



**Co-opetition & Investment for Resilience
in Global Supply Chains**

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Abstract

This paper considers the problem of disruption risk management in global supply chains. We consider a supply chain with two participants, who face interdependent losses resulting from supply chain disruptions such as terrorist strikes and natural hazards. The Harsanyi-Selten-Nash bargaining framework is used to model the choice of supply chain participants of risk mitigation incentives. The bargaining approach allows a framing of both joint financing of mitigation activities before the fact and loss-sharing net of insurance payouts after the fact. The disagreement outcome in the bargaining game is assumed to be the result of the corresponding non-cooperative game. We describe an incentive compatible contract that leads to First Best investment and equal ‘gain’ for all players, when the solution is ‘interior’ (as it almost certainly is in practice). A supplier that has superior security practices (i.e., is inherently safer) exploits its informational advantage by extracting an ‘information rent’ in the usual spirit of incomplete information games. We also identify a special case of this contract, which is robust to moral hazard. The role of auditing in reinforcing investment incentives is also examined.

KEY WORDS: security, supply chain disruptions, risk, bargaining.

1 Introduction

The combined threat posed by natural hazards and purposeful human agents has heightened the riskiness of operations in contemporary business. This has been further exacerbated by the greater complexity and global reach of many supply chains, occasioned by unbundling, outsourcing and globalization that are the hallmarks of economic activity in the 21st century. These developments have led to a heightened interest to identify vulnerabilities in global supply chains, and to design appropriate metrics and management systems to strike a balance between leanness and resilience of supply chains to potential disruptions. The primary reason for the increased interest in disruption management is the awareness, promoted by recent research, regarding the magnitude of losses

resulting from supply chain disruptions. These losses include both direct losses (e.g., physical damage, and lost sales due to shutdown), as well as significant indirect impacts on the company's market value.

The most important empirical evidence on the magnitude of supply disruption costs is the work of Hendricks and Singhal (2005). They use an event study methodology to examine the stock price effects of supply chain disruptions during the period 1989-2000. They found for a sample of major disruptions occurring in the 1990s that the average abnormal stock returns of firms that experienced a disruption was nearly -40% over the two-year period following disruptions. They also found that equity risk was 13.5% higher in the year after the announcement of a significant supply chain disruption as compared to the year prior to the announcement. The magnitude of these effects, and recent losses from natural hazard events, has been a major catalyst in focusing attention on understanding and mitigating potential supply chain disruptions.

One of the major sectors where these trends are evident is the retail sector. The Retail Industry Leaders Association (RILA) is a trade association of US retailers. Its members account for \$1.5 trillion in annual sales. Its goal is to champion causes relevant to the retail industry. One of RILA's committees is devoted to issues in Supply Chain Security. As part of its 2006 public policy agenda, RILA identified 'maintaining a safe, efficient and reliable supply chain,' as a top priority. Their policy rationale states, "Supply chain security is a global issue that cannot be addressed unilaterally...it is important for policy integration within a global framework with foreign trading partners and organizations... ." These notions have been strongly reinforced ever since the 9/11 attacks and the 2002 West Coast port strike (Cooke and Gooley 2002), both of which took a large toll on retail operations. The consequences of these events and the 2004-2005 hurricane seasons have brought home to retailers the critical importance of supply-chain wide visibility and resilience to major disruptions.

The response to these events involves not just private actors, such as Home Depot or Toyota. The problems are also recognized as requiring national and international coordination to assure continuing benefits of cross-border trade while controlling and responding to security threats. For example, the Asia-Pacific Economic Cooperation (APEC), an economic forum for a group of Pacific Rim countries (of which the US is a member), has prepared supply chain security guidelines for the private sector (APEC 2006). The key recommendations of its report suggest that complex, multi-country supply chains demand greater collaboration on security issues. Security inside the

organizational boundaries of individual actors along the supply chain is not sufficient. Collaboration across all the public and private actors, from companies to logistics providers to port authorities, is essential in achieving an efficient solution. The resulting collaboration has the character of co-opetition noted in Brandenburger and Nalebuff (1996), in which competition across supply chains continues, but cooperation is also required to determine workable standards for management systems, for tamper-proof containers, for tracking and clearance methods, and for other procedural issues noted in Kleindorfer and Saad (2005). This collaboration is, in part, the result of the weakest link property of supply chains and networks, and the recognition that a disruption along any link will cause a disruption in the entire supply chain. Hence, risk management strategies require participation of all supply chain partners, in order to be effective. In such a situation it seems natural that ‘cooperation’ (bargaining for redistribution of gains) amongst supply chain partners would be superior to a non-cooperative or uncoordinated approach.

To make these tradeoffs concrete, consider the challenge of justifying investment in Radio Frequency Identification (RFID) technology as a means of increasing supply chain security. RFID is a generic term for technologies that use radio waves to automatically identify people or objects. By improving inventory tracking and accuracy, RFID implementation also results in reduced uncertainty in lead times and demand, and lower pilferage and shrinkage (Uckun et al. 2005, Gaukler et al. 2007). RFID technology has obvious benefits in enhancing security of supply chains; especially pertaining to prevention of container tampering (Lee 2004). A treatment of the broad range of implications of RFID for operations management, can be found in the special issue (2007, 16(4)) of *Production and Operations Management* dedicated to RFID applications.

Consider a retailer in the United States facing the decision of whether to invest in RFID infrastructure or not. Clearly, it would like its supplier to share the cost of setting up the infrastructure, since the supply chain has to be secured end to end. For the supply chain partners, however, the level of RFID investment required for effective implementation of the solution may not be in accordance with their level of risk exposure. A key question to be addressed here is, what share of the total cost (sum of security investment and expected disruption losses) should be borne by each of the participants in the supply chain?

In this paper we formally study a bargaining game that mirrors the characteristics of the problem, including the impact of asymmetric information. We consider a scenario in which one firm, say a downstream retailer, whose type or level of vulnerability to disruptions is known,

bargains with its supply chain partner or upstream supplier, whose level of vulnerability is not known to the retailer. We consider the simplest model here of two supplier types, which we call “safe” and “unsafe.” The retailer’s beliefs regarding the type of the supplier are assumed to be ‘common knowledge.’ The bargaining exercise is undertaken at an *interim* stage, when the retailer has developed prior beliefs pertaining to the probability distribution over the type of the supplier, while the supplier itself is aware of its true type, and the uncertainty pertaining to the disruption has not yet been resolved .

The disagreement outcome of the bargaining game is the corresponding non-cooperative game played between the players. This assumption pre-supposes that the players have established a business relationship; i.e., a certain degree of ‘bilateral monopoly’ is assumed. Thus, strategies such as multi-sourcing, or changing the supply chain network by switching suppliers, are beyond the scope of our model, except to the extent that some fixed alternative sourcing arrangement may be captured in the disagreement or default outcome. As is usual in formal bargaining theory, the nature of the disagreement outcome is an important determinant of predicted bargaining outcomes as it embodies the bargaining power of each player.

We obtain results on the efficient levels of investment for risk mitigation, and on the benefits of cooperation. These provide interesting insights into the challenges of designing supply chain wide security policies, including the following contributions to supply chain security literature:

- i) We use the Harsanyi-Selten-Nash (HSN) axiomatic framework for bargaining under incomplete information, as extended by Myerson (1979), to identify a cost sharing contract which leads to First Best investment in supply chain security, when the solution is interior (i.e., the solution involves some security investment for both supply chain partners and the supplier’s share of losses from a disruption is positive). We highlight the possibility of under-investment for some boundary solutions. We also show that a non-cooperative approach leads to under-investment in security, which is consistent with both intuition and with other results in the bargaining literature.
- ii) For the interior solution described above, we obtain the usual HSN bargaining outcome that the ‘gain’ from bargaining is equal for all players. By ‘gain’ from bargaining, we are referring to the incremental savings over the respective disagreement outcome. This results in the “safe type” of supplier incurring a lower cost than the “unsafe type” in equilibrium. We interpret

this result as the retailer ceding an *information rent* to the safe type of supplier, to ensure that it does not try to mimic the unsafe type in order to obtain more favorable shared-investment treatment. This result is standard in Principal-Agent theory in non-cooperative settings. The interesting aspect of our finding is that *information rent* persists in certain cooperative settings.

- iii) Moral hazard remains a problem, even at the Bargaining Solution. One approach to overcome this problem is to analyze a bargaining contract under which the retailer “buys out” the supplier’s share of losses in return for an upfront transfer payment from the supplier. We can think of this contract as having the retailer establish a captive insurance company that covers the particular supply-chain losses in question, with insurance premiums (upfront transfer payments) paid by all supply chain partners. We show that this contract leads to First-Best outcomes. It is predicated on either investments by the supplier being observable (even though his type may not be) or on the insurance contract being made contingent on an audit verifying that the required investment has been made by the supplier. This same “buy-out” bargaining contract has the property that more precise auditing, which reveals the type of the supplier, is not valuable because the incentives embedded in the type-specific transfer payment of this bargaining contract leave no further efficiencies to be recovered from audit information.
- iv) In the non-cooperative game played under incomplete information, we establish that the unsafe supplier invests more in security at equilibrium, but also incurs a higher expected cost, than the safe type. We also find that, as the likelihood of the supplier being safe goes up, each player’s investment in security also increases, and the expected cost incurred by each player decreases. This suggests that there may be a virtuous cycle of increasing investment and improving security in global supply chains.

The remainder of the paper is organized as follows. Section 2 contains a brief literature review. In Section 3 we introduce the two-player model. We describe the welfare maximizing solution in Section 3.1. In Section 3.2 we characterize the solution to the non-cooperative Bayesian game played between the players; which also serves as the disagreement outcome of the bargaining game. Section 3.3 describes the bargaining game with incomplete information. Section 4 contains conclusions and a discussion of the managerial implications of our results for operations, including the relationship

between our results and the emerging guidelines from APEC on the implementation of best practices for supply chain security.

2 Literature Review

Coping with the risk of disruptions has been an important theme in the literature on disaster risk in Operations Management. The adverse impact of a disruption can be mitigated by taking steps to reduce the probability of the disruption occurring, or by resuming normal operations quickly and thereby curbing losses. A survey of the area is presented in Altay and Green (2006).

The literature on supply chain security/resilience is fairly rich when it comes to strategies related to sourcing and inventory policies. In Figure 1, we cite some research articles, which serve to illustrate the landscape of Supply Chain Security literature. This is by no means a comprehensive compilation, but it provides salient examples of recent work in this area and a sense of the broad scope of on-going research. The closely related area of Homeland Security has also received a great deal of attention from the research community. For instance, an entire issue of *Interfaces* (Volume 36, Issue 6) was devoted to concerns on that front.

Supply Chain Security has also been an extremely active area of development for industry practice (Sheffi 2005). This is partly the result of continuing concerns with terrorism by private companies, but it also reflects the interaction between publicly controlled port authorities and governmental regulators who are providing a variety of carrots and sticks to private companies to motivate them to invest in Supply Chain Security. The scope of recent developments in industrial practice can be seen in the “APEC Private Sector: Supply Chain Security Guidelines,” (APEC 2006) which provides best practice guidelines for strategies in areas such as: physical security, access control, personnel security, education and training, procedural security, documentation and process security, conveyance security, crisis management and disaster recovery. Coupled with audit and bench-marking procedures, Supply Chain Security has become an active area of investment and management system development for a broad spectrum of industries with global reach.

In addition to the concerns about terrorism, supply chain managers have also become much more aware of the consequences of natural disasters and major accidents at their own facilities and those of supply chain partners. This increased awareness is partly the result of increased complexity

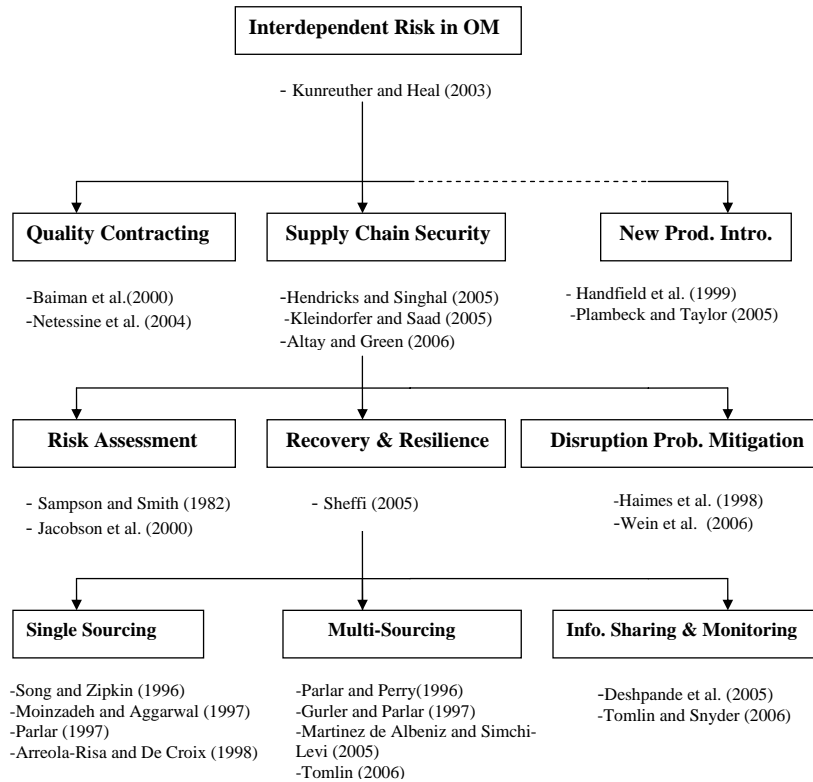


Figure 1: Supply Chain Security/Resilience literature landscape

and exposure to these events of global supply chains. Increased storm activities in recent years have added to these concerns. Finally, the insurance and reinsurance industries have been engaged in a detailed re-examination of business interruption losses related to these events, and pressures on commercial insurance rates and on coverages offered have further brought attention of senior management to the expected costs of supply chain vulnerabilities (GSU-Wharton-iii 2007).

In this paper, we consider the problem of joint investment by supply chain participants in in supply chain security and resilience. The analysis spans issues in ‘Disruption Probability Mitigation’ and ‘Recovery and Resilience’ (see Figure 1). The underlying model that we use is based on the seminal work by Shavell (1984), on modeling risk management. We introduce the single player model in Figure 2 below. An investment of Y dollars in security results in a probability $P(Y)$ of occurrence of a disruptive event. A disruption results in a loss of D dollars. The expected total cost of risk management: $C(Y) = Y + P(Y)D$, and $Y^* \equiv$ optimal investment in supply chain security.

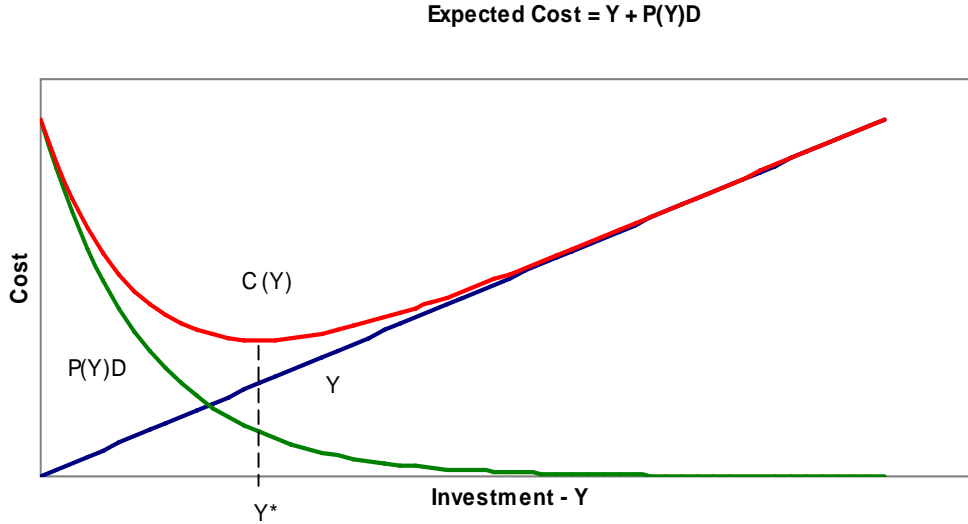


Figure 2: Plot of Expected Cost Vs. Investment for a single player model.

We assume that at most one disruption can occur during the planning horizon, since we are limiting our attention to low frequency, catastrophic disruptions. In the context of this research, the probability of disruption, $P(Y)$ pertains some well defined class of failure modes. Failure mode refers to a general category of events which give rise to a specific form of supply chain failure, regardless of the actual type of disruptive event. Each specific failure could be generated by different causes, such as fires, natural hazard events, explosions, or labor unrest at key intermediary facilities, but the effect on the supply network is nearly the same for failures belonging to the same mode (Rice and Caniato 2003). For instance, no matter what the cause of large-scale physical damage to a supplier's manufacturing plant, the result is a prolonged shut-down and is similar in its impact on the supply chain. For this case, the failure mode would be 'plant shut-down.'

The objective of the risk management exercise in the single player model is to minimize the expected cost of mitigation activities and the residual level of disruption risk, which is driven by the level of mitigation investment, Y^* . By elementary calculus we obtain $Y^* = P'^{-1}(\frac{-1}{D})$, which is the level of investment at which the marginal benefit from investing in security is equal to the marginal cost incurred.

The simple model described above neglects many operational level details, but it captures the main tradeoffs involved in making the security investment decision. Indeed, the model of Shavell

(1984), has engendered a rich stream of theoretical work in risk management and regulation of environmental safety of industrial processes. The Shavell model can be extended to the more complicated setting in this paper. Specifically, this paper considers the two-player model, with interdependent risk, in greater detail. Our two-player model is based on the axiomatic bargaining framework proposed by Nash (1950). The well-known Nash Bargaining Solution entails the maximization of the Nash product as the predicted mutually acceptable outcome of the bargaining game:

$$\max_{x_i} \prod_{i=1}^N (x_i - a_i)$$

where x_i is the outcome of the bargaining game for player i , while a_i is the disagreement outcome, which occurs when the players fail to reach an agreement. In our model, the disagreement outcome is, itself, the solution to a corresponding non-cooperative game. The Nash bargaining solution has found several applications in recent supply chain literature, e.g., Gan et al. (2004), Plambeck and Taylor (2005).

However, participants in a global supply chain may have incentive to hide their true type; i.e., the extent of their vulnerability to disruptions. Disclosing their true type could make the players susceptible to adverse market sentiment (leading to a negative impact on the stock price) or pressure to invest more in security measures than they would otherwise from a purely business perspective. The resulting *adverse selection* issues have been studied in detail in the Principal-Agent literature (Laffont and Martimort 2001).

Harsanyi and Selten (1972) (henceforth referred to as H&S) developed a framework to solve the bargaining problem with incomplete information. Their work was further extended and simplified, including a characterization of the Revelation Principle, in Myerson (1979). We use this as the starting point of our two-player model. There has been some debate in the literature on the appropriateness of the axiomatic approach to bargaining vis-a-vis the strategic approach (sometimes also referred to as the non-cooperative approach to bargaining). We address this debate in Appendix A.

The interdependent nature of risk in supply chains is not a unique phenomenon in the Operations Management context. We conclude our review by identifying two analogous streams of research. Quality Management and New Product Introduction (NPI) present similar challenges pertaining to interdependent risk across supply chain partners. However, the differentiating fea-

ture of catastrophic disruptions is that, unlike in Quality Management, it does not make sense for the players to design contingent contracts that use occurrence of failures as a gauge for performance. Catastrophic disruptions are sufficiently low probability that lagging indicators based on actual catastrophic events would not be useful.

Collaboration for NPI is similar in spirit to the context of a joint venture. Darrough and Stoughton (1989) have looked at profit sharing in joint ventures, using a bargaining approach with incomplete information. Our model differs from theirs in some critical ways. Even though they allow for a continuum of types for the players, as contrasted with our discrete type setup, they assume symmetric partners whose respective types are distributed uniformly over a finite interval. This precludes incorporating publicly available information about the other’s type, in either player’s bargaining strategy. Also, their model does not consider any explicit disagreement outcome (like the non-cooperative game in our model), which can significantly impact the players’ bargaining power and is an important element in global supply chains which typically consist of a mix of large and small firms operating in different environments and with rather different bargaining positions.

3 Two Player Model

We build on the single-player model described in the literature review, to analyze the two-player setting. We incorporate into our model, the framework for incentive compatible bargaining, as described in Myerson (1979). We define a two-echelon supply chain, with a retailer and supplier. The supplier can be of either of two types - {safe,unsafe}, or { s,u }. The type is defined with respect to the level of vulnerability to disruptions. It is determined with accuracy after an exercise in risk assessment, undertaken at an interim stage in the contracting process; i.e., after the supplier knows its true type, but before the occurrence of a disruption.

The probability distribution over the types of the supplier is assumed to be ‘common knowledge’ amongst the participants. If the type of the supplier is represented by τ then:

$$P(\tau = s) = p \quad \text{and} \quad P(\tau = u) = 1 - p,$$

where $0 \leq p \leq 1$. We assume that the players are risk neutral profit maximizers.

In the subsequent analysis, we use the following notation:

- $C_{s\tau}$: The cost incurred by the supplier of type τ , as a result of the bargaining process, for $\tau \in \{s, u\}$.
- C_r : The cost incurred by the retailer as a result of the bargaining process.
- $t_{s\tau}$: The cost incurred by the supplier of type τ , as a result of the non-cooperative Bayesian game, for $\tau \in \{s, u\}$.
- t_r : The cost incurred by the retailer as a result of the non-cooperative Bayesian game.
- $y_{s\tau}$: Security investment made by the supplier of type $\tau \in \{s, u\}$. For the non-cooperative game this investment is represented as $y_{s\tau}^{nc}$.
- y_r : Security investment made by the retailer. For the non-cooperative game this investment is represented as y_r^{nc} .
- $P_{s\tau}(y_{s\tau})$: Probability of a disruption occurring in the supplier's local supply chain, given that the supplier of type τ and investment in risk mitigation is $y_{s\tau}$; where $\tau \in \{s, u\}$.
- $P_r(y_r)$: Probability of disruption occurring in the retailer's local supply chain, given that the investment in risk mitigation is y_r .
- $P_{J\tau}(y_{s\tau}, y_r)$: Probability of a disruption occurring anywhere in a two player supply chain, given that the supplier is of type τ and invests $y_{s\tau}$, while the retailer invests y_r .
- D_J : Damage/Losses in the overall supply chain, resulting from a disruption.
- f : Share of the disruption related losses assigned to the supplier in the non-cooperative Bayesian game. Its value is pre-determined. For instance, its value may be determined based on the contribution of each player to the overall business.
- $f_{s\tau}$: Share of the disruption related losses assigned to the supplier of type τ , in the bargaining game, for $\tau \in \{s, u\}$.
- $\gamma_{s\tau}$: Transfer payment (or side payment) made by the supplier of type τ to the retailer, in the bargaining game, for $\tau \in \{s, u\}$.
- p : Probability that the supplier is of type s . Thus, the probability that the supplier is of type u is $(1 - p)$.

Characterization of ‘Vulnerability Type’

The term ‘type’ embodies a complete description of the player’s relevant characteristics: its preferences, beliefs, abilities and endowments. In the context of this paper, type or level of vulnerability is defined with respect to the possibility of occurrence of a disruption. It is determined by a combination of the kind of infrastructure already in place for risk mitigation, as well as environmental factors such as political turmoil, proximity to a fault line / volcano, etc.

Mathematically, we capture the concept of vulnerability through the supplier’s marginal probability of disruption as a function of investment, i.e., $P_{s\tau}(y_{s\tau})$, or probability of a disruption occurring in the supplier’s local supply chain, given that the supplier of type τ and investment in risk mitigation is $y_{s\tau}$; where $\tau \in \{s, u\}$. $P_r(y_r)$ is the probability of a disruption occurring in the retailer’s local supply chain, given that the investment in risk mitigation is y_r . We define and mathematically characterize the concept of vulnerability in the following assumption:

Assumption 1:

(i) The marginal probabilities of disruption for both, the supplier and retailer, are assumed to be strictly decreasing in investment.

(ii) For the same level of investment, the probability of disruption is greater for the type with higher level of vulnerability:

$$P_{su}(y) > P_{ss}(y), \forall y \in \mathcal{R}_+.$$

(iii) To begin with, an unsafe supplier is more responsive to incremental investment in security:

$$\left| \frac{\partial P_{su}(y)}{\partial y} \right|_{y=0} > \left| \frac{\partial P_{ss}(y)}{\partial y} \right|_{y=0}.$$

We also assume independence of the marginal disruption probabilities at the supplier and retailer locations. Thus, the form of the ‘joint’ probability of a disruption occurring anywhere in the supply chain, with a supplier of type $\tau \in \{s, u\}$, is:

$$P_{J\tau}(y_{s\tau}, y_r) = 1 - (1 - P_{s\tau}(y_{s\tau}))(1 - P_r(y_r)).$$

Exponential Functional Form

For the case of low probability events, such as ones we are studying, the exponential functional form naturally follows from the logistic regression model used for estimating the probability of

disruption, $P(y)$, as a function of the investment level y :

$$\log\left(\frac{P}{1-P}\right) = a - \alpha y,$$

where a and α are non-negative constants. This relationship also forms the basis of models used in the literature to represent the probability of a terrorism-related disruption (Major 2002). Assuming $P \ll 1$ and representing e^a as θ , we obtain:

$$P(y) \approx \theta e^{-\alpha y}.$$

We now obtain:

$$P_{J\tau}(y_{s\tau}, y_r) = \theta_{s\tau} e^{-\alpha_{s\tau} y_{s\tau}} + \theta_r e^{-\alpha_r y_r} - \theta_{s\tau} e^{-\alpha_{s\tau} y_{s\tau}} \theta_r e^{-\alpha_r y_r}. \quad (1)$$

Furthermore, in order to satisfy Assumption 1, we require the following:

$$\alpha_{su} \leq \alpha_{ss}; \theta_{su} > \theta_{ss}; \alpha_{su} \theta_{su} > \alpha_{ss} \theta_{ss}. \quad (2)$$

Since $\theta_{su} \ll 1$, and $\theta_r \ll 1$ (for low probability events), we can safely conclude that $\theta_{su} + \theta_r < 1$. Under this condition it is easy to verify that the ‘joint’ probability of disruption, as expressed in (1), is jointly convex (strictly) in the investments made by the two players. Using (2) we get the following result:

Lemma 1 *An unsafe supplier is always more responsive to incremental investment in security:*

$$\left| \frac{\partial P_{su}(y)}{\partial y} \right| > \left| \frac{\partial P_{ss}(y)}{\partial y} \right|, \forall y \in \mathcal{R}_+.$$

Proof Proofs of all results can be found in Appendix B. □

Finally we note that with some additional mild regularity assumptions, the results in this paper go through without restriction on the particular functional form of $P(y)$, or on the correlation between probabilities of disruption across locations. We discuss this approach in Appendix C.

3.1 Welfare Maximizing Solution

We define the total expected cost of disruption risk management in a supply chain with a safe supplier as W_s , and the corresponding cost in a supply chain with an unsafe supplier as W_u . The

retailer's investment, when the supplier is safe, is y_{rs} . Its investment when the supplier is unsafe is y_{ru} . We let D_J be the damage/loss in the overall supply chain, resulting from a disruption.

Under perfect information, a neutral 3rd party concerned with the welfare of the supply chain participants would optimize the welfare function w.r.t. security investment:

$$W_s^* = \min_{y_{ss}, y_{rs}} [y_{ss} + y_{rs} + P_{Js}(y_{ss}, y_{rs})D_J], \quad (3)$$

$$W_u^* = \min_{y_{su}, y_{ru}} [y_{su} + y_{ru} + P_{Ju}(y_{su}, y_{ru})D_J]. \quad (4)$$

If 'First Best' refers to the investment levels in the absence of any information asymmetry, then optimizing W_s and W_u determines the *First Best security investment levels*. We also define a composite welfare function, W , such that:

$$W^* = \min_{y_{ss}, y_{rs}, y_{su}, y_{ru}} [pW_s + (1 - p)W_u]. \quad (5)$$

It is easy to see that optimizing W leads to the First Best investment levels.

It is possible to model the idiosyncratic losses from a disruption D_s and D_r , for the supplier and retailer respectively, in addition to the overall losses D_J , incurred by the supply chain being analyzed. These would be relevant when the players are involved in other business concerns, besides the one pertaining to the supply chain under study. However, this does not impact the nature of the qualitative insights generated from the analysis presented in this paper, and hence for the sake of expositional clarity we ignore these additional loss terms.

3.2 Non-Cooperative Bayesian Game

In this section we analyze the non-cooperative game, the solution to which is also the disagreement outcome in the bargaining game. In section 3.2.1 we derive some general results which characterize the equilibrium outcome of the non-cooperative game. Then in section 3.2.2 we characterize the sensitivity of the solution to the probability that the supplier is of the safe type.

3.2.1 General Results

In the absence of the option to come to the bargaining table, the supply chain partners default to playing the non-cooperative Bayesian game in which each player invests in security and also incurs a *pre-determined* share of losses in case of a disruption. We define f to be the share of

the disruption related losses assigned to the supplier in the non-cooperative Bayesian game. For instance, the size of f may be determined based on the contribution of each player to the value of the overall business.

We define the total expected costs incurred by the safe supplier, unsafe supplier and retailer by the cost vector $\{t_{ss}, t_{su}, t_r\}$. We have incorporated the superscript - nc , in the notation for the security related investments of the various players, to denote the non-cooperative setting. Then the expected cost functions in this game are:

$$t_{ss} = y_{ss}^{nc} + fP_{Js}(y_{ss}^{nc}, y_r^{nc})D_J, \quad (6)$$

$$t_{su} = y_{su}^{nc} + fP_{Ju}(y_{su}^{nc}, y_r^{nc})D_J, \quad (7)$$

$$t_r = p[y_r^{nc} + (1 - f)P_{Js}(y_{ss}^{nc}, y_r^{nc})D_J] + (1 - p)[y_r^{nc} + (1 - f)P_{Ju}(y_{su}^{nc}, y_r^{nc})D_J]. \quad (8)$$

The players have to decide how much to invest in security. The concavity of the payoff functions (or, convexity of cost), along with the compactness of the action space (the investments are bounded above by D_J and below by 0), guarantees the existence of a pure strategy Bayesian Nash Equilibrium (PSBNE). The reader may note that the notation used to describe the PSBNE is the same as presented in the cost functions above; i.e., any notational indication of optimality is suppressed. This has been done for the sake of convenience in describing the bargaining game, which uses the non-cooperative game as the disagreement outcome.

Proposition 1 *For the non-cooperative game with incomplete information:*

i) *There always exists a unique pure strategy Bayesian Nash Equilibrium:*

$$(y^*, t^*) = (y_{ss}^{nc}, y_{su}^{nc}, y_r^{nc}, t_{ss}, t_{su}, t_r).$$

ii) $y_{su}^{nc} \geq y_{ss}^{nc}$.

iii) $t_{su} \geq t_{ss}$.

The main intuition behind result (ii) is that the unsafe supplier finds its environment more responsive to risk mitigation, hence ends up investing more in security. This is good news from the point of view of reducing the overall vulnerability in the supply chain.

The fact that the unsafe supplier ends up with a higher equilibrium cost, as compared with the safe supplier, is not so intuitive. With higher levels of investment at equilibrium, it is plausible for

the unsafe supplier to drive the probability of disruption to a lower level than the safe supplier in some scenarios. Then it would not have been possible to determine unambiguously which of the two types incurs a greater cost. The key driver of this result is Assumption 1-(ii).

Result (iii) is important for the ‘stability’ of the equilibrium outcome. Observe that, even though the types are exogenously defined, a safe supplier has the flexibility to costlessly transform itself to an unsafe type by dismantling infrastructure or adopting less rigorous management systems, creating intentional negative publicity, etc. Since, in equilibrium, an unsafe supplier incurs a higher cost than a safe one, the latter has no incentive for this perverse behavior.

We are most interested in problems for which the equilibrium solution is interior; i.e., $y_{ss}^{nc} > 0$, $y_{su}^{nc} > 0$ and $y_r^{nc} > 0$. Managerially this is the most interesting situation; i.e., when each player-type combination has reason to invest a non-zero amount in security, as opposed to the boundary case when at least one of the players does not find it economically reasonable to invest in security.

The FOCs, which characterize the interior solution are:

$$\frac{\partial t_{s\tau}}{\partial y_{s\tau}^{nc}} = 1 + \frac{\partial P_{J\tau}(y_{s\tau}^{nc}, y_r^{nc})}{\partial y_{s\tau}^{nc}} f D_J = 0, \quad (9)$$

$$\frac{\partial t_r}{\partial y_r^{nc}} = 1 + \left[p \frac{\partial P_{Js}(y_{ss}^{nc}, y_r^{nc})}{\partial y_r^{nc}} + (1-p) \frac{\partial P_{Ju}(y_{su}^{nc}, y_r^{nc})}{\partial y_r^{nc}} \right] (1-f) D_J = 0. \quad (10)$$

We now plug in the functional forms of the various probabilities of disruption. In order to express the interior solution in closed form, we define the following short-hand notation:

$$\eta = \frac{(1-f)\alpha_r}{f} \left[\frac{p}{\alpha_{ss}} + \frac{1-p}{\alpha_{su}} \right]; \kappa = \alpha_r(1-f)D_J.$$

Equations (9) and (10) imply the following quadratic equation involving $x = P_r(y_r^{nc})$:

$$x^2 - \left(\frac{1 + \kappa - \eta}{\kappa} \right) x + \frac{1}{\kappa} = 0,$$

which has the following roots:

$$x = \frac{1 + \kappa - \eta}{2\kappa} \pm \sqrt{\left(\frac{1 + \kappa - \eta}{2\kappa} \right)^2 - \frac{1}{\kappa}},$$

provided $\eta \leq (1 - \sqrt{\kappa})^2$ (to ensure positive and real roots). If this condition on the parameter values is violated, or one of $P_r \leq \theta_r$ and $P_{s\tau} \leq \theta_{s\tau}$ is violated, then we have a boundary solution, and the Lagrange multipliers associated with the non-negativity constraints on security investment will assume appropriate values to ensure that the modified quadratic equation (incorporating the

Lagrange multipliers) has a feasible solution. We know from Proposition 1 - (i), that this game has a unique Nash Equilibrium. Hence, at most one of the two roots can be a feasible interior solution. The larger root will be the first to violate one of the requirements: $P_r \leq \theta_r$ and $P_{s\tau} \leq \theta_{s\tau}$. Hence, the smaller root is the unique solution required. We can now express the interior solution as:

$$\begin{aligned}
P_r(y_r^{nc}) &= \frac{1 + \kappa - \eta}{2\kappa} - \sqrt{\left(\frac{1 + \kappa - \eta}{2\kappa}\right)^2 - \frac{1}{\kappa}}, \\
y_r^{nc} &= \frac{1}{\alpha_r} \log \left[\frac{\theta_r}{P_r(y_r^{nc})} \right], \\
P_{s\tau}(y_{s\tau}^{nc}) &= \frac{1}{\alpha_{s\tau}(1 - P_r(y_r^{nc}))fD_J}, \\
y_{s\tau}^{nc} &= \frac{1}{\alpha_{s\tau}} \log[\theta_{s\tau}\alpha_{s\tau}(1 - P_r(y_r^{nc}))fD_J],
\end{aligned}$$

where $\tau \in \{s, u\}$.

The closed form expressions for the interior solution offer some insights. It is easy to see that y_{ss}^{nc} is monotonic in θ_{ss} , but θ_{ss} does not impact the value of y_{su}^{nc} and y_r^{nc} . A similar result is true for the way in which y_{su}^{nc} varies with θ_{su} and y_r^{nc} with θ_r , as well. This means that if the base level of threat in the environment of a particular player changes, then its optimal investment in security changes in the same direction, without affecting the investment levels of the other players. The intuition behind this observation is that the marginal cost of investment is always 1, and equating it to the marginal benefit determines the equilibrium probability of disruption. Once this probability is fixed, a player's investment level is determined by the base threat level in it's own environment only.

We can think of exponential rate parameter, α , as the 'sensitivity' of environment to investment in risk mitigation. It can be influenced by the kind of technology in use, or the set of laws and procedures governing trade in a particular country, etc. Applying some basic calculus and algebra, it is easy to see that for $\tau \in \{s, u\}$:

$$\begin{aligned}
\frac{\partial P_r(y_r^{nc})}{\partial \alpha_r} &\leq 0; \quad \frac{\partial P_{s\tau}(y_{s\tau}^{nc})}{\partial \alpha_r} \leq 0; \\
\frac{\partial P_r(y_r^{nc})}{\partial \alpha_{s\tau}} &\leq 0; \quad \frac{\partial P_{s\tau}(y_{s\tau}^{nc})}{\partial \alpha_{s\tau}} \leq 0; \quad \frac{\partial P_{s\tau'}(y_{s\tau'}^{nc})}{\partial \alpha_{s\tau}} \leq 0.
\end{aligned}$$

The interpretation for this finding is that an improvement in 'sensitivity' in one location (i.e., player-type combination) results in a lower equilibrium probability of disruption for every location. This means that a local improvement in the security environment will induce greater safety globally.

The impact of an improved local environment on the level of security investment is ambiguous; the level of required investment to equalize the marginal benefit with the marginal cost of security investment as the local environment improves can either increase or decrease. Similarly, one might conjecture that as the share f of losses allocated to the supplier increases, so does its investment in security. However, it is easy to construct numerical examples where this is not the case. From the expressions for y_{ss}^{nc} and y_{su}^{nc} , we observe that their values are monotonic in $f(1 - P_r(y_r^{nc}))$. As f increases, y_r^{nc} goes down and therefore $(1 - P_r(y_r^{nc}))$ goes down. Hence, the change in $f(1 - P_r(y_r^{nc}))$ may be positive or negative, in certain ranges of parameter values.

Proposition 2 *The non-cooperative Bayesian game leads to ‘under-investment’ in supply chain security, for the case when $f \in (0, 1)$.*

If f is determined based on each supply chain partner’s contribution to the value of the product/business, then we will always have $f \in (0, 1)$. The equilibrium investment in the non-cooperative game is bench-marked against the investment levels determined by a welfare maximizing 3rd party. The result of the proposition is self-explanatory.

3.2.2 Comparative statics with p

Proposition 3 *For the interior solution of the non-cooperative game with incomplete information:*

- i) $\frac{dy_{ss}^{nc}}{dp} \geq 0$; $\frac{dy_{su}^{nc}}{dp} \geq 0$; $\frac{dy_r^{nc}}{dp} \geq 0$,
- ii) $\frac{dy_{su}^{nc}}{dp} \geq \frac{dy_{ss}^{nc}}{dp}$,
- iii) $\frac{dt_{ss}}{dp} \leq 0$; $\frac{dt_{su}}{dp} \leq 0$; $\frac{dt_r}{dp} \leq 0$,
- iv) $\frac{dt_{su}}{dp} \geq \frac{dt_{ss}}{dp}$.

This proposition summarizes the impact of increasing p , the likelihood of the supplier being safe. As p is made higher, the security investments by each player go up and the total expected costs incurred go down. This suggests that both types of supplier have an incentive to convince the retailer that they are of the safe type. Even though, as p is increased, the increase in investment by the unsafe type exceeds that by the safe type of supplier, the cost reduction seen by the safe type is higher.

Result (i) is a bit surprising. Especially since we earlier found that y_r decreases with θ_r , i.e., as the threat of disruption decreases, then so does the investment in security. How is it then, that as p increases, or the overall supply chain becomes safer, the value of y_r increases?

The answer lies in the intuition that the retailer will find that its investment in security is more effective when its supplier partner is safer. Formerly: $\frac{\partial^2 t_r}{\partial y_r \partial p} \leq 0$. The validity of this claim, at equilibrium, can be verified easily. As a result the retailer invests more in security. This, in turn, makes the effectiveness of supplier's investment higher, resulting in increased retailer investment. Thus, Proposition 3 suggests that there may be a virtuous cycle of increasing investment and improving security in global supply chains.

Another interesting analysis is to see the impact of changing p on the expected values of investment and cost, i.e., evaluating the sign of:

$$\begin{aligned}\frac{d\bar{y}}{dp} &= p \frac{dy_{ss}}{dp} + (1-p) \frac{dy_{su}}{dp} + \frac{dy_r}{dp} + y_{ss} - y_{su}, \\ \frac{d\bar{t}}{dp} &= p \frac{dt_{ss}}{dp} + (1-p) \frac{dt_{su}}{dp} + \frac{dt_r}{dp} + t_{ss} - t_{su}.\end{aligned}$$

Using the results in Propositions 1 and 3, it is straightforward to see that $\frac{d\bar{t}}{dp} \leq 0$. However, the sign of $\frac{d\bar{y}}{dp}$ is ambiguous.

3.3 Bargaining with Incomplete Information

The goal of the cooperative contracting process is to come up with a mutually acceptable security investment and cost-sharing arrangement, which induces both players to invest efficiently in security. The contract will henceforth be referred to as the HSN-SCD contract, i.e., Harsanyi-Selten-Nash Supply Chain Disruption contract.

In the subsequent analysis, $C_{s\tau}$ represents the cost incurred by the supplier (of type $\tau \in \{s, u\}$), as a result of the bargaining process; and $t_{s\tau}$ is the payoff of the supplier (of type τ) when the disagreement outcome is reached in the bargaining game. The corresponding costs for the retailer are represented by C_r and t_r . The cost vector $\{t_{ss}, t_{su}, t_r\}$ is the outcome of the non-cooperative game played between the players.

We focus on the case with transferable utility, which is natural between commercial partners who can agree on joint project financing and transfer pricing as means of effecting transfer (side)

payments. We adopt the convention that transfer payments are made by the supplier to the retailer, and define them to be γ_{ss} and γ_{su} for the safe and unsafe supplier respectively. This is without the loss of generality, since we don't impose any sign restriction on the value of transfer payments. Whereas in the non-cooperative setting we used f to represent the pre-determined share of losses borne by each type of the supplier, we now use f_{ss} and f_{su} to represent the negotiated share of losses for the safe and unsafe supplier, respectively. The players' cost functions take the form:

$$C_{ss} = y_{ss} + f_{ss}P_{Js}(y_{ss}, y_{rs})D_J + \gamma_{ss},$$

$$C_{su} = y_{su} + f_{su}P_{Ju}(y_{su}, y_{ru})D_J + \gamma_{su},$$

$$C_r = p[y_{rs} + P_{Js}(y_{ss}, y_{rs})(1 - f_{ss})D_J - \gamma_{ss}] + (1 - p)[y_{ru} + P_{Ju}(y_{su}, y_{ru})(1 - f_{su})D_J - \gamma_{su}].$$

The maximization of the Generalized Nash Product (GNP), as laid out in H&S and Myerson (1979), subject to the incentive compatibility (IC) and individual rationality (IR) constraints, along with other feasibility (Feas) constraints, determines the outcome of the bargaining game. The non-negativity constraints on the security investments are implicit in the following representation:

$$GNP^* = \max_{y, f, \gamma} \{p \log(t_{ss} - C_{ss}) + (1 - p) \log(t_{su} - C_{su}) + \log(t_r - C_r)\}$$

subject to:

$$y_{su} + f_{su}P_{Js}(y_{su}, y_{ru})D_J + \gamma_{su} \geq C_{ss}, \quad (\text{IC s})$$

$$y_{ss} + f_{ss}P_{Ju}(y_{ss}, y_{rs})D_J + \gamma_{ss} \geq C_{su}, \quad (\text{IC u})$$

$$t_{ss} - C_{ss} \geq 0, \quad (\text{IR s})$$

$$t_{su} - C_{su} \geq 0, \quad (\text{IR u})$$

$$t_r - C_r \geq 0, \quad (\text{IR r})$$

$$0 \leq f_{ss} \leq 1; 0 \leq f_{su} \leq 1. \quad (\text{Feas})$$

The solution to the above optimization problem forms the basis for our next result.

Proposition 4 *The HSN-SCD contract is characterized by the solution to the bargaining game. For an interior solution w.r.t. investment and the supplier's negotiated share of losses, i.e., $y_{ss}^*, y_{su}^*, y_r^* > 0$; $f_{ss}^*, f_{su}^* \in (0, 1)$, we obtain:*

- i. *First Best investment in security.*

- ii. *Equal bargaining surplus for each player, i.e., $t_{ss} - C_{ss}^* = t_{su} - C_{su}^* = t_r - C_r^*$. This also implies $C_{su}^* \geq C_{ss}^*$.*

The boundary case, when $f_{ss}^ = 1$, may result in under-investment in security.*

This proposition contains some of the key results of our analysis. We first note that a fundamental implication of the Pareto property of bargaining solutions (including the HSN solution) is that the resulting cooperative approach to security management leads to First Best levels of investment. This contrasts with the non-cooperative approach which leads to under-investment.

Next, note that even though we do not have a convex optimization problem (the objective function is strictly concave, but the feasible region is not convex), the necessary conditions for optimality provide us with the means to determine uniquely, the total expected cost incurred by each player, as well as each player's investment in security. There might be more than one way in which the players agree to split their incurred costs across share of losses from a disruption and upfront transfer payments, to achieve the same total expected cost.

Equally importantly, we establish that at the interior solution (as defined in the Proposition above) each player ends up with an equal surplus from the bargaining game; i.e., equal improvement over the disagreement outcome. This also implies $C_{su}^* \geq C_{ss}^*$, which establishes that the safe supplier incurs a lower cost in the HSN-SCD contract.

In typical 'principal-agent' models, the efficient agent extracts an *information-rent* from the principal, due to its ability to mimic the inefficient type (Laffont and Martimort 2001). In our analysis, the bargaining power of each player is captured in the disagreement outcome; or the non-cooperative equilibrium. Even though the bargaining game results in equal surplus for each player, the greater bargaining ability of the safe supplier (lower cost in the non-cooperative game) ensures that it ends up with a lower cost, as compared to the unsafe type. We interpret this result as the retailer having to cede an *information rent* to the safe supplier, in order to prevent it from mimicking the unsafe type, with associated inefficiencies.

The proof of Proposition (4), which is presented in Appendix B, shows that the boundary case in which $f_{ss}^* = 1$, can lead to under-investment in security. However, we also show that the optimal interior solution leads to a higher objective function (GNP) value than the boundary solution. Hence, the latter will be achieved only if the optimal interior solution cannot be attained feasibly.

If we think of a ‘safer’ supply chain as one in which the likelihood of a safe supplier is higher, i.e., the value of p is greater, we have the following:

Corollary 4.1 *The safe supplier sees greater incremental benefit than the unsafe type, from a ‘safer’ supply chain, i.e., $\frac{\partial C_{ss}}{\partial p} \leq \frac{\partial C_{su}}{\partial p}$.*

Proof The proof follows directly from the results in Proposition 4-(ii) and Proposition 3-(iv). \square

We have thus far implicitly assumed that the investments in security are observable and verifiable by a third party, such as an objective court of law. If not, then the HSN-SCD contract described above is vulnerable to *moral hazard* (Laffont and Martimort 2001). The retailer may not be able to ensure that the supplier invests in security, as agreed upon in the contract. If, on the other hand, the retailer buys out the supplier’s share of losses from a disruption in return for a lump sum transfer payment, then the resulting contract is naturally robust to *moral hazard*.

The special case of the HSN-SCD contract with $f_{ss} = f_{su} = 0$ being pre-determined, achieves this result. We call this contract the robust HSN-SCD contract. Since the analysis proceeds in exactly the same manner as for the HSN-SCD contract, but with f_{ss} and f_{su} set to 0, we state our next result without proof.

Proposition 5 *The robust HSN-SCD contract is unique and results in:*

- i) *First Best levels of investment in security.*
- ii) *Equal expected cost for both types of the supplier, i.e., $C_{ss}^* = C_{su}^*$.*
- iii) *Weakly greater bargaining surplus for the unsafe supplier, i.e., $t_{ss} - C_{ss}^* \leq t_{su} - C_{su}^*$.*

We saw in Proposition 4 that the optimal interior solution results in equal bargaining surplus for each player. The robust HSN-SCD contract results in a weakly greater surplus for the safe supplier, as compared to the unsafe type. Since the robust contract is just a special case of the original contract, it means that the original contract is weakly preferred to the robust version, when investments are observable and verifiable. The reason for the robustness of the contract described in the above proposition, is that the final cost incurred by the supplier, $y_{s\tau}^* + \gamma_{s\tau}^*$, can be replaced by a single upfront transfer payment $\gamma'_{s\tau}$, for $\tau \in \{s, u\}$. Then the retailer has full ownership of disruption related losses, and complete control over investment in security, thus precluding the

possibility of *moral hazard*. This result is similar to ‘sell the firm’ idea in the context of moral hazard in Principal-Agent models (Laffont and Martimort 2001), although in the present context only the disruption risk is transferred to the retailer. Selling share of losses to a larger supply chain partner (the retailer in our model) could be effected by the retailer setting up a captive insurance provider, with premiums for loss coverage equal to the desired bargaining outcome level. This captive provider might further lay off some of the risks associated with these in further insurance or re-insurance contracts. Such captive insurance practices are common for large companies (Adkisson 2006).

The retailer might wish to audit the supplier in order to determine its true type, in the hope of recovering some of the surplus surrendered in the bargaining game. The audit mechanism is described as follows. If the supplier declares its type to be τ , then the retailer audits the supplier with probability q_τ . If the supplier had declared type τ' and the audit reveals its type to be τ , then the retailer imposes a penalty P_τ , which is less than some exogenously specified amount l (e.g., the supplier’s total profits of the retailer’s business). The retailer incurs a cost of auditing which is assumed to be linear in q_τ , i.e., $c_\tau q_\tau$.

Corollary 5.1 *Auditing the supplier’s type, after bargaining and before disruption, does not improve the bargaining outcome for the retailer.*

The intuition behind this result lies in the transferable utility feature of the bargaining game, along with the fact that the (IC) constraints are binding at optimum, even without audit. The retailer would use the threat of audit to deter the supplier from lying about its type and thus, extract concessions. Since, even in the absence of an audit, the IC constraints are binding at optimum, the retailer has no incentive to indulge in costly audit.

The supplier is able to anticipate any other benefit that may accrue to the retailer by audit, and is able to extract this benefit through appropriately adjusting the transfer payment. It is worth noting that this result does not generally hold true for the original bargaining game, i.e., with $f_{ss} \geq 0$ and $f_{su} \geq 0$.

4 Conclusions and Discussion

Designing resilient supply chains that have reduced vulnerabilities to security threats and other major supply chain disruptions has become a central theme in the more complex supply chain

management context ushered in by globalization. Recognizing this imperative, risk managers (both internal to companies and in their insurance partners) are now showing an active interest in efficient strategies and initiatives to identify and mitigate vulnerabilities to disruptions. As we have noted in this paper, the problem is a difficult one because the nature of disruptions as low-probability, high-consequence events makes the use of lagging indicators (such as losses from actual disruptions) ineffective, and because the effectiveness of investments in mitigating such vulnerabilities is difficult to monitor across multiple organization, typical of modern supply chains. This paper has taken a first step in providing a theoretical underpinning for understanding this multi-organizational and interdependent risk problem. We have captured what we view as the essential ingredients of this problem, including incomplete information, moral hazard, and risk interdependency across supply chain partners in a bargaining framework.

Our bargaining analysis establishes the superiority of co-opetition over competition in the context of managing supply chain security. We describe a cooperative contract (HSN-SCD) which leads to efficient investment in security, in contrast to the non-cooperative game, which leads to under-investment. Of course, the non-cooperative default option also underlines the critical features of risk interdependency and the benefits of co-operative behavior in determining whether (as predicted by the Nash bargaining framework) co-operation is likely to emerge. We note also that we have neglected the transactions costs of establishing the bargaining framework as well as the costs of establishing the institutions (such as captive insurance companies) to administer on-going transfer payments and to determine and adjust losses from actual disturbances. Nonetheless, the framework developed arguably captures the essential cost, risk and profit consequences inherent in the supply disruption risk management context. The primary managerial implication of this analysis is that a coordinated effort to establish supply-chain wide standards and to share the costs and residual risks of mitigation activities is critical when interdependent risks are present.

Our analysis highlights the need to compensate safe suppliers, through an *information rent*, for the bargaining advantage they enjoy in case of information asymmetry, by virtue of their ability to mimic less safe types. While the need for collective action in setting standards for security measures amongst trading partners is clearly recognized in the industry (APEC 2006); the idea of rewarding safer suppliers in the form of an *information rent* is equally crucial for the fruitful culmination of joint efforts (i.e., efforts that do not break down and lead to the non-cooperative default outcome).

A natural concern in contractual settings, is the possibility of the trading partners renegeing on

their commitments, in case the security investments are not observable and verifiable. We identify a contract (robust HSN-SCD) which provides a means of getting around these inefficiencies by having the retailer “buy out” the supplier’s share of losses, and also retaining responsibility for security investments at both locations. In this contract, though the unsafe supplier extracts a weakly greater bargaining surplus than the safe supplier, both types incur the same equilibrium expected cost. This implies that no *information rent* is surrendered to the safe supplier.

In our analysis of the non-cooperative Bayesian game, which is also taken as the natural disagreement outcome of the bargaining game, we characterize the sensitivity of the solutions to p - the probability of the supplier being safe, as well as the base threat levels and ‘sensitivity’ to investment. We find that, although a higher value for p is beneficial to all players, a safe supplier sees a greater incremental benefit than an unsafe type.

While the analysis is presented in the framework of risk neutral players, it would essentially remain the same for the case of risk averse players optimizing CVaR (Conditional Value at Risk) or other probabilistic methods of valuing losses beyond some target/acceptable level. While we have only considered here two types, there seems to be no *a priori* reason to believe that the insights presented will change significantly by allowing multiple types (greater than two) for the supplier.

Notwithstanding the importance of inventory control in ensuring smooth operations, the emerging supply chain security guidelines for the private sector (APEC / ISO 28000 etc.) serve as evidence that there are security considerations in operations management, which transcend traditional inventory control, and at the same time have significant implications for the health of global trade. Even a cursory look at the contents of the APEC report is enough to give an idea of the gamut of issues involved; from information sharing between trading partners to education and training of personnel. Together these best practices underline the importance of a cooperative approach among supply chain partners in designing supply-chain wide strategies for disruption risk management.

Our analysis also provides insights into the leveraging effects of public-private partnerships in providing a set of standards that can be used as benchmarks for investments in security and disruption management. In particular, the bargaining framework seems relevant in the light of recent initiatives of the Department of Homeland Security, viz., Container Security Initiative (CSI) and Customs-Trade Partnership Against Terrorism (C-TPAT), which rely heavily on a co-operative approach to securing global supply chains (Bakshi and Gans 2007). Our hope is that the treatment

presented in this paper will spawn greater interest in exploring other opportunities for understanding and reducing vulnerabilities of extended supply chains to major disruptions.

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A Appendix: Axiomatic Vs. Strategic Bargaining Models

Nash (1950) is considered by many to be the starting point of formal bargaining theory. Nash proposed that a bargaining solution should satisfy four plausible axioms. It turns out that there is only one solution that does so, which is known as the Nash Bargaining Solution. Nash does not have any explicit underlying bargaining procedure in his model.

The four axioms specify requirements for: Invariance to equivalent utility representations; Symmetry of the bargaining solution; Independence of Irrelevant Alternatives, and Pareto Efficiency. A detailed explanation of each of these conditions can be found in Nash (1950). We focus on the more controversial Independence of Irrelevant Alternatives, or IIA.

Consider two bargaining problems such that in the first the set of alternatives to choose from is S , while the set of alternatives is T in the other case. Moreover, $S \subset T$. The axiom states that suppose when all the alternatives in T are available, the players agree on an outcome s in the smaller set S . Then it is required that the players agree on the same outcome s when only the alternatives in S are available.

This axiom relates to the (unmodeled) bargaining process (Osborne and Rubinstein 1990). “...if the negotiators gradually eliminate outcomes as unacceptable, until just one remains, then it may be appropriate to assume IIA. Without specifying the details of the bargaining process, it is hard to assess how reasonable the axiom is.”

“In the axiomatic approach, the outcome of bargaining is defined by a list of properties that it is required to satisfy. In the strategic approach, the outcome is an equilibrium of an explicit bargaining process.” Rubinstein (1982) handles the challenge of formulating a tractable model of the bargaining process (from amongst a bewildering variety of such procedures) by focusing on one salient feature of bargaining: the participants’ attitude to delay. In this model, the players alternately propose agreements in each period. The other player may accept the proposed agreement, in which case the game ends, or reject it, in which case the game passes on the next period. The game continues until an agreement is reached, while all the infinite paths (at which an agreement is never reached) are assumed to lead to the same disagreement outcome.

Under a set of assumptions that include time invariant preferences, Binmore et al. (1986) show that the strategic model of bargaining leads to a unique subgame perfect equilibrium. It is also

shown that, as the time between offers goes to zero, the solution from the strategic approach converges to the Nash Bargaining Solution. This result establishes the link between the axiomatic and strategic approaches to bargaining with complete information.

Harsanyi and Selten (1972), or H&S, laid the foundations of the axiomatic approach of bargaining with asymmetric information. In their approach the unique solution to the *two person* bargaining game is obtained by optimizing the Generalized Nash Product (GNP):

$$\max_{x_i} \prod_{i=1}^N (x_i - a_i)^{p_i}$$

where x_i and a_i are as defined in the original Nash product. In addition p_i represents the marginal probability associated with types $1, \dots, K$ of player 1, and $p_{K+m} = q_m$ for the marginal probabilities q_1, \dots, q_M associated with types $1, \dots, M$ of player 2, such that $K + M = N$.

The axiomatic approach to bargaining with asymmetric information is closely linked to relaxing the assumption of symmetric bargaining power in the original Nash model with complete information, (see section 2.5 in Osborne and Rubinstein (1990)). This link is quite intuitive, since the player with the informational advantage would capitalize on it in its bargaining strategy.

The weight associated with bargaining power of each player-type combination turns out to be exactly equal to the marginal probability of that player-type combination, as is shown by H&S. A coarse intuition behind this result is as follows. The maximization of the GNP is equivalent to the maximization of its logarithm, i.e.,

$$\max_{x_i} \sum_{i=1}^N p_i \log(x_i - a_i)$$

Each player has the objective of maximizing its surplus and accordingly chooses a bargaining strategy. However, the strategy should be such that information about the player's type is not revealed. The weighted average, $\sum_{i=1}^K p_i \log(x_i - a_i)$ for player 1 (and a similar term for player 2) in the maximization problem, represent the attempt by the players to balance these two goals.

This axiomatic approach has an explicit and well defined underlying bargaining model - a *multi-stage acceptance model of bargaining*. This explicit model allows one to assess the reasonableness of the axioms stated, as suggested by Osborne and Rubinstein (1990). H&S also point out that under an acceptance model, as bargaining proceeds, each player will declare more and more possible agreements to be acceptable to it, while under a rejection model each player will declare more and

more possible agreements to be unacceptable from its point of view. Thus, acceptance models give clearer expression to the empirical fact that during the bargaining process the two sides often make concessions to each other, rather than make increasingly more extreme demands.

However, the set of axioms that characterize a bargaining solution may not be unique. Myerson (1984) modified the set of axioms in the theory of two person bargaining games with incomplete information. The solutions identified by his proposed set of criteria was termed Neutral Bargaining Solutions, as opposed to the unique solution characterized by H&S. Weidner (1992) showed that, when the types of the two players are independently distributed, the H&S solution *is* the Neutral Bargaining Solution, thus reconciling the two approaches.

The debate between the axiomatic and strategic approaches is best summarized in Nash's words, "The two approaches to the problem, via the negotiation model or via the axioms, are complementary; each helps to justify and clarify the other." (Nash(1953, p129)). The close linkages of the H&S model to the original Nash model, coupled with the latter's linkages to the strategic approach to bargaining, bear out this claim.

Finally, we choose the axiomatic approach for this paper because it is tractable and has the ability to uniquely characterize the outcome of the bargaining game. It is hence easier to measure the impact of asymmetric information on the decision to invest in supply chain security. Unlike some recent work in Bargaining Theory (Ausubel et al. 2002), our focus is clearly not on the process by which the bargaining solution is attained and the inter-temporal considerations therein, but on the final outcome itself.

B Appendix: Proofs

Lemma 1:

Proof We observe that:

$$\left| \frac{\partial P_{s\tau}(y)}{\partial y} \right| = \alpha_{s\tau} \theta_{s\tau} e^{-\alpha_{s\tau} y}$$

Using the results in (2) it is easy to see that:

$$\left| \frac{\partial P_{su}(y)}{\partial y} \right| > \left| \frac{\partial P_{ss}(y)}{\partial y} \right|, \forall y \in \mathcal{R}_+$$

□

Proposition 1:

Proof

i) We can define the payoffs in this game as:

$$\pi_{ss} = -t_{ss}; \pi_{su} = -t_{su}; \pi_r = -t_r$$

We now use Theorem 7 from Cachon and Netessine (2004) which states:

Theorem: *Suppose the strategy space of the game is convex and all payoff functions are quasi-concave. Then if $(-1)^n |H|$ is positive whenever $\frac{\partial \pi_i}{\partial y_i} = 0$, all i , there is a unique Nash Equilibrium.*

The Hessian, in our context, is:

$$H = \begin{bmatrix} \frac{\partial^2 \pi_{ss}}{\partial y_{ss}^2} & \frac{\partial^2 \pi_{ss}}{\partial y_{ss} \partial y_{su}} & \frac{\partial^2 \pi_{ss}}{\partial y_{ss} \partial y_r} \\ \frac{\partial^2 \pi_{su}}{\partial y_{su} \partial y_{ss}} & \frac{\partial^2 \pi_{su}}{\partial y_{su}^2} & \frac{\partial^2 \pi_{su}}{\partial y_{su} \partial y_r} \\ \frac{\partial^2 \pi_r}{\partial y_r \partial y_{ss}} & \frac{\partial^2 \pi_r}{\partial y_r \partial y_{su}} & \frac{\partial^2 \pi_r}{\partial y_r^2} \end{bmatrix}$$

Populating the entries of the Hessian we get:

$$\begin{bmatrix} -\alpha_{ss}^2 P_{ss} (1 - P_r) f D_J & 0 & \alpha_{ss} \alpha_r P_{ss} P_r f D_J \\ 0 & -\alpha_{su}^2 P_{su} (1 - P_r) f D_J & \alpha_{su} \alpha_r P_{su} P_r f D_J \\ p \alpha_{ss} \alpha_r P_{ss} P_r (1 - f) D_J & (1 - p) \alpha_{su} \alpha_r P_{su} P_r (1 - f) D_J & -\alpha_r^2 P_r [1 - p P_{ss} - (1 - p) P_{su}] (1 - f) D_J \end{bmatrix}$$

Taking the determinant of the Hessian we get:

$$|H| = \alpha_{ss}^2 \alpha_{su}^2 \alpha_r^2 P_{ss} P_{su} P_r (1 - P_r) f^2 (1 - f) D_J^3 [p(P_{ss} + P_r - 1) + (1 - p)(P_{su} + P_r - 1)] < 0$$

since $\theta_{su} + \theta_r < 1$. Hence, $(-1)^3 |H| > 0$, as required.

ii) The Nash Equilibrium of the non-cooperative game is computed by evaluating the FOCs. From (9) we infer that at equilibrium: $\frac{\partial P_{su}(y_{su}^{nc})}{\partial y_{su}^{nc}} = \frac{\partial P_{ss}(y_{ss}^{nc})}{\partial y_{ss}^{nc}}$. Then from Lemma 1 and the fact that $\frac{\partial^2 P_{s\tau}(y)}{\partial y^2} > 0$, we conclude that for an interior solution $y_{su}^{nc} \geq y_{ss}^{nc}$.

For a boundary solution it is clear that since the result in Lemma 1 holds and the marginal benefit of investment is always higher for the unsafe type, while the marginal cost of investment is always 1 for both types, the investment of the safe type will go to 0 before that of the unsafe type. Once again $y_{su}^{nc} \geq y_{ss}^{nc}$.

An alternate proof for the above result can be obtained using supermodularity arguments.

iii) Using (9), along with the exponential functional form for disruption probabilities, at equilibrium, for an interior solution, we get:

$$\alpha_{su}P_{su}(y_{su}^{nc}) = \alpha_{ss}P_{ss}(y_{ss}^{nc})$$

Since $\alpha_{su} \leq \alpha_{ss}$, we conclude that $P_{su}(y_{su}^{nc}) \geq P_{ss}(y_{ss}^{nc})$. This relationship, along with the previous result in (ii), implies that $t_{su} \geq t_{ss}$.

For a boundary solution, by arguments similar to the one presented in the proof of (ii), we can easily verify that $P_{su}(y_{su}^{nc}) \geq P_{ss}(y_{ss}^{nc})$ and hence $t_{su} \geq t_{ss}$. \square

Proposition 2:

Proof Using the FOCs of the non-cooperative game, (9) and (10), along with (5), we get for $f \in (0, 1)$:

$$\begin{aligned} \frac{\partial W}{\partial y_{ss}} \Big|_{y_{ss}^{nc}, y_r^{nc}} &= p \left(1 + \frac{\partial P_{Js}(y_{ss}^{nc}, y_r^{nc})}{\partial y_{ss}} D_J \right) = p \left(1 - \frac{1}{f} \right) \leq 0 \\ \frac{\partial W}{\partial y_{su}} \Big|_{y_{su}^{nc}, y_r^{nc}} &= (1-p) \left(1 + \frac{\partial P_{Ju}(y_{su}^{nc}, y_r^{nc})}{\partial y_{su}} D_J \right) = (1-p) \left(1 - \frac{1}{f} \right) \leq 0 \\ \frac{\partial W}{\partial y_{rs}} \Big|_{y_{ss}^{nc}, y_{su}^{nc}, y_r^{nc}} &= p \frac{\partial W_s}{\partial y_{rs}} \Big|_{y_{ss}^{nc}, y_r^{nc}} = p \left(1 + \frac{\partial P_{Js}(y_{ss}^{nc}, y_r^{nc})}{\partial y_{rs}} D_J \right) = p \left(1 - \frac{1}{1-f} \right) \leq 0 \\ \frac{\partial W}{\partial y_{ru}} \Big|_{y_{ss}^{nc}, y_{su}^{nc}, y_r^{nc}} &= (1-p) \frac{\partial W_u}{\partial y_{ru}} \Big|_{y_{su}^{nc}, y_r^{nc}} = (1-p) \left(1 + \frac{\partial P_{Ju}(y_{su}^{nc}, y_r^{nc})}{\partial y_{ru}} D_J \right) = (1-p) \left(1 - \frac{1}{1-f} \right) \leq 0 \end{aligned}$$

Since W is a strictly convex function in the investment vector, we conclude that non-cooperation leads to under-investment, when bench-marked against the welfare optimizing First Best levels of security investment. \square

Proposition 3:

Proof

(i) On applying the Implicit Function Theorem to the FOCs in (9) and (10) we get:

$$\begin{bmatrix} \frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc}} & 0 & \frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_r^{nc}} \\ 0 & \frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc}} & \frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_r^{nc}} \\ p \frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_r^{nc}} & (1-p) \frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_r^{nc}} & p \frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_{su}^{nc}} + (1-p) \frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_{ss}^{nc}} \end{bmatrix} \begin{bmatrix} \frac{dy_{ss}^{nc}}{dp} \\ \frac{dy_{su}^{nc}}{dp} \\ \frac{dy_r^{nc}}{dp} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{\partial P_{Ju}}{\partial y_r^{nc}} - \frac{\partial P_{Js}}{\partial y_r^{nc}} \end{bmatrix}$$

which gives us:

$$\begin{bmatrix} \frac{dy_{ss}^{nc}}{dp} \\ \frac{dy_{su}^{nc}}{dp} \\ \frac{dy_r^{nc}}{dp} \end{bmatrix} = \frac{\left(\frac{\partial P_{Ju}}{\partial y_r^{nc}} - \frac{\partial P_{Js}}{\partial y_r^{nc}} \right)}{Det} \begin{bmatrix} \left(\frac{-\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_r^{nc}} \right) \left(\frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_{ss}^{nc}} \right) \\ \left(\frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_{su}^{nc}} \right) \left(\frac{-\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_r^{nc}} \right) \\ \left(\frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_{su}^{nc}} \right) \left(\frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_{ss}^{nc}} \right) \end{bmatrix}$$

where

$$Det = p \frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_{ss}^{nc}} \left[\left(\frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_r^{nc}} \right) \left(\frac{\partial^2 P_{Js}}{\partial y_r^{nc} \partial y_{su}^{nc}} \right) - \left(\frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_r^{nc}} \right)^2 \right] \\ + (1-p) \frac{\partial^2 P_{Js}}{\partial y_{ss}^{nc} \partial y_{su}^{nc}} \left[\left(\frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_r^{nc}} \right) \left(\frac{\partial^2 P_{Ju}}{\partial y_r^{nc} \partial y_{su}^{nc}} \right) - \left(\frac{\partial^2 P_{Ju}}{\partial y_{su}^{nc} \partial y_r^{nc}} \right)^2 \right]$$

We observe that $Det > 0$, due to the strict joint convexity of $P_{Js}(y_{ss}^{nc}, y_r^{nc})$ and $P_{Ju}(y_{su}^{nc}, y_r^{nc})$. We also note that for $\tau \in \{s, u\}$:

$$\frac{-\partial^2 P_{J\tau}}{\partial y_{s\tau}^{nc} \partial y_r^{nc}} = \alpha_{s\tau} \alpha_r P_{s\tau} P_r > 0 \quad (11)$$

$$\frac{\partial P_{J\tau}}{\partial y_r^{nc}} = -\alpha_r P_r (1 - P_{s\tau}) \quad (12)$$

We showed in the proof of Proposition 1-(iii) that $P_{su} \geq P_{ss}$, at equilibrium. Hence, by (12) we have $\frac{\partial P_{Ju}}{\partial y_r^{nc}} \geq \frac{\partial P_{Js}}{\partial y_r^{nc}}$. Together, the above conditions imply that $\frac{dy_{ss}^{nc}}{dp} \geq 0$, $\frac{dy_{su}^{nc}}{dp} \geq 0$ and $\frac{dy_r^{nc}}{dp} \geq 0$.

(ii) We showed in the paper that the interior solution is:

$$y_{s\tau}^{nc} = \frac{1}{\alpha_{s\tau}} \log[\theta_{s\tau} \alpha_{s\tau} (1 - P_r(y_r^{nc})) f D_J]$$

where $\tau \in \{s, u\}$. Since the only term dependent on p in the above expression is $P_r(y_r^{nc})$, then $\alpha_{su} \leq \alpha_{ss}$ implies that $\frac{dy_{su}^{nc}}{dp} \geq \frac{dy_{ss}^{nc}}{dp}$

(iii) Using the result in (i) along with the FOC equations (9) and (10), we get that at equilibrium:

$$\frac{dt_{s\tau}}{dp} = \left(1 + f D_J \frac{\partial P_{J\tau}}{\partial y_{s\tau}^{nc}} \right) \left(\frac{dy_{s\tau}^{nc}}{dp} \right) + f D_J \left(\frac{\partial P_{J\tau}}{\partial y_r^{nc}} \right) \left(\frac{dy_r^{nc}}{dp} \right) = 0 + f D_J \left(\frac{\partial P_{J\tau}}{\partial y_r^{nc}} \right) \left(\frac{dy_r^{nc}}{dp} \right) \leq 0$$

Now using the result that at equilibrium $P_{su} > P_{ss}$ (shown in the proof of Proposition 1-(iii)) we get:

$$\frac{dt_r}{dp} = (1-f) D_J \left[p \left(\frac{\partial P_{Js}}{\partial y_{ss}^{nc}} \right) \left(\frac{dy_{ss}^{nc}}{dp} \right) + (1-p) \left(\frac{\partial P_{Ju}}{\partial y_{su}^{nc}} \right) \left(\frac{dy_{su}^{nc}}{dp} \right) + (P_{ss} - P_{su}) \right] \leq 0$$

(iv) In part (iii) we saw that:

$$\frac{dt_{s\tau}}{dp} = f D_J \left(\frac{\partial P_{J\tau}}{\partial y_r^{nc}} \right) \left(\frac{dy_r^{nc}}{dp} \right) \leq 0$$

We also showed in the proof of part (i) that: $\frac{\partial P_{Ju}}{\partial y_r^{nc}} \geq \frac{\partial P_{Js}}{\partial y_r^{nc}}$. Hence, we conclude that $\frac{dt_{su}}{dp} \geq \frac{dt_{ss}}{dp}$. \square

Proposition 4

Proof The Lagrangian for the optimization problem is:

$$\begin{aligned} \mathcal{L} = & p \log(t_{ss} - C_{ss}) + (1-p) \log(t_{su} - C_{su}) + \log(t_r - C_r) + \lambda_s [y_{su} + f_{su} P_{Js}(y_{su}, y_{ru}) D_J + \gamma_{su} - C_{ss}] \\ & + \lambda_u [y_{ss} + f_{ss} P_{Ju}(y_{ss}, y_{rs}) D_J + \gamma_{ss} - C_{su}] + \lambda_1 (1 - f_{ss}) + \lambda_2 (1 - f_{su}) + \lambda_3 f_{ss} + \lambda_4 f_{su} \end{aligned}$$

We ignore the (IR) constraints for the moment. The FOCs imply:

$$\frac{\partial \mathcal{L}}{\partial y_{ss}} = 0 :$$

$$\begin{aligned} \frac{p}{t_{ss} - C_{ss}} \left[-1 - f_{ss} \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{ss}} D_J \right] + \frac{p}{t_r - C_r} \left[-\frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{ss}} [(1 - f_{ss}) D_J] \right] \\ - \lambda_s \left[1 + f_{ss} \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{ss}} D_J \right] + \lambda_u \left[1 + f_{ss} \frac{\partial P_{Ju}(y_{ss}, y_{rs})}{\partial y_{ss}} D_J \right] = 0 \quad (13) \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial y_{su}} = 0 :$$

$$\begin{aligned} \frac{1-p}{t_{su} - C_{su}} \left[-1 - f_{su} \frac{\partial P_{Ju}(y_{su}, y_{ru})}{\partial y_{su}} D_J \right] + \frac{1-p}{t_r - C_r} \left[-\frac{\partial P_{Ju}(y_{su}, y_{ru})}{\partial y_{su}} [(1 - f_{su}) D_J] \right] \\ + \lambda_s \left[1 + f_{su} \frac{\partial P_{Js}(y_{su}, y_{ru})}{\partial y_{su}} D_J \right] - \lambda_u \left[1 + f_{su} \frac{\partial P_{Ju}(y_{su}, y_{ru})}{\partial y_{su}} D_J \right] = 0 \quad (14) \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial y_{rs}} = 0 :$$

$$\begin{aligned} \frac{p}{t_{ss} - C_{ss}} \left[-f_{ss} \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{rs}} D_J \right] + \frac{p}{t_r - C_r} \left[-1 - \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{rs}} [(1 - f_{ss}) D_J] \right] \\ - \lambda_s \left[f_{ss} \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{rs}} D_J \right] + \lambda_u \left[f_{ss} \frac{\partial P_{Ju}(y_{ss}, y_{rs})}{\partial y_{rs}} D_J \right] = 0 \quad (15) \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial y_{ru}} = 0 :$$

$$\begin{aligned} \frac{1-p}{t_{su} - C_{su}} \left[-f_{su} \frac{\partial P_{Ju}(y_{su}, y_{ru})}{\partial y_{ru}} D_J \right] + \frac{1-p}{t_r - C_r} \left[-1 - \frac{\partial P_{Ju}(y_{su}, y_{ru})}{\partial y_{ru}} [(1 - f_{su}) D_J] \right] \\ + \lambda_s \left[f_{su} \frac{\partial P_{Js}(y_{su}, y_{ru})}{\partial y_{ru}} D_J \right] - \lambda_u \left[f_{su} \frac{\partial P_{Ju}(y_{su}, y_{ru})}{\partial y_{ru}} D_J \right] = 0 \quad (16) \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \gamma_{ss}} = 0 : \quad \frac{-p}{t_{ss} - C_{ss}} + \frac{p}{t_r - C_r} - \lambda_s + \lambda_u = 0 \quad (17)$$

$$\frac{\partial \mathcal{L}}{\partial \gamma_{su}} = 0 : \quad -\frac{1-p}{t_{su} - C_{su}} + \frac{1-p}{t_r - C_r} + \lambda_s - \lambda_u = 0 \quad (18)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial f_{ss}} = 0 : \\ \frac{p}{t_{ss} - C_{ss}} [-P_{Js}(y_{ss}, y_{rs})D_J] + \frac{p}{t_r - C_r} [P_{Js}(y_{ss}, y_{rs})D_J] \\ - \lambda_s [P_{Js}(y_{ss}, y_{rs})D_J] + \lambda_u [P_{Ju}(y_{ss}, y_{rs})D_J] - \lambda_1 + \lambda_3 = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial f_{su}} = 0 : \\ \frac{1-p}{t_{su} - C_{su}} [-P_{Ju}(y_{su}, y_{ru})D_J] + \frac{1-p}{t_r - C_r} [P_{Ju}(y_{su}, y_{ru})D_J] \\ + \lambda_s [P_{Js}(y_{su}, y_{ru})D_J] - \lambda_u [P_{Ju}(y_{su}, y_{ru})D_J] - \lambda_2 + \lambda_4 = 0 \end{aligned} \quad (20)$$

On adding (17) and (18) we get:

$$\frac{p}{t_{ss} - C_{ss}} + \frac{1-p}{t_{su} - C_{su}} = \frac{1}{t_r - C_r} \quad (21)$$

This relationship holds for all optima, and for it to be well defined the (IR) constraints cannot be binding at optimum, for any player-type combination. The only other possibility being that the players reach the disagreement outcome. We now consider two cases, in order to complete the proof:

Case 1: Both, f_{ss}^* and f_{su}^* are strictly less than 1.

This implies that λ_1 and λ_2 are equal to 0. Then from (17) and (19) we get:

$$\lambda_u [P_{Ju}(y_{ss}, y_{rs}) - P_{Js}(y_{ss}, y_{rs})]D_J + \lambda_3 = 0$$

Since in Assumption 1 - (ii) we have assumed that $P_{Ju}(y_1, y_2) > P_{Js}(y_1, y_2) \forall (y_1, y_2)$, and the Lagrange multipliers are non-negative; we conclude that $\lambda_u = \lambda_3 = 0$. Similarly, using (18) and (20), we can show that $\lambda_s = \lambda_4 = 0$, provided $f_{su}^* > 0$. This implies that the incentive compatibility (IC) constraints are strictly satisfied at optimum. Using (17) and (18), we conclude that:

$$t_{ss} - C_{ss}^* = t_{su} - C_{su}^* = t_r - C_r^* \quad (22)$$

Using the above results in (13), (14), (15) and (16), we get:

$$-\frac{1}{D_J} = \frac{\partial P_{J_s}(y_{ss}, y_{rs})}{\partial y_{ss}} = \frac{\partial P_{J_u}(y_{su}, y_{ru})}{\partial y_{su}} = \frac{\partial P_{J_s}(y_{ss}, y_{rs})}{\partial y_{rs}} = \frac{\partial P_{J_u}(y_{su}, y_{ru})}{\partial y_{ru}}$$

The above relationships imply the First Best, or welfare maximizing investment in security. (We get First Best investment even for the case when $f_{su}^* = 0$.) From (22) and Proposition 3, we can conclude that $C_{su}^* \geq C_{ss}^*$.

From the expression for the welfare function W , we now know that:

$$W = pC_{ss} + (1 - p)C_{su} + C_r$$

For an interior solution the above equation, along with (22) uniquely determines the total cost incurred by each player as:

$$C_{ss}^* = \frac{W^* - (1 - p)(t_{su} - t_{ss}) - (t_r - t_{ss})}{2} \quad (23)$$

$$C_{su}^* = C_{ss}^* + (t_{su} - t_{ss}) \quad (24)$$

$$C_r^* = C_{ss}^* + (t_r - t_{ss}) \quad (25)$$

NOTE: For the remaining subcase when $f_{su}^* = 0$ and $\lambda_4 > 0$, we have $\lambda_s > 0$, or (IC s) is binding. For this to hold, we would require (IC s) to be binding at the First Best levels of investment. For these cases the result in (22) may not hold. Instead we have, $C_{ss}^* = C_{su}^*$. This implies:

$$t_{ss} - C_{ss}^* \leq t_{su} - C_{su}^*$$

We show later in this proof, as part of the analysis for Case 2, that subject to the (IC) constraints being strictly satisfied, the optimal solution implies:

$$t_{ss} - C_{ss}^* = t_{su} - C_{su}^* = t_r - C_r^*$$

Hence, if a feasible solution exhibiting the above property along with First Best investment, exists, then it would rule out the case when $f_{su}^* = 0$ and $\lambda_4 > 0$, as a candidate for optimality.

Case 2: Either f_{ss}^* , and/or f_{su}^* is equal to 1.

First, consider the case when $f_{ss}^* = 1$ and $f_{su}^* < 1$. This implies that $\lambda_1 \geq 0$ and $\lambda_2 = 0$. Then by (22) we see that:

$$\lambda_u [P_{J_u}(y_{ss}, y_{rs}) - P_{J_s}(y_{ss}, y_{rs})] D_J = \lambda_1$$

which implies $\lambda_u > 0$ only when $\lambda_1 > 0$, and vice versa. By equation (20), we see that:

$$\lambda_s [P_{Js}(y_{su}, y_{ru}) - P_{Ju}(y_{su}, y_{ru})] D_J + \lambda_4 = 0$$

which implies $\lambda_s f_{su} = 0$. Then using (13) and (17), we get:

$$-\frac{p}{t_r - C_r} \left[1 + \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{ss}} D_J \right] + \lambda_u \left[\frac{\partial P_{Ju}(y_{ss}, y_{rs})}{\partial y_{ss}} - \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{ss}} \right] D_J = 0$$

From Lemma 1 we have $\frac{\partial P_{Ju}(y_1, y_2)}{\partial y_1} < \frac{\partial P_{Js}(y_1, y_2)}{\partial y_1} \forall (y_1, y_2)$, and when $\lambda_u > 0$, we conclude that:

$$1 + \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{ss}} D_J < 0$$

By similar analysis we also get that:

$$1 + \frac{\partial P_{Js}(y_{ss}, y_{rs})}{\partial y_{rs}} D_J < 0$$

$$1 + \frac{\partial P_{Ju}(y_{su}, y_{ru})}{\partial y_{su}} D_J = 0$$

$$1 + \frac{\partial P_{Ju}(y_{su}, y_{ru})}{\partial y_{ru}} D_J = 0$$

This means that while the values of y_{su} and y_{ru} are the same as in the First Best case, the values of y_{ss} and y_{rs} are below their corresponding First Best values. When $\lambda_u = 0$ we still get First Best investment.

For the situation when $f_{ss}^* < 1$ and $f_{su}^* = 1$, we have $\lambda_1 = \lambda_4 = 0$. Then we get the following relationships:

$$\lambda_u [P_{Ju}(y_{ss}, y_{rs}) - P_{Js}(y_{ss}, y_{rs})] D_J + \lambda_3 = 0$$

$$\lambda_s [P_{Js}(y_{su}, y_{ru}) - P_{Ju}(y_{su}, y_{ru})] D_J - \lambda_2 = 0$$

which together imply $\lambda_u = \lambda_3 = \lambda_s = \lambda_2 = 0$. This leads to the same optimal solution as that for an interior solution to the bargaining game.

We now look at the case when $f_{ss}^* = 1$ and $f_{su}^* = 1$. The given condition would imply $\lambda_3 = \lambda_4 = 0$. Using these in (20), we see that:

$$\lambda_s [P_{Js}(y_{su}, y_{ru}) - P_{Ju}(y_{su}, y_{ru})] D_J - \lambda_2 = 0$$

which cannot be the case for non-negative lagrange multipliers unless $\lambda_s = \lambda_2 = 0$. Also from (19):

$$\lambda_u [P_{Ju}(y_{ss}, y_{rs}) - P_{Js}(y_{ss}, y_{rs})] D_J = \lambda_1$$

The analysis now would be similar to that for the case with $f_{ss}^* = 1$ and $f_{su}^* < 1$, which means there is a possibility of under-investment.

Next, we show that if there exists a feasible solution which leads to First Best investment, then an alternate feasible solution leading to under-investment cannot be optimal. Consider the optimization problem:

$$\max_{C_{ss}, C_{su}, C_r} [p \log(t_{ss} - C_{ss}) + (1-p) \log(t_{su} - C_{su}) + \log(t_r - C_r)]$$

subject to:

$$\begin{aligned} \frac{p}{t_{ss} - C_{ss}} + \frac{1-p}{t_{su} - C_{su}} &= \frac{1}{t_r - C_r} \\ pC_{ss} + (1-p)C_{su} + C_r &= W \end{aligned}$$

The lagrangian of the above optimization problem looks like:

$$\begin{aligned} \mathcal{L} &= p \log(t_{ss} - C_{ss}) + (1-p) \log(t_{su} - C_{su}) + \log(t_r - C_r) \\ &+ \mu_1 \left[\frac{p}{t_{ss} - C_{ss}} + \frac{1-p}{t_{su} - C_{su}} - \frac{1}{t_r - C_r} \right] + \mu_2 [pC_{ss} + (1-p)C_{su} + C_r - W] \end{aligned}$$

The FOC imply:

$$\frac{\partial \mathcal{L}}{\partial C_{ss}} = 0:$$

$$\frac{-p}{t_{ss} - C_{ss}} + \mu_1 \frac{p}{(t_{ss} - C_{ss})^2} + \mu_2 p = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_{su}} = 0:$$

$$-\frac{1-p}{t_{su} - C_{su}} + \mu_1 \frac{1-p}{(t_{su} - C_{su})^2} + \mu_2 (1-p) = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_r} = 0:$$

$$-\frac{1}{t_r - C_r} - \mu_1 \frac{1}{(t_r - C_r)^2} + \mu_2 = 0$$

The above conditions can be satisfied only at $\mu_1 = 0$, which implies:

$$t_{ss}^* - C_{ss}^* = t_{su}^* - C_{su}^* = t_r^* - C_r^*$$

and hence:

$$C_{ss}^* = \frac{W - (1-p)(t_{su} - t_{ss}) - (t_r - t_{ss})}{2}$$

$$C_{su}^* = C_{ss}^* + (t_{su} - t_{ss})$$

$$C_r^* = C_{ss}^* + (t_r - t_{ss})$$

The proves that the optimal solution is decreasing in W , which is minimized at First Best investment. \square

Corollary 5.1

Proof The analysis of the robust HSN-SCD contract, with audit, is as follows:

$$GNP^* = \max_{y, \gamma, q, P} \{p \log(t_{ss} - C_{ss}) + (1 - p) \log(t_{su} - C_{su}) + \log(t_r - C_r - pc_s q_s - (1 - p)c_u q_u)\}$$

subject to:

$$y_{s\tau} + \gamma_{s\tau} + q_\tau P_\tau \geq y_{s\tau'} + \gamma_{s\tau'} \quad (\text{IC } \tau)$$

$$t_{s\tau} - C_{s\tau} \geq 0 \quad (\text{IR } \tau)$$

$$t_r - C_r - pc_s q_s - (1 - p)c_u q_u \geq 0 \quad (\text{IR } r)$$

$$P_\tau \leq l \quad (\text{MP } \tau)$$

$$q_\tau \geq 0 \quad (\text{NNG } \tau)$$

where $\tau \in \{s, u\}$, and (MP) stands for maximum penalty, while (NNG) stands for non-negativity of q . We now have a strictly concave objective function, which is to be maximized over a convex feasible region. Hence, the optimization problem has a unique solution.

As before, we assume an interior solution w.r.t. the security investments y . We observe that (MP s) and (MP u) will be binding at optimum because the values of P_s and P_u do not directly affect the objective, but higher values for P_τ results in lower q_τ . Moreover, the objective function is increasing in q_τ . The Lagrangian of the problem (ignoring the (IR) constraints for the moment) is:

$$\begin{aligned} \mathcal{L} = & p \log(t_{ss} - C_{ss}) + (1 - p) \log(t_{su} - C_{su}) + \log(t_r - C_r - pc_s q_s - (1 - p)c_u q_u) \\ & + \lambda_s [y_{ss} + \gamma_{ss} + q_s l - y_{su} - \gamma_{su}] + \lambda_u [y_{su} + \gamma_{su} + q_u l - y_{ss} - \gamma_{ss}] + \mu_s q_s + \mu_u q_u \end{aligned}$$

Taking the FOCs w.r.t. q_s, q_u we get:

$$\frac{-pc_s}{t_r - C_r - pc_s q_s - (1 - p)c_u q_u} + \lambda_s l + \mu_s = 0 \quad (26)$$

$$\frac{-(1 - p)c_u}{t_r - C_r - pc_s q_s - (1 - p)c_u q_u} + \lambda_u l + \mu_u = 0 \quad (27)$$

Equations (26) and (27) imply that:

$$\frac{pc_s}{\lambda_s l + \mu_s} = \frac{(1-p)c_u}{\lambda_u l + \mu_u} \quad (28)$$

Since $c_\tau > 0$, the following 4 cases arise:

Case 1: $\mu_s = \mu_u = 0 \Rightarrow q_s^* > 0, q_u^* > 0$.

Then (28) gives us:

$$\frac{pc_s}{\lambda_s} = \frac{(1-p)c_u}{\lambda_u}$$

which leads to a contradiction, since, $q_s^* > 0$ and $q_u^* > 0$, we cannot have both:

$$y_{ss}^* + \gamma_{ss}^* > y_{su}^* + \gamma_{su}^*$$

$$y_{su}^* + \gamma_{su}^* > y_{ss}^* + \gamma_{ss}^*$$

Case 2: $\mu_s > 0, \mu_u = 0 \Rightarrow q_s^* = 0, q_u^* > 0$.

Then (28) gives us:

$$\frac{pc_s}{\lambda_s l + \mu_s} = \frac{(1-p)c_u}{\lambda_u l}$$

The situation in which $\lambda_s > 0$ and $\lambda_u > 0$ is ruled out because of the logic used in Case 1. The case in which $\lambda_s = 0$ and $\lambda_u > 0$ is ruled out because it results in the following conditions:

$$y_{ss}^* + \gamma_{ss}^* > y_{su}^* + \gamma_{su}^*$$

$$y_{su}^* + \gamma_{su}^* + q_u^* l = y_{ss}^* + \gamma_{ss}^*$$

which again leads to a contradiction. Hence, Case 2 is ruled out.

Case 3: $\mu_s = 0, \mu_u > 0 \Rightarrow q_s^* > 0, q_u^* = 0$. This case is ruled out by an argument analogous to the one used to rule out Case 2.

Case 4: $\mu_s > 0, \mu_u > 0 \Rightarrow q_s^* = 0, q_u^* = 0$. This is the only feasible case. □

C Appendix: Generalized Probability of Disruption

Most of our results still go through (except Proposition 3-(ii) and uniqueness of the PSBNE), when we relax the requirement of an exponential functional form on the marginal disruption probabilities, and even allow the disruption probabilities at the retailer and supplier locations to be correlated; subject to the following six assumptions:

$$\left| \frac{\partial P_{Ju}(y_s, y_r)}{\partial y_s} \right| > \left| \frac{\partial P_{Js}(y_s, y_r)}{\partial y_s} \right|, \forall (y_s, y_r) \in \mathcal{R}_+^2 \quad (\text{A1})$$

This implies that the marginal benefit of the supplier's investment is higher if it is of the unsafe type. This suggests that high investment in risk mitigation is a viable strategy for the highly vulnerable supplier. The situations where the highly vulnerable environment is not more responsive to investment in risk mitigation can be considered to be outside the scope of this model.

$$P_{Ju}(y_s, y_r) > P_{Js}(y_s, y_r), \forall (y_s, y_r) \in \mathcal{R}_+^2 \quad (\text{A2})$$

This means that for the same level of investment, the joint probability of disruption is higher for the unsafe supplier.

$$\left| \frac{\partial P_{Ju}(y_{su}, y_r)}{\partial y_{su}} \right| = \left| \frac{\partial P_{Js}(y_{ss}, y_r)}{\partial y_{ss}} \right| \Rightarrow P_{Ju}(y_{su}, y_r) > P_{Js}(y_{ss}, y_r) \quad (\text{A3})$$

This implies that when the marginal benefit of investment is made equal for both types of the supplier (keeping the retailer's investment same), then at that level of investment, the probability of disruption is greater for the unsafe supplier. This property of the joint distributions ensures that the non-cooperative equilibrium results in a higher cost for the unsafe supplier, as compared to the safe type. It is also relevant in ensuring that (A2) is reasonable.

$$P_{J\tau}(y_s, y_r) \in \mathcal{C}^2 \quad (\text{A4})$$

This means that the combined probability of disruption decreases with increasing investment by either player, i.e. $\frac{\partial P_{J\tau}}{\partial y_s} < 0$ and $\frac{\partial P_{J\tau}}{\partial y_r} < 0$. It is also reasonable to expect that the marginal benefit of investment by either player (proportional to the negative slope of the joint probability) decreases with increasing investment. This would imply that $\frac{\partial^2 P_{J\tau}}{\partial y_s^2} > 0$ and $\frac{\partial^2 P_{J\tau}}{\partial y_r^2} > 0$.

$$\frac{\partial^2 P_{J\tau}(y_s, y_r)}{\partial y_s \partial y_r} < 0 \quad (\text{A5})$$

This assertion also implies that $\frac{\partial^2 t_{J\tau}(y_s, y_r)}{\partial y_s \partial y_r} < 0$, hence y_s and y_r are strategic complements. This means that the marginal benefit of investment by the supplier is higher for higher levels of investment by the retailer, and vice versa. To better illustrate this notion we once again refer to the case of RFID implementation. The marginal benefit to the supplier from investment in RFID would be higher if the retailer also implemented RFID at its location (due to the weakest link property of supply chains), and vice versa.

$$\left| \frac{\partial P_{Ju}(y_{su}, y_r)}{\partial y_{su}} \right| = \left| \frac{\partial P_{Js}(y_{ss}, y_r)}{\partial y_{ss}} \right| \Rightarrow \left| \frac{\partial P_{Ju}(y_{su}, y_r)}{\partial y_r} \right| < \left| \frac{\partial P_{Js}(y_{ss}, y_r)}{\partial y_r} \right| \quad (\text{A6})$$

This implies that when the marginal benefit of investment is made equal for both types of the supplier, then the marginal benefit of investment by the retailer, is higher for the safe supplier than the unsafe type. When the environments of both types of the supplier have been brought to a state when they are equally responsive to the supplier's investment in risk mitigation, it is reasonable for the marginal benefit of an additional dollar of retailer investment, to be higher for the safe type of supplier, since her environment is more conducive for risk mitigation.