

Envy and Portfolio Allocation in Defined Contribution Pension Plans

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Abstract

We examine the effect of envy on the portfolio allocation of a worker in a defined contribution (DC) pension plan. For instance, if a worker's DC plan performs better than his co-worker's, he may gloat; on the other hand, if his DC plan performs worse, he may feel envy. We model such anticipated envy when a worker makes his portfolio allocation. In equilibrium, workers will mimic their co-worker's allocation to eliminate the disutility from envy. In some instances, this allocation will result in a riskier portfolio than that of a worker who does not exhibit envy.

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1 Introduction

Over the last several decades there has been a movement in retirement savings plans offered by employers from defined benefit (DB) to defined contribution (DC). In DB plans, the plan sponsor promises workers a specified annuity benefit for their retirement, is liable for such a promise, and is responsible for asset management of funds in the plan. DC plans allow employees to decide in which assets their retirement investment will be allocated. Investment choice, within a menu offered by the sponsor, and capital market risk is moved from the employer to the employee in this plan.

The introduction of DC plans to retirement savings has been of great concern lately as capital market volatility has affected the performance of many plan participants' portfolios. Therefore, it is important to understand how investors will make investment decisions as their behavior could significantly alter their retirement wealth. Additionally, investment behavior is significant as "the aggregate outcome of a cluster of nonpochable decisions - such as insuring appropriately, planning for retirement . . . - will be more heavily influenced by a behavioral component" (Zeckhauser, 1986, S444). For instance, Muermann et al (2005) find that if a individual experiences regret, then his portfolio allocation will be different than that of a worker concerned only with risk. It is possible that an individual could feel regret when comparing his DC pension plan performance with that of his co-worker. For example, a worker could regret his portfolio allocation decision if his own individually-managed investments in his pension plan does not perform as well as those of his co-workers. Such regret, we call here "envy". Alternatively, a worker may experience some sort of rejoicing with regard to his allocation decision, if his own DC performance were better than that of his co-worker. This type of rejoicing we call here "gloating". Workers may then anticipate the disutility from envy when making their portfolio allocation decisions. This paper concentrates on different functional forms with which to model envy and the effect that anticipated envy has on the portfolio allocation within a worker's DC pension. We show that, in equilibrium, envy-averse workers will choose the same allocation to eliminate the disutility from envy. In certain instances, this portfolio is a riskier one than that chosen by a worker that does not exhibit envy.

But what exactly is envy? Adam Smith defines it as "passion which views with malignant dislike the superiority of those who are really entitled to all the superiority they possess" (1790, p 244). When a

co-worker has more DC pension wealth and he or she deservedly has it, the envious worker will feel he could have had the same wealth had he chosen the same portfolio allocation. Envy is different from resentment, as resentment is concerned with seeing another's good fortune which is undeserved; envy is more concerned with others having something we want (Ben Ze-ev, 1992). In this framework, envy is applicable to the DC pension portfolio allocation decision.

Other work has been done on envy, both theoretical and experimental. Goel and Thakor (2005) model optimal contracting in the labor market when agents exhibit envy and the principal is risk-neutral. They find that envy causes the wage of one agent to depend on the performance of other agents, and that an agent's utility can be increased or decreased depending on the outcomes of other agents. As for the principal, envy causes agents to work hard so incentives are easier to give, but envy also reduces the expected utility of agents so the principal has to compensate with higher wages. Dubey et al (2005) demonstrate that envy and pride by workers can explain preferences for prizes (bonuses) over wages. Envy has also been shown, through survey data, to be one reason why people care about relative status; more than 50% of participants preferred a state where they had half the purchasing power but higher relative income, than in another state (Solnick and Hemenway, 1998). Experimental evidence demonstrates the distributional conflicts and distortions that envy can create such as preventing subjects from choosing innovations that are Pareto-improving (Cason and Mui, 2002).¹

In the DC pension setting, behavioral attributes such as envy and gloating are important as most plan participants do not appear to actively manage their retirement accounts once making an initial asset allocation. Any distortions to the allocation could have significant impact on retirement wealth as participants do not re-allocate and adjust their portfolios. For instance, Ameriks and Zeldes show that approximately 47 percent of TIAA-CREF participants made no changes in their 401(k) portfolio allocation from 1986-1996. During the same time period, an additional 21 percent made only one change. Therefore, it appears that portfolio performance is depended on the initial asset allocation made by the worker and subsequent returns. Furthermore, Madrian and Shea (2001) find that employees exhibit procrastination with regard to portfolio allocations in the 401(k) plan of a large U.S. corporation that switched to automatic enrollment in the plan.

¹Somewhat related, Fehr and Schmidt (1999) model inequity aversion in market games and Bolton and Ockenfels (2000) develop a model which explains behavioral phenomena found from experiments.

Before automatic enrollment, employee procrastination was seen as 401(k) participation rates were initially lower than observed under automatic enrollment, but then increased over time. With automatic enrollment, procrastination is seen as a large percent of 401(k) participants initially had the default contribution rate and asset allocation, but this fraction decreased over time. Mitchell et al (2006) find in a two year period that 80 percent of 401(k) participants in over 1500 plans initiate no trades and an additional 11 percent make only one trade. If envy and gloating cause individuals to choose a riskier portfolio and workers do not change their allocations over time, there is a great risk that workers will have less wealth at retirement than if they were not envious, especially if returns are low.²

In what follows, we review the literature that motivates envy in portfolio allocation and discuss other areas that are related to our work. We propose a model that allows individuals to have preferences that depend on a co-worker's DC plan performance. We examine how anticipating the disutility from envy a worker may feel when using his co-worker's DC plan performance as a benchmark affects his retirement account allocation between risky and risk-free assets. Several functional forms are considered.

Motivation and Related Literature

In understanding the investment behavior of individuals, it has become evident that workers discuss their investment strategies with each other. Duflo and Saez (2003) show that employees of a large university discussed their portfolio allocation in retirement savings plans among themselves. While only a subset of department employees participated in a benefits information fair to learn about the Tax Deferred Account (TDA) retirement plan, after the fair, TDA enrollment increased across the board. Indeed, the increase was almost as large for individuals in "treated" departments who received information, as for those who did not attend the fair. Thaler and Bernartzi (2004) examine the implementation of Save More Tomorrow (SMarT), a prescriptive savings plan, at Philips Electronics and find a spillover effect among employees. Even those employees who did not join the SMarT plan had an increased saving rate, greater than that observed for the control group, which was employees at plants that did not introduce the SMarT plan. Furthermore, peer effects with regard to retirement savings participation decisions have been examined by Duflo and Saez

²Further evidence of procrastination by pension plan participants includes Caplin and Leahy (2001) where they argue that avoidant behavior has practical implications for retirement savings as many households from the baby boom generation have inadequately saved for their retirement. Bernheim (1995) notes that such low levels of accumulated wealth are representative of psychological issues such as psychological impediments to adequate planning; such impediments could include envy.

(2002) and Madrian and Shea (2000). The former considered TDA participation while the latter analyzed a 401(k) and Employee Stock Purchase Plan (ESPP) at a large corporation; significant positive peer effects were found in both studies.

Since employees discuss their retirement decisions with each other, it is of interest to determine how such interaction might affect their investments. Hong et al (2004) examine the Health and Retirement Study and find that a "social" investor participates more in the market if his peers participate. Therefore, in the context of DC pension plans, one might think that a worker will invest safely (risky) if his co-workers are investing safely (risky). In support of this idea, Patel et al (1991) argue that "relative valuations could lead decision makers to distort their own decisions, say in a 'keep up with the Joneses' effort, or an attempt to move with the herd as a mechanism of protection" (233). Additionally, Hong et al (2005) investigate mutual-fund manager behavior and discover that managers are more likely to hold (or buy or sell) a certain stock during a quarter, if other managers in the same city are also holding (or buying or selling) that same stock. Workers appear to not only discuss their investment strategies with each other, but they also tend to follow the strategies of their peers.

Similarly, it has been argued that individuals' preferences are dependent not only on their own wealth, but also on the wealth of others. Veblen (1899) noted that one's relative standing would influence one's preferences and reasoned that individuals were concerned with their stature in society. In this way, even if an individual's wealth increased, his utility would only truly rise if the wealth of those around him remained relatively the same. Easterlin (1998) also found that an individual's happiness was correlated with income; yet one's happiness only increases when his or her income rises relative to those around him. Luttmer (2005) finds that, controlling for an individual's own income, having neighbors with higher earnings is associated with lower levels of self-reported happiness. This association is not driven by changes in the way that individuals define happiness, and the effects on happiness derive mainly from changes in satisfaction with one's own financial situation. Therefore, when a worker is aware of his co-workers' investments, it is likely that his utility will depend not only on his own DC plan performance but also on his co-worker's DC plan performance. If individuals have preferences that depend on their relative wealth compared to that of their neighbors or co-workers, it is possible that co-worker envy could have a significant effect on workers' portfolio

allocations in their DC pension plans.

There has been some previous work on how such dependent preferences can be modeled. Duesenberry (1949) proposed the "demonstration effect" which holds that after some minimum income is reached, the frequency and strength of impulses to increase expenditures for one individual depends on the ratio of his expenditures to those with whom he associates. Thus, consumption is influenced by the consumption of neighbors. Abel (1990) allows preferences to depend on lagged cross-sectional average levels of consumption; without such "keeping up with the Jones" preferences and habit formation, this model represents power utility for the agent. Gali (1994) investigates dependent preferences and portfolio choice; he allows consumption to depend not only on one's own consumption, but also on the average per capita consumption in the economy. Again, without such dependent preferences, his model represents power utility. In this paper, a more generalized model with different parameterization is used; the advantage here is that, without dependent preferences, our model does not specify a specific form for the utility function. We also modify the model from Abel to accommodate for envy in our setting and obtain a somewhat similar result; envious workers will choose identical portfolios.

Related to our work is the literature on habit formation. Habit formation involves the concept that one's reported well-being is often more related to changes in consumption from yesterday to today, rather than only on one's level of consumption today. To explain the equity premium puzzle, Constantinides (1990) suggested a model where one's current consumption was dependent on one's past consumption history. Campbell and Cochrane (1999) add habit formation to the standard power utility function and allow consumption to depend on the history of aggregate consumption to predict dynamic asset pricing phenomena such as the long-horizon predictability of excess stock returns. The lack of international diversification in equity portfolios can also be explained by habit formation, as shown by Shore and White (2003); they allow utility to depend on the difference between one's own consumption and a "standard of living," defined as a backward-looking average of aggregate consumption of the agent's perceived peer group.³ Some of the literature focuses on the difference between current consumption and a benchmark which accounts for habit, while others use a ratio format for the comparison. Our modification of Abel's model uses a ratio of one's consumption to other's

³Also related is work by Ryder and Heal (1973) and Sundaresan (1989).

consumption. Due to framing concerns of individuals, we propose that a ratio might better account for the feeling of envy and knowing one's relative standing than a differencing approach (Tversky and Kahneman, 1981). Yet, we also modify a differencing model to account for envy and see what results are derived from such a model.

2 Modeling Envy

In this section we introduce the set-up for envious individuals. We explain how we will model their behavior in a individually-managed pension plan and show how we plan to investigate the optimal portfolio allocation for an envious investor. We also show that the purely risk-averse (non-envious) investor is a special case of our model.

Envy can be experienced if one makes a decision that is suboptimal, relative to the decision made by a co-worker. In this way, ex-post utility could be increased if one's decision allowed for greater wealth, compared to a co-worker. We take several approaches to examine the impact of envy on the employee's ex-ante allocation of wealth with regard to a co-worker's DC pension plan wealth.

We assume an investor has initial wealth, w_0 , which he can choose to allocate between a risk-free and risky asset. The return of the risk-free asset is deterministic and given by r_f , whereas the return on the risky asset is a random variable, R . R is distributed according to a cumulative distribution function, F . The worker allocates his wealth between the two assets so that his expected utility of final wealth is maximized. We let α_i^* denote the optimal fraction of wealth invested in the risky asset for investor i , and we add a subscript, e , if the investor is envious. Final wealth for investor i is given by

$$\begin{aligned} w_i &= w_0(\alpha_i(1 + R) + (1 - \alpha_i)(1 + r_f)) \\ &= w_0(1 + \alpha_i R + (1 - \alpha_i)r_f), \end{aligned}$$

and a similar equation exists for the co-worker. We consider co-workers that have the same initial wealth, as envy is more relevant when comparing oneself to someone with a wealth level which is similar (Ben-Ze'ev, 1992).

2.1 Regret Theory Approach

We follow Braun and Muermann (2004), who model investor preferences that include regret with a two-attribute utility function. This is useful since envy is, in a way, a specific form of regret. Regret Theory was formulated theoretically by Bell (1982, 1983) initially and Loomes and Sugden (1982). Bell (1982) demonstrated that Regret Theory can depict preferences for both insurance and gambling. Braun and Muermann (2004) find that incorporating regret into preferences can explain the observed preferences for low deductibles in personal insurance markets. Muermann et al (2005) investigate portfolio allocation with regret-averse investors and find that if individuals anticipate the disutility of regret ex-ante in decision making, then they will invest more in stocks than an individual not considering regret, for a zero risk premium. Yet, for a sufficiently high risk premium, an individual considering regret will invest less in stocks compared to an individual not exhibiting regret (Muermann et al, 2005).

Bell (1982, 1983) and Loomes and Sugden (1982) model regret by allowing individuals to optimize a "modified" utility function which has a second attribute that represents the regret the individual experiences as a result of having made the choice he did versus some alternative choice he could have made. In adapting this model for envy, let utility for individual i depend on the wealth of a co-worker, denoted by w_j , be given by the following:

$$V(w_i) = u(w_i) - e \cdot g(u(w_j) - u(w_i)) \text{ where } j \neq i.$$

The first attribute, $u(w_i)$, represents the risk aversion of the individual with $u' > 0$ and $u'' < 0$. The second attribute, $eg(u(w_j) - u(w_i))$, corresponds to the fact that the individual exhibits envy, where the $g(\cdot)$ function measures the amount of "envy" that the investor experiences. This is dependent on the difference between the utility he would have if he had his co-worker's DC plan performance, w_j , which he could have obtained, and the utility he derives from his actual level of final wealth. Here, e , represents the relative importance of "envy" compared to the first attribute of risk aversion for the individual.

We assume that $g(0) = 0$, $g(x) > 0$ if $x > 0$ which corresponds to cases of envy, and $g(x) < 0$ if $x < 0$ as this corresponds to cases of gloating. Additionally, $g(\cdot)$ is increasing for all values (that is, $g' > 0$). Kahneman and Tversky (1979) show individuals exhibit risk-loving behavior over losses and risk-averse

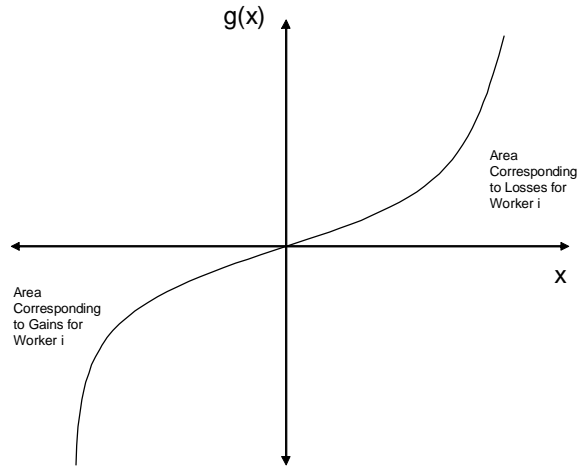


Figure 1: Representation of $g(\cdot)$. The figure demonstrates the properties of the function that measures the amount of envy or gloating worker i experiences. Here, x represents the difference between worker i 's utility of his co-worker's wealth (worker j) minus the utility of his own wealth. The function is convex over positive values of x as these values correspond to losses for worker i . Additionally, the function is concave over negative values of x as these values correspond to gains for worker i .

preferences over gains. This behavior implies that utility functions for individuals should be concave over gains in wealth and convex over losses in wealth. Positive values with regards to the second attribute occur when the co-worker's wealth is greater than worker i 's wealth. This level of wealth actually corresponds to a loss for individual i so g would then be convex over positive values ($g''(x) > 0$ for $x > 0$). Furthermore, negative values of the g function correspond to gains, which implies that g is concave over negative values ($g''(x) < 0$ for $x < 0$). Figure 1 depicts how the second attribute would look when these assumptions are taken into account. Initially, we assume that $g'' > 0$ for all values to simplify matters. Later we will allow for a more sophisticated form of the g function.

It is also possible that individuals exhibiting envy do not evaluate the difference in utilities but merely evaluate the difference in wealth levels between their own wealth and that of their co-worker. In their study of regret, Shefrin and Statman (1984) find that regret aversion results in investors preferring to consume

from dividends rather than from capital, which they show explains the observed preference of investors for cash dividends. Their model, however, considers differences in absolute levels of wealth.⁴ Therefore we allow the two-attribute utility function to take the following form as well:

$$V(w_i) = u(w_i) - e \cdot g(w_j - w_i) \text{ where } j \neq i.$$

2.1.1 Risk-Averse Individual

A purely risk-averse investor can be modeled by letting $e = 0$. We let α_0^* denote the optimal fraction of wealth invested in the risky asset for such a risk-averse investor, whose maximization problem is then determined by the following:

$$\max_{\alpha \in [0,1]} E[u(w)]$$

$$\text{where } w = w_0(1 + \alpha R + (1 - \alpha)r_f).$$

The optimal asset allocation in the DC pension plan will depend on the worker's initial wealth (w_0), the risk premium ($E[R] - r_f$), and his preferences. This problem is a standard one with the usual results that the risk-averse investor will invest a positive fraction of his wealth in the risky asset as long as the risk premium is positive (i.e. as long as the expected return on the risky asset is greater than the risk-free rate).⁵

Therefore, the risk-averse investor will hold all bonds when the risk premium is nonpositive but all stocks when the risk premium is sufficiently high. We use this asset allocation as the benchmark portfolio to compare against the optimal allocation of a worker exhibiting envy.

2.2 Dependent Preferences Approach

Building on prior literature, we consider the effect of dependent preferences and habit formation on consumption. Following Abel (1990) and Campbell and Cochrane (1999), we adapt their models to include envy in

⁴Goel and Thakor (2005) also implement differences in wealth levels in their work on optimal contracting with envy-averse agents.

⁵For more detail see, for example, Gollier (2001).

our portfolio allocation setting. These models implement utility functions that benchmark an individual's consumption to average aggregate per capita consumption or what they define as "habit" in the economy. To modify the models to include envy, we benchmark the worker's utility against the DC pension wealth of a co-worker having the same initial wealth. Initial wealth is held constant because envy is more relevant in situations where a worker's domain for comparison is small; we use a single co-worker as a benchmark (as opposed to an average of co-workers' wealth) because envy is also more relevant when the individual compares himself to a specific other person (Ben-Ze'ev, 1992). We consider these models because their approach is different than the regret-type model we implement and we are interested in knowing if our results are consistent across models. As mentioned earlier, dependent preference models can either have a differencing or a ratio approach; that is, differencing models look at the difference between current consumption and a benchmark, while ratio models consider the ratio of current consumption to a benchmark. The Abel model is a ratio type of model while Campbell and Cochrane (CC) is a differencing type model so we consider one of each type.⁶

2.2.1 Abel Model

In his paper, Abel (1990) proposes a utility function that allows for time-separable utility, dependent preferences (as in "keeping up with the Joneses"), and habit formation. This utility function takes the following form

$$U_t \equiv \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, v_{t+j})$$

$$\text{where } v_t \equiv [c_{t-1}^D C_{t-1}^{1-D}]^\gamma.$$

In this model, c_t is a consumer's own consumption at time t and C_t is aggregate per capita consumption at time t . Parameter values are $\gamma \geq 0$ and $D \geq 0$, where D allows for a combination of both dependent

⁶The ratio model from Gali (1994) was also modified to account for envy. This model can be derived from the Abel model with certain parameter specifications. The results obtained from the Gali model with regards to envy are exactly the same as the Abel model and therefore, are not included here.

preferences and habit formation. Abel suggest the following form for one period utility:

$$u(c_t, v_t) = \frac{(c_t/v_t)^{1-\alpha}}{1-\alpha}, \quad \alpha > 0.$$

To adjust this model to be consistent notationally with our model, we let c_t be w_i , the worker's own DC pension wealth, and C_t becomes w_j , the co-worker's DC pension wealth. We assume $D = 0$ as we do not allow for habit formation and change γ to e as this parameter will account for envy with regards to the co-worker's wealth. Furthermore, as α is part of w_i and w_j we change this to γ . Therefore, our modified model will have the following form:

$$u(w_i, w_j) = \frac{(w_i/w_j^e)^{1-\gamma}}{1-\gamma}$$

where $e \geq 0$. Note that if $e = 0$, the worker is non-envious and the utility function becomes the standard power utility (or constant relative risk aversion) function with γ as the coefficient of relative risk aversion. We determine the Nash equilibrium allocation if both workers exhibit envy using this utility function and compare this allocation to the optimal allocation of a non-envious worker with this utility.

2.2.2 Campbell and Cochrane Model

Campbell and Cochrane (1999) have a "slow-moving habit, or time-varying subsistence level, added to the basic power utility function" (206). They define lifetime utility as

$$U(C_t, X_t) = E \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}.$$

Here, C_t is consumption for one individual at time t , X_t is the level of habit, δ is a subjective time discount factor, and γ is a parameter for risk aversion. For one period, we would have the following utility expression:

$$U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}.$$

In adjusting this model to conform with our notation, we replace C_t with w_i , the worker's own DC pension wealth, and X_t with w_j , the co-worker's DC pension wealth. Additionally, we add e in front of the

variable for the co-worker’s DC pension wealth as it measures the intensity of envy. Therefore, the modified CC model is:

$$U(w_i, w_j) = \frac{(w_i - ew_j)^{1-\gamma}}{1-\gamma}.$$

We allow $e \geq 0$ as done previously. Again, if $e = 0$, the worker is non-envious and the utility function reduces to the power utility function with γ as the coefficient of relative risk aversion. The expected value of this utility is maximized to find the optimal portfolio allocation if both workers are envious. This allocation is compared to the optimal allocation of a non-envious worker to determine the impact of envy.

We first examine the optimal allocation of envious workers with the modified regret model to see how envy causes workers to allocate assets differently from non-envious workers. Then we maximize the modified models of Abel and CC as shown here to determine if we obtain the same result with regards to the effect of envy on portfolio allocation.⁷

3 Effect of Envy on Portfolio Allocation in the Regret Theory

Approach

For this part of the paper, we investigate the optimal allocation of workers using the model modified from Regret Theory. First we assume that the second attribute is convex over all values and look at situations when the co-worker is both non-envious and envious. Then we relax the convexity assumption on the second attribute and allow it to be concave over gains. Again we consider the optimal allocation when the co-worker is both envious and non-envious.

3.1 Envy Attribute is Convex

In this section of the paper, for simplicity, we follow the assumption from Braun and Muermann (2004) that the second attribute of the utility function is convex for all values. Under this assumption, workers are risk seeking over both gains and losses compared to their co-worker.

⁷We also use a very general model for utility, $U(w_i, w_j) = v_1(w_i)v_2(w_j)$ where $U_i > 0$, $U_{ii} < 0$, and $U_{ij} > 0$. We are able to show that if both workers are envious the optimal allocation is to have the same portfolio; however, we are unable to have a definitive conclusion on how this allocation compares to that of a non-envious worker.

3.1.1 Envy Attribute Shows Differences in Utility

Initially, we assume the worker evaluates envy by comparing the utility of his own wealth and the utility he would have had if he had his co-worker's portfolio. This is demonstrated by the following equation:

$$V(w_i) = u(w_i) - e \cdot g(u(w_j) - u(w_i)) \text{ where } j \neq i.$$

Co-Worker is Non-envious First assume that the worker exhibiting envy, worker i , is comparing himself to a non-envious co-worker. Therefore, co-worker j is purely risk-averse and his optimal portfolio allocation is that of a risk-averse individual, $\alpha_j^* = \alpha_0^*$. The worker exhibiting envy solves the following problem in trying to maximize his own expected wealth:

$$\begin{aligned} & \max_{\alpha_{ei} \in [0,1]} E[V(w_i)] \\ \text{s.t. } & \alpha_j^* \in \arg \max_{\alpha_j \in [0,1]} E[u(w_j)] \end{aligned}$$

The solution to the above maximization problem is given in the following proposition.

Proposition 1 *If the second attribute of the utility function is convex over all values, models differences in utility levels, and the co-worker is non-envious, then the envious individual's optimal asset allocation is identical to that of his co-worker. That is, $\alpha_{ei}^* = \alpha_j^* = \alpha_0^*$.*

Proof. See Appendix. ■

In order to eliminate the disutility from envy, the envious worker's optimal allocation is to invest the same as his co-worker. Since the co-worker is non-envious ($e = 0$), the optimal allocation for the envious individual is the optimal allocation for the risk-averse individual.

Co-Worker is Envious: A Nash Equilibrium Setting Now we suppose that both worker i and worker j exhibit preferences with envy. In this case, worker i solves the following maximization problem:

$$\max_{\alpha_{ei} \in [0,1]} E[V(w_i)]$$

where $\alpha_{ej}^* \in \arg \max_{\alpha_{ej} \in [0,1]} E[V(w_j)]$.

Worker j solves a similar maximization problem (interchange all i 's with j 's and vice versa). In Nash equilibrium, both workers choose their allocations simultaneously and an optimal allocation is made.

Proposition 2 *If the second attribute of the utility function is convex over all values, models differences in utility levels, and both workers exhibit preferences with envy, the optimal allocation is a Nash equilibrium where $\alpha_{ei}^* = \alpha_{ej}^*$. That is, both workers have identical portfolio allocations.*

Proof. See Appendix. ■

By choosing such an allocation, both workers eliminate the change in expected utility resulting from envy or gloating. This occurs because the second attribute of the utility function reduces to $eg(0)$ when both workers have the same allocation, which equals zero by assumption. However, is this allocation different from that made by a risk-averse individual? By choosing the same allocation as their co-workers, are individuals choosing a riskier (or less risky) allocation? Additionally, is this Nash equilibrium unique? The answers are revealed in the following proposition.

Proposition 3 *If the second attribute of the utility function is convex over all values, models differences in utility levels, and both workers exhibit preferences with envy, there exists a unique Nash equilibrium where $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$. That is, both workers choose the same allocation, which is also the optimal allocation of a purely risk-averse individual.*

Proof. See Appendix. ■

In the current setting, each worker's optimal allocation is a function of the other worker's portfolio allocation (i.e. $\alpha_{ei} = \alpha_{ei}(\alpha_{ej})$). To help distinguish if we have a unique Nash equilibrium, we plot both workers' best reply functions to see where they intersect. Mutual best replies are Nash equilibria. If we

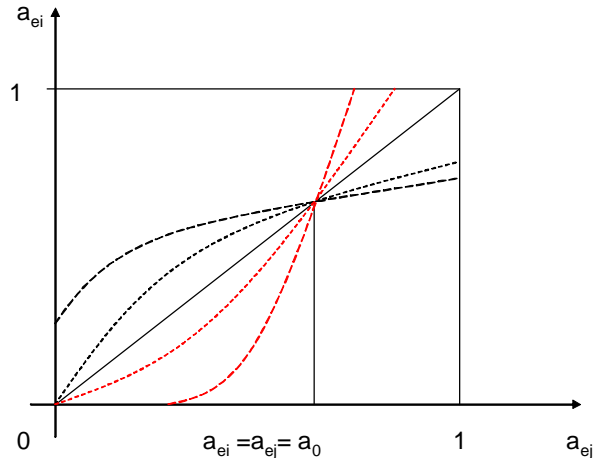


Figure 2: Best Reply Functions When Modeling with Differences in Utility. If both workers exhibit envy, then they choose the same allocation. This allocation is the same as the optimal allocation of a non-envious worker. The thicker dashed lines represent the best reply functions when the equity premium is positive and the thinner lines represent best replies when the equity premium is zero. The two lines that start below the 45 degree line and slope upwards represent best replies for worker j and those above the 45 degree line represent best replies for worker i .

have only one intersection then we have a unique solution. We find the following type of function as shown in Figure 2. On the x-axis is worker j 's investment in the risky asset, and along the y-axis is worker i 's investment in the risky asset. We see that the functions intersect at only one point. This point corresponds to both workers investing the same amount in the risky-asset and is the same allocation as the risk-averse investor's optimal portfolio. Details on the shape of the best reply functions are given in the Appendix.

In this part of the analysis, being envious does not cause an investor to choose a portfolio which is riskier or less risky than that chosen by a purely risk-averse (non-envious) investor.⁸

⁸Additionally, we have investigated how an increase in envy would affect the portfolio allocation of a worker, but thus far have no definitive conclusion.

3.1.2 Envy Attribute Shows Differences in Wealth Levels

It is possible that individuals exhibiting envy do not evaluate the difference in utilities but merely evaluate the difference in wealth levels between their own wealth and that of their co-worker. In this section, we assume the worker exhibiting envy has a utility function with the following form:

$$V(w_i) = u(w_i) - e \cdot g(w_j - w_i) \text{ where } j \neq i.$$

Co-Worker is Non-Envious First, we assume that the worker exhibiting envy, worker i , is comparing himself to a co-worker that is non-envious. Therefore, worker j , is risk-averse and his optimal portfolio allocation is given by that of a risk-averse individual, $\alpha_j^* = \alpha_0^*$. The optimal portfolio for the risk-averse investor is the same as we had in the previous section. The worker exhibiting envy solves the following maximization problem:

$$\begin{aligned} & \max_{\alpha_{ei} \in [0,1]} E[V(w_i)] \\ \text{s.t. } & \alpha_j^* \in \arg \max_{\alpha_j \in [0,1]} E[u(w_j)]. \end{aligned}$$

The solution to the above problem is given in the following proposition.

Proposition 4 *If the second attribute of the utility function is convex over all values, models differences in wealth levels, the co-worker is non-envious, and the equity premium is positive, then the envious individual's optimal allocation is one which is riskier than that of his co-worker. If the equity premium is zero, however, the envious individual's optimal allocation is identical to that of his co-worker. That is, if $E[R] - r_f > 0$ then $\alpha_{ei}^* > \alpha_j^* = \alpha_0^*$ and if $E[R] - r_f = 0$ then $\alpha_{ei}^* = \alpha_j^* = \alpha_0^*$.*

Proof. See Appendix. ■

In this setting, for a positive equity premium, a worker exhibiting envy will choose a riskier portfolio than that of his purely risk-averse co-worker.

Co-Worker is Envious: A Nash Equilibrium Setting Now suppose both worker i and worker j exhibit preferences with envy. The maximization problem for worker i is

$$\max_{\alpha_{ei} \in [0,1]} E[V(w_i)]$$

where $\alpha_{ej}^* \in \arg \max_{\alpha_{ej} \in [0,1]} E[V(w_j)]$.

Worker j solves a similar maximization problem (interchange all i 's with j 's and vice versa). In equilibrium, both workers solve their maximization problems simultaneously and an optimal allocation is made.

Proposition 5 *If the second attribute of the utility function is convex over all values, models differences in wealth levels, and both workers exhibit preferences with envy, then the optimal allocation is a Nash equilibrium where $\alpha_{ei}^* = \alpha_{ej}^*$. That is, both workers have the same portfolio allocation.*

Proof. See Appendix. ■

In order to eliminate the effects of envy each worker chooses the allocation of his co-worker. Again, we are interested in knowing whether this equilibrium is different from the optimal allocation of a purely risk-averse worker, and whether this equilibrium is unique.

Proposition 6 *If the second attribute of the utility function is convex over all values, models differences in wealth levels, both workers exhibit preferences with envy, and the equity premium is positive, then there exists a unique Nash equilibrium where both workers choose the same allocation. This allocation is riskier than the optimal allocation of a non-envious investor. If the equity premium is zero, however, there is a unique equilibrium where both workers choose the same allocation which is also the optimal allocation of a non-envious investor. That is, if $E[R] - r_f > 0$ then $\alpha_{ei}^* = \alpha_{ej}^* > \alpha_0^*$ and if $E[R] - r_f = 0$ then $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.*

Proof. See Appendix. ■

Again, each worker's optimal allocation is a function of the other worker's portfolio allocation (i.e. $\alpha_{ei} = \alpha_{ei}(\alpha_{ej})$). To distinguish whether we have a unique Nash equilibrium, we plot each worker's best reply functions to see where they intersect as shown in Figure 3. Mutual best replies are Nash equilibria and if we have only one intersection, then we have a unique solution. We see that the functions intersect at one point

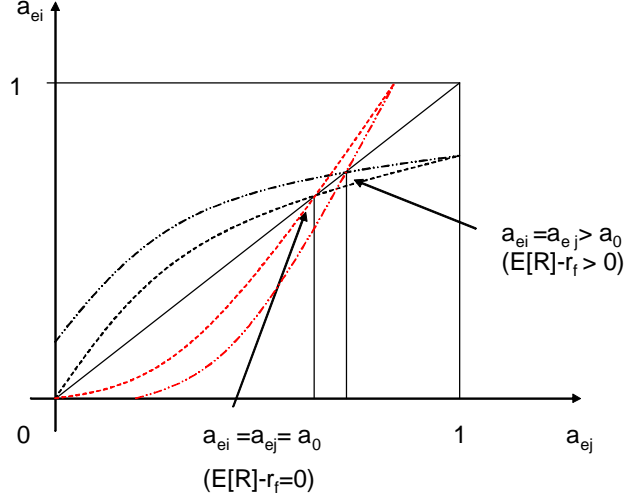


Figure 3: Best Reply Functions When Modeling with Differences in Wealth Levels. If both workers exhibit envy, then they choose the same allocation. When the equity premium is zero, this allocation is the same as a non-envious worker; yet, when the equity premium is positive, this allocation is riskier than a non-envious worker’s allocation. Please see the description of Figure 2 for details on what each dashed line represents.

when the equity premium is positive and at another point when risk premium is zero. When the equity premium is zero this intersection corresponds to both workers investing the same amount in the risky-asset which is also the same allocation as the non-envious investor’s optimal portfolio. If the equity premium is positive, however, the intersection represents both workers investing the same amount in the risky asset and this allocation is riskier than that chosen by the non-envious worker. Details on the shape of the best reply functions are given in the Appendix.⁹

⁹We also investigate wealth effects in two ways. First we look directly at comparative statics with respect to initial wealth for both differences in utility levels and differences in wealth levels for the $g(\cdot)$ function, but are unable to make any conclusive comments. Second, it has been suggested that the intensity of envy for an investor could be dependent on his wealth level; therefore, we allow the envy parameter to be an increasing function of initial wealth. That is, e becomes $e(w_0)$ with $e' > 0$. We then look at comparative statics with respect to initial wealth for both differences in utility levels and differences in wealth levels with this new form for the envy parameter. Again, we are unable to make any conclusive comments.

3.2 Envy Attribute is Convex over Positive Values and Concave over Negative Values

Next we assume, as in Kahneman and Tversky, that the second attribute is risk-seeking over losses and risk-averse over gains. In our model, this implies that the second derivative of $g(\cdot)$ is positive over positive values and negative over negative values. That is, $g''(x) > 0$ for $x > 0$ and $g''(x) < 0$ for $x < 0$.

As the equations and maximization problems remain the same, and only the assumption on the second derivative of the second attribute has changed, the first order conditions for the problems remain the same for both differences in utility and wealth levels in the second attribute. However, the second order condition can now not be assumed to be less than zero for all values. Therefore, the same results hold as in the previous section but the optimal solutions cannot be termed global maxima when we assume the co-worker is non-*envious*. Rather, the solutions from the previous section are local maxima. When assuming the co-worker also exhibits *envy*, however, we still obtain the same Nash equilibrium solutions (for both forms of the second attribute - differences in utility levels and differences in wealth levels). See the Appendix for more detail.

3.2.1 Specific Example

To illustrate the second attribute, we investigate a specific form of the utility function implemented by Gollier and Salanie (2005) and representing the same preferences suggested by Kahneman and Tversky. Gollier and Salanie (2005) consider the impact of regret on asset prices and specify a utility function that exhibits "multiplicative regret"; this is opposed to the model used in Braun and Muermann (2004), advocated by Bell and Loomes and Sugden, which we adapted to exhibit *envy* as shown earlier in this paper. Gollier and Salanie (2005) specify the utility function as

$$U^i(x, y) = \frac{x^{1-\gamma}y^\alpha}{1-\gamma}$$

where x is the consumption of agent i and the function, $U^i(x, y)$, is increasing and concave in x . That is, $U_x^i > 0$ and $U_{xx}^i < 0$. The maximal feasible consumption that could have been obtained is denoted by y and

γ is a positive scalar. In this model, α is positive scalar which measures the intensity of regret and $U_{xy} \geq 0$. The marginal utility of consumption is increasing in the foregone alternative.

To adapt this model to include envy, we assume the utility function takes the same form. However, we replace x with w_i , the wealth of investor i , and y with w_j , the wealth of the co-worker (or investor j). That is we have,

$$U^i(w_i, w_j) = \frac{w_i^{1-\gamma} w_j^e}{1-\gamma}$$

$$\text{where } w_i = w_0(1 + \alpha_i R + (1 - \alpha_i)r_f)$$

$$w_j = w_0(1 + \alpha_j R + (1 - \alpha_j)r_f).$$

We assume initial wealth is the same for both investors, namely w_0 , and α_i (α_j) denotes the fraction of wealth invested in the risky asset for worker i (j). We also assume that the utility function is increasing and concave in w_i so that $U_w^i > 0$ and $U_{ww}^i < 0$. Again, γ is a positive scalar and e is a positive scalar measuring the intensity of envy. A purely risk-averse (non-envious) investor would have $e = 0$ which would imply that U^i will depend only on w_i . Additionally, γ represents the coefficient of relative risk aversion.

Again, we are interested in knowing if there is an equilibrium when both investors exhibit envy and if such an equilibrium reflects a riskier allocation than that selected by a non-envious investor. The maximization problem for worker i is now

$$\max_{\alpha_{ei} \in [0,1]} E[U^i(w_i, w_j)]$$

$$\text{s.t. } \alpha_{ej}^* \in \arg \max_{\alpha_{ej} \in [0,1]} E[U^j(w_j, w_i)].$$

Worker j has a similar maximization problem (reverse all i 's and j 's). Both workers solve their maximization problems at the same time and an optimal allocation is derived.

Proposition 7 *Using a "multiplicative" model, where preferences are allowed to be concave over gains and convex over losses, both workers exhibit envy, and assuming $0 < \gamma < 1$, then there exists a Nash equilibrium where both workers choose the same allocation, which is a riskier allocation than the optimal allocation of a*

non-envious investor. That is, $\alpha_{ei}^* = \alpha_{ej}^* > \alpha_0^*$. If $\gamma < e$, then this equilibrium is unique.

Proof. See Appendix. ■

In order to eliminate the disutility from envy, it is optimal for a worker exhibiting envy to choose the same allocation as his envious co-worker. In this example, the optimal allocation is a riskier one than that chosen optimally by a non-envious worker.

4 Modeling Envy and Effect on Portfolio Allocation: Dependent Preferences Approach

Dependent preferences have been modeled by others as either a ratio of consumption for worker i to average consumption of other workers, or the difference between consumption for worker i and some average. We select one of each of these model types, one by Abel and the other by Campbell and Cochrane, modify them to incorporate portfolio allocation for an individually-managed pension plan, and adjust to account for workers who exhibit envy. These models are more specific than the regret models with regard to including risk-aversion parameters and their functional form. We find that, under certain parameter specifications, the wealth ratio model provides similar results as derived in the previous section for differences in wealth levels; that is, envious workers' optimal allocation is different from the optimal allocation of a non-envious investor. For the wealth differencing model, we obtain similar results to the difference in utility levels model; envy-averse workers' optimal allocation is the same as that of a non-envious worker.

4.1 Abel Model

As shown previously, the utility that Abel proposed can be modified for our setting as follows:

$$u(w_i, w_j) = \frac{(w_i/w_j^e)^{1-\gamma}}{1-\gamma}.$$

Again, $e \geq 0$ and when $e = 0$, the function reduces to power utility with a coefficient of relative risk aversion of γ . We are interested in knowing the optimal allocation when both workers exhibit envy and if this

allocation is different from the optimal allocation of a non-envious worker. The maximization problem for worker i is now

$$\begin{aligned} & \max_{\alpha_{ei} \in [0,1]} E[u(w_i, w_j)] \\ \text{s.t. } & \alpha_{ej}^* \in \arg \max_{\alpha_{ej} \in [0,1]} E[u(w_j, w_i)]. \end{aligned}$$

Worker j maximizes a similar problem (interchange all i 's and j 's). The two workers maximize their problem simultaneously to derive an optimal allocation, which is explained in the following proposition.

Proposition 8 *Using a modified utility from Abel, if both workers are envious and $0 < \gamma < 1$, then there exists a unique Nash equilibrium where both workers choose the same allocation, which is a less risky allocation than the optimal allocation of a non-envious investor. That is, $\alpha_{ei}^* = \alpha_{ej}^* < \alpha_0^*$.*

Proof. See Appendix. ■

In this model, it is optimal for envious workers to hold the same portfolio in order to eliminate any disutility from envy. Again, this allocation is different than the optimal allocation of non-envious worker; yet, in this model, the envious workers choose a safer allocation.

4.2 Campbell and Cochrane (CC) Model

The CC model modified to incorporate envy takes the following form:

$$U(w_i, w_j) = \frac{(w_i - ew_j)^{1-\gamma}}{1-\gamma}.$$

We assume that $e \geq 0$ and γ is the coefficient of relative risk aversion. When we are in a case where $e = 0$ (i.e. a non-envious worker) utility is the standard power utility function. Again, two workers, i and j , maximize their expected utility simultaneously to make a portfolio allocation. Worker i 's maximization

problem looks like the following:

$$\begin{aligned} & \max_{\alpha_{ei} \in [0,1]} E[U(w_i, w_j)] \\ \text{s.t. } \alpha_{ej}^* & \in \arg \max_{\alpha_{ej} \in [0,1]} E[U(w_j, w_i)]. \end{aligned}$$

The maximization problem for worker j is the same when the i 's and j 's are interchanged. The optimal allocation for the workers and its comparison to the non-envious worker's allocation is in next proposition.

Proposition 9 *Using a modified utility from Campbell and Cochrane, if both workers are envious, then there exists a unique Nash equilibrium where both workers choose the same allocation. This allocation is the same allocation as the optimal allocation of a non-envious investor; that is, $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.*

Proof. See Appendix. ■

Using the CC utility function, envious workers eliminate the negative effect of envy by choosing the same portfolio allocation. This portfolio is the same as the optimal portfolio of a non-envious worker.

In sum, if we use ratio and differencing comparisons for dependent preferences, we find that, in equilibrium, envious workers will choose the same portfolio. In the ratio model, this allocation is different from the optimal allocation of a non-envious worker; yet, in this case, envious workers choose a safer portfolio. The differencing model obtains the same result as the regret-approach model with differences in utility levels; workers exhibiting envy choose the same portfolio as workers who are not envious.

5 Relating the Regret Theory Approach and Dependent Preferences Approach

Because we obtain different results with the regret theory and dependent preferences models, it is of interest to see if they can be related by specifying a functional form for the $u()$ and $g()$ functions in the regret type models. As the models stand now, relating them has proven difficult. It appears the reason the models do not relate is because the regret-type models evaluate the absolute levels of DC pension wealth and then add

(subtract) utility if there is a gain (loss) compared to a co-worker's DC pension wealth. Yet, the models from Abel and CC only evaluate the relative position. In this section, we adapt the Abel and CC models further so that both the absolute and relative position of DC pension wealth are evaluated. Doing so allows us to see more clearly how the types of models relate to each other.

5.1 Abel Model

Recall that the utility function adapted for envy from Abel is given by the following ratio-type model:

$$u(w_i, w_j) = \frac{(w_i/w_j^e)^{1-\gamma}}{1-\gamma}.$$

We further adapt this model to become

$$u(w_i, w_j) = \frac{1}{1-\gamma} \left[\frac{w_i}{\left(\frac{w_j}{w_i}\right)^e} \right]^{1-\gamma}$$

so that the absolute level of DC pension wealth is evaluated and then is increased (decreased) if the relative position of the worker's DC performance is greater (less) than that of his co-worker's. Again, two envious workers maximize their expected utility simultaneously to make an optimal allocation. We then compare this allocation to that of a non-envious worker.

Proposition 10 *Using the modified Abel model (for both absolute and relative positioning) and assuming both workers are envious, then there is a Nash equilibrium where both workers choose the same allocation, which is also the optimal allocation of a non-envious worker. That is, $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$. This solution is unique if $\gamma > 1$.*

Proof. See Appendix. ■

Now consider the regret-type model which has differences in utility within the $g()$ function:

$$V(w) = u(w_i) - eg(u(w_j) - u(w_i))$$

which has a unique Nash equilibrium of $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$ as shown earlier in the paper. Allow $u(w) = \ln(w)$ and $g(w) = w$. Then the regret-type models becomes

$$\begin{aligned} V(w) &= \ln(w_i) - e(\ln(w_j) - \ln(w_i)) \\ &= \ln(w_i) - \ln\left(\frac{w_j}{w_i}\right)^e \\ &= \ln\left[\frac{w_i}{\left(\frac{w_j}{w_i}\right)^e}\right]. \end{aligned}$$

This is equivalent to the adapted ratio model from Abel with $\gamma = 0$. Both models predict that $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$ although this equilibrium is only unique for the modified Abel model if $\gamma > 1$.

5.2 Campbell and Cochrane (CC)

Consider the model from CC which was adapted for envy

$$U(w_i, w_j) = \frac{(w_i - ew_j)^{1-\gamma}}{1-\gamma}.$$

and modify it further to become

$$V(w) = \frac{1}{1-\gamma} (w_i - e(w_j - w_i))^{1-\gamma}$$

The absolute level of DC pension wealth is evaluated and utility is increased (decreased) if the worker's DC performance is better (worse) than that of his co-worker. We allow two workers to be envious and maximize their expected utilities simultaneously to obtain the optimal allocation.

Proposition 11 *Using the modified CC model (for both absolute and relative positioning) and assuming both workers are envious, then there is a unique Nash equilibrium where both workers choose the same allocation, which is also the optimal allocation of a non-envious worker. That is, $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.*

Proof. See Appendix. ■

Now we try to relate this model to one of the regret-type models. Unfortunately, neither of the current forms for the regret-type models relate to this modified CC function. However, if we allow the model that evaluates differences in wealth levels to take the following form

$$V(w) = u(w_i - eg(w_j - w_i))$$

a relation can be made. First a proposition about the optimal allocation of two envious workers with this type of utility.

Proposition 12 *Using the modified regret theory model which considers differences in wealth levels and assuming both workers are envious, there is a unique Nash equilibrium where both workers choose the same allocation, which is also the optimal allocation of a non-envious worker. That is, $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.*

Proof. See Appendix. ■

Using this modified regret theory model with differences in wealth levels, let $u(w) = \frac{1}{1-\gamma}w^{1-\gamma}$ and $g(w) = w$. Then the modified regret model becomes

$$V(w) = \frac{1}{1-\gamma} (w_i - e(w_j - w_i))^{1-\gamma}$$

which is the modified CC model. Both models predict that both workers choose the same allocation, which is also the optimal allocation for a non-envious worker.

The model of most interest from the regret-theory approach is the one that shows differences in wealth levels; this model has a unique Nash equilibrium where both envious workers choose the same allocation which is also a riskier allocation than that of a non-envious worker. Unfortunately, we have been unable to relate this model to current forms of habit formation and dependent preference models. Experiments that could give evidence that envy leads to riskier allocations would support this functional form.

6 Conclusion

Over time employers are increasingly offering DC rather than DB pension plans. Although DC plans allow employees more flexibility in maintaining their pension wealth, they also cause employees to bear market risk. As a result, it is important to understand how employees make financial decisions with regards to their pensions and the effect of their behavior on their allocations.

This paper demonstrates how envy can alter workers' DC plan portfolio allocations. In equilibrium, workers who exhibit envy will choose the same allocation. Using a range of functional forms for utility, we find that this allocation is different from the optimal allocation of non-envious workers for some cases. In some instances, the envious worker will have a riskier portfolio than the non-envious worker; in others, the envious worker will have a safer portfolio than his non-envious counterpart. As previous literature has shown that employees often procrastinate when managing their pension portfolios, employees' initial allocations in their DC plans might be maintained over the entire work life. If envy causes workers to choose a riskier allocation than that chosen by non-envious individuals, and workers do not rebalance their portfolios, a market shock could have a substantial impact on pension wealth. This may prompt employers to limit the number of high risk options offered to employees in their DC plan menus.

Future work will take several forms. It would be interesting to test the predictions of the model empirically. Anticipated envy could be an explanation for employees to hold similar portfolios. Also, experiments could be run to determine whether envy causes individuals to hold riskier portfolios than those who are not envious. Such work could aid in ascertaining which functional forms are more representative of investor behavior.

7 Appendix

7.1 Proof of Proposition 1

Consider the first order condition of (1) which is

$$E \left[w_0 (R - r_f) u' (w_i) (1 + e g' (u (w_j^*) - u (w_i))) \right] = 0.$$

The second order condition of (1) is

$$\begin{aligned} & E [w_0^2 (R - r_f)^2 u''(w_i) (1 + eg'(u(w_j^*) - u(w_i)))] \\ & - E [w_0^2 (R - r_f)^2 u'^2(w_i) eg''(u(w_j^*) - u(w_i))]. \end{aligned}$$

Considering the second order condition we see that it is always negative. This implies that the optimal portfolio allocation for the envious individual is characterized by the first order condition. Thus,

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei} = \alpha_j^*} &= E [w_0 (R - r_f) u'(w_j^*) (1 + eg'(0))] \\ &= (1 + eg'(0)) E [w_0 (R - r_f) u'(w_j^*)] \\ &= 0. \end{aligned}$$

The last equality statement is made as the second term in the previous line is the first order condition of the purely risk-averse worker which we know equals zero. Therefore, the first order condition for the envious worker when evaluated at the optimal portfolio for the risk-averse worker is zero. That implies there is a global maximum where the solution is $\alpha_{ei}^* = \alpha_j^* = \alpha_0^*$.

7.2 Proof of Proposition 2

Consider the first order condition of worker i 's maximization problem which is

$$E [w_0 (R - r_f) u'(w_i) (1 + eg'(u(w_j^*) - u(w_i)))] = 0.$$

Similarly, the first order condition of worker j 's maximization problem is

$$E [w_0 (R - r_f) u'(w_j) (1 + eg'(u(w_i^*) - u(w_j)))] = 0.$$

For a Nash equilibrium both equations above need to hold. Setting the two equations above equal to each other, we find that for a Nash equilibrium we would need $\alpha_{ei}^* = \alpha_{ej}^*$.

7.3 Proof of Proposition 3

Consider the first order condition:

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei} = \alpha_{ej} = \alpha} &= E [w_0 (R - r_f) u'(w_0(1 + \alpha R + (1 - \alpha)r_f)) (1 + eg'(0))] \\ &= (1 + eg'(0)) E [w_0 (R - r_f) u'(w_0(1 + \alpha R + (1 - \alpha)r_f))]. \end{aligned}$$

Evaluating the above at the optimal allocation for the risk-averse individual, α_0^* , we see that the term becomes zero as the second term in the above equation is the first order condition for the risk-averse investor. That is,

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei} = \alpha_{ej} = \alpha_0^*} = 0.$$

Therefore $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$. We examine the best reply functions for this problem to determine whether this solution is unique. In the current setting, each worker's optimal allocation is a function of the other worker's portfolio allocation (i.e. $\alpha_{ei} = \alpha_{ei}(\alpha_{ej})$). Taking the total differential of the first order condition

in (5) with respect to α_{ej} and rearranging terms we find

$$\alpha'_{ei}(\alpha_{ej}) = \frac{-E \left[\frac{w_0^2 (R - r_f)^2 u' (w_0 (1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) u' (w_j)}{\cdot eg'' (u (w_j) - u (w_0 (1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)))} \right]}{E \left\{ \cdot \left[\begin{array}{c} \frac{w_0^2 (R - r_f)^2}{\left(\begin{array}{c} u'' (w_0 (1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) \\ \cdot (1 + eg' (u (w_j) - u (w_0 (1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f))) \end{array} \right)} \right) \\ - \left(\begin{array}{c} u'^2 (w_0 (1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) \\ \cdot eg'' (u (w_j) - u (w_0 (1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f))) \end{array} \right) \end{array} \right] \right\}}$$

The numerator is always negative and the denominator is always negative which implies that $\alpha'_{ei}(\alpha_{ej}) > 0$. As worker j 's problem is the same as worker i 's we therefore find that $\alpha'_{ej}(\alpha_{ei}) > 0$ as well (we would obtain the same equation as above but replace all i 's with j 's and vice versa).

First we consider the first order condition for worker i assuming that worker j invests everything in the risk-free asset ($\alpha_{ej} = 0$) and find

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(0)} = E \left[\frac{w_0 (R - r_f) u' (w_0 (1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0))r_f))}{\cdot (1 + eg' (u (w_0 (1 + r_f)) - u (w_0 (1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0))r_f)))} \right]$$

which when we evaluate at the point where worker i also invests everything in the risk-free asset we obtain

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(0)=0} &= E [w_0 (R - r_f) u' (w_0 (1 + r_f)) (1 + eg'(0))] \\ &= w_0 (1 + eg'(0)) u' (w_0 (1 + r_f)) (E[R] - r_f). \end{aligned}$$

Therefore, if the equity premium is positive (i.e. $E[R] - r_f > 0$) then the above is positive which implies that $\alpha_{ei}^*(0) > 0$. Additionally, if the equity premium is positive then $\alpha_{ej}^*(0) > 0$.

From the first order condition for worker i we see that if worker j invests everything in the risky asset we have

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(1)} = E \left[\frac{w_0 (R - r_f) u' (w_0 (1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1))r_f))}{\cdot \left(1 + eg' \left(\begin{array}{c} u (w_0 (1 + R)) \\ -u (w_0 (1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1))r_f)) \end{array} \right) \right)} \right]$$

which when we also assume worker i invests everything in the risky asset becomes

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(1)=1} &= E [w_0 (R - r_f) u' (w_0 (1 + R)) (1 + eg'(0))] \\ &= w_0 (1 + eg'(0)) [(E[R] - r_f) E[u' (w_0 (1 + R))] + Cov(R, u' (w_0 (1 + R)))]. \end{aligned}$$

The preceding equation implies that if $E[R] - r_f \geq \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))]}$ then $\alpha_{ei}^*(1) = 1$ and if $E[R] - r_f < \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))]}$ then $\alpha_{ei}^*(1) < 1$. Similarly, for worker j we obtain that if $E[R] - r_f \geq \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))]}$ then $\alpha_{ej}^*(1) = 1$ and if $E[R] - r_f < \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))]}$ then $\alpha_{ej}^*(1) < 1$.

Now we consider the situation when both workers allocate everything in the risk-free asset. We find

$$\begin{aligned} \alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=0} &= \frac{-E \left[w_0^2 (R - r_f)^2 u'^2 (w_0 (1 + r_f)) eg''(0) \right]}{E \left[w_0^2 (R - r_f)^2 (u'' (w_0 (1 + r_f)) (1 + eg'(0)) - u'^2 (w_0 (1 + r_f)) eg''(0)) \right]} \\ &= \frac{-u'^2 (w_0 (1 + r_f)) eg''(0)}{u'' (w_0 (1 + r_f)) (1 + eg'(0)) - u'^2 (w_0 (1 + r_f)) eg''(0)} \\ &< 1. \end{aligned}$$

Additionally, $\alpha'_{ej}(\alpha_{ei}) \Big|_{\alpha_{ej}=\alpha_{ei}=0} < 1$. The slope of the best reply function is less than one at the point where both workers invest everything in the risk-free asset. If we simply look at the point where both workers

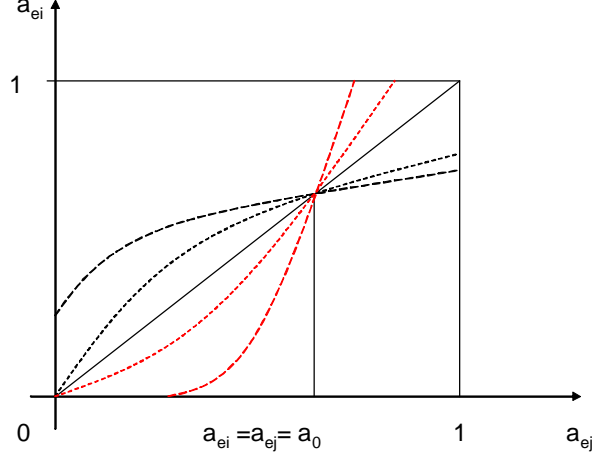


Figure 4: Best Reply Functions When Modeling with Differences in Utility. If both workers exhibit envy, then they choose the same allocation. This allocation is the same as the optimal allocation of a non-envious worker.

have the same allocation we obtain

$$\begin{aligned}
\alpha'_{ei}(\alpha_{ej})|_{\alpha_{ei}=\alpha_{ej}=\alpha} &= \frac{-E \left[w_0^2 (R - r_f)^2 u'^2 (1 + \alpha R + (1 - \alpha) r_f) eg''(0) \right]}{E \left[w_0^2 (R - r_f)^2 \left(\frac{u'' (1 + \alpha R + (1 - \alpha) r_f) (1 + eg'(0))}{-u'^2 (1 + \alpha R + (1 - \alpha) r_f) eg''(0)} \right) \right]} \\
&= \frac{-w_0^2 eg''(0) E \left[(R - r_f)^2 u'^2 (1 + \alpha R + (1 - \alpha) r_f) \right]}{w_0^2 (1 + eg'(0)) E \left[(R - r_f)^2 u'' (1 + \alpha R + (1 - \alpha) r_f) \right]} \\
&\quad - \frac{w_0^2 eg''(0) E \left[(R - r_f)^2 u'^2 (1 + \alpha R + (1 - \alpha) r_f) \right]}{w_0^2 (1 + eg'(0)) E \left[(R - r_f)^2 u'' (1 + \alpha R + (1 - \alpha) r_f) \right]} \\
&< 1.
\end{aligned}$$

Again, we also obtain $\alpha'_{ej}(\alpha_{ei})|_{\alpha_{ej}=\alpha_{ei}=\alpha} < 1$. The slope of the best reply function is also less than one at the point where both workers have the same portfolio allocation (that is, along the 45 degree line of a plot of both best reply functions, the slope of such functions will be less than one). Summarizing the results of the best reply functions in the graph below we see that the functions intersect at only one point. There is an unique Nash Equilibrium, $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.

7.4 Proof of Proposition 4

The first order condition for the maximization problem for the envious investor is

$$E \left[w_0 (R - r_f) (u'(w_i) + eg'(w_j^* - w_i)) \right] = 0.$$

The second order condition is found to be

$$E \left[w_0^2 (R - r_f)^2 (u''(w_i) - eg''(w_j^* - w_i)) \right].$$

The second order condition is strictly less than zero which implies that the optimal allocation for the envious worker is characterized by the first order condition.

Evaluating the first order condition where worker i chooses his co-worker's optimal allocation, which is the optimal allocation for a risk-averse investor we have

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_j^*=\alpha_0^*} &= E [w_0 (R - r_f) (u'(w_0(1 + \alpha_0^*R + (1 - \alpha_0^*)r_f)) + eg'(0))] \\ &= w_0 eg'(0) (E[R] - r_f) + E [w_0 (R - r_f) u'(w_0(1 + \alpha_0^*R + (1 - \alpha_0^*)r_f))] \\ &= w_0 eg'(0) (E[R] - r_f). \end{aligned}$$

The second to last line has the first order condition for the risk-averse worker as the second term which is equal to zero. The last line implies that if $E[R] - r_f > 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_j^*=\alpha_0^*} > 0$ which implies that $\alpha_{ei}^* > \alpha_j^* = \alpha_0^*$. Additionally, if $E[R] - r_f = 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_j^*=\alpha_0^*} = 0$ which implies that $\alpha_{ei}^* = \alpha_j^* = \alpha_0^*$.

7.5 Proof of Proposition 5

The first order condition of worker i 's maximization problem which is

$$E [w_0 (R - r_f) (u'(w_i) + eg'(w_j^* - w_i))] = 0.$$

Similarly, the first order condition of worker j 's maximization problem is

$$E [w_0 (R - r_f) (u'(w_j) + eg'(w_i^* - w_j))] = 0.$$

For a Nash equilibrium, both equations above need to hold. Setting the two equations above equal to each other, we find that for a Nash equilibrium we would need $\alpha_{ei}^* = \alpha_{ej}^*$.

7.6 Proof of Proposition 6

Consider the first order decision evaluated at the optimal allocation of $\alpha_{ei}^* = \alpha_{ej}^* = \alpha$:

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{eu}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} &= E [w_0 (R - r_f) (u'(w_0(1 + \alpha R + (1 - \alpha)r_f)) + eg'(0))] \\ &= w_0 eg'(0) (E[R] - r_f) + E [w_0 (R - r_f) u'(w_0(1 + \alpha R + (1 - \alpha)r_f))] \end{aligned}$$

Evaluating the above at the optimal allocation for the risk-averse individual, α_0^* , we see that the second term becomes zero as it is the first order condition for the risk-averse investor. Therefore, this implies that if $E[R] - r_f > 0$ then $\frac{\partial E}{\partial \alpha_{eu}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha_0^*} > 0$ which implies $\alpha_{ei}^* = \alpha_{ej}^* > \alpha_0^*$. Additionally, if $E[R] - r_f = 0$ then $\frac{\partial E}{\partial \alpha_{eu}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha_0^*} = 0$ which implies $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.

We examine the best reply functions for this problem to determine if the solution is unique. Taking the total differential of the first order condition in (5) with respect to α_{ej} and rearranging terms we find

$$\alpha'_{ei}(\alpha_{ej}) = \frac{E \left[w_0^2 (R - r_f)^2 eg''(w_j - w_0(1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) \right]}{E \left[w_0^2 (R - r_f)^2 \left(\begin{array}{c} eg''(w_j - w_0(1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) \\ -u''(w_0(1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) \end{array} \right) \right]}.$$

The numerator and denominator are always positive which implies that $\alpha'_{ei}(\alpha_{ej}) > 0$. Similarly, $\alpha'_{ej}(\alpha_{ei}) > 0$.

Consider the first order condition for worker i assuming that worker j invests everything in the risk-free asset ($\alpha_{ej} = 0$) and find

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(0)} = E \left[w_0 (R - r_f) \left(\begin{array}{c} u' (w_0(1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0))r_f)) \\ +eg' \left(\begin{array}{c} w_0(1 + r_f) \\ -w_0(1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0))r_f) \end{array} \right) \end{array} \right) \right].$$

Evaluating the above equation where worker i allocates all his wealth in the risk-free asset we have

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(0)=0} &= E [w_0 (R - r_f) (u' (w_0 (1 + r_f)) + eg'(0))] \\ &= w_0 (u' (w_0 (1 + r_f)) + eg'(0)) (E[R] - r_f). \end{aligned}$$

If $E[R] - r_f > 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(0)=0} > 0$ which implies $\alpha_{ei}^*(0) > 0$. If $E[R] - r_f = 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(0)=0} = 0$ which implies $\alpha_{ei}^*(0) = 0$. Also, if the equity premium is positive then $\alpha_{ej}^*(0) > 0$ and if the equity premium is zero then $\alpha_{ej}^*(0) = 0$.

From the first order condition for worker i we see that if worker j invests everything in the risky asset we have

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(1)} = E \left[w_0 (R - r_f) \left(\begin{array}{c} u' (w_0(1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1))r_f)) \\ +eg' \left(\begin{array}{c} w_0(1 + R) \\ -w_0(1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1))r_f) \end{array} \right) \end{array} \right) \right]$$

which when we evaluate at the point where worker i also invests everything in the risky asset we obtain

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(1)=1} &= E [w_0 (R - r_f) (u' (w_0 (1 + R)) + eg'(0))] \\ &= w_0 (E[R - r_f]E [u' (w_0 (1 + R)) + eg'(0)] + Cov (R, u' (w_0 (1 + R)))). \end{aligned}$$

If $E[R] - r_f \geq \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))+eg'(0)]}$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(1)=1} \geq 0$ which implies $\alpha_{ei}^*(1) = 1$. If $E[R] - r_f < \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))+eg'(0)]}$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(1)=1} < 0$ which implies $\alpha_{ei}^*(1) < 1$. Doing the same analysis for worker j we obtain that if $E[R] - r_f \geq \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))+eg'(0)]}$ then $\alpha_{ej}^*(1) = 1$ and if $E[R] - r_f < \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))+eg'(0)]}$ then $\alpha_{ej}^*(1) < 1$.

Considering the slope of the best reply function when both workers allocate everything in the risk-free asset we find

$$\begin{aligned} \alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=0} &= \frac{E [w_0^2 (R - r_f)^2 eg''(0)]}{E [w_0^2 (R - r_f)^2 (eg''(0) - u'' (w_0 (1 + r_f)))]} \\ &= \frac{eg''(0)}{eg''(0) - u'' (w_0 (1 + r_f))} \\ &< 1. \end{aligned}$$

Additionally, $\alpha'_{ej}(\alpha_{ei}) \Big|_{\alpha_{ej}=\alpha_{ei}=0} < 1$. Therefore, the slope of the best reply function is less than one at the point where both workers invest everything in the risk-free asset. If we simply look at the point where both workers have the same allocation we obtain

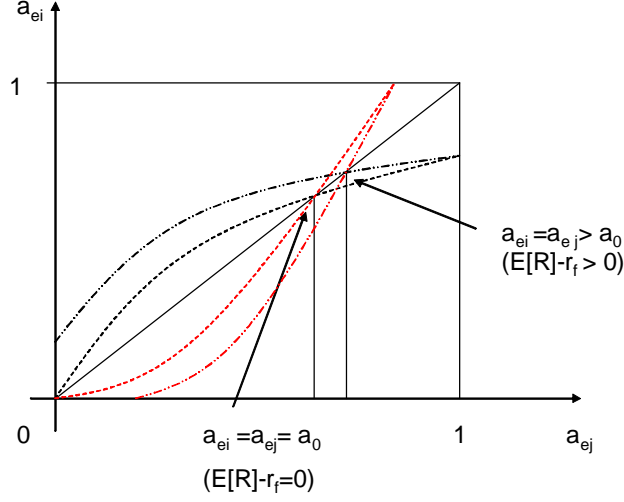


Figure 5: Best Reply Functions When Modeling with Differences in Wealth Levels. If both workers exhibit envy, then they choose the same allocation. When the equity premium is zero, this allocation is the same as a non-envious worker; yet, when the equity premium is positive, this allocation is riskier than a non-envious worker's allocation.

$$\begin{aligned}
\alpha'_{ei}(\alpha_{ej})|_{\alpha_{ei}=\alpha_{ej}=\alpha} &= \frac{E \left[w_0^2 (R - rf)^2 eg''(0) \right]}{E \left[w_0^2 (R - rf)^2 (eg''(0) - u''(w_0(1 + \alpha R + (1 - \alpha)rf))) \right]} \\
&= \frac{E \left[w_0^2 (R - rf)^2 eg''(0) \right]}{E \left[w_0^2 (R - rf)^2 eg''(0) \right] - E \left[w_0^2 (R - rf)^2 u''(w_0(1 + \alpha R + (1 - \alpha)rf)) \right]} \\
&< 1.
\end{aligned}$$

Again, we also obtain $\alpha'_{ej}(\alpha_{ei})|_{\alpha_{ej}=\alpha_{ei}=\alpha} < 1$. The slope of the best reply function is also less than one at the point where both workers have the same portfolio allocation (that is, along the 45 degree line of a plot of both best reply functions, the slope of such functions will be less than one). Summarizing the results of the best reply functions in the graph below we see that the functions intersect at only one point. There is a unique Nash Equilibrium which corresponds to both workers investing the same amount in the risky-asset and this allocation is a riskier portfolio than the risk-averse investor's optimal portfolio allocation if the equity premium is positive. If the equity premium is zero, the unique Nash equilibrium occurs where both workers choose the same allocation, which is the optimal allocation of a risk-averse investor.

7.7 Proof of Proposition 7

The first order condition for worker i is given by

$$E \left[w_0 (R - r_f) (w_i)^{-\gamma} (w_j^*)^e \right] = 0$$

and the second order condition is given by

$$E \left[-\gamma w_0^2 (R - r_f)^2 (w_i)^{-\gamma-1} (w_j^*)^e \right] < 0.$$

A similar first and second order condition exist for worker j (reverse the i 's and j 's in the above equations). For a Nash equilibrium the first order conditions for both workers need to hold. Setting the first order condition for each worker equal to each other we see that the solution is $\alpha_{ei}^* = \alpha_{ej}^*$.

Setting $e = 0$ we see that their maximization problem for a non-envious investor would be

$$\max_{\alpha_i \in [0,1]} E \left[\left(\frac{1}{1-\gamma} \right) (w_i)^{1-\gamma} \right]$$

which has a first order condition of

$$E \left[w_0 (R - r_f) (w_i)^{-\gamma} \right] = 0.$$

The second order condition is

$$E \left[-\gamma w_0^2 (R - r_f)^2 (w_i)^{-\gamma-1} \right] < 0$$

The first order condition characterizes the solution to the problem.

Evaluating the first order condition for the envious worker at the point where both workers choose the same allocation we have

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} &= E \left[\frac{w_0 (R - r_f) (w_0 (1 + \alpha R + (1 - \alpha)r_f))^{-\gamma}}{(w_0 (1 + \alpha R + (1 - \alpha)r_f))^e} \right] \\ &= E \left[w_0 (R - r_f) (w_0 (1 + \alpha R + (1 - \alpha)r_f))^{-\gamma} \right] E [(w_0 (1 + \alpha R + (1 - \alpha)r_f))^e] \\ &\quad + Cov \left((R - r_f) (w_0 (1 + \alpha R + (1 - \alpha)r_f))^{-\gamma}, (w_0 (1 + \alpha R + (1 - \alpha)r_f))^e \right). \end{aligned}$$

Note that the first term of the covariance term is increasing in R if $0 < \gamma \leq 1$ and the second term is increasing in R as $e > 0$. Therefore the covariance term is positive for $0 < \gamma < 1$. Evaluating the above at the optimal allocation for the risk -averse investor, α_0^* , we see that the first term contains the first-order condition for a risk-averse worker which is zero. Therefore we have $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha_0^*} > 0$ which implies that $\alpha_{ei}^* = \alpha_{ej}^* > \alpha_0^*$ for $0 < \gamma < 1$.

To determine if this equilibrium is unique, we examine the best reply functions for workers i and j . Taking the total differential of the first order condition with respect to α_{ej} and rearranging terms we find

$$\alpha'_{ei}(\alpha_{ej}) = \frac{\gamma E \left[(R - r_f)^2 (w_0 (1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f))^{-\gamma-1} (w_j)^e \right]}{e E \left[(R - r_f)^2 (w_0 (1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f))^{-\gamma} (w_j)^{e-1} \right]}$$

which is always positive so we know that $\alpha'_{ei}(\alpha_{ej}) > 0$ and similarly, $\alpha'_{ej}(\alpha_{ei}) > 0$. Evaluating the above at the point where both workers choose the same allocation we have

$$\alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} = \frac{\gamma}{e} > 0.$$

Additionally, $\alpha'_{ej}(\alpha_{ei}) \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} > 0$. Therefore, whenever the best reply function crosses the 45 degree line of a plot of the best reply functions against each other the slope must be greater than zero.

Looking at the first order condition, first we suppose that the co-worker invests everything in the risk-free asset. We then have

$$\begin{aligned}\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(0)} &= E \left[w_0 (R - r_f) (w_0 (1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0))r_f))^{-\gamma} (w_0(1 + r_f))^e \right] \\ &= (w_0(1 + r_f))^e E \left[w_0 (R - r_f) (w_0 (1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0))r_f))^{-\gamma} \right].\end{aligned}$$

Letting worker i invests everything in the risk-free asset we have

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(0)=0} = (w_0(1 + r_f))^{e-\gamma} (E[R] - r_f).$$

Therefore if the equity premium is positive, $E[R] - r_f > 0$, then $\alpha_{ei}^*(0) > 0$ and if the equity premium is zero, $\alpha_{ei}^*(0) = 0$. Similarly we have if $E[R] - r_f > 0$, then $\alpha_{ej}^*(0) > 0$ and if $E[R] - r_f = 0$, then $\alpha_{ej}^*(0) = 0$.

Evaluating the first order condition assuming the co-worker invests everything in the risky asset we have

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(1)} = E \left[w_0 (R - r_f) (w_0 (1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1))r_f))^{-\gamma} (w_0(1 + R))^e \right].$$

Letting worker i invests everything in the risky asset we have

$$\begin{aligned}\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}(1)=1} &= E \left[w_0 (R - r_f) (w_0(1 + R))^{-\gamma+e} \right] \\ &= w_0 \left\{ E[R - r_f] E \left[(w_0(1 + R))^{-\gamma+e} \right] + Cov \left(R, (w_0(1 + R))^{-\gamma+e} \right) \right\}.\end{aligned}$$

Therefore if $E[R] - r_f \geq \frac{-Cov(R, (w_0(1+R))^{-\gamma+e})}{E[(w_0(1+R))^{-\gamma+e}]}$ then $\alpha_{ei}^*(1) = 1$ and if $E[R] - r_f < \frac{-Cov(R, (w_0(1+R))^{-\gamma+e})}{E[(w_0(1+R))^{-\gamma+e}]}$

then $\alpha_{ei}^*(1) < 1$. Similarly, for worker j , we have if $E[R] - r_f \geq \frac{-Cov(R, (w_0(1+R))^{-\gamma+e})}{E[(w_0(1+R))^{-\gamma+e}]}$ then $\alpha_{ej}^*(1) = 1$ and

if $E[R] - r_f < \frac{-Cov(R, (w_0(1+R))^{-\gamma+e})}{E[(w_0(1+R))^{-\gamma+e}]}$ then $\alpha_{ej}^*(1) < 1$.

Summarizing the information about the best reply functions derived above, in a graph we see that the solution, $\alpha_{ei}^* = \alpha_{ej}^* > \alpha_0^*$ for $0 < \gamma < 1$ is unique if the slope of the best reply functions is less than one whenever it crosses the 45 degree line. This implies that we need $\gamma < e$. With this assumption, we see that the solution is an unique Nash equilibrium.

7.8 Proof of Proposition 8

The first order condition for worker i is

$$E \left[w_0 (R - r_f) (w_j)^{-e} \left(\frac{w_i}{(w_j)^e} \right)^{-\gamma} \right] = 0$$

and the second order condition is

$$E \left[-\gamma w_0^2 (R - r_f)^2 (w_j)^{-2e} \left(\frac{w_i}{(w_j)^e} \right)^{-\gamma-1} \right] < 0.$$

Worker j has similar first and second order conditions. Setting the first order condition for both workers equal to each other we find that the Nash equilibrium is $\alpha_{ei}^* = \alpha_{ej}^*$.

A non-envious worker solves the following:

$$\max_{\alpha \in [0,1]} E \left(\left(\frac{1}{1-\gamma} \right) (w_0(1 + \alpha R + (1 - \alpha)r_f)^{1-\gamma}) \right)$$

with first order condition

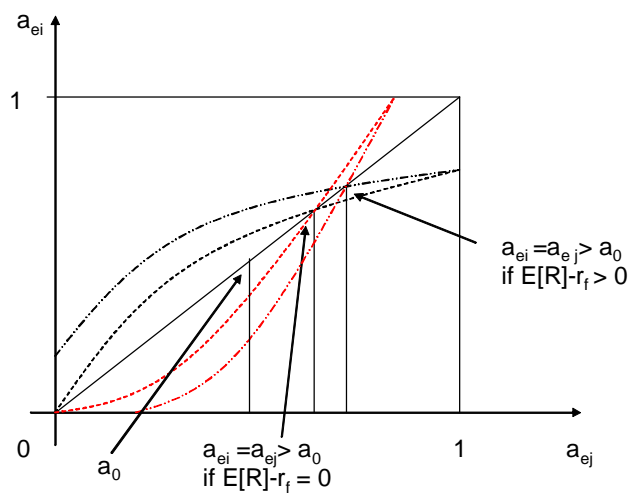


Figure 6: Modified Gollier and Salanie Model. Assuming $0 < \gamma < 1$ and $\gamma < e$ then the envy-averse agents choose the same allocation and this allocation is riskier than that chosen by a non-envious agent.

$$E \left[w_0 (R - r_f) (w_0(1 + \alpha R + (1 - \alpha)r_f)^{-\gamma}) \right] = 0$$

and second order condition

$$E \left[-\gamma w_0^2 (R - r_f)^2 (w_0(1 + \alpha R + (1 - \alpha)r_f)^{-\gamma-1}) \right] < 0.$$

Therefore,

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} &= E \left[w_0 (R - r_f) (w_0(1 + \alpha R + (1 - \alpha)r_f)^{-e} \left(\frac{w_0(1 + \alpha R + (1 - \alpha)r_f)}{(w_0(1 + \alpha R + (1 - \alpha)r_f)^e} \right)^{-\gamma} \right) \right] \\ &= E \left[w_0 (R - r_f) (w_0(1 + \alpha R + (1 - \alpha)r_f)^{-\gamma} \right] E \left[(w_0(1 + \alpha R + (1 - \alpha)r_f)^{e(\gamma-1)}) \right] \\ &\quad + Cov \left((R - r_f) (w_0(1 + \alpha R + (1 - \alpha)r_f)^{-\gamma}, (w_0(1 + \alpha R + (1 - \alpha)r_f)^{e(\gamma-1)}) \right) \\ &= \frac{\partial E}{\partial \alpha_0} E \left[(w_0(1 + \alpha R + (1 - \alpha)r_f)^{e(\gamma-1)}) \right] \\ &\quad + Cov \left((R - r_f) (w_0(1 + \alpha R + (1 - \alpha)r_f)^{-\gamma}, (w_0(1 + \alpha R + (1 - \alpha)r_f)^{e(\gamma-1)}) \right) \end{aligned}$$

The first part of the covariance term is increasing in R if $0 < \gamma < 1$ and the second term is increasing in R if $\gamma > 1$ (that is, decreasing if $0 \leq \gamma \leq 1$). Thus, if $0 < \gamma < 1$ the covariance term will be negative. Evaluating the above equation at the optimal allocation for the non-envious worker implies $\alpha_i^* = \alpha_j^* < \alpha_0^*$ if $0 < \gamma < 1$.

Taking the total differential of the first order condition for an envious worker with respect to the co-worker's allocation to the risky asset, rearranging terms, and evaluating where both workers invest the same in the risky asset we find

$$\alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} = \frac{e(\gamma-1)}{\gamma}$$

and also

$$\alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=0} = \frac{e(\gamma-1)}{\gamma}$$

Thus, for $0 < \gamma < 1$ we have $\alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} < 0$ and $\alpha'_{ej}(\alpha_{ei}) \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} < 0$.

Considering the first order condition where the co-worker invests everything in the risk-free asset we have

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(0)} = w_0 (w_0(1 + r_f))^{-e} E \left[(R - r_f) \left(w_i (w_0(1 + r_f))^{-e} \right)^{-\gamma} \right].$$

If worker i also invests everything in the risk-free asset the above reduces to

$$w_0 (w_0(1 + r_f))^{-e-\gamma+e\gamma} E(R - r_f)$$

Therefore if $E[R] - r_f > 0$ then $\alpha_{ei}^*(0) > 0$ and if $E[R] - r_f = 0$ then $\alpha_{ei}^*(0) = 0$. Similar analysis for investing everything in the risky asset yields that if $E[R] - r_f \geq \frac{-Cov(R, (w_0(1+R))^{-e-\gamma+e\gamma})}{E[(w_0(1+R))^{-e-\gamma+e\gamma}]}$ then $\alpha_{ei}^*(1) < 1$ and if $E[R] - r_f < \frac{-Cov(R, (w_0(1+R))^{-e-\gamma+e\gamma})}{E[(w_0(1+R))^{-e-\gamma+e\gamma}]}$ then $\alpha_{ei}^*(1) = 1$.

Summarizing the above information in the plot below of best reply functions, we see that, assuming $0 < \gamma < 1$, there is a unique Nash equilibrium where $\alpha_{ei}^* = \alpha_{ej}^* < \alpha_0^*$.

7.9 Proof of Proposition 9

Worker i 's maximization problem is

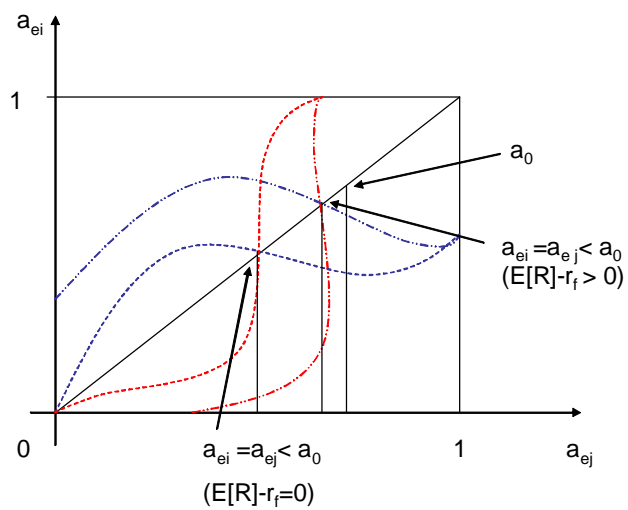


Figure 7: Modified Abel Model. Assuming $0 < \gamma < 1$ envy-averse agents choose the same allocation, which is a less risky allocation than a non-envious worker.

$$\max_{\alpha_{ei} \in [0,1]} E \left[\left(\frac{1}{1-\gamma} \right) (w_i - ew_j)^{1-\gamma} \right]$$

which has first order condition

$$E \left[w_0 (R - r_f) (w_i - ew_j)^{-\gamma} \right] = 0$$

and second order condition

$$E \left[-\gamma w_0^2 (R - r_f)^2 (w_i - ew_j)^{-\gamma-1} \right] < 0.$$

Worker j has a similar problem with a similar first and second order condition (interchange i and j in above equations). Setting the first order conditions for the two workers equal to each other we find that the Nash equilibrium is $\alpha_{ei}^* = \alpha_{ej}^*$.

For the non-envious worker ($e = 0$) the maximization problem is

$$\max E \left(\left(\frac{1}{1-\gamma} \right) (w_i)^{1-\gamma} \right)$$

with first order condition

$$E \left[w_0 (R - r_f) (w_i)^{-\gamma} \right] = 0$$

and second order condition

$$E \left[-\gamma w_0^2 (R - r_f)^2 (w_i)^{-\gamma-1} \right] < 0.$$

Therefore we have

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei} = \alpha_{ej} = \alpha} &= E \left[w_0 (R - r_f) (w_0(1 + \alpha R + (1 - \alpha)r_f - ew_0(1 + \alpha R + (1 - \alpha)r_f))^{-\gamma} \right] \\ &= E \left[w_0 (R - r_f) (1 - e)^{-\gamma} (w_0(1 + \alpha R + (1 - \alpha)r_f))^{-\gamma} \right] \\ &= (1 - e)^{-\gamma} \frac{\partial E}{\partial \alpha_0} \end{aligned}$$

Evaluating the above at the optimal allocation for a non-envious worker implies $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.

Differentiating the first order condition for envious worker with respect to α_{ej} we obtain

$$\alpha'_{ei}(\alpha_{ej}) = e > 0$$

and similarly,

$$\alpha'_{ei}(\alpha_{ej}) = e > 0.$$

Consider the first order condition where the co-worker invests everything in the risk-free asset and then letting worker i also invests everything in the risk free we find if $E[R] - r_f > 0$ then $\alpha_{ei}^*(0) > 0$ and if $E[R] - r_f = 0$ then $\alpha_{ei}^*(0) = 0$. A similar analysis for investing everything in the risky asset finds that if $E[R] - r_f \geq \frac{-Cov(R, (w_0(1+R))^{-\gamma})}{E[(w_0(1+R))^{-\gamma}]}$ then $\alpha_{ei}^*(1) = 1$ and if $E[R] - r_f \geq \frac{-Cov(R, (w_0(1+R))^{-\gamma})}{E[(w_0(1+R))^{-\gamma}]}$ then $\alpha_{ei}^*(1) < 1$.

Plotting the above information in the graph below of best reply functions we see that the Nash equilibrium is unique. Therefore $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.

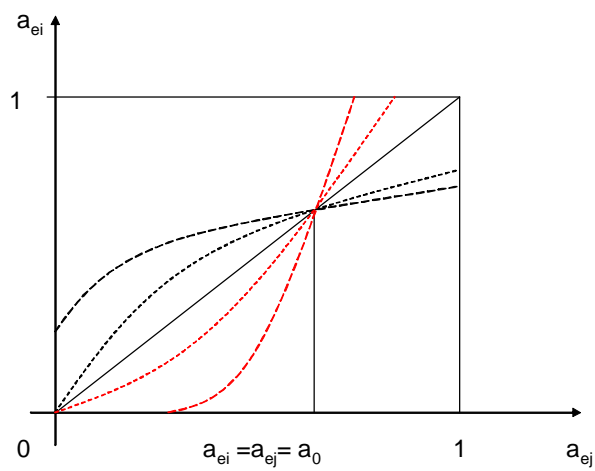


Figure 8: Modified Campbell and Cochrane Model. Workers exhibiting envy-averse preferences optimally choose the same allocation, which is also the same allocation as that made by a non-envious worker.

7.10 Proof of Proposition 10

Worker i 's maximization problem is

$$\max_{\alpha_{ei} \in [0,1]} E \left[\left(\frac{1}{1-\gamma} \right) \left(\frac{w_i}{\left(\frac{w_j}{w_i} \right)^e} \right)^{1-\gamma} \right]$$

which has first order condition

$$E \left[w_0 (R - r_f) (1 + e) \left(\frac{w_i}{w_j} \right)^e \left(\frac{w_i^{1+e}}{w_j^e} \right)^{-\gamma} \right] = 0$$

and second order condition

$$E \left[(e - \gamma - e\gamma) w_0^2 (R - r_f)^2 (1 + e) w_j^{e(\gamma-1)} w_i^{e-e\gamma-\gamma-1} \right]$$

which is less than zero if $\gamma > 1$ (which will need to assume later anyway). Worker j has a similar problem and solving them simultaneously we find that the Nash equilibrium is $\alpha_{ei}^* = \alpha_{ej}^*$.

For a non-vious worker, ($e = 0$), the maximization problem is

$$\max_{\alpha_i \in [0,1]} E \left[\left(\frac{1}{1-\gamma} \right) (w_i)^{1-\gamma} \right]$$

which has first order condition

$$E \left[w_0 (R - r_f) (w_i)^{-\gamma} \right] = 0$$

and second order condition

$$E \left[-\gamma w_0^2 (R - r_f)^2 w_i^{-\gamma-1} \right] < 0.$$

Evaluate the first order condition for the envious worker at the equilibrium allocation

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} &= E \left[w_0 (R - r_f) (1 + e) (w_0(1 + \alpha R + (1 - \alpha)r_f))^{1-\gamma} \right] \\ &= (1 + e) E \left[w_0 (R - r_f) (w_0(1 + \alpha R + (1 - \alpha)r_f))^{-\gamma} \right] \\ &= (1 + e) \frac{\partial E}{\partial \alpha_0}. \end{aligned}$$

Evaluating the above at the optimal allocation for a non-vious worker implies $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$. To see if this equilibrium is unique we look at the best reply functions and find that $\frac{\partial \alpha_{ei}}{\partial \alpha_{ej}} > 0$ if $\gamma > 1$ and $\frac{\partial \alpha_{ei}}{\partial \alpha_{ej}} \Big|_{\alpha_{ei}=\alpha_{ej}} < 1$ if $\gamma > 1$. Additionally, we use the same method as in the other proofs to consider what the functions look like when investing everything or nothing in the risky asset for both workers. Summarizing all this in a best-replies plot as done previously reveals that this equilibrium is unique if $\gamma > 1$.

7.11 Proof of Proposition 11

Worker i 's maximization problem is

$$\max_{\alpha_{ei} \in [0,1]} E \left[\left(\frac{1}{1-\gamma} \right) (w_i - e(w_j - w_i))^{1-\gamma} \right]$$

which has first order condition

$$E \left[w_0 (R - r_f) (1 + e) (w_i - e(w_j - w_i))^{-\gamma} \right] = 0$$

and second order condition

$$E \left[-\gamma w_0^2 (R - r_f)^2 (1 + e)^2 (w_i - e(w_j - w_i))^{-\gamma-1} \right] < 0.$$

Worker j has a similar problem and solving them simultaneously we find that the Nash equilibrium is $\alpha_{ei}^* = \alpha_{ej}^*$.

For a non-envious worker, ($e = 0$), the maximization problem is

$$\max_{\alpha_i \in [0,1]} E \left[\left(\frac{1}{1 - \gamma} \right) (w_i)^{1-\gamma} \right]$$

which has first order condition

$$E \left[w_0 (R - r_f) (w_i)^{-\gamma} \right] = 0$$

and second order condition

$$E \left[-\gamma w_0^2 (R - r_f)^2 w_i^{-\gamma-1} \right] < 0.$$

Evaluate the first order condition for the envious worker at the equilibrium allocation

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei} = \alpha_{ej} = \alpha} &= E \left[w_0 (R - r_f) (1 + e) (w_0(1 + \alpha R + (1 - \alpha)r_f) - eg(0))^{-\gamma} \right] \\ &= E \left[w_0 (R - r_f) (1 + e) (w_0(1 + \alpha R + (1 - \alpha)r_f))^{-\gamma} \right] \\ &= (1 + e) \frac{\partial E}{\partial \alpha_0}. \end{aligned}$$

Evaluating the above at the optimal allocation for a non-envious worker implies $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$. To see if this equilibrium is unique we look at the best reply functions and find that $\frac{\partial \alpha_{ei}}{\partial \alpha_{ej}} > 0$ and $\frac{\partial \alpha_{ei}}{\partial \alpha_{ej}} \Big|_{\alpha_{ei} = \alpha_{ej}} < 1$. Additionally, we use the same method as in the other proofs to consider what the functions look like when investing everything or nothing in the risky asset for both workers. Summarizing all this in a best-replies plot as done previously reveals that this equilibrium is unique.

7.12 Proof of Proposition 12

Worker i 's maximization problem is

$$\max_{\alpha_{ei} \in [0,1]} E [u(w_i - eg(w_j - w_i))]$$

which has first order condition

$$E [w_0 (R - r_f) (1 + eg'(w_j - w_i)) u'(w_i - eg(w_j - w_i))] = 0$$

and second order condition

$$E \left[\begin{array}{l} w_0^2 (R - r_f)^2 (1 + eg'(w_j - w_i))^2 u''(w_i - eg(w_j - w_i)) \\ -w_0^2 (R - r_f)^2 eg''(w_j - w_i) u'(w_i - eg(w_j - w_i)) \end{array} \right] < 0.$$

Worker j has a similar problem and maximizing both expected utilities simultaneously we find the Nash equilibrium is $\alpha_{ei}^* = \alpha_{ej}^*$.

For a non-envious worker, ($e = 0$), the maximization problem is

$$\max_{\alpha_i \in [0,1]} E [u(w_i)]$$

which has first order condition

$$E [w_0 (R - r_f) u'(w_i)] = 0$$

and second order condition

$$E \left[w_0^2 (R - r_f)^2 u''(w_i) \right] < 0.$$

Evaluating the first order condition for the envious worker at the equilibrium allocation we have

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} &= E [w_0 (R - r_f) (1 + eg'(0)) u'(w_0(1 + \alpha R + (1 - \alpha)r_f) - eg(0))] \\ &= (1 + eg'(0)) \frac{\partial E}{\partial \alpha_0}. \end{aligned}$$

Evaluating the above at the optimal allocation for a non-envious worker implies $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$. To see if this equilibrium is unique we look at the best reply functions and find that $\frac{\partial \alpha_{ei}}{\partial \alpha_{ej}} > 0$ and $\frac{\partial \alpha_{ei}}{\partial \alpha_{ej}} \Big|_{\alpha_{ei}=\alpha_{ej}} < 1$. Additionally, we use the same method as in the other proofs to consider what the functions look like when investing everything or nothing in the risky asset for both workers. Summarizing all this in a best-replies plot as done previously reveals that this equilibrium is unique.

7.13 Envy Attribute is Convex over Positive Values and Concave over Negative Values

7.13.1 Envy Attribute Shows Differences in Utility Levels

Here we assume the worker evaluates envy by examining the differences in utilities between his own wealth and that of his co-worker as demonstrated by the following equation:

$$V(w_i) = u(w_i) - e \cdot g(u(w_j) - u(w_i)) \text{ where } j \neq i.$$

Co-Worker is Non-envious If worker j is risk-averse then worker i 's maximization problem becomes

$$\max_{\alpha_{ei} \in [0,1]} E [u(w_i) - eg(u(w_j^*) - u(w_i))]$$

and α_j^* is determined by the maximization problem for the non-envious individual or

$$\alpha_j^* \in \arg \max_{\alpha_j \in [0,1]} E [u(w_j)].$$

Recall that the first order condition for a non-envious individual implies that α_j^* is such that it solves

$$E [w_0(R - r_f)u'(w_j)] = 0.$$

The first order condition for the envious investor is

$$E [w_0(R - r_f)u'(w_i) (1 + eg'(u(w_j^*) - u(w_i)))] = 0.$$

The second order condition is

$$\begin{aligned} &E [w_0^2(R - r_f)^2 u''(w_i) (1 + eg'(u(w_j^*) - u(w_i)))] \\ &- E [w_0^2(R - r_f)^2 u'^2(w_i) eg''(u(w_j^*) - u(w_i))]. \end{aligned}$$

Evaluating the second order condition at the point where both investors choose the same allocation, that is $\alpha_{ei} = \alpha_j^*$, we obtain

$$\begin{aligned} \frac{\partial^2 E}{\partial \alpha_{ei}^2} \Big|_{\alpha_{ei}=\alpha_j^*} &= E [w_0^2(R - r_f)^2 u''(w_j^*) (1 + eg'(0))] - E [w_0^2(R - r_f)^2 u'^2(w_j^*) eg''(0)] \\ &= E \{ w_0^2(R - r_f)^2 [u''(w_j^*) (1 + eg'(0)) - u'^2(w_j^*) eg''(0)] \} \end{aligned}$$

As $g''(0) > 0$, $u'' < 0$, and $(1 + eg'(0)) > 0$ then $\frac{\partial^2 E}{\partial \alpha_i^2} |_{\alpha_{ei} = \alpha_j^*} < 0$ so we have local concavity. Thus,

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} |_{\alpha_{ei} = \alpha_j^*} &= E [w_0 (R - r_f) u' (w_j^*) (1 + eg'(0))] \\ &= (1 + eg'(0)) E [w_0 (R - r_f) u' (w_j^*)] \\ &= 0. \end{aligned}$$

The second term in the middle line of equations above is the first order condition for the risk-averse worker, which equals zero. Therefore we have that $\frac{\partial E}{\partial \alpha_{ei}} |_{\alpha_{ei} = \alpha_j^*} = 0$ which implies there is a local maximum where $\alpha_{ei}^* = \alpha_j^* = \alpha_0^*$.

Co-Worker is Envious: A Nash Equilibrium Setting Suppose both worker i and worker j exhibit preferences with envy. In this case worker i solves the following maximization problem:

$$\begin{aligned} &\max_{\alpha_{ie} \in [0,1]} E [u(w_i) - eg (u(w_j^*) - u(w_i))] \\ \text{s.t. } \alpha_{ej}^* &\in \arg \max_{\alpha_{ej} \in [0,1]} E [u(w_j) - eg (u(w_i^*) - u(w_j))] \end{aligned}$$

which has a first order condition of

$$E [w_0 (R - r_f) u' (w_i) (1 + eg' (u(w_j^*) - u(w_i)))] = 0$$

as seen before. Worker j solves a similar maximization problem (interchange all i 's with j 's and vice versa) with a similar first order condition of

$$E [w_0 (R - r_f) u' (w_j) (1 + eg' (u(w_i^*) - u(w_j)))] = 0$$

For a Nash Equilibrium both first order conditions above need to hold. Setting the first order conditions equal to each other we find that for a Nash equilibrium we would need $\alpha_{ei}^* = \alpha_{ej}^*$. Additionally,

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} |_{\alpha_{ei} = \alpha_{ej} = \alpha} &= E [w_0 (R - r_f) u' (w_0 (1 + \alpha R + (1 - \alpha) r_f)) (1 + eg'(0))] \\ &= (1 + eg'(0)) \cdot E [w_0 (R - r_f) u' (w_0 (1 + \alpha R + (1 - \alpha) r_f))] \end{aligned}$$

Evaluating the above equation at $\alpha = \alpha_0^*$ we see that the second term becomes zero as it is the first order condition for the purely risk-averse individual. Therefore,

$$\frac{\partial E}{\partial \alpha_{ei}} |_{\alpha_{ei} = \alpha_{ej} = \alpha_0^*} = 0$$

which implies $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.

Each worker's optimal allocation is a function of the other worker's portfolio allocation (i.e. $\alpha_{ei} = \alpha_{ei}(\alpha_{ej})$). Taking the total differential of the first order condition for the envious worker with respect to α_{ej} and rearranging terms we find

$$\alpha'_{ei}(\alpha_{ej}) = \frac{-E \left[\frac{w_0^2 (R - r_f)^2 u' (w_0 (1 + \alpha_{ei}(\alpha_{ej}) R + (1 - \alpha_{ei}(\alpha_{ej})) r_f)) u' (w_j)}{\cdot eg'' (u(w_j) - u(w_0 (1 + \alpha_{ei}(\alpha_{ej}) R + (1 - \alpha_{ei}(\alpha_{ej})) r_f)))} \right]}{E \left\{ \left[\begin{array}{c} \frac{w_0^2 (R - r_f)^2}{\left(\begin{array}{c} u'' (w_0 (1 + \alpha_{ei}(\alpha_{ej}) R + (1 - \alpha_{ei}(\alpha_{ej})) r_f)) \\ \cdot (1 + eg' (u(w_j) - u(w_0 (1 + \alpha_{ei}(\alpha_{ej}) R + (1 - \alpha_{ei}(\alpha_{ej})) r_f))) \end{array} \right)} \right) \\ - \left(\begin{array}{c} u'^2 (w_0 (1 + \alpha_{ei}(\alpha_{ej}) R + (1 - \alpha_{ei}(\alpha_{ej})) r_f)) \\ \cdot eg'' (u(w_j) - u(w_0 (1 + \alpha_{ei}(\alpha_{ej}) R + (1 - \alpha_{ei}(\alpha_{ej})) r_f))) \end{array} \right) \end{array} \right] \right\}}$$

Therefore,

$$\alpha'_{ei}(\alpha_{ej})|_{\alpha_{ei}=\alpha_{ej}=\alpha} = \frac{-E \left[w_0^2 (R - r_f)^2 u'^2 (w_0 (1 + \alpha R + (1 - \alpha) r_f)) eg''(0) \right]}{E \left[w_0^2 (R - r_f)^2 \left(\begin{array}{c} u'' (w_0 (1 + \alpha R + (1 - \alpha) r_f)) (1 + eg'(0)) \\ - (u'^2 (w_0 (1 + \alpha R + (1 - \alpha) r_f)) eg''(0)) \end{array} \right) \right]}.$$

The numerator is always negative and the denominator is always negative which implies that $\alpha'_{ei}(\alpha_{ej})|_{\alpha_{ei}=\alpha_{ej}=\alpha} > 0$. Additionally, we can see that $\alpha'_{ei}(\alpha_{ej})|_{\alpha_{ei}=\alpha_{ej}=\alpha} < 1$. Similarly, $0 < \alpha'_{ej}(\alpha_{ei}) < 1$. The best reply functions have a slope between zero and one when it crosses the 45 degree line of a plot of both best replies. Furthermore,

$$\begin{aligned} \alpha'_{ei}(\alpha_{ej})|_{\alpha_{ei}=\alpha_{ej}=0} &= \frac{-E \left[w_0^2 (R - r_f)^2 u'^2 (w_0 (1 + r_f)) eg''(0) \right]}{E \left[w_0^2 (R - r_f)^2 (u'' (w_0 (1 + r_f)) (1 + eg'(0)) - u'^2 (w_0 (1 + r_f)) eg''(0)) \right]} \\ &= \frac{-u'^2 (w_0 (1 + r_f)) eg''(0)}{u'' (w_0 (1 + r_f)) (1 + eg'(0)) - u'^2 (w_0 (1 + r_f)) eg''(0)} \\ &< 1. \end{aligned}$$

Additionally, $\alpha'_{ej}(\alpha_{ei})|_{\alpha_{ej}=\alpha_{ei}=0} < 1$.

Considering the first order condition for worker i assuming that worker j invests everything in the risk-free asset ($\alpha_{ej} = 0$) we find

$$\frac{\partial E}{\partial \alpha_{ei}}|_{\alpha_{ei}=\alpha_{ei}(0)} = E \left[\cdot \left(\begin{array}{c} w_0 (R - r_f) u' (w_0 (1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0)) r_f)) \\ 1 + eg' \left(\begin{array}{c} u (w_0 (1 + r_f)) \\ -u (w_0 (1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0)) r_f)) \end{array} \right) \end{array} \right) \right]$$

which when we evaluate at the point where worker i also invests everything in the risk-free asset we obtain

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}}|_{\alpha_{ei}(0)=0} &= E [w_0 (R - r_f) u' (w_0 (1 + r_f)) (1 + eg'(0))] \\ &= w_0 (1 + eg'(0)) u' (w_0 (1 + r_f)) (E [R] - r_f). \end{aligned}$$

Therefore, if the equity premium is positive (i.e. $E[R] - r_f > 0$) then the above is positive which implies that $\alpha_{ei}^*(0) > 0$. If we did the same analysis for worker j we would also see that if the equity premium is positive then $\alpha_{ej}^*(0) > 0$. From the first order condition for worker i we see that if worker j invests everything in the risky asset we have

$$\frac{\partial E}{\partial \alpha_{ei}}|_{\alpha_{ei}=\alpha_{ei}(1)} = E \left[\cdot \left(\begin{array}{c} w_0 (R - r_f) u' (w_0 (1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1)) r_f)) \\ 1 + eg' \left(\begin{array}{c} u (w_0 (1 + R)) \\ -u (w_0 (1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1)) r_f)) \end{array} \right) \end{array} \right) \right]$$

which when we also assume worker i invests everything in the risky asset becomes

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}}|_{\alpha_{ei}(1)=1} &= E [w_0 (R - r_f) u' (w_0 (1 + R)) (1 + eg'(0))] \\ &= w_0 (1 + eg'(0)) [(E[R] - r_f) E[u' (w_0 (1 + R))] + Cov (R, u' (w_0 (1 + R)))] . \end{aligned}$$

The preceding equation implies that if $E[R] - r_f \geq \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))]}$ then $\alpha_{ei}^*(1) = 1$ and if $E[R] - r_f < \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))]}$ then $\alpha_{ei}^*(1) < 1$. Doing the same analysis for worker j we obtain that if $E[R] - r_f \geq \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))]}$ then $\alpha_{ej}^*(1) = 1$ and if $E[R] - r_f < \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))]}$ then $\alpha_{ej}^*(1) < 1$.

Summarizing the results of the best reply functions in a graph (same graph as with g function convex over all values) we see that the functions intersect at only one point. There is a unique Nash Equilibrium where $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.

7.13.2 Envy Attribute Shows Differences in Wealth Levels

Here, we assume the worker exhibiting envy has a utility function with the following form:

$$V(w_i) = u(w_i) - e \cdot g(w_j - w_i) \text{ where } j \neq i.$$

Co-Worker is Non-envious Assuming that the worker exhibiting envy has a co-worker that is risk-averse (non-envious) the optimization problem for worker i becomes

$$\begin{aligned} & \max_{\alpha_{ei} \in [0,1]} E [u(w_i) - eg(w_j^* - w_i)] \\ \text{s.t. } \alpha_j^* & \in \arg \max_{\alpha_j \in [0,1]} E [u(w_j)]. \end{aligned}$$

The first order condition for this maximization problem is

$$E [w_0(R - r_f) (u'(w_i) + eg'(w_j^* - w_i))] = 0.$$

The second order condition for this maximization problem is

$$E [w_0^2(R - r_f)^2 (u''(w_i) - eg''(w_j^* - w_i))].$$

Again we do not have the second order condition strictly less than zero. Evaluating the second order condition where the investment in the risky asset is the same for both workers, we find we have local concavity. Thus,

$$\begin{aligned} \frac{\partial^2 E}{\partial \alpha_{ei}^2} \Big|_{\alpha_{ei}=\alpha_j^*} &= E [w_0^2(R - r_f)^2 (u''(w_j^*) - eg''(0))] \\ &< 0. \end{aligned}$$

Considering the first order condition where worker i chooses his co-worker's optimal allocation we have

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_j^*} &= E \{w_0(R - r_f) [u'(w_j^*) + eg'(0)]\} \\ &= w_0 eg'(0) (E[R] - r_f) + E [w_0(R - r_f) u'(w_j^*)] \\ &= w_0 eg'(0) (E[R] - r_f). \end{aligned}$$

If $E[R] - r_f = 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_j^*} = 0$ which implies $\alpha_{ei}^* = \alpha_j^* = \alpha_0^*$ and $E[R] - r_f > 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_j^*} > 0$ which implies $\alpha_{ei}^* > \alpha_j^* = \alpha_0^*$. This is our local maximum.

Co-Worker is Envious: A Nash Equilibrium Setting Suppose now both workers exhibit preferences with envy. Worker i 's maximization problem becomes

$$\begin{aligned} & \max_{\alpha_{ei} \in [0,1]} E [u(w_i) - eg(w_j^* - w_i)] \\ \text{s.t. } \alpha_{ej}^* & \in \arg \max E [u(w_j) - eg(w_i^* - w_j)]. \end{aligned}$$

Worker j has a similar maximization problem (interchange the i 's and j 's in the above equation). The first order condition for worker i is

$$E \{w_0(R - r_f) [u'(w_i) + eg'(w_j^* - w_i)]\} = 0$$

and for worker j is

$$E \{w_0(R - r_f) [u'(w_j) + eg'(w_i^* - w_j)]\} = 0.$$

Setting the first order conditions for both workers equal to each other we find that the Nash equilibrium is $\alpha_{ei}^* = \alpha_{ej}^*$. Thus,

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} &= E \{w_0(R - r_f) [u'(w_0(1 + \alpha R + (1 - \alpha)r_f)) + eg'(0)]\} \\ &= w_0(E[R] - r_f)eg'(0) + E[w_0(R - r_f)u'(w_0(1 + \alpha R + (1 - \alpha)r_f))] \end{aligned}$$

Evaluating the above at the optimal allocation for the risk-averse (non-envious) worker, α_0^* , we find that the second term becomes zero. Therefore if $E[R] - r_f > 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha_0^*} > 0$ which implies $\alpha_{ei}^* = \alpha_{ej}^* > \alpha_0^*$. Additionally, if $E[R] - r_f = 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha_0^*} = 0$ which implies $\alpha_{ei}^* = \alpha_{ej}^* = \alpha_0^*$.

Each worker's optimal allocation is a function of the other worker's portfolio allocation (i.e. $\alpha_{ei} = \alpha_{ei}(\alpha_{ej})$). Taking the total differential of the first order condition for the envious worker with respect to α_{ej} and rearranging terms we find

$$\alpha'_{ei}(\alpha_{ej}) = \frac{E \left[w_0^2 (R - r_f)^2 eg''(w_j - w_0(1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) \right]}{E \left[w_0^2 (R - r_f)^2 \left(\begin{array}{c} eg''(w_j - w_0(1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) \\ -u''(w_0(1 + \alpha_{ei}(\alpha_{ej})R + (1 - \alpha_{ei}(\alpha_{ej}))r_f)) \end{array} \right) \right]}.$$

and

$$\alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} = \frac{E \left[w_0^2 (R - r_f)^2 eg''(0) \right]}{E \left[w_0^2 (R - r_f)^2 (eg''(0) - u''(w_0(1 + \alpha R + (1 - \alpha)r_f))) \right]}$$

The numerator and denominator are always positive which implies that $\alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} > 0$. Additionally, we can see that $\alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=\alpha} < 1$. As worker j 's problem is the same as worker i 's we find that $0 < \alpha'_{ej}(\alpha_{ei}) \Big|_{\alpha_{ej}=\alpha_{ei}=\alpha} < 1$. Furthermore,

$$\begin{aligned} \alpha'_{ei}(\alpha_{ej}) \Big|_{\alpha_{ei}=\alpha_{ej}=0} &= \frac{E \left[w_0^2 (R - r_f)^2 eg''(0) \right]}{E \left[w_0^2 (R - r_f)^2 (eg''(0) - u''(w_0(1 + r_f))) \right]} \\ &= \frac{eg''(0)}{eg''(0) - u''(w_0(1 + r_f))} \\ &< 1. \end{aligned}$$

Additionally, $\alpha'_{ej}(\alpha_{ei}) \Big|_{\alpha_{ej}=\alpha_{ei}=0} < 1$.

Considering the first order condition for worker i assuming that worker j invests everything in the risk-free asset ($\alpha_{ej} = 0$) we find

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(0)} = E \left[w_0(R - r_f) \left(\begin{array}{c} u'(w_0(1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0))r_f)) \\ +eg' \left(\begin{array}{c} w_0(1 + r_f) \\ -w_0(1 + \alpha_{ei}(0)R + (1 - \alpha_{ei}(0))r_f) \end{array} \right) \end{array} \right) \right].$$

Evaluating the above equation where worker i allocates all his wealth in the risk-free asset we have

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(0)=0} &= E [w_0(R - r_f) (u'(w_0(1 + r_f)) + eg'(0))] \\ &= w_0 (u'(w_0(1 + r_f)) + eg'(0)) (E[R] - r_f). \end{aligned}$$

If $E[R] - r_f > 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(0)=0} > 0$ which implies $\alpha_{ei}^*(0) > 0$. If $E[R] - r_f = 0$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(0)=0} = 0$

which implies $\alpha_{ei}^*(0) = 0$. If we did the same analysis for worker j we would also see that if the equity premium is positive then $\alpha_{ej}^*(0) > 0$ and if the equity premium is zero then $\alpha_{ej}^*(0) = 0$.

From the first order condition for worker i we see that if worker j invests everything in the risky asset we have

$$\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}=\alpha_{ei}(1)} = E \left[w_0 (R - r_f) \left(\begin{array}{c} u' (w_0(1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1))r_f)) \\ + eg' \left(\begin{array}{c} w_0 (1 + R) \\ -w_0(1 + \alpha_{ei}(1)R + (1 - \alpha_{ei}(1))r_f) \end{array} \right) \end{array} \right) \right]$$

which when we evaluate at the point where worker i also invests everything in the risky asset we obtain

$$\begin{aligned} \frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(1)=1} &= E [w_0 (R - r_f) (u' (w_0 (1 + R)) + eg'(0))] \\ &= w_0 (E[R - r_f]E [u' (w_0 (1 + R)) + eg'(0)] + Cov (R, u' (w_0 (1 + R)))). \end{aligned}$$

If $E[R] - r_f \geq \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))+eg'(0)]}$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(1)=1} \geq 0$ which implies $\alpha_{ei}^*(1) = 1$. If $E[R] - r_f < \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))+eg'(0)]}$ then $\frac{\partial E}{\partial \alpha_{ei}} \Big|_{\alpha_{ei}(1)=1} < 0$ which implies $\alpha_{ei}^*(1) < 1$. Doing the same analysis for worker j we obtain that if $E[R] - r_f \geq \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))+eg'(0)]}$ then $\alpha_{ej}^*(1) = 1$ and if $E[R] - r_f < \frac{-Cov(R, u'(w_0(1+R)))}{E[u'(w_0(1+R))+eg'(0)]}$ then $\alpha_{ej}^*(1) < 1$.

Summarizing the results of the best reply functions in a graph (same as that shown when g function is convex over all values) we see that the functions intersect at only one point. There is an unique Nash Equilibrium which corresponds to both workers investing the same amount in the risky-asset and this allocation is a riskier portfolio than the risk-averse (non-envious) investor's optimal portfolio allocation if the equity premium is positive. If the equity premium is zero, the unique Nash equilibrium occurs where both workers choose the same allocation, which is the optimal allocation of a non-envious investor.

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