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1. Introduction

It is known that the Bertrand outcome can be avoided when sellers are capacity constrained (Edgeworth 1897); when products are differentiated (d'Aspremont, Gabszewicz, and Thisse 1979); when the model is changed from one of short-run competition to long-run competition (Chamberlin 1929); when consumers are not perfectly informed or if it is costly for them to obtain information (Salop 1977). Jain and Kannan [2002] show that undifferentiated providers of online databases can make positive profits if their marginal costs are large enough and consumers are risk averse and uncertain about their usage. We show another way that duopolists selling information goods may use to earn monopoly profits even when sellers’ marginal costs are negligible and buyers are risk-neutral and informed about their usage. Specifically, when there are two segments of buyers “suited” to different pricing schemes, then the firms’ adopt distinct pricing schemes to differentiate themselves and obtain substantial profits². A pricing scheme is better “suited” to a set of buyers if the scheme can extract greater monopoly profits from these buyers compared to other pricing schemes. There is extensive literature in the IS area that examines the optimal pricing of information goods without addressing why sellers may use multiple pricing schemes. Recent papers include Sundararajan [2003], Bhargava and Choudhary [2001].

2. Model

Our model is set in the context of information goods such as movies, online newspapers and journal archives that are available on a per-unit basis as well as subscription. In particular, we focus on software products that can be purchased by firms on a per-user basis or as a site license. A site license provides access to an unlimited number of users while per-unit pricing provides access to a single user.

We model two undifferentiated sellers with perfectly substitutable products with no marginal costs of production and unlimited capacity. Buyer firms are perfectly informed about prices and there are no transactions costs. There exist two segments of buyers, one with a declining marginal willingness to pay (WTP) for multiple units and the second with a cumulative WTP for a fixed number of units. For example a firm may have the usual declining marginal WTP for Adobe Acrobat software, purchasing licenses for fewer employees when prices are higher. On the other hand, firms that plan to use Acrobat as part of their enterprise system for process automation and work flow require certain employees to use the software and therefore need a fixed number of licenses. We refer to buyers with declining marginal WTP as k-type buyers and assume that they constitute a fraction k of the total number of buyers. The remaining (1-k) buyers are referred to as type (1-k) buyers. Each k-type buyer has a declining marginal willingness to pay for multiple units which can be expressed in terms of the inverse demand function for each buyer:

\[ p_{unit} = a - q \]

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² Some parallels may be drawn between our model and the horizontal differentiation model, although our model has no transportation costs and the firms’ products are perfectly substitutable.
where $p_{\text{unit}}$ is the per unit price of the software, $q$ the quantity of licenses bought by each buyer, and $a$ the willingness to pay for the first license. The net surplus for a type $k$ buyer for $q$ units is:

$$\int_0^q (a - x) \, dx - p_{\text{unit}} \cdot q \quad \text{Or} \quad \int_0^q (a - x) \, dx - p_{\text{site}}$$

where $p_{\text{site}}$ is the price for the site license. When buying a site license, there is a fixed price for the first license and additional licenses are free. Therefore, the surplus from buying a site license is $\left(\frac{a^2}{2}\right) - p_{\text{site}}$. If the site license or subscription involves a stream of payments then $p_{\text{site}}$ is simply the discounted present value of the payment stream.

The cumulative WTP for each of the (1-$k$) type buyers is $v \cdot n$ where $v$ is the average benefit per user and $n$ the number of users. These buyers yield a discrete demand function where they buy $n$ licenses for a price up to $v \cdot n$. Across this segment of buyers, let $n$ be uniformly distributed over $[0, 1]$.

We begin by determining conditions under which a monopoly constrained to offer only one pricing scheme to two segments of buyers would find it optimal to address only one segment. If a different pricing scheme addresses a different buyer segment then a separating equilibrium may exist where each duopolist sells to an uncontested segment of buyers.

3. Monopoly Constrained to Site Licensing

If the monopolist offers only a site licensing policy, then all $k$-type buyers are willing to pay up to $(a^2/2)$ and the indifferent (1-$k$) type buyer is the one with benefit $v \cdot n = p_{\text{site}}$. Hence the fraction of (1-$k$) type buyers who buy at price $p_{\text{site}}$ is given by $(v - p_{\text{site}})/(v)$. The profit function may be expressed as:

$$\pi_{\text{site}} = \begin{cases} 
(p_{\text{site}} \cdot k) + \frac{(v - p_{\text{site}})}{v} p_{\text{site}} (1 - k) & \text{when} \quad p_{\text{site}} \leq v \quad \text{and} \quad p_{\text{site}} \leq \frac{a^2}{2} \\
p_{\text{site}} \cdot k & \text{when} \quad v < p_{\text{site}} \leq \frac{a^2}{2} \\
\frac{(v - p_{\text{site}})}{v} p_{\text{site}} (1 - k) & \text{when} \quad \frac{a^2}{2} < p_{\text{site}} \leq v
\end{cases}$$

Let $p_{\text{site}}^*$ be the price that maximizes $\pi_{\text{site}}$. Lemma 1 states the conditions under which all k-type buyers are covered while none of the (1-k) type buyers participate.

**Lemma 1** When $v < a^2/2$ and either (i) $k < 0.5$ or (ii) $v < 2k(1-k)a^2$ then $p_{\text{site}}^* = a^2/2$, $\pi_{\text{site}}^* = ka^2/2$, all k-type buyers are covered while none of the (1-k) type buyers participate.
A monopoly with site licensing compared to unit pricing compared to site licensing. Then
\[ (1-k) \text{ type buyers are willing to pay} \]
\[ \frac{v}{2} \text{ with each buying} \]
\[ n \text{ licenses. The} \ k \text{ type buyers are willing to pay up to} \]
\[ a \text{ with each buying} \]
(\(a-p_{\text{unit}}\)) licenses at price \(p_{\text{unit}}\). The profit function may be expressed as:
\[ \pi_{\text{unit}} = \begin{cases} p_{\text{unit}}k(a - p_{\text{unit}}) + p_{\text{unit}}(1-k) & \text{when } p_{\text{unit}} \leq v \text{ and } p_{\text{unit}} \leq a \\ p_{\text{unit}}k(a - p_{\text{unit}}) & \text{when } v < p_{\text{unit}} \leq a \\ p_{\text{unit}}(1-k)/2 & \text{when } a < p_{\text{unit}} \leq v \end{cases} \]

Let \(p_{\text{unit}}^*\) be the price that maximizes \(\pi_{\text{unit}}\). The following proposition states conditions under which the firm covers all (1-k) type buyers without the participation of any k-type buyers.

**Lemma 2** When \(v > a\) and either (i) \(\frac{1-k}{2k} > a\) or (ii) \(v > \frac{(2ak + (1-k))^2}{8k(1-k)}\) then \(p_{\text{unit}}^* = v\).

\(\pi_{\text{unit}}^* = (1-k)v/2\), all (1-k) type buyers are covered while none of the k-type buyers participate.

5. Duopoly

Having established lemmas 1 and 2, we can obtain conditions under which a duopoly operates as two distinct monopolies, using different pricing schemes. The model consists of a two stage game where the duopolists choose a pricing scheme in the first stage and the price to be charged in the second stage. In stage one, each firm may decide to use the per-unit pricing policy, or the site licensing policy or both.

When the two firms adopt the same pricing scheme or if one adopts both schemes, Bertrand competition ensues, forcing all prices to zero. Hence we have the following payoff matrix (table 1) where neither seller earns a profit if there is any overlap in their pricing schemes.

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3 The two stages are necessary to ensure the existence of an equilibrium that supports positive profits for the sellers. The cost of changing the price is much smaller than the cost of changing the pricing scheme which includes the cost of making consumers aware of the new pricing policy; retraining employees; and adjusting cash registers and other systems.
Table 1: Payoff from adopting various pricing schemes

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<tr>
<th></th>
<th>Per-unit pricing</th>
<th>Site license pricing</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per-unit pricing</td>
<td>0, 0</td>
<td>(\pi_{\text{unit}}, \pi_{\text{site}})</td>
<td>0, 0</td>
</tr>
<tr>
<td>Site license pricing</td>
<td>(\pi_{\text{site}}, \pi_{\text{unit}})</td>
<td>0, 0</td>
<td>0, 0</td>
</tr>
<tr>
<td>Both</td>
<td>0, 0</td>
<td>0, 0</td>
<td>0, 0</td>
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When the conditions stated in lemmas 1 and 2 are satisfied, \(\pi_{\text{site}}, \pi_{\text{unit}}\) equals monopoly profits under the respective pricing scheme.

**Proposition 1** When (i) \(v > a\), (ii) \(k < 0.5\), (iii) \(v < 2k(1-k)a^2\) and (iv) \(v > \frac{(2ak + (1-k))^2}{8k(1-k)}\), then \(p_{\text{site}}^A = \frac{a^2}{2}\) and \(p_{\text{unit}}^B = v\) is the pareto-dominant pure strategy equilibrium of the two stage game. One of the firms \((A)\) offers a site license based pricing policy with price \(a^2/2\) thus extracting the entire surplus from all \(k\)-type buyers and earning a profit of \(\pi_{\text{site}}^A = ka^2/2\). The other firm \((B)\) offers a per unit pricing policy with price \(v\), thus extracting the entire surplus from all \((1-k)\)-type buyers and earning a profit of \(\pi_{\text{unit}}^B = (1-k)v/2\).

Is it possible for one (or both) firm(s) to set prices that attract some buyers from each segment and still earn monopoly profits? In other words, is a separating equilibrium necessary? To see that this is necessary, note that the profit from setting a price equal to the WTP of \(k\)-type buyers provides a lower-bound on A’s monopoly profits. Similarly, the monopoly profit of B is never less than the profit from setting a price equal to the WTP of \((1-k)\) type buyers. In order for A and B to obtain these profit lower-bounds, there must be no deadweight loss. If any \(k\)-type buyer buys from B (strictly positive price), then there will be some dead-weight loss hence all \(k\)-type buyers must buy from A. It follows that for B to achieve its profit lower bound it must sell to all \((1-k)\)-type buyers. Hence the conditions stated in proposition 1 are necessary to ensure that the duopolists \((A\) and \(B)\) receive the monopoly profits stated in lemmas 1 and 2.

The existence of a feasible region has been verified (see Fig 2).

![Figure 2(a) Feasible region for Proposition 1 \((k=0.25)\), (b) Profit \((\pi_A, \pi_B)\) as a function of \(k\) plotted within the feasible region \((v=24, a=10)\).](image)

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4 In addition, each seller playing “both” is also a pure strategy equilibrium resulting in zero profits for each seller; and there exist mixed strategy equilibria.
Proposition 2: The conditions stated in proposition 1 are necessary and sufficient to ensure that the duopolists obtain profits equal to that of two monopolists, each using a different pricing scheme as stated in lemmas 1 and 2. There exists a feasible region, where these conditions are satisfied.

6. Conclusion

We have shown that duopolists selling information goods may be able to achieve monopoly profits by using different pricing policies – per unit pricing and site-license (or subscription) based pricing. From the firm’s perspective, the per-unit pricing scheme is better suited to (1-k) type buyers since a monopoly using the unit pricing scheme can extract the entire surplus from (1-k) type buyers while the site licensing policy would yield only half as much profit. Similarly, the site licensing policy is better suited to k-type buyers since a monopoly using the site license based pricing can extract the entire surplus whereas the unit pricing policy would yield half the profit.

Such pricing schemes are commonly observed for information goods. Websense.com, a software firm in the Employee Internet Management market offers only per-seat pricing while its competitor Apreo.com offers a fixed price for 100 licenses. Many software publishers such as Oracle and Microsoft have pricing policies that include a number of additional dimensions such as the number of CPUs, the number of developers etc. which could provide additional means of differentiating pricing policies. The existence of such equilibria may raise interesting policy questions. The existence of multiple firms in the same (otherwise undifferentiated) market may not be enough to ensure a competitive outcome. The absence of barriers to entry would also be insufficient if the entrants can foresee the Bertrand outcome that would result from competing within certain pricing schemes.

References


