subscription fees determined by Firms to trade off the risk of underutilized capacity against unit capacity costs. Retailers’ optimal portfolios are shown to follow a merit order (or greedy) shopping rule, under which contracts are signed following a newsvendor strategy in order of a certain index of the capital intensity of Firms’ bids.

The rest of this paper is organized as follows. The next section presents the basic model structure and reviews our earlier Single-Firm theoretical results (Wu, Kleindorfer and Zhang, 1999), which should also serve to introduce our general framework in the simpler world where there is only one Firm serving the Retail sector. Section 3 generalizes these results to the Multi-Firm, Single-Retailer case, solving this case completely. Section 4 generalizes these results to the Multi-Firm, Multi-Retailer case, again solving this case completely. Section 5 concludes with a number of suggestions for future research.

Insert Figure 1 Here.

2 Review of Single Firm Structure and Results

This section presents our basic model. We build on the earlier framework of Wu, Kleindorfer and Zhang (hereafter, WKZ, 1999), beginning in this section with the simplest case in which there are only two agents: a Firm and a Retailer. This will pave the way for the general case treated later. The capacity for Firm $i$ is $K_i$; the demand function for the Retailer is $D_i(P_s(\omega))$, where $P_s(\omega)$ is the spot market price at the state of the world $\omega$, which is assumed to be uninfluenced by either the Firm or the Retailer. Contracts are signed before the market price $P_s(\omega)$ is known. For convenience of analysis, assume there is no transportation cost or capacity constraint; these could be introduced with no difficulty (see WKZ, 1999). Figure 2 illustrates the Single-Firm-Single-
Retailer supply chain with a back-up spot market, which constitutes our basic model.

Insert Figure 2 Here.

The cost of Firm $i$ is characterized by the parameter pair $(b_i, \beta_i)$, where $b_i$ is the short-run marginal cost of providing a unit at the Firm's door, and $\beta_i$ is unit capacity cost per period. To avoid dominated technology types (see Crew and Kleindorfer, 1986, p.44), we assume $b_1 < b_2 < \ldots < b_i, \beta_1 > \beta_2 \ldots > \beta_i$. If Firm $i$ produces $z$ units when capacity is $K_i$, its total cost per period is

$$c_i(K_i, z) = \beta_i K_i + b_i z, \quad z \leq K_i. \quad (1)$$

Firm $i$'s contract with the Retailer has the two-part tariff form $[s_i, g_i]$, where $s_i =$ subscription fee per unit of capacity per period.

g_i = contract price, the price per unit the firm charges to the Retailer if the Retailer exercises its contract with the firm at a later date.

$Q_i(s_i, g_i) =$ the long-term contract amount the retailer signs with Firm $i$ when the Firm bids $[s_i, g_i]$.

$q(P_s(\omega)) =$ the actual non-storable commodity consumed by the Retailer as provided by the Firm under long-term contract $Q(s, g)$. $q = \min[Q(s, g), D(P_s(\omega))] \chi(P_i(\omega) - g)$, where $\chi(\cdot)$ is the indicator function (which takes the value of 1 if its argument is positive and 0 else).

### 2.1 The Retailer's Optimal Consumption Portfolio

Using the standard quasi-linear form of utility (Crew and Kleindorfer, 1986), Retailer $i$'s utility at $P_s$ is given by

$$V_i(D_i, Q_i, P_s) = U_i(D_i) - s_i Q_i - g_i q_i(P_s, g_i, Q_i, D_i) - P_s x_i$$

$$= U_i(D_i) - P_s D_i + (P_s - g_i)^+ \min[D_i, Q_i] - s_i Q_i \quad (2)$$

where the first term is its Willingness-To-Pay (WTP) at $P_s$, evaluated at the realized demand $D_i(P_s)$, the second and the third term is the payment for the non-storable commodity delivered.
under the long-term contract, the fourth term is the payment for non-storable commodity \( x \) purchased in the spot market.

The Retailer’s problem can be solved in two stages. Starting at the end (the second-stage), once \( P_s \) is known and \( Q_i \) is set, the Retailer solves the problem for \( q_i \) and \( x_i \). Knowing these solutions, the Retailer can solve for \( Q_i \) (the first-stage) by maximizing the expected value of \( V_i(D_i, Q_i, P_s) \) in (2). In keeping with decreasing marginal utility of consumption, we will assume that the Retailer’s WTP \( U_i(x) \) is strictly concave and increasing so that

\[
U_i'(x) > 0, \quad U_i''(x) < 0, \quad \text{for } x \geq 0, \quad \forall i. \tag{3}
\]

The Retailer’s contracting problem is to find an optimal long-term contract size \( Q_i \) in order to maximize its expected utility, i.e.,

\[
\max_{Q_i \geq 0} \int V_i(D_i, Q_i, P_s)f(P_s)dP_s
\]

s.t.

\[Q_i(s_i, g_i) \leq K_i.\]

where \( f(P_s) \) is the probability density function of spot market price \( P_s \), assumed to be common knowledge among all market participants.

The Retailer’s optimal contracting strategy can be shown to be (see WKZ, 1999):

\[
Q_i^*(s_i, g_i) = \begin{cases} 
0 & s_i + G(g_i) \geq G(U_i'(0)) \\
U_i^{-1}(G^{-1}(s_i + G(g_i))) & \text{otherwise}
\end{cases} \tag{4}
\]

where \( G(x) \) is defined as the “risk hedging function”

\[
G(g) = \int_0^g (1 - F(y))dy = E\{(P_s - g)^+\} \tag{5}
\]

and \( G^{-1} \), respectively \( U_i^{-1} \), is the inverse function of \( G \), respectively \( U_i' \). From (5) we note that the risk hedging function gives the expected benefit of facing a contract execution fee \( g \) on the day rather than facing the spot price \( P_s \), since the former will be used whenever \( g < P_s \).
2.2 The Firm’s Optimal Bidding Strategies

Given any Retailer’s contracting strategy \( Q_i(s_i, g_i) \), the Firm will find an optimal bidding strategy \([s^*_i, g^*_i]\) that maximizes his expected profit.

\[
\max_{s_i, g_i} \int \Pi(s_i, g_i; P_s, Q_i(s_i, g_i)) f(P_s) dP_s
\]

s.t.

\[Q_i(s_i, g_i) \leq K_i.\]

The profit function \( \Pi_i(s_i, g_i; P_s, Q_i(s_i, g_i)) \) for the Firm at \( P_s \) from the long-term contract and from the spot market is given as

\[
\Pi_i(s_i, g_i; P_s, Q_i(s_i, g_i)) = s_iQ_i(s_i, g_i) + g_iqu_i - (\beta_iK_i + b_iq_i) + (P_s - b_i)^+m(K_i - q_i)
\] (6)

The first two terms on the right hand side of equation (6) represent the Firm’s revenue from the contract, the third term is the Firm’s cost, and the fourth term is the Firm’s profit from the spot market. We assume that “on the day” the Firm can only sell a percentage \((0 \leq m \leq 1)\) of its residual output in the spot market. The factor \( m \), which is assumed to be fixed here, provides the incentive for the Firm to sign a contract with the Retailer. This factor represents the difficulty of lining up appropriate customers at the last minute, scheduling of production and transportation, as well as potential transportation/transmission constraints, and other barriers that might hinder participation in the spot market. If there are no such barriers or constraints, and the costs of scheduling on the day are no greater than the costs of scheduling in advance, then there will be no incentive for the Firm to engage in long-term contracting.\(^2\)

When Firm \( i \)'s capacity \( K_i \) is fixed over the contract time horizon, then the optimal bidding strategy for Firm \( i \) is truth telling, i.e., to bid its production cost, and is given by (see WKZ,

\(^2\)While we do not treat here the problem of multiple contracting periods, one might expect that the estimate of \( m \) will improve as one approaches real time. Thus, the incentives to contract could change over time as the estimates of \( m \) change, and as the estimates of spot price change. Modeling this more complex framework is beyond the scope of this paper.
1999):

\[
\begin{align*}
  s_i^* &= \begin{cases} 
  \frac{m(P_s - G(b_i))}{1 - \epsilon_s(Q_i)} & Q_i < K_i \\
  G(U'(K_i)) - G(b_i) & Q_i \geq K_i 
  \end{cases} \\
  g_i^* &= b_i
\end{align*}
\]

where \( \epsilon_s(Q_i) \) is the demand elasticity w.r.t. to the subscription charge

\[
\epsilon_s(Q_i) = \frac{s_i}{Q_i} \left| \frac{\partial Q_i}{\partial s_i} \right| = -\frac{s_i}{Q_i} \frac{\partial Q_i}{\partial s_i}
\]

When facing multiple Retailers, it can be shown that Firm \( i \)'s optimal bidding strategy is to use Retailers' aggregated contract demand, essentially treating all Retailers as a single large wholesaler (see WKZ, 1999):

\[
\begin{align*}
  s_i^* &= \frac{m(P_s - G(b_i))}{1 - \frac{1}{\epsilon_s(Y^*)}} \\
  g_i^* &= b_i
\end{align*}
\]

where \( Y^* \) is the aggregate contract demand from the Retailers,

\[
Y^* = \sum_{j=1}^{J} Q_j^*
\]

where \( Q_j^* \) is the optimal contract of Retailer \( j \) (\( j = 1, \ldots, J \)).

Figure 2 illustrates the Single-Firm-Multi-Retailer, radial network supply chain with a back-up open spot market.

Insert Figure 2 Here.

3 Multi-Firm, Single-Retailer, Radial Network

This section considers the Multi-Firm-Single-Retailer radial network case. Assume there are \( I \) Firms (suppliers) and one Retailer. Firm \( i \) bids the following information on an electronic bulletin board, \( [s_i, g_i, L_i], \quad i = 1, \ldots, I \); Retailer \( j \) posts its demand information on the bulletin board, \( [D(P_s(\omega), s_i, g_i, Q_i), Q_i, g_i], \quad j = 1, \ldots, J \). As noted, we assume that the demand function of the Retailer is common knowledge among Firms. Figure 3 illustrates the Multi-Firm-Single-Retailer, radial network supply chain with a back-up open spot market.

Insert Figure 3 Here.
3.1 Retailer’s Optimal Consumption Portfolio

The problem confronting the Retailer is to choose an optimal portfolio of contracts from those available on the bulletin board. Some of the contract offers carry high subscription rates $s_i$ but low execution fees $g_i$. These must be compared to other offers carrying lower subscription fees but higher execution fees. This problem has the characteristics of the diverse technology problem originally solved by Crew and Kleindorfer (1986) and the solution to it is basically to rank contracts in increasing order of a certain index reflecting the cost of reserving capacity at price $s_i$ plus the cost of maintaining the option to execute the contract at price $g_i$ rather than utilizing the back stop spot market.

Without loss of generality, assume that Firms’ offers are indexed so that $g_1 < g_2 < \ldots < g_I$. Letting $D(\cdot)$ be the Retailer’s demand function (which we want to derive as a function of various parameters), we express the Retailer’s utility as

$$V(D, Q; x, P, s, g) = U(D) - \sum_{i=1}^{I} s_i Q - \sum_{i=1}^{I} g_i q_i - P s x$$

and

$$x = D - \sum_{i=1}^{I} q_i$$

where

$$q_i = \min[(D - \sum_{j=1}^{i-1} q_j), Q_i] \chi(P_s - g_i), \quad i = 1, \ldots, I.$$ 

since

$$D \geq \min[D, Q_1] \geq \min[D, Q_1] \chi(P_s - g_1) = q_1$$

hence,

$$q_2 = \min[D - q_1, Q_2] \chi(P_s - g_2)$$

by the same logic,

$$q_i = \min[D - \sum_{j=1}^{i-1} q_j, Q_i] \chi(P_s - g_i)$$
therefore

\[
V(D, Q, q, x, P_s, s, g) = U(D) - \sum_{i=1}^{I} s_i Q_i - \sum_{i=1}^{I} g_i q_i - P_s (D - \sum_{i=1}^{I} q_i)
\]

\[
= - \sum_{i=1}^{I} s_i Q_i + \sum_{i=1}^{I} \{ (P_s - g_i)^+ \min[D - \sum_{j=1}^{i-1} g_j, Q_i] \} + U(D) - P_s D
\]

**Lemma 1:** Let \( \phi = (P_s, s, g, Q, i = 1, \ldots, I) \) be given. Define the Retailer’s demand function \( D(\phi) \) (as the solution to \( \max\{V(D, \phi)|D \geq 0\} \)). Without loss of generality, assume that Firms’ offers are indexed so that \( g_1 < g_2 < \ldots < g_I \). Suppose contract \( k \) provides the last unit of output to the Retailer. Then \( D(\phi) \) is given by the following:

1. if \( P_s < g_k \), then \( D(\phi) = U^{-1}(P_s) \)
2. if \( U'(\sum_{j=1}^{k} Q_j) < g_k \leq P_s \), then \( D(\phi) = U^{-1}(g_k) \)
3. if \( g_k \leq U'(\sum_{j=1}^{k} Q_j) \leq P_s \), then \( D(\phi) = \sum_{j=1}^{k} Q_j \)

Proof, omitted here, is analogous to that of Lemma 1 in WKZ (1999). The intuition is straightforward. On the day, the Retailer is only concerned with the execution prices available to the Retailer, based on previously committed contracts and the spot market price \( P_s \). Depending on where \( P_s \) is located relative to the ascending order (“merit order”) of contract execution fees \( g_1 < g_2 < \ldots < g_I \), the Retailer consumes available contract capacities in order of increasing \( g_i \) until the spot market price is reached as the next available option or until marginal WTP is less than the committed contracting capacity. If contract \( j \) provides the last unit of output to the Retailer, then \( U'(D) = g_j \); if the spot market provides the last unit of output, then \( U'(D) = P_s \). The various possibilities are enumerated in Lemma 1.

**Theorem 1 (Retailer’s Optimal Contract Portfolio):** Given (3), Greedy Contracting is optimal for the Retailer, i.e.,

\[
Q_i = \min\{[U'^{-1} G^{-1}(s_i + G(g_i)) - \sum_{k=1}^{i-1} L_k]^+, L_i\}, \quad i = 1, \ldots, I.
\]
where $s_1 + G(g_1) < \ldots < s_I + G(g_I)$ is the ranking or the “merit order” the Retailer made according to the Firms’ bids on the electronic bulletin board and $G(·)$ is the “risk hedging function” as defined in (5).

Proof: See Appendix A.

The idea in the proof can be neatly illustrated using Figure 4, where line AD is

$$Q_1 + Q_2 = U''^{-1}(G^{-1}(s_1 + G(g_1))),$$

and line BC is

$$Q_1 + Q_2 = U''^{-1}(G^{-1}(s_2 + G(g_2))).$$

We are looking for the maximum $V$ in the quadrilateral ABCD, and the arrow on each side indicates the direction in which $V$ increases, therefore the maximum $V$ appears at the right lower corner of the quadrilateral ABCD, i.e., point D. Combining the constrains on both $Q_1$ and $Q_2$ gives Theorem 1.

Insert Figure 4 Here.

The structure of the optimal portfolio captured in Theorem 1 is relatively simple. It calls for the Retailer to rank all offers in terms of a single index $s_i + G(g_i)$ and then to pull off as much capacity as allowed by Firm $i$, proceeding in rank order of the contract index until the marginal WTP is exceeded by the contract index. Since $G$ (and therefore $G^{-1}$) is strictly increasing, and since by concavity (see (3)) $U''^{-1}$ is strictly decreasing, $Q_j$ is strictly decreasing in the contract index. Before WTP is exceeded, the Retailer takes all capacity offered by Firms from whom it contracts. Of course, WTP may be exceeded with the first Firm and may, in fact, sign no contracts whatsoever (if $G(U'(0)) < s_1 + G(g_1)$).

The contract index $s_i + G(g_i)$ reflects the cost of reserving a unit of capacity plus the cost of maintaining the option to execute the contract at the price $g_i$ rather than using the spot market at
price $P_s$. This captures the industrial marketing practice in paying the cost of reserving capacity, which is the payment to the Firm for the opportunity cost of their committing their capacity to a specific contract. Optimal contracting for the Retailer is determined by adding to this reservation price the risk hedging benefit of maintaining the option to purchase at the fixed execution fee $g_i$ rather than facing spot market volatility. From the noted properties of $U$ and $G$, Retailer’s contract amounts are decreasing in both $s_i$ and $g_i$ as expected.

It is interesting to note some examples of the type of contracting captured in Theorem 1. In consolidators for airline tickets or in hotel convention planning, a block of seats or rooms is reserved by a travel planner. The cost of this is the subscription fee $s_i$, which gives the travel planner a considerably lower execution fee $g_i$ than would be obtained in the spot market (showing up on the day and requesting a seat or a room). When the travel planner is relatively certain of being able to (re-)sell the seats or rooms, the benefits are clear. It is also clear that providing the travel planner with a contracted number of seats or rooms with no subscription fee would put all the risk on the Firm, an unacceptable outcome for the Firm. Bilateral contracts in electricity planning also have this two-part tariff structure to assure that both the opportunity cost of capacity reservation and the cost of execution are covered. Notwithstanding these standard practices, to the best of our knowledge this is the first characterization of the structure of the optimal portfolio of contracts for a Retailer.

3.2 Firms’ Optimal Bidding Strategies

Lemma 2 (Optimal Bidding Capacity): Each Firm will be truth telling when bidding its capacity, i.e., $L_i^* = K_i, \ i = 1, \ldots, J$.

Straightforward, proof omitted. The rationale for this result is that the Firm’s profit function is unchanged by the amount of capacity bid on the bulletin board. Bidding a larger capacity into the contract market relaxes the constraint region (for pricing its capacity subject to a capacity constraint) and therefore can only increase profits.
Lemma 3 (Optimal Bidding for Contracting): Each Firm will be truth telling when bidding its contract price, i.e., \( g_i^* = b_i \), \( i = 1, \ldots, I \).

Proof: See Appendix B.

Lemma 4 (Lower Bound for Subscription Charge): Each Firm will never bid a subscription fee \( s_i^* \) that is lower than \( s_i \), i.e., \( s_i^* \geq s_i \) where

\[
s_i \overset{\text{def}}{=} \max\{\beta_i, m_i(P_s - G(b_i))\} \quad i = 1, 2, \ldots, I
\]

Proof: See Appendix C.

The rationale for Lemma 3 is that there is a tradeoff for Firms between charging higher \( s_i \) and higher \( g_i \), depending on their market power relative to competitors. It turns out that charging higher \( g_i \) erodes the options value (recall the contract index \( s_i + G(g_i) \)) of the benefit Retailers see from contracting more quickly than the marginal benefits associated with increases in \( s_i \). The lowest level for \( g_i \), namely \( b_i \), is therefore the result.

The rationale for Lemma 4 is that the Firm does not want to make any less per unit of capacity than it could by committing to sell its capacity into the spot market (although access to the spot market is imperfect and occurs on the day only with probability \( m \)). Equating the two possibilities leads to Lemma 4. From Lemma 4, we know that when there is a long-term contract market, plant \( i \) could still be used even if \( \beta_i > m_i(P_s - G(b_i)) \), as long as plant \( i \) could recover its capacity investment cost. However, plant \( i \) will never be used if such a contracting market does not exist, since the expected profit generated from the spot market cannot offset the marginal capacity investment cost \( \beta_i \).

Lemma 4 demonstrates the value of the contract market. Note that, from (5),

\[
P_s - G(b_i) = \int_{b_i}^{\infty} (P_s - b_i) f(P_s) dP_s
\]

is the expected unit profit on the spot market. Intuitively, what Lemma 4 says is the following. If plant \( i \)'s unit capacity cost is less than its expected proportional unit profit on the spot market, then
the minimum incentive for plant $i$ to sign contract is that the Retailer would cover its opportunity cost on the spot market. Otherwise, plant $i$ would demand the Retailer to cover its entire capacity investment cost.

### 3.3 von Stackelberg Leader-Follower Equilibrium

We are now in a position to derive the market equilibrium prices in the long-term contract market. Several definitions of market equilibrium might be used. The approach we take is to assume that Firms all know the demand function of the Retailer, consistent with sophisticated Firms and well-developed markets. Firms only know their own costs and capacities and bid these into the market via an electronic bulletin board. Firms adjust their bids until they achieve a Nash equilibrium in this market. We refer to this equilibrium as a von Stackelberg Leader-Follower Equilibrium (vSLFE) to account for the assumption that Firms anticipate Retailer responses to their actions, given what they observe other Firms to be bidding on the electronic bulletin board. Our primary result is the following Theorem.

**Theorem 2 (von Stackelberg Leader-Follower Equilibrium (vSLFE)):** Given the definition of $s_i$ in (8), the vSLFE is determined as the following.

**Step 1:** Each Firm ($i$) will initially bid $s_i = s_i$ but truthfully reveal its production cost by bidding $g_i = b_i$.

**Step 2:** The Retailer contracts with Firms according to index $s_i + G(b_i)$. W.o.l.g., assume the ranking is

$$s_1 + G(b_1) < s_2 + G(b_2) < \ldots < s_I + G(b_I)$$

**Step 3:** Find the Firm (denoted as Firm $k$) that contracts the last unit to the Retailers: For $i = 1$ to $I$ do
1. Compute $Q_i^o$: the Retailer's optimal contract capacity with Firm $i$ using the following implicit equation (WKZ, 1999):

$$Q_i^o U'' \left( \sum_{i=1}^{i-1} K_i + Q_i^o \right) \left[ 1 - F(U'' \left( \sum_{i=1}^{i-1} K_i + Q_i^o \right) \right] + G(U' \left( \sum_{j=1}^{i-1} K_j + Q_i^o \right)) - m_i \tilde{p} - (1 - m_i)G(b_i) = 0.$$

2. If $i = I$ and $Q_i^o \geq K_i$ then $k = I$ and Exit.

else if $Q_i^o < K_i$ then $k = i$ and Exit (Firm $k$ provides the last unit of contract capacity to Retailers).

**Step 4:** Suppose Firm $k$ contracts the last unit to the Retailer.

1. if $k = I$ and $Q_k^o \geq K_I$ then

$$\forall j \leq I, \quad s_j^* = G(U'(\sum_{i=1}^{j} K_i)) - G(b_j)$$

2. else

$$s_k^* = G(U'(\sum_{i=1}^{k-1} K_i + Q_k^o)) - G(b_k); \quad \forall j < k, \quad s_j^* = G(U'(\sum_{i=1}^{k-1} K_i)) - G(b_j)$$

Proof, omitted, is not difficult. The rationale for Theorem 2 will be illustrated in several examples below. Basically, the Retailer is known to use the greedy shopping rule and therefore to shop in order of the contract index $s_i + G(g_i)$. Firms have a dominant strategy for $g_i$, i.e., $g_i = b_i$. Thus, the effective contract index, the object of competitive bidding among Firms, becomes $s_i + G(b_i)$. The game becomes therefore which Firm can afford to bid the lowest $s_i$, which is in equilibrium then driven down, close to $s_i$. The remainder of Theorem 2 derives from the fact that Firm $i$ actually can move its $s_i$ bid to take the advantage that its full capacity has been backed by the Retailer, i.e., to $G(U'(\sum_{i=1}^{k-1} K_i)) - G(b_j)$. The proof demonstrates that this simple approach cannot be improved on by any Firm. Hence, this is the SLFE.
3.4 Numerical Examples

Example 1: Only Firm 1 contracts in equilibrium. We assume $b_1 < b_2$, $\beta_1 > \beta_2$ so that neither Firm is a priori dominated. Further, we assume $m_1 > m_2$ so that Firm 2 has a stronger incentive to contract with the Retailer. We assume the spot market has the following distribution: $f(P_s) = P_se^{-P_s}$, therefore the mean of the spot market price is $\bar{P}_s = 2$. The following are the specific parameters used.

Firm 1: $b_1 = 5$, $\beta_1 = 0.01$, $m_1 = 0.9$, $K_1 = 1$,
Firm 2: $b_2 = 6$, $\beta_2 = 0.005$, $m_2 = 0.85$, $K_2 = 1$,
Retailer: $U(x) = 10(1 - e^{-x})$

Firm 1 is the bid winner since $m_1\bar{P}_s + (1 - m_1)G(b_1) < m_2\bar{P}_s + (1 - m_2)G(b_2)$. The lower bound for subscription charge is

$$s_\bot = m_1(\bar{P}_s - G(b_1)) = 0.0424 > \beta_1$$

Solving equation

$$-10Q_1^e e^{-Q_1^e} [1 - F(10e^{-Q_1^e})] + G(10e^{-Q_1^e}) - 1.8 - 0.1G(5) = 0$$

gives us $Q_1^e = 0.161 < K_1$, therefore

$$s_1^e = G(10e^{-Q_1^e}) - G(b_1) = 0.0451 > s_\bot$$

\text{vSLFE is}

$$s_1^* = s_1^e = 0.0451, \quad g_1^* = b_1 = 5, \quad Q_1^* = Q_1^e = 0.161$$

$$Q_2^* = 0$$

Example 2: Both Firms contract in equilibrium. The parameters are the same as in Example 1 except that Firm 1 has a much smaller capacity.

Firm 1: $b_1 = 5$, $\beta_1 = 0.01$, $m_1 = 0.9$, $K_1 = 0.1$, $K_2 = 0.1$.
Firm 2: \( b_2 = 6, \beta_2 = 0.005, m_2 = 0.85, K_2 = 1, \)

Retailer: \( U(x) = 10(1 - e^{-x}) \)

Firm 1 is the bid winner since \( m_1 \bar{P}_s + (1 - m_1)G(b_1) < m_2 \bar{P}_s + (1 - m_2)G(b_2) \). The lower bound for subscription charges are

\[
\begin{align*}
\bar{s}_1 &= m_1(\bar{P}_s - G(b_1)) = 0.0424 > \beta_1 \\
\bar{s}_2 &= m_2(\bar{P}_s - G(b_2)) = 0.0169 > \beta_2
\end{align*}
\]

Solving equations

\[
-10Q_1^2 e^{-Q_1^2}(1 - F(10e^{-Q_1^2})) + G(10e^{-Q_1^2}) - 1.8 - 0.1G(5) = 0 \\
-10Q_2^2 e^{-(0.1+Q_2)}[1 - F(10e^{-(0.1+Q_2)})] + G(10e^{-(0.1+Q_2)}) - 1.7 - 0.15G(6) = 0
\]

gives us \( Q_1^* = 0.161 > K_1 = 0.1 \) and \( Q_2^* = 0.058 < K_2 = 1 \). vSLFE is

\[
\begin{align*}
\bar{s}_1^* &= G(10e^{-K_1}) - G(b_1) = 0.0459, & g_1^* &= b_1 = 5, & Q_1^* &= K_1 = 0.1 \\
\bar{s}_2^* &= G(10e^{-(K_1+Q_2^*)}) - G(b_2) = 0.0178, & g_2^* &= b_2 = 6, & Q_2^* &= Q_2^* = 0.058
\end{align*}
\]

4 Multi-Firm, Multi-Retailer, General Network

This section considers the Multi-Firm-Multi-Retailer general network case, containing the most general results in this paper. Figure 5 illustrates the Multi-Firm-Multi-Retailer, general network supply chain with a back-up open spot market.

Insert Figure 5 Here.

We first characterize each Retailer’s Optimal Portfolio, under the assumption that each Retailer can choose from the electronic bulletin board any mix of contracts posted there. The result is that, as expected from Theorem 1, greedy shopping is still optimal. We then show the Firms’ optimal bidding strategies and show that at the vSLFE a result analogous to Theorem 2 obtains. Thereafter, we describe an algorithm to compute the vSLFE characterized in the Theorem. Some examples illustrate the results.
4.1 Retailer’s Optimal Portfolio

Without loss of generality, assume $g_1 < g_2 < \ldots < g_I$. Mimicking the previous section, Retailer $j$’s utility is given by

$$V_j = U_j(D_j) - \sum_{i=1}^{I} s_i Q_{ij} - \sum_{i=1}^{I} g_i q_{ij} - P_s x_j$$

where

$$x_j = D_j - \sum_{i=1}^{I} q_{ij}$$

and

$$q_{ij} = \min[(D_j - \sum_{k=1}^{i-1} q_{kj})^+, Q_{ij}] \chi(P_s - g_i), \quad i = 1, \ldots, I.$$ 

by the same logic as in the previous section,

$$q_{ij} = \min[D_j - \sum_{k=1}^{i-1} q_{kj}, Q_{ij}] \chi(P_s - g_i), \quad i = 1, \ldots, I.$$ 

Therefore

$$V_j = U_j(D_j) - \sum_{i=1}^{I} s_i Q_{ij} - \sum_{i=1}^{I} g_i q_{ij} - P_s (D_j - \sum_{i=1}^{I} q_{ij})$$

$$= - \sum_{i=1}^{I} s_i Q_{ij} + \sum_{i=1}^{I} \{(P_s - g_i)^+ \min[D_j - \sum_{k=1}^{i-1} q_{kj}, Q_{ij}]\}$$

$$+ U_j(D_j) - P_s D_j$$

Lemma 5: Let $\phi_j = (P_s, s_i, g_i, Q_{ij} \ (i = 1, \ldots, I))$ be given. Define Retailer $j$’s demand function $D_j(\phi_j)$ as the solution to $\max\{V_j(D_j, \phi_j) | D_j \geq 0\}$. Then $D_j(\phi_j)$ is given by Lemma 1.

The rationale for Lemma 5 is identical to that of Lemma 1. From the Retailer’s point of view, the existence of other Retailers does not affect its own demand. Lemma 5 is essentially the same as Lemma 1.

Since each Retailer’s problem remains as the same as in the Multi-Firm-Single-Retailer case, Theorem 1 carries over to the Multi-Firm-Multi-Retailer case. We restate it as Corollary 1.
Corollary 1 (Retailer’s Optimal Consumption Portfolio): Given (3), *Greedy Contracting* is optimal for every Retailer $j$, as specified in Theorem 1.

### 4.2 Firms’ Optimal Bidding Strategies

Both Lemma 2, Lemma 3 and Lemma 4 in the previous sections hold since the Firms’ problems remain the same.

### 4.3 von Stackelberg Leader-Follower Equilibrium

Theorem 3 (von Stackelberg Leader-Follower Equilibrium (SLFE)): The vSLFE is determined by the following “Algorithm vSLFE”.

**Algorithm vSLFE**

**Step 1:** Each Firm $(i)$ will initially bid $s_i = s_i$ but truthfully reveal its production cost by bidding $g_i^* = b_i$.

**Step 2:** Retailers contract with the Firms according to index $s_i + G(b_i)$. W.o.l.g., assume the rank is

$$s_1 + G(b_1) < s_2 + G(b_2) < \ldots < s_I + G(b_I).$$

**Step 3:** Find the Firm (denoted as Firm $k$) that contracts the last unit to the Retailers: For $i = 1$ to $I$ do

1. Compute $Q_{ij}^*$, Retailer $j$’s optimal contract capacity with Firm $k$ using the following implicit equation (WKZ, 1999):

$$Q_{ij}^* U_j''\left(\sum_{i=1}^{i-1} K_i + Q_{ij}^*\right)[1 - F(U_j'(\sum_{i=1}^{i-1} K_i + Q_{ij}^*))] + G(U_j'(\sum_{i=1}^{i-1} K_i + Q_{ij}^*))$$

$$- m_i\bar{P}_c - (1 - m_i)G(b_i) = 0.$$
2. Compute $Y_i^o$, the total contract capacity for Firm $i$ using

$$Y_i^o = \sum_{j=1}^{J} Q_{ij}^o$$

(9)

3. If $i = I$ and $Y_i^o \geq K_i$ then $k = I$ and Exit

else if $Y_i^o < K_i$ then $k = i$ and Exit (Firm $k$ provides the last unit of contract capacity to Retailers).

Step 4: Suppose Firm $k$ contracts the last unit to the Retailers.

1. if $k = I$ and $Y_i^o \geq K_I$ then

$$\forall i \leq I, \quad s_i^* = \frac{m_i(\bar{P}_s - G(b_i))}{1 - \frac{1}{\epsilon_{i_k}(\sum_{j=1}^{I} K_j)}}$$

2. else

$$s_k^* = \frac{m_k(\bar{P}_s - G(b_k))}{1 - \frac{1}{\epsilon_{k}(\sum_{i=1}^{k-1} K_i + Y_k)}}; \quad \forall i < k, \quad s_i^* = \frac{m_i(\bar{P}_s - G(b_i))}{1 - \frac{1}{\epsilon_{i}(\sum_{j=1}^{k-1} K_j)}}$$

Proof, omitted, is a direct generalization of Theorem 2 by using the Single-Firm-Multi-Retailer result from WKZ (1999) and treating all Retailers as a single "Retail Sector". The reader will note that the solution to the Multi-Firm-Multi-Retailer general network case is obtained by having Firms aggregate all demand and having Retailers doing greedy shopping. The solution to the general network case results from using the demand and equilibrium contracting building blocks of the Multi-Firm, Single-Retailer case for the Retailer and the Single-Firm, Multi-Retailer case for the Firm, the latter developed in WKZ (1999).

4.4 Numerical Examples for 2 Firms and 2 Retailers

Example 3: Only Firm 1 contracts in equilibrium. The parameters are the same as in Example 1 except that there is a second retailer with its own WTP function.

Firm 1: \quad b_1 = 5, \quad \beta_1 = 0.01, \quad m_1 = 0.9, \quad K_1 = 1,
Firm 2: \( b_2 = 6, \beta_2 = 0.005, m_2 = 0.85, K_2 = 1, \)

Retailer 1: \( U_1(x) = 10(1 - e^{-x}) \)

Retailer 2: \( U_2(x) = 5(1 - e^{-2x}) \)

Firm 1 is the bid winner since \( m_1 \bar{P}_s + (1 - m_1)G(b_1) < m_2 \bar{P}_s + (1 - m_2)G(b_2) \). The lower bound for subscription charge is

\[
s_1 = m_1(\bar{P}_s - G(b_1)) = 0.0424 > \beta_1
\]

Solving equations

\[
m_1(\bar{P}_s - G(b_1)) = s + \frac{Q_{11} + Q_{12}}{\frac{\partial(G(Q_{11}) + G(Q_{12}))}{\partial s}}
\]

\[
s = G(U'_1(Q_{11})) - G(b_1) = G(U'_2(Q_{12})) - G(b_1)
\]

gives us \( Q^*_1 = 0.161 \) and \( Q^*_2 = 0.080 \). Since \( Q^*_1 + Q^*_2 = 0.241 < K_1 = 1 \), vSLFE is

\[
s^*_1 = G(U'_1(Q^*_{11})) - G(b_1) = G(U'_2(Q^*_{12})) - G(b_1) = 0.0451, \quad g^*_1 = b_1 = 5
\]

\[
Q^*_{11} = Q^*_{12} = 0.161, \quad Q^*_{12} = Q^*_{12} = 0.080
\]

\[
Q^*_{21} = Q^*_{22} = 0
\]

**Example 4: Both Firms contract in equilibrium.** The parameters are the same as in Example 1 except that there is a second retailer with its own WTP function and Firm 1 reduced its capacity by 90%.

Firm 1: \( b_1 = 5, \beta_1 = 0.01, m_1 = 0.9, K_1 = 0.1, \)

Firm 2: \( b_2 = 6, \beta_2 = 0.005, m_2 = 0.85, K_2 = 1, \)

Retailer 1: \( U_1(x) = 10(1 - e^{-x}) \)

Retailer 2: \( U_2(x) = 5(1 - e^{-2x}) \)

Firm 1 is the bid winner since \( m_1 \bar{P}_s + (1 - m_1)G(b_1) < m_2 \bar{P}_s + (1 - m_2)G(b_2) \). The lower bound for subscription charges are

\[
s_1 = m_1(\bar{P}_s - G(b_1)) = 0.0424 > \beta_1
\]

\[
s_2 = m_2(\bar{P}_s - G(b_2)) = 0.0169 > \beta_2
\]
Solving equations

\[ m_1(P_e - G(b_1)) = s_1 + \frac{Q_{11} + Q_{12}}{\partial (Q_{11} + Q_{12})/\partial s_1} \]

\[ s_1 = G(U_1'(Q_{11})) - G(b_1) = G(U_2'(Q_{12})) - G(b_1) \]

gives us \( Q_{11}^* = 0.161 \) and \( Q_{12}^* = 0.080 \). \( Q_{11}^* + Q_{12}^* = 0.241 > K_1 = 0.1 \). Since \( U_1'(0.1) > U_2'(0.1) \), we allocate all capacity to Retailer 1, since his WTP evaluated at \( K_1 \) is larger. Therefore vSLFE for Firm 1 is

\[ s_1^* = G(10e^{-K_1}) - G(b_1) = 0.0459, \quad g_1^* = b_1 = 5, \quad Q_{11}^* = K_1 = 0.1, \quad Q_{12}^* = 0 \]

Solving equations

\[ m_2(P_e - G(b_2)) = s_2 + \frac{K_1 + Q_{21} + Q_{22}}{\partial (K_1 + Q_{21} + Q_{22})/\partial s_2} \]

\[ s_2 = G(U_1'(K_1 + Q_{21})) - G(b_2) = G(U_2'(Q_{22})) - G(b_2) \]

gives us \( Q_{21}^* = 0.058 \) and \( Q_{22}^* = 0.079 \). \( Q_{21}^* + Q_{22}^* = 0.137 < K_2 = 1 \). Therefore vSLFE for Firm 2 is

\[ s_2^* = G(10e^{-(K_1+Q_{21})}) - G(b_2) = G(10e^{-2Q_{22}}) - G(b_2) = 0.0178, \quad g_2^* = b_2 = 6, \quad Q_{21}^* = Q_{21}^* = 0.058, \quad Q_{22}^* = Q_{22}^* = 0.079 \]

5 Summary

This paper has provided a general solution to the Multi-Firm, Multi-Retailer supply chain contracting problem for non-storable commodities. This solution has important applications in a number of industrial and service sector contexts where capital intensity and non-storability are essential characteristics of supply chain operations. Non-storability (including cases of JIT delivery) is important to our analysis as is the implicit assumption throughout of capital intensity of the production process. Absent either of these and inventory or rapid scale-up of production would be suitable substitutes for long-term contracting or spot market purchases. A number of
models have been developed for contracting where inventory is possible, but none of them with the richness of the framework developed here, including in particular heterogeneous production costs across potential suppliers.

The fact that the good in question is a commodity deserves further comment. One reason for long-term contracting is to allow customization of intermediate products to take place (either in product attributes or in delivery or other service attributes). In the case of idiosyncratic goods, the framework developed here is applicable if JIT delivery is required, where one would interpret \( m \) in this context as the probability of being able to customize the output and delivery requirements to the buyer’s needs at the last minute (where last minute is relative to the production planning/ordering horizon of the buyer).

A number of future research topics are evident. First and foremost, it would be interesting to explore the opportunities for both market research and Firm/Retailer contracting support systems based on these results in sectors of interest. On the theoretical side, it would important to extend this framework to the case of limited storability of goods, as well as to the case where contracting or re-contracting could take place along a temporal continuum. In the latter case, options pricing results can be expected to reflect the changes in information about the value of the spot price as time progresses to the day of physical delivery. We would expect, based on the results of this paper, that not only will the stochastic evolution of the spot market price be important to valuing such options, but also the evolution of predicted access to the market by various suppliers (the \( m_i \) of our framework). Since one might expect that spot market price and \( m_i \) are likely to be correlated (depending on congestion and on search intensity of Retailers), the resulting framework could be quite interesting. In particular, it could link to the effects of internet access and shopping since such access is likely to increase the probability of finding customers at the last minute.
A Sketch of Proof of Theorem 1

For notation simplicity, we use the two-firm case to illustrate our idea. However, this idea works for the general case of Multi-Firms. A complete proof can be obtained from the first author.

W.o.l.g., assume \( s_1 + G(g_1) \leq s_2 + G(g_2) \).

1. if \( U'(Q_1) < g_1 \) then \( V_{Q_1} = -s_1, V_{Q_2} = -s_2 \), therefore \( Q_1 = Q_2 = 0 \).

2. if \( g_1 < U'(Q_1 + Q_2) < g_2 < U'(Q_1) \) then \( V_{Q_1} = G(g_2) - (s_1 + G(g_1)), V_{Q_2} = -s_2 \), therefore \( Q_2 = 0 \), which contradicts the assumptions, hence rejected.

3. if \( g_1 < g_2 < U'(Q_1 + Q_2) \) then \( V_{Q_1} = G(U'(Q_1 + Q_2)) - (s_1 + G(g_1)), V_{Q_2} = G(U'(Q_1 + Q_2)) - (s_2 + G(g_2)) \), therefore

   (a) if \( s_1 + G(g_1) < s_2 + G(g_2) \), then \( Q_1 = U'^{-1}(G^{-1}(s_1 + G(g_1))) \), \( Q_2 = 0 \).

   (b) if \( s_1 + G(g_1) = s_2 + G(g_2) \), then \( Q_1 + Q_2 = U'^{-1}(G^{-1}(s_1 + G(g_1))) \).

B Sketch of Proof of Lemma 3

The expected profit of Firm \( i \) is

\[
\Pi_i = s_iQ_i + (1 - m_i\chi(g_i - b_i))Q_i(g_i - b_i)(1 - F(g_i)) - m_iQ_i(P - G(\max(g_i, b_i))) + m_iK_i(P - G(b_i) - \beta_i K_i)
\]

We notice that

\[
\frac{\partial Q_i}{\partial g_i} = \frac{(1 - F(g_i))}{\partial s_i} \frac{\partial Q_i}{\partial s_i}
\]

is true for the \( Q_i \)'s in equations (7). The total derivative of \( \Pi_i \) w.r.t. \( g_i \) along the curve, \( s_i(g_i) = \text{const.} - G(g_i) \), is

\[
\frac{d\Pi_i}{dg_i} = \frac{\partial \Pi_i}{\partial g_i} + \frac{\partial \Pi_i}{\partial s_i} ds_i = -(1 - m_i\chi(g_i - b_i))Q_i(g_i - b_i)f(\bullet(g_i))
\]

A necessary condition of optimal bidding is

\[
\frac{d\Pi_i}{dg_i} = 0, \quad \text{i.e.} \quad g_i = b_i
\]
C Sketch of Proof of Lemma 4

Under the condition of optimal contract price $g^*_i = b_i$, the expected profit of Firm $i$ is

$$\Pi_i = (s_i - m_i(\bar{P}_s - G(b_i)))Q_i + (m_i(\bar{P}_s - G(b_i)) - \beta_i)K_i \quad (10)$$

If $\beta_i > m_i(\bar{P}_s - G(b_i))$, setting (10) $> 0$ results

$$s_i \geq m_i(\bar{P}_s - G(b_i)) + (\beta_i - m_i(\bar{P}_s - G(b_i))) + \frac{K_i}{Q_i} > m_i(\bar{P}_s - G(b_i)) + (\beta_i - m_i(\bar{P}_s - G(b_i))) \Rightarrow \beta_i.$$

Otherwise if $\beta_i < m_i(\bar{P}_s - G(b_i))$, a positive expected profit on the contract secures a positive expected net profit for Firm $i$, i.e., $\Pi_i > 0$ if $s_i > m_i(\bar{P}_s - G(b_i))$. Hence the proof.
References


Figure 1: Basic Model Structure: One-Firm-One-Retailer

Figure 2: One-Firm-Multi-Retailer Radial Network Supply Chain

Figure 3: Multi-Firm-One-Retailer Radial Network Supply Chain

Figure 4: A sketch of Proof of Theorem 1
DEPARTMENT OF ECONOMICS AND FINANCE

WORKING PAPER SERIES

* Working Paper No. 1 - No. 73 are out of print

No. 74  Common Predictable Components in Regional Stock Markets
Yin-wong Cheung (University of California at Santa Cruz), Jia He and Lilian
Kheng-lian Ng (CityU) (December 1995)

No. 75  Asset Pricing Specification Errors and Performance Evaluation
Jia He, Lilian Kheng-lian Ng (CityU) and Chu Zhang (University of Alberta)
(February 1996)

No. 76  The Changing Economic Environment in People's Republic of China
Kui-wai Li (March 1996)

No. 77  The Productivity of Financial Capital in China's Economic Reform: A Simple
Regression Analysis
Kui-wai Li (March 1996)

No. 78  Some Thoughts on China's 1995 Bank Reform
Kui-wai Li (May 1996)

No. 79  Bid-Ask Spread and Arbitrage Profitability: A Study of the Hong Kong Index
Futures and Options Market
Kee-hong Bae (CityU), Kalok Chan (Hong Kong University of Science &
Technology and Arizona State University) and Stephen Yan-leung Cheung (CityU)
(May 1996)

No. 80  The Performance of Trading Rules on Four Asian Currency Exchange Rates
Yin-wong Cheung (University of California at Santa Cruz) and Clement Yuk-pang
Wong (CityU) (June 1996)

No. 81  International Evidence on the Stock Market and Aggregate Economic Activity
Yin-wong Cheung (University of California) and Lilian Kheng-lian Ng (CityU)
(June 1996)
No. 82  Decision Rights, Residual Claim and Performance: A Theory of How the Chinese State Enterprise Reform Works  
Weiying Zhang (Peking University and CityU) (June 1996)

No. 83  An Analysis of Import Protection as Export Promotion  
Anming Zhang (University of Victoria and CityU) and Yimin Zhang (University of New Brunswick) (June 1996)

No. 84  An Analysis of Fortress Hubs in Airline Networks  
Anming Zhang (University of Victoria and CityU) (June 1996)

No. 85  The Effects of Airline Codesharing Agreements on Firm Conduct and International Air Fares  
Tae-hoon Oum, Jong-hun Park (University of British Columbia) and Anming Zhang (CityU) (June 1996)

No. 86  On Existence of an "Optimal Stock Price": Evidence from Stock Splits and Reverse Stock Splits in Hong Kong  
Lifan Wu and Bob Yau-ching Chan (June 1996)

No. 87  Stability of Nash Equilibrium: The Multiproduct Case  
Anming Zhang (University of Victoria and CityU) and Yimin Zhang (University of New Brunswick) (June 1996)

No. 88  Further Investigation of the Uncertain Unit Root in GNP  
Yin-wong Cheung (University of California and CityU) and Menzie D. Chinn (University of California) (July 1996)

No. 89  A New Stochastic Duration Measure by the Vasicek and CIR Term Structure Theories  
Xueping Wu (July 1996)

No. 90  Stock Market Volatility and Fractional Integration  
Yin-wong Cheung (University of California and CityU) (July 1996)
No. 91  Money, Inflation and Growth  
Keith Blackburn (University of Manchester) and Victor Tin-yau Hung (CityU)  
(August 1996)

No. 92  Effects of Merger and Foreign Alliance: An Event Study of the Canadian Airline Industry  
Anming Zhang (CityU and University of Victoria) and Derek Aldridge (University of Victoria)  
(August 1996)

No. 93  The Invariance of Best Reply Correspondences in Two-Player Games  
Andy Luchuan Liu (October 1996)

J. Colin H. Jones (University of Victoria) and Anming Zhang (CityU and University of Victoria)  
(January 1997)

No. 95  A Theory of Growth, Financial Development and Trade  
Keith Blackburn (University of Manchester) and Victor Tin-yau Hung (CityU)  
(January 1997)

No. 96  Foreign Exchange Exposure, Risk and the Japanese Stock Market  
Jia He (Chinese University of Hong Kong), Lilian Kheng-lian Ng and Xueping Wu  
(CityU) (January 1997)

No. 97  Optimal Demand for Operating Lease of Aircraft  
Tae-hoon Oum (University of British Columbia), Anming Zhang (CityU and University of Victoria) and Yimin Zhang (University of New Brunswick)  
(February 1997)

No. 98  A Principal-agent Theory of the Public Economy  
Weiyiing Zhang (Peking University and CityU) (April 1997)

No. 99  Capital Structure and Socially Optimal Capacity in Oligopoly: The Case of Airline Industry  
Tae-hoon Oum (University of British Columbia), Anming Zhang (CityU and University of Victoria) and Yimin Zhang (University of New Brunswick) (April 1997)
No. 100  Horizontal Mergers in an Open Economy
        Anming Zhang (CityU and University of Victoria) (April 1997)

No. 101  An Analysis of Import Protection as Export Promotion under Economies of Scale
        Anming Zhang (CityU and University of Victoria) and Yimin Zhang (University of New Brunswick) (April 1997)

No. 102  The Performance of Trading Rules on Four Asian Currency Exchange Rates
        Yin-wong Cheung (University of California and CityU) and Clement Yuk-pang Wong (CityU) (May 1997)

No. 103  Individual Learning in Normal Form Games: Some Laboratory Results
        Yin-wong Cheung (University of California and CityU) and Daniel Friedman (University of California) (May 1997)

No. 104  Bandwidth Selection, Prewhitening and the Power of the Phillips-Perron Test
        Yin-wong Cheung (University of California and CityU) and Kon S. Lai (California State University) (May 1997)

No. 105  Foreign Exchange Markets in Hong Kong, Tokyo, and Singapore
        Yin-wong Cheung (University of California and CityU) and Clement Yuk-pang Wong (CityU) (May 1997)

No. 106  On Cross-Country Differences in the Persistence of Real Exchange Rates
        Yin-wong Cheung (University of California and CityU) and Kon S. Lai (California State University) (May 1997)

No. 107  Macroeconomic Determinants of Long-Term Stock Market Comovements Among Major EMS Countries
        Yin-wong Cheung (University of California and CityU) and Kon S. Lai (California State University) (May 1997)

No. 108  Parity revision in real exchange rates during the post-Bretton Woods period
        Yin-wong Cheung (University of California and CityU) and Kon S. Lai (California State University) (May 1997)
| No. 109 | Why Are Entrepreneurs Liquidity Constrained?  
         | Weiying Zhang (Peking University and CityU) (June 1997) |
|---------|--------------------------------------------------------|
| No. 110 | China’s Financial Reform: The Relation between State Banks and State Enterprises  
         | Edgardo Barandiaran (The World Bank and CityU) (July 1997) |
| No. 111 | Concession Revenue and Optimal Airport Pricing  
         | Anming Zhang (CityU and University of Victoria) and Yimin Zhang (CityU and University of New Brunswick) (August 1997) |
| No. 112 | The Optimal Number of Contracts in Cross- or Delta-Hedges  
         | Piet Sercu (University of Leuven) and Xueping Wu (CityU) (October 1997) |
| No. 113 | Reliability Differentiation of Electricity Transmission  
         | Chi-keung Woo (Energy and Environmental Economics), Ira Horowitz (University of Florida and CityU) and Jennifer Martin (Energy and Environmental Economics) (November 1997) |
| No. 114 | Conditional Multifactor Explanation of Return Momentum  
         | Xueping Wu (February 1998) |
| No. 115 | Industrial Reform and Air Transport Development in China  
         | Anming Zhang (CityU and University of Victoria) (April 1998) |
| No. 116 | On the Effects of Strategic Alliances on Partners’ Output  
         | Jong-hun Park (CityU) and Anming Zhang (CityU and University of Victoria) (April 1998) |
| No. 117 | Competition and Institutional Change: Privatization in China  
         | Shaomin Li (CityU), Shuhe Li (CityU) and Weiying Zhang (Peking University) (May 1998) |
| No. 118 | Corruption and Contract Enforcement  
         | Shuhe Li and Victor Tin-yau Hung (May 1998) |