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Strategic Inventories in Vertical Contracts

Krishnan Anand
Wharton School, University of Pennsylvania, Philadelphia, PA 19104-6340, anandk@wharton.upenn.edu

Ravi Anupindi
Stephen M. Ross School of Business, University of Michigan, Ann Arbor, MI 48109, anupindi@umich.edu

Yehuda Bassok
Marshall School of Business, University of Southern California, Los Angeles, LA 90089, bassok@marshall.usc.edu

Classical reasons for carrying inventory include fixed (non-linear) production or procurement costs, lead times, non-stationary or uncertain supply / demand, and capacity constraints. The last decade has seen active research in supply chain coordination focusing on the role of incentive contracts to achieve first-best levels of inventory. An extensive literature in Industrial Organization that studies incentives for vertical controls largely ignores the effect of inventories. Does the ability to carry inventory influence the problem of vertical control? Conversely, can inventories arise purely due to incentive effects? This paper explicitly considers both incentives and inventories, and their interplay, in a *dynamic* model of an upstream firm (supplier) and a downstream firm (buyer) who can carry inventories. In our model none of the classical reasons for carrying inventory exists. Yet (as we prove) the buyer's optimal strategy in equilibrium is to carry inventories, and the supplier is unable to prevent this. These inventories arise out of purely *strategic* considerations not identified before in the literature, and have a significant impact on the equilibrium solution as well as supplier, buyer and channel profits.

We prove that strategic inventories play a pivotal role under arbitrary contractual structures, general (arbitrary) demand functions and general (finite or infinite) horizon lengths. As one example, two-part tariff contracts do not lead to optimal channel performance, nor can the supplier extract away all of the channel profits, in our dynamic model. Our results imply that firms can and must carry inventories strategically, and that optimal vertical contracts must take the possibility of inventories into account.

Key words: Contracts; Inventories; Industrial Organization, Supply Chain Coordination

1. Introduction

Inventory may be carried for a number of well-documented reasons (*cf.* Zipkin (2000), Nahmias (2004), and Anupindi *et al.* (2006)). First, due to economies of scale in procurement or production, it may be cheaper to procure or produce in quantities larger than what is immediately needed, resulting in *cycle inventory.* Second, *pipeline inventory* may be carried to ensure availability of goods in the face of delivery or production delays. Third, a firm may carry *safety inventory* to cope with uncertainty in demand or supply. Fourth, firms may carry *speculative inventory* to hedge against price fluctuations. Finally, while economies of scale lead to cycle inventory, a firm with production diseconomies (e.g., increasing marginal production cost) and fluctuating demand may carry inventory to smooth production and thus lower production costs; see Holt *et al.* (1960).

Using models that incorporate one or more of the above characteristics, much of the literature on supply chain coordination has focused on developing incentive contracts to achieve optimal levels of inventory. Tsay, Nahmias, and Agarwal (1998) give a review of the supply chain contracting literature including both deterministic and stochastic demand models. Cachon (2003) is a more recent review of the literature that focuses exclusively on models with demand uncertainty. Supply chain coordination continues to be an active area of research with several papers published every year. In contrast, much of the literature on contract theory has focused on models that don't take inventories into account.[1] The primary focus of this body of literature has been on incentives for motivating the right action under moral hazard or asymmetric information. Likewise, the extensive literature in Industrial Organization on incentives for vertical controls also ignores the effect of inventories: This body of work studies various mechanisms for vertical control with sales in a period originating from *current* production in that period (Tirole 1990).

The classical reasons for holding inventories, discussed above, arise even in a non-competitive context, due to the economics of matching supply and demand. Vertical control issues in such environments have been studied, for example, by Deneckere et al (1996). Some researchers have demonstrated that inventories may play a strategic role under horizontal competition. Specifically, Saloner (1986) considers a two-period duopoly model in which firms produce and sell in the first period but may salvage inventory carried over to the next period. Although no inventories are carried in equilibrium, the firm with the first-mover advantage at the production stage is unable to

[1] For instance, the index of the monumental Laffont and Tirole (1993), devoted to procurement contracts and regulation, does not even mention inventories.

achieve the Stackelberg outcome, because it cannot credibly commit to selling its entire production in the first period. Rotemberg and Saloner (1989) consider a duopoly model in which colluding firms keep inventories to sustain collusive profits by the threat of reversion to competitive behavior. Similarly, a firm may invest in inventories to pre-empt its competitor's future production (*cf.* Mollgaard et al. (2000) and references therein).

As in the papers on horizontal competition cited above, none of the classical reasons for holding inventories exist in our model as well. However, our focus is on a *vertical* value chain with a supplier and a buyer. We demonstrate in our dynamic model that *strategic inventories* are an artifact of contractual structure, and significantly alter the contractual outcome (the equilibrium solution, as well as supplier, buyer and channel profits, consumer surplus and welfare). Hence, firms can and must carry inventories strategically, and optimal contract design must take the possibility of inventories into account. [2]

The Static Problem

The classical static (single-period) version of our model, with linear costs and prices, is analyzed in many economics textbooks (*cf.* Tirole (1990)). Consider a vertical value chain with a single supplier and a solitary buyer. The supplier has constant economies of scale in procurement (or, production), and sells to the buyer at a linear wholesale price. The buyer in turn sells her purchased quantity in a market with a known (linear) demand curve. When supplier and buyer maximize their individual profits, the resulting incentive misalignment leads to the well-known *double marginalization*;[3] i.e., the supplier and buyer add their margins to the price quoted downstream, and the final (retail) price is higher than what is optimal for the channel. The quantity sold is correspondingly lower, and total channel profits fall due to double marginalization.

The Dynamic Problem

[2] A similar strategic interaction between a buyer and a supplier in the context of capacity is explored by Erhun *et al.* (2000). Here the supplier 'sells' manufacturing capacity, and the buyer can purchase capacity options at different times to produce, once, at the end of the horizon.

[3] See Spengler (1950) for an early reference.

In this paper we focus on the dynamic version of the classical model, first in a two-period setting and later extending it to arbitrary horizon lengths. The buyer may carry inventories across periods, while incurring holding costs. None of the classical reasons for holding inventories exist in our model. Furthermore, there is no lateral competition (between buyers) as in Rotemberg and Saloner (1989) and Mollgaard et al. (2000). Therefore, we would expect that the equilibrium solution for the dynamic problem simply mimics the static solution in each period, and that no inventories are carried. We prove that this "intuition" does not hold. In fact, in the two-period model, the buyer *does* carry inventories in equilibrium. We postulate that the inventories carried in our model are motivated purely by strategic considerations that override any additional (procurement or holding) costs.

In our model, the buyer uses strategic inventories to force the supplier to lower his second-period wholesale price. Since inventories, via holding costs, are a drain on channel profits (and perhaps also on supplier and buyer profits), the supplier might be willing to directly offer the lower second-period wholesale price in exchange for reduced inventories, if this were credible. We explore such a *commitment contract*, where the supplier quotes wholesale prices for both periods at the beginning of the first period, and the buyer then decides on his purchase quantities. As expected, we find that such commitment contracts eliminate inventories from the system, and the equilibrium for the dynamic game mimics the solution for the one-period game in each period. Surprisingly, however, we find that the profits do not move in the direction we would expect. For a wide range of holding costs, buyer, supplier and channel profits are *greater* under the short-term (dynamic) contracting mechanism than under the commitment contract; thus, the commitment contract is in fact Pareto-dominated by the dynamic contract. Further, consumer surplus and welfare are both greater under the dynamic contract than under the commitment contract, even though inventory holding costs are incurred in the former. These results imply that strategic inventories, far from being just a drain on channel profits through the holding costs, have actually *improved* channel coordination.

Several questions arise at this point. (a) Are strategic inventories, and the Pareto-improvement induced by them, a consequence only of the restrictive contract space of linear wholesale prices, which the possibility of carrying inventories expands? (b) Do the results for linear demand curves extend to more general demand functions? (c) Are they an artifact of "end-of-period" effects arising from the two-period formulation? If not, do strategic inventories arise under longer horizons? To address the first issue, we relax the contract space by extending the vertical contract to two-part tariffs which we discuss next. Later, we extend the analysis to general (arbitrary) dynamic contracts. To address the second issue, we relax the linear demand curve to consider convex, piecewise-linear demand functions with linear wholesale contracts and show that strategic inventory is always carried.[4] Finally, to respond to the third question, we analyze the problem for longer horizons under the relaxed contract space and demand structure.

Two-part tariffs

Two-part tariffs are widely used in Business-to-Business contracts, where their popularity is second only to linear prices. They have been extensively studied by researchers in Economics and Business. Within the Operations and Marketing literatures, they are of particular interest as a form of quantity discounting (*cf.* Jeuland and Shugan (1983), Moorthy (1987), and Weng (1995)). In the static problem, the supplier quotes a fixed fee in addition to a linear wholesale price. As is well-known, two-part tariffs lead to the first-best solution, and the supplier extracts all of the channel profits (Tirole 1990). In a two-period setting, we study both dynamic and commitment contracts that follow a two-part tariff structure. We find that inventories continue to play a pivotal strategic role in the equilibrium outcomes as well as individual and channel profits. Specifically, the first-best solution cannot be implemented in either case. Further, under the two-part tariff contract with commitment, the buyer makes positive residual profits that the supplier is unable to extract.

General demand functions, arbitrary dynamic contracts, longer horizons

We first relax the demand function to consider a general convex, piecewise-linear demand curve. Under linear wholesale price contracts, inventory is always carried in equilibrium in the two-period problem. We then further relax the contract and demand structures– allowing both very general

[4] Note that any demand function can be approximated to any desired degree by an appropriately chosen piecewise-linear demand function.

demand functions and the most general (arbitrary) *dynamic* contracts, under arbitrary (finite or infinite) horizon lengths. We show that the buyer's ability to carry inventories continues to affect the equilibrium outcome. Specifically, the supplier cannot enforce the first-best solution and simultaneously extract all of the buyer's residual profits. We prove a similar result for commitment contracts under linear or quantity-discount pricing schemes. However, under general commitment contracts, the supplier can attain first-best profits.

The rest of the paper is organized as follows. In the next section, we present the analysis and implications of the main model (with linear wholesale prices). In section 3 we analyze the impact of two-part tariff wholesale contracts. Then, in section 4, we first show that, in equilibrium, inventory is carried for piecewise-linear convex demand functions under dynamic linear wholesale prices. We also extend our results to general horizons, arbitrary contracts and general demand functions. Finally, we summarize and conclude with a discussion of future research extensions in section 5. Proofs of all results are provided in an online supplement to this manuscript.

2. Model and Analysis with Linear Wholesale Prices

We formulate a dynamic (two-period) model of vertical contracting under full information and no uncertainty.[5] The buyer may carry inventories from the first period to the second. Following Rotemberg and Saloner (1989) and many other precedents in the academic literature (see Tirole (1990)), we (initially) focus on models with linear demand curves. Further, we assume that the slope and intercept of the demand curve are known to both parties, and identical for both periods.

Our initial model assumes linear wholesale prices, which are widely observed in practice. They have been extensively used in economic models as well, and their use helps the supplier forestall buyer arbitrage (Tirole 1990). Later, we will study the performance of two-part tariff wholesale contracts, and eventually extend the analysis to arbitrarily general vertical contracts.

We first summarize the analysis of the well-known static (single-period) model to facilitate comparisons with the dynamic model. In the static model, the supplier first quotes a linear wholesale

[5] As will become clear in the analysis, these assumptions help to isolate the *strategic* interactions between buyer and supplier via inventories, without muddying the waters through other effects that are not the focus of this paper.

price w, and the buyer responds with his purchase quantity Q. The buyer then sells the quantity q in the market, at a price given by the linear demand curve $p(q) = a - bq$. The unit production cost for the supplier is normalized to zero. To eliminate arbitrage, we assume here and throughout this paper that the salvage value of the good is zero. Hence the buyer will sell all of the purchased goods in the market (i.e., $q = Q$), at the price $p(q)$. Both buyer and supplier maximize their individual profits. For this game, the unique equilibrium outcome is: wholesale price, $w = a/2$; purchase (and sales) quantity, $Q = q = a/4b$; and the retail price, $p = 3a/4$. The corresponding profits[6] are $\Pi_B = a^2/16b; \Pi_S = a^2/8b$; and $\Pi_C = \Pi_B + \Pi_S = 3a^2/16b$.

Observe that had the channel been owned by a single player, the optimal outcome would be $q^{fb} = a/2b; p^{fb} = a/2$; and $\Pi_C^{fb} = a^2/4b$, where the superscript fb denotes the $f$irst-$b$est solution: this maximizes channel profits. When supplier and buyer maximize their individual profits, double marginalization leads to a retail price higher than optimal for the channel, and a correspondingly lower sales quantity. The loss for the channel from double marginalization is $\Pi_C^{fb} - \Pi_C = a^2/16b$, which is a full *quarter* of the first-best profits.

2.1. A Two-period Model

We extend the static model to two periods, where the demand curve, $p(q) = a - bq$, is identical in each period. Further, the buyer can hold part or all of his first-period purchases as inventories, to sell in the second period.[7] We assume that the buyer's per-period holding cost is linear, at h per unit where $0 < h < a/4$.[8] Since there are two periods, we consider two types of contracts depending on when the supplier announces the prices, viz., a *dynamic* and a *commitment contract.* Under the dynamic contract, the supplier sequentially announces the wholesale prices at the beginning of each period. Specifically, the supplier quotes a wholesale price w_1 in the first period, and the

[6] Throughout this paper, we use the subscripts B, S and C to denote the Buyer, the Supplier and the Channel respectively.

[7] Since the unit production cost is identical in the two periods and production is immediate, the supplier does not ever carry inventories.

[8] The lower bound ensures that carrying inventory is costly while the upper bound ensures that carrying inventories is feasible. Since the demand intercept (and hence, the maximum possible price) is a, this assumption is consistent with most settings. When the holding cost is too high for inventories to be feasible, the dynamic problem decouples in effect into two separate single-period problems.

buyer responds with his purchase quantity Q_1 for immediate delivery. The buyer sells the quantity $q_1 \le Q_1$, at the price $p(q_1)$, and carries inventory $I = Q_1 - q_1$. In the second period, the supplier quotes the wholesale price w_2. Then the buyer purchases and immediately acquires Q_2 and sells $q_2 = (Q_2 + I)$.

On the other hand, under the commitment contract, the supplier commits to a price sequence at the beginning of the horizon. Specifically, the supplier quotes wholesale prices w_1 and w_2 at the beginning of the horizon. The buyer responds with his actions in periods 1 and 2 as before. While the dynamic game has a unique subgame-perfect equilibrium which is asymmetric across periods and does *not* simply mimic the static solution, the equilibrium under the commitment contract game mimics the static solution.

THEOREM 1. *The unique subgame-perfect equilibrium under the two contracts are as follows:*

1. *Under the dynamic contract (i) $w_1 > w_2$, and* yet *(ii) $Q_1 > Q_2$ and (iii) $I > 0$. Also, (iv) $q_1 < q_2$, and (v) $p_1 > p_2$.*

2. *Under the commitment contract, the equilibrium mimics the static solution in each period. Specifically, no inventories are carried in equilibrium.*

	Dynamic	Commitment
Wholesale prices $\{w_1, w_2\}$	$\left\{\frac{9a-2h}{17}, \frac{6a+10h}{17}\right\}$	$\left\{\frac{a}{2}, \frac{a}{2}\right\}$
Purchase Quantities $\{Q_1, Q_2\}$	$\left\{\frac{13a-18h}{34b}, \frac{3a+5h}{17b}\right\}$	$\left\{\frac{a}{4b}, \frac{a}{4b}\right\}$
Inventory, I	$\frac{5(a-4h)}{34b}$	0
Sales Quantities $\{q_1, q_2\}$	$\left\{\frac{4a+h}{17b}, \frac{11a-10h}{34b}\right\}$	$\left\{\frac{a}{4b}, \frac{a}{4b}\right\}$
Retail Prices $\{p_1, p_2\}$	$\left\{\frac{13a-h}{17}, \frac{23a+10h}{34}\right\}$	$\left\{\frac{3a}{4}, \frac{3a}{4}\right\}$
Buyer's Profits, Π_B	$\frac{155a^2-118ah+304h^2}{1156b}$	$\frac{a^2}{8b}$
Supplier's Profits, Π_S	$\frac{9a^2-4ah+8h^2}{34b}$	$\frac{a^2}{4b}$

Table 1 **Dynamic vs. Commitment contracts under linear wholesale prices**

Table 1 provides the equilibrium solution and profits. Consider the dynamic contract. Observe that:

1. The model assumes no fixed (or non-linear) costs. In the absence of economies of scale, no *cycle* inventories need be carried.

2. Zero lead times preclude any *pipeline inventory* in the model.

3. The absence of demand or supply uncertainty precludes *safety inventory*.

4. There are no exogenous price-effects. In fact, since the endogenously determined wholesale prices are related as $w_1 > w_2$, the inventory carried was procured at the more costly first-period wholesale price, and cannot be attributed to *forward buying*.[9]

5. In addition, the buyer incurs positive holding costs of h per unit for his inventories.

Thus none of the classical reasons for carrying inventories (identified in the Introduction) exist and yet inventories are carried in equilibrium. Why is this so?

In equilibrium, the buyer *anticipates* the supplier's second-period wholesale price as a *strategic response* to his first-period actions (purchased quantities). Hence, the strategic value of carrying inventories to the buyer must have overridden the drawbacks of higher unit prices and holding costs. We use the term *strategic inventories* for inventories that arise purely from such strategic considerations.

When the buyer carries strategic inventories, he forces the supplier to price only for the buyer's residual demand, leading to a lower second-period wholesale price w_2. Thus, the buyer curtails the supplier's monopoly power in the second period, by inducing (Cournot-like) supply-side competition between the supplier and the buyer's inventories. Since the supplier anticipates this, he wants to discourage the buyer from carrying inventories. Hence he raises the first-period wholesale price. The problem with raising w_1 is that this reduces profits from sales in the first period; thus, the supplier sets w_1 to balance these two considerations. In equilibrium, this leads to $w_1 > w_2$. As a result, retail prices are higher in the first period (i.e., $p_1 > p_2$), and the sales quantities are related inversely (i.e., $q_1 < q_2$).

Under an alternative contracting mechanism – one that allowed for credible *ex ante* commitment by the supplier to a second-period wholesale price (independent of the buyer's inventories) – one would expect that strategic inventories would be eliminated. We find that this is indeed so, and

[9] *Forward buy* is a term used in the retail industry to denote inventory carried forward in response to an announced or anticipated future price increase.

inventories are not carried in equilibrium under the commitment contract.

The next section compares the prices and profits (for supplier, buyer and channel) under the dynamic and commitment contracts.

2.2. Comparisons

From Table 1 it is easy to see that the first (respectively, second) period wholeprice and retail prices under the dynamic contract are larger (respectively, smaller) than the wholesale and retail prices in the commitment contract. In order to compare the contracts over the two-period horizon, we consider weighted averages of the wholesale and retail prices for the dynamic contract, where the weights are the buyer's purchase and sales quantities respectively. Using the wholesale prices and quantities listed in Table 1, $w^d_{avg} = \frac{9a^2-4ah+8h^2}{19a-8h}$ and $w^c_{avg} = a/2$. [10] It is straightforward to show that $w^d_{avg} < w^c_{avg}$ for all $h < a/4$. (For $h \geq a/4$, inventories are too costly to be feasible, and dynamic and commitment equilibria are identical.) Thus we conclude that *strategic inventories force the supplier to reduce wholesale prices.*

We find that the lower wholesale price translates to a lower average retail price. In fact the behavior of retail prices as h varies mimics that of the wholesale prices: $p^d_{avg} = \frac{461a^2-84ah-104h^2}{34(19a-8h)}$ is bounded from above by $p^c_{avg} = 3a/4$, which is attained only at $h = a/4$. While the double marginalization effect persists under both contracts (since retail prices are greater than $p^{fb} = a/2$), it is diminished under dynamic contracts (since $p^{fb} < p^d_{avg} < p^c$), especially for smaller values of h. Hence we conclude that *strategic inventories reduce the double marginalization effect.*[11]

For sufficiently small h, inventory costs become negligible, and channel profits are greater under dynamic contracts than under commitment contracts due to diminished double marginalization. However, as h increases, inventory holding costs reduce the channel profits under dynamic contracts. For large enough h, the channel does better under commitment contracts.

[10] We will use the following superscripts throughout: 'd' for dynamic contract, 'c' for commitment contract, 'fb' for first-best and 's' for the static equilibrium.

[11] The double marginalization effect is seen whenever the price charged to the consumer is greater than the first-best price. It usually results from the desire of each player in the channel to secure a margin. The magnitude of the effect will thus depend on both the retail and the wholesale prices.

We now study how the two parties fare *individually* under each type of contract. Intuitively, we would expect that the buyer is better off under the dynamic contract because he uses strategic inventories to reduce the supplier's second-period monopoly power and obtain lower second-period wholesale prices (The buyer induces competition between the supplier and his own inventories.). On the other hand, for precisely these reasons, the supplier might be better off under the commitment contract, where he preempts the buyer by committing early to the second-period wholesale price. The following Proposition, which summarizes the profit comparisons under the two kinds of contracts, demonstrates that this 'intuition' does not hold.

PROPOSITION 1. *The channel profits are higher under the dynamic contract for* $h < 55a/288 \approx 0.19a$*; otherwise the channel does better under the commitment contract. The buyer prefers the dynamic contract for* $h < 21a/152 \approx 0.14a$*. Finally, the supplier* always *prefers the dynamic contract.*

Recall that h is a per-period holding cost. So, for example, if our period length is a month, a value of $h = 0.14a$ implies that even for a wholesale price of $a/2$ (which is the maximum wholesale price observed under either contract) the holding cost rate is $0.14/0.5 = 0.28$ or 28 *percent per month.* This value is extremely high, suggesting that for most reasonable values of h, the dynamic contract is Pareto-dominating.

The results above are driven by a surprising dynamic inadvertently induced by strategic inventories. We know that linear prices lead to channel and welfare losses, due to both unextracted consumer surplus and dead-weight loss (customers who value the good above its marginal cost are not served). Double marginalization exacerbates the dead-weight loss and the welfare loss. However, under the dynamic contract, by carrying inventories in the first period, the buyer can in effect source for the second period at two different prices: *(i)* from the supplier, at his second period wholesale price, or *(ii)* from his own inventories at a different price. Although the supplier anticipates this and prices accordingly, he is unable to entirely eliminate inventories from the channel. *Thus, strategic inventories relax (expand) the effective space of vertical contracts, in effect inducing* *non-linear wholesale pricing in the second period– lowering average wholesale and retail prices, and reducing the level of double-marginalization.* However, a second, countervailing effect is the drain on buyer (and channel) profits due to inventory holding costs. In most cases (small to medium values of h), the *contract-space-expansion* effect dominates the *inventory-drain* effect, and supplier, buyer and channel profits are greater under dynamic pricing. However, when h is high enough, the inventory-drain effect outweighs the contract-space-expansion effect of the equilibrium, and the buyer and channel begin to do better under commitment contracts. The supplier controls the level of the buyer's inventories through the first-period wholesale price; in effect, he implements non-linear pricing in the second period (through w_1 *and* w_2). Thus the contract-expansion-effect works in his favor; the costs of inventory are in any case borne by the buyer. Hence the supplier is always better off under the dynamic contract.

Finally, Proposition 2 compares the consumer surplus and welfare under the two contracts.

Proposition 2. *The total consumer surplus and welfare are always higher under the dynamic contract.*

Since the average retail price is lower under the dynamic contract, the consumer surplus under the dynamic contract is greater than that under the commitment contract. Welfare is also greater because, overall, the *contract-space-expansion* effect dominates the *inventory-drain* effect for all values of the holding cost.

To summarize, compared to the commitment contract, the dynamic contract usually leads to higher buyer and channel profits, and *always* results in higher supplier profits, greater consumer surplus and improved welfare. Given these findings, the question that arises is whether strategic inventories are of consequence in our model only because of the overly restrictive nature of the linear wholesale pricing. In other words, will inventories vanish when the contract space is relaxed, diminishing the contract-expansion effect? In the following section, we analyze one such relaxation– two-part tariff vertical contracts.

3. Two-Part Tariffs

A two-part tariff is an affine pricing schedule of the form $P(q) = K + w \cdot q$, where K is the fixed fee and w is the marginal wholesale price. The two-part tariff reverts to a linear pricing schedule upon setting $K = 0$. Two-part tariffs are widely used in Business-to-Business contracts, where their popularity is second only to linear prices. They have been extensively studied by researchers in Economics and Business. Within the Operations and Marketing literatures, they are of particular interest as a form of quantity discounting. Sometimes, such quantity discounts mirror actual scale economies for the supplier (*cf.* Zipkin (2000)). Channel coordination using two-part tariffs and quantity discounts have been studied in a series of papers (*cf.* Jeuland and Shugan (1983), Moorthy (1987), and Weng (1995)). Two-part tariffs find application in a number of different settings, including franchising in vertical relationships, licensing of patented technologies, pricing of complementary goods (such as razor blades and razors or camera film and cameras), and tie-in sales and quantity discounts for second-degree price discrimination [Tirole (1990) provides an extensive survey of the related academic literature].

Conventional wisdom holds that the two-part tariff does as well as more complex contracts in a deterministic setting. Quoting Tirole (1990) [Page 171] again, "*In a deterministic environment ... there is no loss in adopting a two-part tariff (i.e., a franchise fee plus a fixed marginal price) so there is no point in considering more complex nonlinear prices. This vindicates the focus on franchise fees* [coupled with linear prices; i.e., two-part tariffs]." Indeed, in the static problem, the supplier can both implement the first-best solution, and extract all of the residual channel profits, by setting $(K, w) = (a^2/4b, 0)$. We examine whether inventories can play any (strategic) role under two-part tariff vertical contracts, in a dynamic setting.

The two-part tariff scheme has two possible interpretations under the static setting: *(i)* The supplier 'sells the firm' to the buyer at the price K (which allows the marginal production cost, 0 here, to pass through); and *(ii)* The supplier offers quantity (or volume) discounts to the buyer, via this instrument– the average purchase price is decreasing in the purchased quantity. These two interpretations are equivalent under the static setting.

In the dynamic context, however, the two approaches ('selling the firm' and 'volume discounting') are not equivalent. The supplier can sell his production/procurement facility to the buyer at a price equal to the expected channel profits over the horizon. Then the erstwhile buyer, who now owns the entire channel, implements the first-best solution. For reasons well-documented in the literature, selling the firm is often not a viable strategy; after all, multiple firms do exist in practice.[12] Hence, in this section, we focus on the more interesting case of using two-part tariffs as a volume discount instrument. We address the following issues: *(i)* Are inventories carried under dynamic two-part tariff vertical contracts? *(ii)* If so, what is their effect on supplier, buyer, and channel profits? *(iii)* What would be the impact of the supplier's early price commitment on supplier, buyer, and channel profits? *(iv)* Finally, do inventories play any role under commitment two-part tariff contracts?

3.1. The Two-period Model

We assume that the demand and cost structures are the same as before. As we did for linear wholesale prices, we analyze both *dynamic* and *commitment* two-part tariff contracts.

Under the dynamic contract, the supplier first quotes a price schedule (K_1, w_1) for the first period, where K_1 is the fixed fee payable upon purchase of non-zero quantities, and w_1 is the linear, marginal wholesale price. The buyer can purchase $Q_1 > 0$ units for immediate delivery at the price $K_1 + w_1 \cdot Q_1$, or elect not to purchase any quantity at all and pay nothing. He can then sell $q_1 \leq Q_1$ in the market, and carry inventory $I = Q_1 - q_1$ into the second period. In the second period, the supplier quotes the price schedule (K_2, w_2). The buyer can purchase (and immediately acquire) $Q_2 > 0$ units at the price $K_2 + w_2 \cdot Q_2$, or set $Q_2 = 0$ and pay nothing. He sells $Q_2 + I$ in the second period.

Under the commitment contract, in a manner analogous to the linear commitment contract, the supplier announces his comprehensive price schedule $[(K_1, w_1), (K_2, w_2)]$ at the outset. Then the buyer decides on purchase quantities Q_1 and Q_2, and inventory I. Thus the sales quantities in each period are $Q_1 - I$ and $Q_2 + I$, for which revenues are realized.

[12] Furthermore, 'selling the firm' will fail to garner the first-best profits to the supplier under a host of reasonable relaxations of the problem - the obvious ones being any kind of demand uncertainty coupled with asymmetric information and/or risk aversion. As discussed previously, we prefer to isolate the phenomenon of strategic inventories by not introducing these confounding issues.

The game with either dynamic or commitment contract under the two-part tariff structure has a unique subgame-perfect equilibrium, characterized by Theorem 2 below.

THEOREM 2. *The unique subgame-perfect equilibrium under the two contracts are as follows:*

1. *Under dynamic two-part tariffs, the supplier is unable to implement the first-best solution, although he extracts the entire channel profits. The buyer carries inventory in equilibrium, even though his residual profit is driven to zero.*

2. *Under two-part tariffs with commitments, the supplier is* never *able to implement the first-best solution.*

(a) *For lower values of the holding cost* $(0 \leq h < (1-2/\sqrt{6})a\ )$ *(i) the supplier is able to extract the entire residual profits of the channel, but (ii) the buyer carries inventory in equilibrium (although he makes zero profit);*

(b) *For* $h \geq (1-2/\sqrt{6})a$*, (i) the supplier is not able to extract the entire channel profits, and (ii) the buyer does not carry inventory in equilibrium, but he makes positive residual profits.*

Table 2 provides the equilibrium prices, quantities, inventory and profits.

	Dynamic	Commitment	
		$h<(1-2/\sqrt{6})a$	$h \geq (1-2/\sqrt{6})a$
Wholesale prices $\{w_1, w_2\}$	$\{h, 0\}$	$\{0,\text{N.A.}\}$	$\left\{\frac{a-h}{2}, 0\right\}$
Fixed Fees $\{K_1, K_2\}$	$\left\{\frac{a^2}{2b} - \frac{3ah}{2b} + \frac{5h^2}{4b}, \frac{h^2}{b}\right\}$	$\left\{\frac{a^2}{4b} + \frac{(a-h)^2}{4b}, \text{Large}\right\}$	$\left\{\frac{(a+h)^2}{16b}, \frac{a^2}{4b} - \frac{(a-h)^2}{16b}\right\}$
Purchase Quantities $\{Q_1, Q_2\}$	$\left\{\frac{2a-3h}{2b}, \frac{h}{b}\right\}$	$\left\{\frac{2a-h}{2b}, 0\right\}$	$\left\{\frac{a+h}{4b}, \frac{a}{2b}\right\}$
Inventory, I	$\frac{a-2h}{2b}$	$\frac{a-h}{2b}$	0
Sales Quantities $\{q_1, q_2\}$	$\left\{\frac{a-h}{2b}, \frac{a}{2b}\right\}$	$\left\{\frac{a}{2b}, \frac{a-h}{2b}\right\}$	$\left\{\frac{a+h}{4b}, \frac{a}{2b}\right\}$
Retail Prices $\{p_1, p_2\}$	$\left\{\frac{a+h}{2}, \frac{a}{2}\right\}$	$\left\{\frac{a}{2}, \frac{a+h}{2}\right\}$	$\left\{\frac{3a-h}{4}, \frac{a}{2}\right\}$
Buyer's Profits, Π_B	0	0	$\frac{(a-h)^2}{16b}$
Supplier's Profits, Π_S	$\frac{a^2}{2b} - \frac{ah}{2b} + \frac{3h^2}{4b}$	$\frac{a^2}{2b} - \frac{ah}{2b} + \frac{h^2}{4b}$	$\frac{3a^2+2ah-h^2}{8b}$

Table 2 Dynamic vs. Commitment contracts under two-part tariffs

Under the dynamic contract, by carrying inventories, the buyer forestalls first-best channel profits. As before, inventories force the supplier to price to residual demand in the second period. Furthermore, if the buyer carries sufficient inventory, he can skip paying the fixed fee in the second period by not purchasing any quantity. This threat forces the supplier to *(i)* charge a non-zero marginal wholesale price in the first period ($w_1 = h$) to discourage inventories, and *(ii)* lower his fee schedule in the second period to encourage purchases. While successfully inducing the buyer to make purchases in both periods, this pricing scheme further erodes channel profits through double-marginalization.

For the commitment contract, Theorem 2 reflects the tradeoff that the supplier has to make between channel losses due to double marginalization and those due to strategic inventories. For lower values of h, the supplier is unable to prevent strategic inventories, so he sets the marginal cost $w_1 = 0$ to avoid double marginalization (and extracts the residual profits through K_1). In this range, all of the buyers second-period sales come from his inventory ($Q_2 = 0$). For higher h, the supplier successfully forestalls inventory by raising w_1, and induces the buyer to purchase quantities in both periods– however, this leads to double marginalization losses. Finally, if h were so high as to render inventories a non-credible threat for the buyer, the supplier would obtain first-best profits by implementing the static solution in each period. As before, the buyer's ability to carry inventories constitutes a dual threat to the supplier: *(i)* inventories force the supplier to price to residual demand in the second period; and *(ii)* if the buyer carries inventory, he can skip paying the fixed fee in the second period by not purchasing any quantity. This threat is credible in inverse proportion to the holding cost; hence, the prices, quantities and profits in equilibrium are all functions of the holding cost h.

3.2. Comparisons

Proposition 3 compares the channel, buyer and supplier profits for the two kinds of two-part tariff contracts analyzed above.

Proposition 3. *Consider two-part tariffs. For low to intermediate values of the holding cost (specifically, $h < (1 - 2/\sqrt{6})a$), the dynamic contract* Pareto-dominates *the commitment contract: both the supplier and the channel are strictly better off under the dynamic contract, while the buyer is indifferent between the two options. For $h \geq \frac{3-\sqrt{2}}{7}a$, the commitment contract Pareto-dominates the dynamic contract. In the intermediate range $(1 - 2/\sqrt{6})a \leq h < \frac{3-\sqrt{2}}{7}a$, the supplier prefers the dynamic contract, while both the buyer and the channel are better off under commitment contracts.*

We saw earlier that for linear wholesale contracts, the *contract expansion effect* favored the supplier more than the buyer (Proposition 1). Furthermore, contract expansion had a bigger impact under dynamic contracts, and for lower values of h. These intuitions continue to hold under two-part tariffs, driving the results of Proposition 3.

We now compare the channel performance under all four vertical contracts analyzed previously. Building on Proposition 1 (that compares the two linear contracts) and Proposition 3 (that compares the two contracts under two-part tariffs), we have the following result:

PROPOSITION 4. *Comparing the four kinds of vertical contracts – dynamic linear pricing, commitment linear pricing, dynamic two-part tariffs and commitment two-part tariffs – we conclude the following:*

- *channel profits are highest under two-part tariff contracts. Both dynamic and commitment two-part tariffs dominate their respective linear counterparts.*
- *the supplier always prefers a two-part tariff contract (either dynamic or commitment) over any linear contract. Specifically, the supplier prefers the two-part dynamic contract for* $h < \frac{3-\sqrt{2}}{7}a$*; otherwise he prefers the two-part commitment contract;*
- *the buyer always prefers a linear contract (either dynamic or commitment) over a two-part tariff contract. Specifically, the buyer prefers the linear dynamic contract for* $h < 21a/152$*; otherwise he prefers the linear commitment contract.*

The result that the supplier always prefers a two-part tariff contract over any linear contract is once again driven by the *contract expansion effect* (recall the discussion following Proposition 1 on Page 10). The effective choice-set of vertical contracts for the supplier is clearly larger under two-part tariffs than under linear contracts (even accounting for strategic inventories, which, after all, the buyer can carry in *either* setting). Hence the supplier can pick a superior contract (one that maximizes his own surplus, both by reducing double-marginalization and extracting away the buyers surplus) using two-part tariffs rather than linear wholesale prices. This naturally leads to a reverse set of contract preferences for the buyer. Since the contract expansion effect reduces double-marginalization, channel profits are greater under dynamic or commitment two-part tariffs than under their linear counterparts.

To summarize, we find that first-best is *never* achieved in the dynamic (two-period) setting, even with two-part tariff vertical contracts. Strategic inventories (or the threat of carrying them) are a powerful weapon in the hands of the buyer; this threat is inversely proportional to his holding costs. The supplier's optimal response (via his price-schedule) leads to double marginalization or inventory drain or both, hence the channel does not ever attain first-best profits.

4. General demand functions, arbitrary dynamic contracts, longer horizons

In this section, we relax some of the assumptions on the demand structure, linearity of contracts and horizon length to demonstrate the robustness of our results. We first explore if inventory is carried for general demand functions. We next show that end-of-period effects *do not* play much of a role in driving our results. Later we relax the space of contracts to consider the most general dynamic contracts under a general demand structure, and prove that when the buyer can carry inventories strategically, the supplier cannot simultaneously enforce the first-best solution and extract all of the buyer's residual profits.

4.1. Dynamic Contracts

We first analyze the two-period problem with linear wholesale prices but generalize the demand function to consider arbitrary piecewise linear convex (and concave) demand functions. Since any convex (or concave) demand function can be approximated by a piecewise linear function, the results we derive should hold for the more general case. Subsequently, we will analyze the effect of strategic inventories on the general horizon game and for a general (non-parametric) demand curve under any arbitrary dynamic contract.

4.1.1. Piecewise Linear Demand Functions In this subsection, we analyze a two-period horizon problem, where a buyer's demand curve is stationary, convex and piece-wise linear. We assume that the demand curve is made up of a finite number of line-segments. For simplicity, we will assume that the holding cost $h = 0$. Our central result is that the buyer will carry inventory in equilibrium. We will also show (essentially by appealing to continuity arguments) that the buyer will carry inventory for small enough $h > 0$.[13] Assume a piecewise-linear, convex demand function $P(q)$ consisting of $N > 1$ serially-connected line segments numbered $1, 2, ..., N$, and $(N+1)$ kinks labeled $0, 1, 2, ..., N$. Kink 0 is the intersection of the demand curve with the price (y-)axis, and kink N is the point of intersection of the demand curve with the quantity (x-) axis. Kink i ($i = 1, ..., N-1$) is the intersection of the i^{th} and $(i+1)^{th}$ line-segments. Let the quantities corresponding to these kinks be $k_0, k_1, ..., k_N$. We define the demand curve to be

$$P(q) = a_i - b_i q, \text{ if } k_{i-1} \le q \le k_i \qquad \text{for } i = 1, \ldots, N.$$

We note that by definition, $k_0 = 0$, $k_N = a_N/b_N$, and $k_i = \frac{a_i - a_{i+1}}{b_i - b_{i+1}}$ for $i = 1, ..., N-1$. It is easy to see that: $a_1 > a_2 > ... > a_N$ and $a_1/b_1 < a_2/b_2 < ... < a_N/b_N$, to ensure that the demand curve is convex and well-defined (i.e., consecutive line-segments intersect). These conditions imply that $b_1 > b_2 > ... > b_N$.

Thus, the *composite* demand curve is made up of N line-segments. We will refer to the linear-demand curve $P(q) = a_i - b_i q$ as the 'i -curve'. Thus, for the composite demand curve, the feasible quantities on the i-curve are the range $[k_{i-1}, k_i]$. We know that for the static (one-period) problem, where demand is characterized by the i-curve $P(q) = a_i - b_i q$, the supplier's optimal wholesale price $w_i^* = a_i/2$, which induces the buyer's optimal response $q_i^* = a_i/4b_i$. Of course, *the tuple* (w_i^*, q_i^*) *may or may not be feasible for the composite (convex) demand curve.* More precisely, it is feasible *iff* $q_i^* \in [k_{i-1}, k_i]$. If $q_i^* < k_{i-1}$ or $q_i^* > k_i$, (w_i^*, q_i^*) is infeasible as an equilibrium on the composite demand curve (for the static problem).

The following proposition provides a structural result for the static game that will serve as a key building block for the dynamic analysis.

[13] It is obvious that for large enough h, the buyer will not carry inventory. In essence, the dynamic problem will be broken up into a sequence of static problems.

Proposition 5. *In the single-period game with the composite demand curve, there exists a unique wholesale price w^* (inducing a buyer-purchase of q^*) that optimizes the supplier's profits. Further,*

(i) $w^* = w_i^* = a_i/2$ *for some* $i \in \{1, 2, ..., N\}$*; and*

(ii) $q^* = q_i^* = a_i/4b_i$*, where the quantity* $q_i^* = a_i/4b_i \in (k_{i-1}, k_i)$ *and is therefore optimal for the buyer, given the wholesale price* w_i^**.*

(iii) The supplier's maximal profit is $\pi_S = a_i^2/8b_i$*.*

Essentially, the proposition states that the solution to the static problem cannot be at a kink. Next we show that the profits of the supplier in the static problem is a lower bound on his equilibrium profits even when the buyer carries inventory across the two periods.

Proposition 6. *Suppose the unique optimal solution to the static problem is* $w^* = a_i/2$ *for some* $i \in \{1, 2, ..., N\}$*, with the corresponding induced purchase quantity being* $q^* = a_i/4b_i$ *(We know by Proposition 5 that such a unique solution exists). Then, over the two periods, the supplier can ensure a profit* π^S *of at least* $(2 \cdot a_i^2/8b_i)$*. In this solution, the buyer carries inventory across the two periods.*

Proposition 6 leads to the principal result: that in a dynamic setting, the buyer will carry inventory across periods in equilibrium.

Theorem 3 **(Inventory in equilibrium)**. *In the two-period problem, with a convex, piecewise-linear, decreasing demand function in each period, the buyer always carries inventory in equilibrium.*

Now consider the case of $h > 0$. Proposition 5 is unaffected by h. In Proposition 6, for $h = 0$, we argued that the supplier is able to ensure himself a total of two-period static contract profits and the buyer carries inventory. Since the static solution is an interior solution (Proposition 5) and appealing to continuity arguments, for a small perturbation in h away from zero, the solution will remain an interior solution and the buyer will carry inventory (in anticipation of price declines in the second period), while the supplier ensures himself the minimum profits as outlined. Given this and appealing to Theorem 3, we conclude that inventory will be carried in equilibrium for small enough $h > 0$.

Since any convex demand function can be approximated by $P(q)$ above, Theorem 3 shows that strategic inventory arises in more general convex demand functions.

Analogously, one can construct an arbitrary piecewise linear concave demand function. For this class of functions, however, inventory is not always carried in equilibrium. In fact, in a 2-period problem with dynamic linear wholesale price contracts one can show (we skip the proof here) that under certain conditions (on the intercept and slopes of the linear segments), the equilibrium solution could be at a "kink" and inventory is not carried. Even so, the *threat* of inventories can affect the equilibrium outcomes and profits. In fact, even when the contract space is relaxed, as we show later (see Theorem 4), inventory continues to play a strategic role for a broad class of revenue functions including those induced by convex and concave demand functions.

4.1.2. Longer Horizons To test the robustness of our results, we explicitly derived the equilibrium solution for a 3-period game under linear dynamic contracts. We observe that for small to medium values of the holding cost, inventory is carried across every period, for intermediate range of holding cost inventory is carried only from the second period to the third, while for large values of holding cost, inventory is never carried. The detailed expressions for the equilibrium and their derivations are quite cumbersome and add little further insight, so we skip the details. Proofs are available from the authors for the interested reader.

4.1.3. General Dynamic Contracts For the general (finite or infinite) horizon, with number of periods equal to n, closed-form solutions are analytically intractable. Based on the analysis of the two- and three-period problems, one may conjecture that inventory will continue to be carried in longer horizon games.[14]

[14] For example, consider the following piecewise-linear demand function incorporating quantity discounts:

$$p = \begin{cases} p_1 & \text{if } 0 \le q \le q_1 \\ p_2 & \text{if } q_1 < q \le q_2, \end{cases}$$

However, we seek to demonstrate that the buyer's ability to carry inventories *strategically* affects the equilibrium outcome and buyer/supplier profits in a general horizon game. We allow for a general (not necessarily linear) demand structure. We also allow for the most general (arbitrary; not necessarily linear) dynamic contracts whereby in each period the supplier announces a price schedule for purchase quantities and the buyer places an order for immediately delivery.

Consider the case of a general demand function, stationary across periods, given by $P(q)$, where $P(\cdot)$ is the price as a function of quantity. The revenue function, $R(q)$, is given by $R(q) = P(q)\cdot q$. In our analysis, we work with the revenue function. We assume that future periods are discounted, and model this via a per-period multiplicative discount factor of δ; without loss of generality, $0 < \delta \leq 1$. Thus, M dollars received in period k are worth $\delta \cdot M$ dollars in period $k-1$. The undiscounted case corresponds to $\delta = 1$. Our results also extend to the infinite horizon, provided $\delta < 1$ strictly: this condition ensures that total discounted profits over the infinite horizon are bounded. To minimize technical distractions, we assume that the revenue function $R(q)$ satisfies the following properties:

1. $R(\cdot)$ is differentiable everywhere and has a unique maximum at q_{fb}; furthermore $R'(q_{fb}) = 0$.
2. $R(0) = 0$ (selling zero quantities will yield zero revenues) and $\delta \cdot R'(0) > h$. The latter condition states that the *maximum* marginal revenue from carrying inventory to the next period, discounted to the current period, is greater than the marginal holding cost, a very reasonable assumption.

Most non-pathological demand functions can be characterized or at least approximated by revenue functions that satisfy the preceding properties. We use the single-period, first-best solution as a benchmark. Assuming as before that the marginal production cost is zero [15], the *unique* optimal solution is $q_{fb} : R'(q_{fb}) = 0$.[16]

where $p_1 > p_2 > 0$. We can show that for any arbitrarily long finite horizon problem: (i) the number of periods in which strategic inventory is carried increases as the marginal holding cost decreases; and (ii) as the marginal holding cost per period falls to zero, strategic inventory is carried in every period.

[15] Relaxing this assumption is straight-forward but omitted in the interests of clarity of the presentation. The condition we really need is that the net profit function, given by $R(\cdot) - C(\cdot)$ satisfies the properties specified above for $R(\cdot)$, where $C(\cdot)$ is the production cost function.

[16] In the special case of the linear demand curve given by $P(q) = a - bq$, the marginal revenue curve is given by $R'(q) = a - 2bq$. Hence, $q_{fb} = a/2b$. The per-period sales revenue from the first-best solution is $R(q_{fb}) = R(a/2b) = a^2/4b$.

Theorem 4 below proves that inventories play a strategic role under any general, stationary demand function, for general horizon lengths, and in the space of the most general (arbitrary) dynamic vertical contracts (not confined to linear pricing or two-part tariffs). The Theorem shows that the supplier *cannot* implement a contract with the buyer that will give him the first-best profits. Two possible outcomes frustrate the supplier: *(i)* Either the buyer makes residual profits using strategic inventories, or *(ii)* The supplier reduces the buyer's profits to zero, but the outcome in this case is different from the first-best outcome and hence generates lower profits.

THEOREM 4. *Consider the finite horizon model (n-period repeated game) under dynamic contracting, with a general demand (and revenue) function as specified above. When the buyer can carry inventories, there exists no dynamic vertical contract using which the supplier can simultaneously implement the first-best solution and extract away all of the buyer's residual profits.*

COROLLARY 1. *Strategic inventories ensure that the supplier can never make first-best profits, even if he manages to extract away all of the buyer's residual profits.*

Theorem 4 applies to both undiscounted and discounted finite horizons (i.e., $\delta \leq 1$), as well as the discounted infinite horizon ($\delta < 1$): The latter restriction for the infinite horizon is to ensure that total profits are not unbounded.

To establish that it is the buyer's ability to carry inventories that drives the result of Theorem 4, consider the case that the buyer (for whatever reason) *cannot* carry inventories[17]. Here, the periods are effectively decoupled, and so the supplier can easily structure a dynamic contract that implements the first-best solution and extracts all of the buyer's residual profits. For example, the two-part tariff contract in each period, with a fixed fee of $R(q_{fb})$ and marginal-cost unit pricing (0 in this case), ensures first-best profits for the supplier. Thus, under dynamic contracts, the strategic role of inventories is *not* an artifact of horizon length, or other restrictions such as linear or two-part tariff contracts or the linear demand function. We now study the effect of strategic inventories under commitment contracts, for the general horizon length.

[17] The assumption that $\delta \cdot R'(0) > h$ is not applicable in this case.

4.2. Commitment Contracts: general horizon lengths

In Theorem 2, we showed that inventories play a strategic role under a quantity discount commitment contract for the two-period problem. Theorem 5 below extends this result to the general n-period problem.

THEOREM 5. *Consider the finite horizon model (n-period repeated game) under commitment contracting, where the vertical contract in each period is a quantity discount scheme implemented using two-part tariffs. We assume a general demand (and revenue) function as specified above. When the buyer can carry inventories, there exists no two-part tariff commitment contract using which the supplier can simultaneously implement the first-best solution and extract away all of the buyer's residual profits.*

COROLLARY 2. *Strategic inventories ensure that the supplier can never make first-best profits, even if he manages to extract away all of the buyer's residual profits.*

Theorem 5 applies to both undiscounted and discounted finite horizons (i.e., $\delta \leq 1$), as well as the discounted infinite horizon ($\delta < 1$).

Once again, it is easily established that the result of Theorem 5 is driven by the buyer's ability to carry inventories: When the buyer cannot carry inventories, the supplier can obtain first-best profits by proposing a simple two-part tariff contract in each period with a fixed fee of $R(q_{fb})$ and marginal price of 0.

Since linear commitment contracts are a special case of two-part tariffs, with the fixed fee set to zero, Theorem 5 also applies to this case. The following Theorem studies the case of *general* commitment contracts (not confined to linear pricing or two-part tariffs).

THEOREM 6. *In the space of the most general of commitment contracts, the supplier can always achieve first-best profits, i.e., simultaneously implement the first-best solution and extract away all of the buyer's residual profits, even when the buyer can carry inventories strategically.*

Theorem 6 holds for any arbitrary (finite or infinite) horizon length, and for any arbitrary demand (and revenue) function. The proof of Theorem 6 is by construction of a contract that actually implements first-best profits for the supplier. Contracts that ensure first-best profits to the supplier are variants of the idea of 'selling the firm' to the buyer, at a fixed fee equal to the value of the combined firm.

To summarize the results of this section, inventories play an unassailable strategic role for the buyer under dynamic vertical contracts for general (finite or infinite) horizon lengths; they are by no means an artifact of "edge-effects" in the two-period model. The *ability* to carry inventories affects the equilibrium outcome, regardless of whether inventories are actually carried by the buyer in equilibrium or not. The intuition is that, however rich the dynamic vertical contract constructed by the supplier is, the contract's effectiveness is restricted to *within* the period of its validity. The buyer maneuvers around the string of intra-period dynamic contracts by exploiting the inter-period link enabled by his inventories. This inter-period dynamic of inventories (wherein inventories acquired in earlier periods serve as a deterrent to the supplier's monopoly power in later periods) is less effective when the supplier can design a commitment contract that anticipates this inter-period dynamic. The strategic effect of inventories persists when the form of contracting is restricted to the widely used linear or quantity-discount commitment contracts (Theorem 5). However, as Theorem 6 shows, when the space of commitment contracts is expanded to allow for the most general (arbitrary) contracts, the supplier manages to garner first-best profits. The result of such commitment contracts is in fact equivalent to "selling the firm" to the buyer at an appropriate fee that (barely) satisfies the buyer's participation constraint.[18]

5. Postscript: Taking Stock

In traditional inventory management, the buyer told the supplier when and how much of the good he needed, and the supplier attempted to meet those demands. This "arms-length" relationship, characterized by contracting on inputs alone, incentive misalignment and upstream variability, led to excess inventories and costs. More creative vertical relationships such as *Vendor Managed*

[18] It is worth emphasizing that such 'selling the firm' solutions will fail under many plausible model relaxations; e.g., demand uncertainty coupled with asymmetric information and/or risk aversion. These relaxations will induce *safety* and/or *speculative* inventories in addition to strategic inventories.

Inventories (VMI), *holding cost subsidies* and *consignment* are now gaining in popularity. Under VMI, the supplier uses the available consumer demand data and his own product expertise to actively manage the retailer's inventory. Holding cost subsidies enable suppliers to share risk with retailers if, for example, a product does not move quickly enough off the display aisles. Such subsidies can be a device for the supplier to credibly signal his confidence in the demand for his products. Under consignment, the supplier absorbs the entire financial holding costs for his products by charging the retailer only after sales are registered.

A commonly cited benefit of vertical relationships such as VMI and consignment is that the supplier, by gaining direct access to sales data, improves his ability to forecast demand. Coupled with better control of inventory and replenishment, the overall performance of the channel improves. Our analysis, however, would suggest that even in the absence of any forecasting benefits, vertical arrangements such as VMI and consignment can add value by eliminating strategic inventories from the channel.

An assumption implicit in our model is that inventories are either (a) not observable by the supplier, or (b) observable but not contractible; hence the vertical contract was on the buyer's purchase quantities. Inventories will not play a strategic role if the buyer commits *a priori* in a credible fashion to his inventory levels, or if the supplier can write a contract contingent on the buyer's inventories (or sales quantities). Thus, one reinterpretation of our model is in an incomplete contract framework where the level of inventories is not contractible. Strategic inventories, and the concomitant rent that the supplier has to leave to the retailer, would then be artifacts of the incompleteness of contracts.

One consistent finding in our research was the *persistence* of the impact of strategic inventories in a vertical relationship. Two results in particular highlight this robustness. First, we proved that for any horizon length and for very general specifications of the demand function, the mere possibility of strategic inventories ensures that the supplier can never make first-best profits under *any* dynamic contract. In other words, the equilibrium solution as well as supplier, buyer and channel profits are affected by the threat of strategic inventories. Second, we showed that in the two-period model, inventories are always carried in equilibrium when the demand curve is piecewise-linear and convex. Nevertheless, an interesting open question is the generalization of the latter result (that strategic inventories arise in equilibrium) to general (non-parametric) continuous-demand functions. Our intuition is that, at a minimum, additional structural constraints would need to be imposed on the demand function. In a bilateral monopoly setting such as ours, albeit in a single-period model with demand uncertainty, Song *et al.* (2007) establish (in a very different research context) that both the curvature of the demand curve (convex/concave) and the nature of the demand uncertainty (additive/multiplicative) are pivotal in deriving structural insights. Both Song *et al.* (2007) and Granot *et al.* (2005) show that the multiplicative uncertainty model is conducive to gaining insights. Further, Song *et al.* (2007) provide an illuminating illustration of the *invalidity* of their structural results under *additive* demand. This negative result is relevant to our dynamic model with stationary demand and no uncertainty, since the cumulative demand over multiple periods is obtained by the *horizontal summation* of the single-period demand curves. In effect, it is impossible to escape an induced "additive demand" scenario in a dynamic model such as ours.

Extending our model (under non-parametric demand) to settings with demand uncertainty obviously runs into the same difficulties. Inventories compound the problem, since the aggregate demand curve when inventories are endogenously determined is no longer a simple composite of the single-period demand curves. In the present paper, it was important to assume no uncertainty in demand in order to eliminate safety inventory, and focus instead on its *strategic* role. An interesting avenue for future research is to study the interactions between safety and strategic inventories under demand uncertainty.

Previous academic research has shown that under demand uncertainty, information interacts with inventories in subtle but important ways (Anand 2008). Hence, an interesting extension of the present research would be to asymmetric information, with the buyer having more accurate demand information than the supplier. In essence, the supplier tries to infer the demand structure for future periods from the buyer's orders. Inventories enable the buyer to decouple his demand from his orders to the supplier, and thus to conceal his actual demand. On the other hand, by building up adequate levels of inventories, the consequences to the buyer of revealing his expected future demand may be less significant. We speculate that if the latter effect dominates, the entire channel may be better off with inventories, and in fact the supplier may want to offer holding cost subsidies to induce revelation.

Another interesting extension would be to horizontal competition among retailers sourcing from one or several suppliers; we speculate that inventory would play other kinds of strategic roles in this setting. For example, in addition to using inventories to reduce suppliers' monopoly power, as in our research, retailers may use inventories to sustain collusive behavior (as in Rotemberg and Saloner (1989)) or preempt competitors' future production (as in Molgaard *et al.* (2000)). The interaction among these different kinds of strategic uses of inventories should be a fruitful avenue for future research.

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Proofs of Statements

Proof of Theorem 1:

Part 1 of Theorem 1 follows from the results outlined in the column for Dynamic contracts in Table 1. We now prove these results.

In the analysis of this sequential game, we start with the second period, assuming that the buyer carries inventories I and that the supplier has quoted a wholesale price w_2. The buyer's problem is now the quadratic:

$$\max_{Q_2}(a-b(Q_2+I))(Q_2+I)-w_2Q_2.$$

It is straightforward to see that $Q_2=\max\{\frac{a-w_2}{2b}-I,0\}$.

• **Case (i),** $Q_2=0$**:** We first analyze the case when $Q_2=0$. Notice that the suppliers second period wholesale price w_2 is irrelevant. The buyer's second period profit function is simply $(a-bI)I$. Across the two periods, the buyer then solves the following problem:

$$\max_{q_1,I\geq 0}(a-bq_1)q_1-w_1(q_1+I)-hI+(a-bI)I$$

The optimal sales and inventory are:

$$q_1=\frac{a-w_1}{2b};\qquad I=\frac{a-(w_1+h)}{2b}.$$

Since there is no quantity bought in the second period, the supplier chooses w_1 to maximize $w_1(q_1+I)$. The optimal wholesale price, $w_1=\frac{a}{2}-\frac{h}{4}$ and the supplier's profits are $\frac{(2a-h)^2}{16b}$.

• **Case (ii),** $Q_2>0$**:** Next consider the case when $Q_2>0$. The supplier's problem in the second period is:

$$\max_{w_2} w_2\left(\frac{a-w_2}{2b}-I\right).$$

The optimal $w_2=\frac{a}{2}-bI$. This implies that $Q_2=\frac{a}{4b}-\frac{I}{2}$. The second-period profit functions of the supplier and the buyer as a function of buyer's inventories are given by

$$\Pi_{B,2}(I) = \frac{1}{4b}[a-w_2]^2 + w_2 I = \frac{1}{4b}\left[\frac{a}{2}+bI\right]^2 + \left[\frac{a}{2}-bI\right]I, \text{ and} \quad \text{(EC.1)}$$

$$\Pi_{S,2}(I) = \frac{1}{2b}\left[\frac{a}{2}-bI\right]^2. \quad \text{(EC.2)}$$

Now, moving to the first period, when the supplier quotes a wholesale price w_1, the buyer's problem becomes

$$\max_{q_1, I\geq 0}(a-bq_1)q_1 - w_1(q_1+I) - hI + \frac{1}{4b}\left[\frac{a}{2}+bI\right]^2 + \left[\frac{a}{2}-bI\right]I, \quad \text{(EC.3)}$$

where the buyer buys $Q_1 = q_1 + I$ in the first period, sells q_1 and carries inventory I to sell in the second period. Using the Lagrangian and complementary slackness conditions, it is rather straightforward to show that the solution exhibits two cases:

(i) $w_1 + h \geq \frac{3a}{4}$: $\quad I = 0 \quad Q_1 = q_1 = \frac{a-w_1}{2b}$.

(ii) $w_1 + h < \frac{3a}{4}$: $\quad I = \frac{2}{3b}\left[\frac{3a}{4} - w_1 - h\right]; \quad q_1 = \frac{a-w_1}{2b}; \quad Q_1 = \frac{1}{b}\left[a - \frac{7}{6}w_1 - \frac{2}{3}h\right]$.

Substituting into the supplier's profit functions for the first and second periods, we get

$$\Pi_S = \begin{cases} w_1\left(\frac{a-w_1}{2b}\right) + \frac{a^2}{8b}, & \text{if } w_1 + h > \frac{3a}{4}; \\ \frac{w_1}{b}\left[a - \frac{7}{6}w_1 - \frac{2}{3}h\right] + \frac{2}{9b}(w_1+h)^2, & \text{otherwise.} \end{cases}$$

The supplier determines the optimal w_1 and the corresponding profits for each of the two cases $Q_2 > 0$ and $Q_2 = 0$. His profits are maximized for the case when $Q_2 > 0$ and $w_1 + h \leq \frac{3a}{4}$. The optimal $w_1 = \frac{9a-2h}{17}$, with profits equal to $\frac{9a^2-4ah+8h^2}{34b}$.

Part 2 of Theorem 1 follows from the results outlined in the Commitment contract column of Table 1. We now prove these results.

We first solve for the buyer's optimal response to a given w_1 and w_2. Since his decision variables are Q_1, Q_2 and I, the buyer's decision problem is

$$\max_{Q_1\geq I\geq 0, Q_2\geq 0}(a - b(Q_1-I)(Q_1-I) + (a - b(Q_2+I))(Q_2+I) - hI - w_1Q_1 - w_2Q_2.$$

Subsequently, we solve for the supplier's optimal pricing decision.

In solving the problem, we initially ignore the constraint that $Q_2 \geq I$; we will check that that it holds in the solution to the relaxed problem. The solution reduces to two possible cases, which are:

(i) $w_1 + h \geq w_2$: $\quad Q_1 = q_1 = \frac{a-w_1}{2b}; \quad Q_2 = q_2 = \frac{a-w_2}{2b}; \quad I = 0.$

(ii) $w_1 + h < w_2$: $\quad Q_1 = \frac{2(a-w_1)-h}{2b}; \quad q_1 = \frac{a-w_1}{2b}; \quad Q_2 = 0; \quad q_2 = I = \frac{a-w_1-h}{2b}.$

Under both these cases, the constraint $Q_1 \geq I$ is satisfied; so the solution is both feasible and optimal.

The supplier profits (over *both* periods) are $\Pi_S = w_1 Q_1 + w_2 Q_2$, which reduce to

$$\Pi_S = \begin{cases} w_1\left(\frac{a-w_1}{2b}\right) + w_2\left(\frac{a-w_2}{2b}\right), \text{ if } & w_1 + h \geq w_2; \\ w_1\left(\frac{2(a-w_1)-h}{2b}\right), & \text{otherwise.} \end{cases}$$

Now the supplier optimizes his profit function over w_1 and w_2. For the first case (assuming $w_1 + h \geq w_2$), we get that the optimal prices are $w_1 = w_2 = \frac{a}{2}$ giving the supplier optimal profits of $\frac{a^2}{4b}$. For the second case (assuming $w_1 + h < w_2$), we derive $w_1 = \frac{a}{2} - \frac{h}{4}$ with optimal profits of $\frac{(2a-h)^2}{16b}$. It is straightforward to see that the supplier makes a higher profit by implementing the prices given by the first case. ∎

Proof of Proposition 1:

Follows by straightforward comparison of the appropriate profit expressions given in Theorem 1 (see Table 1). ∎

Proof of Proposition 2:

Formally, the consumer surplus in any given period is given by $\frac{a}{2}q_i^2$ where q_i is the sales quantity in period i. Thus the total consumer surplus (CS) across both periods for the dynamic contract is:

$$CS^d = \frac{185a^2 - 188ah + 104h^2}{2312b}$$

and for the commitment contract is $CS^c = \frac{a^2}{16b}$. The difference between the two is:

$$CS^d - CS^c = \frac{81a^2 - 376ah + 208h^2}{4624b}$$

which is always greater than zero for $h < a/4$. The difference in total welfare between the dynamic and commitment contract is computed as:

$$\frac{191a^2 - 1392ah + 2512h^2}{4624b}$$

which can be shown to be always non-negative for $h < a/4$. ∎

Proof of Theorem 2: (dynamic contract)

The equilibrium result for dynamic contracts in Theorem 2 follows from the results outlined in the column for dynamic contracts in Table 2 which we now prove. We proceed in a backward fashion beginning from the last period. Let $\mathbf{1}_{\mathcal{X}}$ be an indicator function which takes the value one if the condition represented by $\mathcal{X}$ is true; otherwise it takes the value zero. Given K_2,w_2, and I, the buyer solves the following optimization problem for period 2.

$$\max_{Q_2 \geq 0} \Pi_{B,2}(Q_2; I, K_2, w_2) = (a - b(Q_2 + I))(Q_2 + I) - w_2 Q_2 - K_2 \mathbf{1}_{Q_2 > 0}. \tag{EC.4}$$

Given the buyer's response to K_2 and w_2, the supplier solves

$$\max_{w_2, K_2} \; K_2 \mathbf{1}_{Q_2 > 0} + w_2 Q_2(w_2, I) \tag{EC.5}$$

s.t.

$$\Pi_{B,2}(I, K_2, w_2) \geq 0$$

The following Lemma describes the optimal policy structure for the second period:

Lemma EC.1. *The optimal second period two-part tariff is* $(K_2^*(I), w_2^*)$ *where*

$$K_2^*(I) = \frac{a^2}{4b} - (a - bI)I$$

$$w_2^* = 0$$

The optimal purchase quantity for the buyer is $Q_2^* = \max\{\frac{a}{2b} - I, 0\}$. *The corresponding optimal profits of the buyer and supplier are as follows:*

$$\Pi_{B,2}(I) = (a - bI)I$$

$$\Pi_{S,2}(I) = \frac{a^2}{4b} - (a - bI)I.$$

Proof: : Solving for the optimal purchase quantity for the buyer (and ignoring K_2 for the moment), we get:

$$Q_2 = \max\left\{\frac{a-w_2}{2b} - I, 0\right\} \tag{EC.6}$$

If $Q_2 > 0$, then the optimal second period profits for the buyer are:

$$\Pi_{B,2}(I,K_2,w_2) = \left(a - b\left(\frac{a-w_2}{2b}\right)\right)\left(\frac{a-w_2}{2b}\right) - w_2\left(\frac{a-w_2}{2b} - I\right) - K_2.$$

Clearly, $Q_2 > 0$ if and only if, $\Pi_{B,2}$ above is at least as large as the profits that the buyer would earn with $Q_2 = 0$ and selling from inventory I. That is, we require that

$$\Pi_{B,2}(I,K_2,w_2) \geq (a - bI)I.$$

Substituting for $\Pi_{B,2}(I,K_2,w_2)$ and simplifying, we get that

$$K_2 \leq \frac{(a-w_2)^2}{4b} - (a - w_2 - bI)I.$$

Define

$$K_2(I) \equiv \frac{(a-w_2)^2}{4b} - (a - w_2 - bI)I.$$

Then we have that if $K_2 \leq K_2(I)$, the buyer will purchase a non-negative quantity in period 2.

The supplier, in the second period, solves the following problem:

$$\max_{K_2,w_2} \quad K_2(I)\mathbf{1}_{Q_2>0} + w_2 Q_2$$

$$\text{s.t.}$$

$$\Pi_{B,2}(I,K_2,w_2) \geq (a - bI)I$$

where Q_2 is given by (EC.6). Notice that if the supplier sets w_2 such that $\frac{a-w_2}{2b} \leq I$ then $Q_2 = 0$ and the supplier makes zero profits in the second period. So let us assume that $w_2 < a - 2bI$ such that $Q_2 > 0$. Substituting for $K_2(I)$ and Q_2 in the supplier's profit function and optimizing for w_2, we get that $w_2 = 0$. Of course, we need $I \leq \frac{a}{2b}$ for $Q_2 > 0$. Since $w_2 = 0$, it is straightforward to see that the buyer will obey this constraint; else he loses $(w_1 + h)$ per unit at the margin. Thus,

$$K_2(I) = \frac{a^2}{4b} - (a - bI)I,$$

and

$$Q_2 = \frac{a}{2b} - I.$$

Substituting into the expressions for the profits of the buyer and the supplier, we get the buyer's profits to be:

$$\Pi_{B,2}(I) = (a - bI)I \tag{EC.7}$$

and the supplier's profits as:

$$\Pi_{S,2}(I) = \frac{a^2}{4b} - (a - bI)I. \tag{EC.8}$$

∎

We now solve for the first period prices and quantities. In the first period, the supplier announces K_1 and w_1. The buyer then decides to purchase Q_1, sells $q_1 \leq Q_1$, and perhaps carries inventory of I into the next period. He solves the following optimization problem:

$$\max_{Q_1 \geq I \geq 0} \ (a - b(Q_1 - I))(Q_1 - I) - hI - w_1 Q_1 - K_1 \mathbf{1}_{Q_1 > 0} + \Pi_{B,2}(I) \tag{EC.9}$$

Then the buyer's first period strategy (assuming for the moment that K_1 is small enough to induce purchases) is given by the following Lemma which is straightforward to derive (we skip the proof).

Lemma EC.2. *The optimal purchase quantity Q_1 and optimal inventory I are given as follows:*

$$(Q_1, I) = \begin{cases} \left(\frac{2(a-w_1)-h}{2b}, \frac{a-w_1-h}{2b}\right) & \text{if } w_1 < a - h \\ \left(\frac{a-w_1}{2b}, 0\right) & \text{if } w_1 \geq a - h \end{cases}$$

The supplier needs to determine K_1 and w_1. he will do so to extract all of the buyer's current and future profits, where the latter is given by $\Pi_{B,2}(I)$. Thus, he solves the following problem:

$$\max_{K_1, w_1} \ K_1 \mathbf{1}_{Q_1 > 0} + w_1 Q_1(w_1) + \Pi_{S,2}(I(w_1)) \tag{EC.10}$$

s.t.

$$\Pi_B \geq \Pi_{B,2}(I = 0)$$

where $Q_1(w_1)$ and $I(w_1)$ are as given by Lemma EC.2.

The supplier's optimal response is derived as follows. First consider the constraint in (EC.10). Recall that, if the buyer chooses $Q_1 > 0$, then

$$\Pi_B = (a - b(Q_1 - I))(Q_1 - I) - hI - w_1 Q_1 - K_1 + \Pi_{B,2}(I).$$

Also observe that $\Pi_{B,2}(I = 0) = 0$. Substituting for $\Pi_{B,2}(I)$ from Lemma EC.1, the constraint in (EC.10) can written as:

$$K_1 \leq (a - b(Q_1 - I))(Q_1 - I) - hI - w_1 Q_1 + (a - bI)I.$$

Since the supplier is maximizing profits, he will set

$$K_1 = (a - b(Q_1 - I))(Q_1 - I) - hI - w_1 Q_1 + (a - bI)I.$$

Substituting for K_1 (we will write it as $K_1(w_1)$ since Q_1 and I are functions of w_1) in the optimization problem of the supplier, we get that the supplier solves the following problem:

$$\max_{w_1} K_1(w_1) + w_1 Q_1(w_1) + \Pi_{S,2}(I(w_1)).$$

From Lemma EC.2 we need to consider two cases. First suppose $w_1 < a - h$. Then substituting for $Q_1(w_1)$ and $I(w_1)$ and simplifying, we write the supplier's objective function as:

$$\max_{w_1} \frac{2a^2 - w_1^2}{4b} - h\left(\frac{a - w_1 - h}{2b}\right).$$

Equating the first order condition with respect to w_1 to zero, we get $w_1 = h$. Since we require that $w_1 < a - h$, this price is feasible whenever $h < a/2$ which holds. Substituting this price into expressions for $Q_1(w_1)$ and $I(w_1)$, we get

$$I = \frac{a - 2h}{2b} \quad \text{and} \quad Q_1 = \frac{2a - 3h}{2b}.$$

So $Q_1 - I = \frac{a-h}{2b}$. Substituting into $K_1(w_1)$, we get

$$\begin{aligned} K_1 &= (a - b(Q_1 - I))(Q_1 - I) - hI - w_1 Q_1 + (a - bI)I \\ &= \frac{a^2}{2b} - \frac{3ah}{2b} + \frac{5h^2}{4b}. \end{aligned}$$

Substituting for the optimal I into $K_2(I)$ and $Q_2(I)$ in Lemma EC.1, we get $K_2 = h^2/b$ and $Q_2 = h/b$. Therefore, the supplier's overall profits given by

$$K_1 + K_2 + w_1 Q_1 + w_2 Q_2 = \frac{a^2}{2b} - \frac{ah}{2b} + \frac{3h^2}{4b},$$

which is less than the first-best profits of $a^2/2b$. The buyer makes zero profits.

From Lemma EC.2, the other case is $w_1 \geq a - h$ with $Q_1 = \frac{a-w_1}{2b}$ and $I = 0$. Working out this case, we see that the unconstrained optimal $w_1 = 0$ is infeasible as we require that $w_1 \geq a - h$. So the supplier sets $w_1 = a - h$. Then, $Q_1 = h/2b$ and $I = 0$. Substituting to get the fixed fees, we get that $K_1 = h^2/4b$ and $K_2 = a^2/4b$. The supplier's total profits are

$$K_1 + K_2 + w_1 Q_1 = \frac{a^2 - h^2 + 2ah}{4b}.$$

It is easy to see this case is dominated by the earlier case discussed (when $Q_1 > 0$ and $I > 0$) for $h < a/4$.

So in equilibrium the supplier sets $K_1 = \frac{a^2}{2b} - \frac{3ah}{2b} + \frac{5h^2}{4b}$, $K_2 = h^2/b$, $w_1 = h$ and $w_2 = 0$. ∎

Proof of Theorem 2: (commitment contract)

The equilibrium results for the commitment contract in Theorem 2 follows from the results outlined in the column for commitment contracts in Table 2. We now prove these results.

We derive the optimal policy structure working backwards starting with the buyer's response. The buyer solves the following problem:

$$\max_{Q_1 \geq I \geq 0, Q_2 \geq 0} \; (a - b(Q_1 - I))(Q_1 - I) - w_1 Q_1 - K_1 \mathbf{1}_{Q_1 > 0}$$
$$+ (a - b(Q_2 + I))(Q_2 + I) - hI - w_2 Q_2 - K_2 \mathbf{1}_{Q_2 > 0} \qquad \text{(EC.11)}$$

The buyer's optimal response is given by the following Lemma.

LEMMA EC.3. *Given (K_1, w_1) and (K_2, w_2), the buyer chooses one of the following actions:*

No.	*Actions*			*Buyer's Profit*	*Necessary Conditions on w_1, w_2, h*
	Q_1	I	Q_2		
1	0	0	$\frac{a-w_2}{2b}$	$\Pi_B^{(1)} = -K_2 + \frac{(a-w_2)^2}{4b}$	$w_2 < a$
2	$\frac{a-w_1}{2b}$	0	0	$\Pi_B^{(2)} = -K_1 + \frac{(a-w_1)^2}{4b}$	$a - h \leq w_1 < a$
3	$\frac{2(a-w_1)-h}{2b}$	$\frac{a-w_1-h}{2b}$	0	$\Pi_B^{(3)} = -K_1 + \frac{(a-w_1)^2}{4b} + \frac{(a-w_1-h)^2}{4b}$	$w_1 < a - h$
4	$\frac{a-w_1}{2b}$	0	$\frac{a-w_2}{2b}$	$\Pi_B^{(4)} = -K_1 - K_2 + \frac{(a-w_1)^2}{4b} + \frac{(a-w_2)^2}{4b}$	$\{w_1, w_2\} < a$, *and* $w_1 + h \geq w_2$

Proof: : The above table identifies the necessary conditions on w_1, w_2, and h for each of the actions specified to be implemented. Obviously, for the supplier to implement a particular action, he also needs to choose the right values of K_1 and K_2, which we will address when we solve the supplier's problem.

There are three possible strategies for the buyer assuming that the participation constraint of non-negative profit is met. Either (i) $Q_1 = 0$, $Q_2 > 0$; or (ii) $Q_1 > 0$, $Q_2 = 0$; or (iii) $Q_1 > 0$, $Q_2 > 0$. We now consider each of these cases.

Case (i) $Q_1 = 0$, $Q_2 > 0$: In this case the optimization problem for the buyer is

$$\max_{Q_2} -K_2 - w_2 Q_2 + (a - bQ_2)Q_2.$$

Solving for Q_2 gives $Q_2 = \frac{a-w_2}{2b}$ which is non-zero only when $w_2 < a$. Substituting back into the profit function gives us that

$$\Pi_B = -K_2 + \frac{(a-w_2)^2}{4b}.$$

Case (ii) $Q_1 > 0$, $Q_2 = 0$: In this case the buyer does not purchase any quantity in the second period. This implies that he may choose to carry over inventory from the first period and sell it in the second. Thus the buyer's objective function is:

$$\max_{Q_1, I} -K_1 - w_1 Q_1 + (a - b(Q_1 - I))(Q_1 - I) - hI + (a - bI)I.$$

The optimal inventory is given as $I = \frac{Q_1}{2} - \frac{h}{4b}$. Thus if $Q_1 \leq \frac{h}{2b}$ then $I = 0$ and no inventory is carried; otherwise $I > 0$.

Now suppose $I = 0$; substituting back into the profit function and solving for optimal Q_1, we see that $Q_1 = \frac{a-w_1}{2b}$. First we require that $w_1 < a$ for $Q_1 > 0$. Furthermore, we need that $I = 0$ which implies that $Q_1 \leq \frac{h}{2b}$. This implies that $w_1 \geq a - h$.

Next consider the situation when $I > 0$. Substituting the expression for I in the buyer's objective function and solving for the optimal Q_1 by equating the first order condition w.r.t. Q_1 to zero, we get that,

$$Q_1 = \frac{2(a-w_1)-h}{2b}; \qquad I = \frac{a-w-h}{2b}.$$

Notice that for $Q_1 > \frac{h}{2b}$ we would need that $w_1 < a - h$.

Thus case (ii) has two solutions: if $w_1 \geq a - h$ then $Q_1 = \frac{a-w_1}{2b}$ and $I = 0$; else if $w_1 < a - h$ then $Q_1 = \frac{2(a-w_1)-h}{2b}$ and $I = \frac{a-w-h}{2b}$. Substituting back into the profit function, we derive that the buyer's profits in the former case is $\Pi_B = -K_1 + \frac{(a-w_1)^2}{4b}$ and in the latter it is $\Pi_B = -K_1 + \frac{(a-w_1)^2}{4b} + \frac{(a-w_1-h)^2}{4b}$.

Case (iii) $Q_1 > 0$*,* $Q_2 > 0$: In this case the buyer's optimization problem is:

$$\begin{aligned}\max_{Q_1,Q_2,I} \quad & -K_1 - K_2 - w_1 Q_1 - w_2 Q_2 \\ & +[a - b(Q_1 - I)](Q_1 - I) + [(a - b(Q_2 + I)](Q_2 + I) - hI\end{aligned}$$

Given that $Q_1, Q_2 > 0$, the analysis of this case is similar to the linear contract under commitments. Recall that for that case we derived that if $w_1 + h \geq w_2$, then $Q_1 = \frac{a-w_1}{2b}$, $Q_2 = \frac{a-w_2}{2b}$, and $I = 0$ giving

$$\Pi_B = -K_1 - K_2 + \frac{(a-w_1)^2}{4b} + \frac{(a-w_2)^2}{4b}.$$

If $w_1 + h < w_2$ then the buyer's optimal action would be $Q_1 > 0$ and $Q_2 = 0$ and we are back to case (ii).

To summarize, the buyer acts to choose one of four outcomes– one outcome derived in case (i), two in case (ii), and one in case (iii)– subject to the participation constraint for non-negative profits. These correspond to the four cases in Lemma EC.3. ∎

The supplier then optimizes the following objective function

$$\max_{K_1,K_2,w_1,w_2} K_1 \mathbf{1}_{Q_1>0} + w_1 Q_1 + K_2 \mathbf{1}_{Q_2>0} + w_2 Q_2$$

subject to the buyer maximizing his own profit function as per the choices listed in Lemma EC.3. Now observe that the buyer's action (1) is a special case of his action (4), which Pareto-dominates it. That is, the supplier will always choose to force the buyer to pick (4) instead of (1). Similarly, buyer's action (2) is also Pareto-dominated by his action (4). Thus the supplier only needs to consider the buyer's actions (3) and (4) in deciding his pricing scheme.

Let $\Pi_B^{(k)}$ denote buyer's profits under action (k) outlined in Lemma EC.3. Now consider the situation that the supplier wishes to enforce buyer's action (3). Then he solves the following problem:

$$\max_{K_1, w_1} \quad K_1 + w_1 \frac{2(a-w_1)-h}{2b} \tag{EC.12a}$$

$$\text{s.t.} \quad \Pi_B^{(3)} \geq 0 \tag{EC.12b}$$

$$\Pi_B^{(3)} \geq \Pi_B^{(1)} \tag{EC.12c}$$

$$\Pi_B^{(3)} \geq \Pi_B^{(2)} \tag{EC.12d}$$

$$\Pi_B^{(3)} > \Pi_B^{(4)} \tag{EC.12e}$$

$$w_1 < a-h \tag{EC.12f}$$

Notice that,

$$\text{Constraint (EC.12b)} \Rightarrow K_1 \leq \frac{(a-w_1)^2}{4b} + \frac{(a-w_1-h)^2}{4b}.$$

Since the supplier is maximizing over K_1, he will choose

$$K_1 = \frac{(a-w_1)^2}{4b} + \frac{(a-w_1-h)^2}{4b}.$$

Substituting this into Constraint (EC.12c), we have

$$\text{Constraint (EC.12c)} \Rightarrow K_2 \geq \frac{(a-w_2)^2}{4}.$$

Constraint (EC.12d) is trivially satisfied. Finally,

$$\text{Constraint (EC.12e)} \Rightarrow K_2 \geq \frac{(a-w_2)^2}{4b} - \frac{(a-w_1-h)^2}{4b}$$

Thus the constraints (EC.12b)- (EC.12e) reduce to

$$\begin{aligned} K_1 &= \frac{(a-w_1)^2}{4b} + \frac{(a-w_1-h)^2}{4b} \\ K_2 &\geq \frac{(a-w_2)^2}{4b}. \end{aligned}$$

Substituting for K_1 into the objective function and setting a large enough value for K_2, the supplier's optimization problem to implement action (3) reduces to:

$$\max_{w_1 < a-h} \frac{(a-w_1)^2}{4b} + \frac{(a-w_1-h)^2}{4b} + w_1 \frac{2(a-w_1)-h}{2b}.$$

The optimal solution is given by:

$$w_1 = 0; \quad K_1 = \frac{a^2}{4b} + \frac{(a-h)^2}{4b},$$

and the buyer chooses

$$Q_1 = \frac{2a-h}{2b}; \quad I = \frac{a-h}{2b}.$$

Since inventory is carried, first-best is not achieved. The supplier extracts all of the channel profits with $\Pi_S = K_1 = \frac{a^2}{4b} + \frac{(a-h)^2}{4b}$.

Now consider the situation when the supplier wishes to enforce buyer's action (4). Then he solves the following problem:

$$\max_{K_1,K_2,w_1,w_2} \quad K_1 + K_2 + w_1 \frac{a-w_1}{2b} + w_2 \frac{a-w_2}{2b} \tag{EC.13a}$$

$$\text{s.t.} \quad \Pi_B^{(4)} \geq 0 \tag{EC.13b}$$

$$\Pi_B^{(4)} \geq \Pi_B^{(1)} \tag{EC.13c}$$

$$\Pi_B^{(4)} \geq \Pi_B^{(2)} \tag{EC.13d}$$

$$\Pi_B^{(4)} \geq \Pi_B^{(3)} \qquad \text{if } w_1 + h \leq a \tag{EC.13e}$$

$$w_1 + h \geq w_2 \tag{EC.13f}$$

$$w_1 \leq a, \qquad w_2 \leq a \tag{EC.13g}$$

Notice that,

$$\text{Constraint (EC.13c)} \Rightarrow K_1 \leq \frac{(a-w_1)^2}{4b}$$

$$\text{Constraint (EC.13d)} \Rightarrow K_2 \leq \frac{(a-w_2)^2}{4b}$$

$$\text{Constraints (EC.13c) and (EC.13d)} \Rightarrow \text{Constraint (EC.13b)}$$

$$\text{Constraint (EC.13e)} \Rightarrow K_2 \leq \frac{(a-w_2)^2}{4b} - \frac{(a-w_1-h)^2}{4b} \qquad \text{if } w_1+h \leq a$$

This implies that the supplier will choose the following fixed fees:

$$K_1 = \frac{(a-w_1)^2}{4b} \tag{EC.14}$$

$$K_2 = \begin{cases} \frac{(a-w_2)^2}{4b} & \text{if } w_1+h > a \\ \frac{(a-w_2)^2}{4b} - \frac{(a-w_1-h)^2}{4b} & \text{otherwise} \end{cases} \tag{EC.15}$$

The supplier's optimization problem to implement action (4) is then given by the objective function (EC.13a) with constraints (EC.14)- (EC.15) and (EC.13f)- (EC.13g). Depending on the value of w_1, the supplier solves two optimization problems by substituting the appropriate value of K_1 and K_2 into the objective funtion and then optimizing over w_1 and w_2. The optimal solution for the two cases that arise is as follows:

- $w_1 + h > a$: The supplier chooses the following:

$$w_1 = a - h; \quad w_2 = 0; \quad K_1 = \frac{h^2}{4b}; \quad K_2 = \frac{a^2}{4b}.$$

The buyer's actions are:

$$Q_1 = \frac{h}{2b}; \quad Q_2 = \frac{a}{2b}; I = 0.$$

The supplier's profits are: $\Pi_S = \frac{a^2}{4b} + \frac{ah}{2b} - \frac{h^2}{4b}$.

- $w_1 + h \leq a$: The supplier chooses the following:

$$w_1 = \frac{a-h}{2b}; \quad w_2 = 0; \quad K_1 = \frac{(a+h)^2}{16b}; \quad K_2 = \frac{a^2}{4b} - \frac{(a-h)^2}{16b}.$$

The buyer's actions are:

$$Q_1 = \frac{a+h}{4b}; \quad Q_2 = \frac{a}{2b}; \quad I = 0.$$

The supplier's profits are: $\Pi_S = \frac{3a^2+2ah-h^2}{8b}$.

Comparing the supplier's strategies under actions (3) and (4), we get that the supplier will prefer to implement action (3) when $h < (1 - 2/\sqrt{6})a$ and action (4) otherwise. The equilibrium solution is summarized in the following table.

	Supplier					Buyer			
	w_1	w_2	K_1	K_2	Π_S	Q_1	I	Q_2	Π_B
$h < (1-2/\sqrt{6})a$	0	N.A.	$\frac{a^2}{4b} + \frac{(a-h)^2}{4b}$	large	K_1	$\frac{2a-h}{2b}$	$\frac{a-h}{2b}$	0	0
$h \geq (1-2/\sqrt{6})a$	$\frac{a-h}{2}$	0	$\frac{(a+h)^2}{16b}$	$\frac{a^2}{4b} - \frac{(a-h)^2}{16b}$	$\frac{3a^2+3ah-h^2}{8b}$	$\frac{a+h}{4b}$	0	$\frac{a}{2b}$	$\frac{(a-h)^2}{16b}$

∎

Proof of Proposition 3:

The buyer clearly is indifferent between the dynamic and commitment contracts for $h < (1-2/\sqrt{6})a$ and prefers the commitment contract for $h \geq (1-2/\sqrt{6})a$.

Similarly it is easy to see that the supplier prefers the dynamic contract for $h < (1-2/\sqrt{6})a$. For $h \geq (1-2/\sqrt{6})a$, it is straightforward to show that the supplier prefers the dynamic contract for $h < \frac{3-\sqrt{2}}{7}a$ and the commitment contract otherwise. Since we require that $h < a/4$, we have the following situation for the supplier: he prefers the dynamic 2-part tariff contract for $h < \frac{3-\sqrt{2}}{7}a$ and the commitment 2-part tariff contract for $h \in [\frac{3-\sqrt{2}}{7}a, \frac{a}{4}]$.

The channel profits under the dynamic contract are given by:

$$\Pi_C = \frac{a^2}{2b} - \frac{ah}{2b} + \frac{3h^2}{4b} \tag{EC.16}$$

Similarly, the channel profits under the commitment contract are

$$\Pi_C = \Pi_B + \Pi_S = \begin{cases} \frac{a^2}{2b} - \frac{ah}{2b} + \frac{h^2}{4b} & h < (1-2/\sqrt{6})a \\ \frac{7a^2}{16b} + \frac{ah}{8b} - \frac{h^2}{16b} & h \geq (1-2/\sqrt{6})a \end{cases} \tag{EC.17}$$

It is then straightforward to show that the channel profit is higher under dynamic 2-part tariff whenever $h < (1-2/\sqrt{6})a$ and under the commitment 2-part tariff whenever $h \geq (1-2/\sqrt{6})a$. ∎

Proof of Proposition 4: Recall that the channel preferred dynamic contracts whenever $h < 55a/288$. In contrast, with two-part tariffs, the channel preferred the commitment contract for $h < (1-2/\sqrt{6})a$ and commitment contract otherwise (Proposition 3). However, comparing the linear and two-part tariff contracts for the channel, we conclude that the channel is always better off with the commitment 2-part tariff contract than with either linear contract. Therefore, the channel profits are maximized as per the results of the 2-part tariff contract - dynamic for $h < (1-2/\sqrt{6})a$ and commitment otherwise.

Similarly, we conclude that the buyer's profit is always maximized as per the linear contract with the buyer preferring the dynamic contract for $h < 21a/152$ and the commitment contract otherwise.

Finally, the supplier profits are maximized as per the 2-part tariff contracts. ∎

Proof of Proposition 5:

Before proceeding with the proof of the Proposition, we show the following lemma which states that the optimal solution for at least one i-curve is an interior solution.

LEMMA EC.4. *There always exists at least one* $q_i^* \in (k_{i-1}, k_i)$, *for* $i = 1, \ldots, N$.

Proof: Consider some $i \in [2, N-1]$. If $q_i^* \in (k_{i-1}, k_i)$ we are done. Suppose $q_i^* \geq k_i$. Recall that $q_i^* > q_{i+1}^*$ which implies that $q_{i+1}^* > k_i$. Then either there exists some $j > i$ such that $q_j^* \in (k_{j-1}, k_j)$, in which case we have shown the lemma, or $q_j^* \geq k_j$ for all $j < N$. Now consider $j = N$. Since $q_{N-1}^* \geq k_{N-1}$, we must have that $q_N^* > k_{N-1}$. However, by definition, we have that $q_N^* < k_N$, which implies that $q_N^* \in (k_{N-1}, k_N)$ which proves the lemma.

Alternately, suppose that $q_i^* \leq k_{i-1}$. Observe then that $q_{i-1}^* < k_{i-1}$. Then either there exists some $j < i$ such that $q_j^* \in (k_{j-1}, k_j)$, in which case we have shown the lemma, or $q_j^* \leq k_{j-1}$ for all $j > 1$. Now consider $j = 2$. Since $q_2^* \leq k_1$, we have that $q_1^* < k_1$. However, by definition, we have that $q_1^* > k_0 = 0$, which implies that $q_1^* \in (k_0, k_1)$ which proves the lemma. ∎

Proof of Proposition: For a given wholesale price w, the buyer's optimization problem can be determined by solving N (constrained) optimizations on the i-curve, for $i = 1, \ldots, N$ and selecting the one that gives maximal profit. The buyer's (constrained) optimization problem for any i-curve can be stated as $\max_{k_{i-1} \leq q \leq k_i} (a_i - b_i q)q - wq$.

Let $\pi_B^i(q)$ be the buyer's profit when he chooses a quantity q on the i-curve. Let $\pi_S^i(w)$ be the supplier's profits when the wholesale price is w and the buyer buys a quantity by optimizing on the i-curve. Let w_i^* and q_i^* be the unconstrained equilibrium wholesale price and quantity for the i-curve. From linearity of the i-curve, we have that $w_i^* = \frac{a_i}{2}$ and $q_i^* = \frac{a_i}{4b_i}$.

We will prove the proposition by contradiction. Let the optimal price and quantity be w^* and

$q^*(w^*)$. Without loss of generality assume $q^*(w^*) \in (k_{i-1}, k_i]$. Now suppose that $w^* \neq \frac{a_i}{2}$. Since w_i^* is the equilibrium solution when buyer optimizes on the i-curve, we have that $\pi_S^i(w^*) < \pi_S^i(w_i^*)$. Now if $q_i^* \in (k_{i-1}, k_i)$, then $w^* \neq \frac{a_i}{2}$ cannot be the optimal price. Otherwise, either $q_i^* \geq k_i$ or $q_i^* \leq k_{i-1}$. We consider each of these separately.

- $q_i^* \geq k_i$: Since $k_N = \frac{a_N}{b_N}$ and $q_N^* < k_N$. We have a contradiction for $i = N$. Therefore, $i \leq N-1$.

Then we must have that $q^*(w^*) = k_i$. Now we need to consider two cases.

Case (a): The buyer optimizes on the i-curve to choose k_i. Notice that $w = a_i - 2b_ik_i$ induces $q = k_i$. Since the supplier's profit function is concave on the i-curve, we must have that

$$\pi_S^i(w^*) \leq \pi_S^i(a_i - 2b_ik_i).$$

However, it is not clear that at this price ($w = a_i - 2b_ik_i$) the buyer will indeed choose $q = k_i$ if he were to optimize on the (i+1)-curve given by $P^{i+1}(q) = a_{i+1} - b_{i+1}q$. To evaluate this recall that on the (i+1)-curve, the buyer will choose a quantity $q = \frac{a_{i+1}-w}{2b_{i+1}}$ for a wholesale price w. For $w = a_i - 2b_ik_i$ and using the fact that $k_i = \frac{a_{i+1}-a_i}{b_{i+1}-b_i}$ and $b_i > b_{i+1}$, we see that

$$q = \frac{a_{i+1} - w}{2b_{i+1}} = \frac{(b_{i+1} + b_i)k_i}{2b_{i+1}} > k_i.$$

Thus,

$$\pi_B^i(k_i) < \pi_B^{i+1}\left(\frac{(b_{i+1} + b_i)k_i}{2b_{i+1}}\right)$$

and the buyer will rather optimize on the (i+1) curve when $w = a_i - 2b_ik_i$. But what about the supplier? Since the buyer buys a larger quantity on the (i+1)-curve, we must have that

$$\pi_S^i(a_i - 2b_ik_i) \leq \pi_S^{i+1}(a_i - 2b_ik_i).$$

But, appealing to concavity of the profit function, we have already established that

$$\pi_S^i(w^*) \leq \pi_S^i(a_i - 2b_ik_i).$$

Combining this with the optimality of $w_{i+1}^* = \frac{a_{i+1}}{2}$ for the (i+1)-curve, we have that

$$\pi_S^i(w^*) \leq \pi_S^{i+1}(a_i - 2b_ik_i) \leq \pi_S^{i+1}(w_{i+1}^*).$$

So when $w = a_i - 2b_i k_i$, it is neither in the interest of the buyer nor the supplier to optimize on the i-curve; it is Pareto optimal to be on the (i+1)-curve which, as shown, will lead to a quantity different from k_i. Then $q^*(w^*) \notin (k_{i-1}, k_i]$ which is a contradiction to the assumption. Notice that, in the process, we have eliminated the solution being at the kink k_i when optimizing on the i-curve.

Case (b): The buyer optimizes on the (i+1)-curve to choose k_i; observe that this will only happen if $q^*_{i+1} \le k_i$. Now when the buyer optimizes on the (i+1) curve, he chooses a quantity $q = \frac{a_{i+1}-w}{2b_{i+1}}$ which by assumption equals k_i. We will now show that, if the supplier sets w such that $q = k_i$ while the buyer optimizes on the (i+1)-curve, at this w the buyer can do better by optimizing on the i-curve instead. That is,

$$\pi_B^i\left(\frac{a_i - w}{2b_i}\right) > \pi_B^{i+1}(k_i).$$

Observe that for a given w the buyer's optimal profit when optimizing on the i-curve

$$\pi_B^i\left(\frac{a_i - w}{2b_i}\right) = \frac{(a_i - w)^2}{4b_i}.$$

The buyer's profits from choosing k_i while optimizing on the (i+1)-curve is given by

$$\begin{aligned}\pi_B^{i+1}(k_i) &= (a_{i+1} - b_{i+1}k_i)k_i - wk_i \\ &= \frac{a_i - a_{i+1}}{b_i - b_{i+1}}\left(\frac{a_{i+1}b_i - a_i b_{i+1}}{b_i - b_{i+1}} - w\right).\end{aligned}$$

But w is such that $\frac{a_{i+1}-w}{2b_{i+1}} = k_i$ which implies that $w = a_{i+1} - \left(\frac{a_i - a_{i+1}}{b_i - b_{i+1}}\right) 2b_{i+1}$. Substituting for w in the simplified expresssions for the inequality $\pi_B^i(\frac{a_i-w}{2b_i}) > \pi_B^{i+1}(k_i)$, we see that the inequality is true as long as $(b_i - b_{i+1})^2 > 0$ which always holds. Thus, if the supplier chooses a wholesale price such that $q^*(w^*) = k_i$, the buyer is better off optimizing on the i-curve rather than on the (i+1)-curve; notice that this implies the solution cannot be at the kink k_i. Either the unconstrained optimal quantity $q_i^* = \frac{a_i}{4b_i}$ is feasible which will prove the proposition or $q_i^* \le k_{i-1}$ (for $i > 1$) [19] which implies that $q^*(w^*) \notin (k_{i-1}, k_i]$ as assumed and hence a contradiction.

[19] For $i = 1$, then $q_1^* > k_0$.

- $q_i^* \leq k_{i-1}$: For $i = 1$, we know that $q_1^* > k_0 = 0$, so we have a contradiction. So $i > 1$. From the concavity of the profit function, if the buyer optimizes on the i-curve, $q^*(w^*) = k_{i-1}$. But this means that $q^*(w^*) \notin (k_{i-1}, k_i]$ and we again have a contradiction.

From the lemma we know that there always exists at least one $q_i^* \in (k_{i-1}, k_i)$, for $i = 1, \ldots, N$; suppose that the maximum $\pi_B^i(q_i^*)$ over all such feasible q_i^* is obtained on the k-curve. Then $q^*(w^*) = q_k^*$ and $w^* = \frac{a_k}{2}$. Notice that throughout we have ruled out the solution being at a kink. Each time we suppose that the solution is at a kink, k_i, the buyer and supplier find it better to move to either the (i-1) or the (i+1)-curve. ∎

Proof of Proposition 6:

Suppose that the supplier offers $w_1 = \frac{a_i}{2}$ in the first period. We know from our previous analysis of the dynamic problem under linear demand that the buyer will purchase a quantity $q_1 = \frac{a_i}{4b_i} + I$, where $I > 0$, sell the quantity $\frac{a_i}{4b_i}$ this period and carry inventory I. Now suppose that the supplier offers $w_2 = \frac{a_i}{2}$ in the second period (This is feasible, but not optimal.). Then the buyer will purchase the quantity $q_2 = \frac{a_i}{4b_i} - I$, and sell $\frac{a_i}{4b_i}$ this period. The supplier's profits from this strategy are $\pi^S = \frac{a_i}{2} \cdot \left(\frac{a_i}{4b_i} + I\right) + \frac{a_i}{2} \cdot \left(\frac{a_i}{4b_i} - I\right) = 2 \cdot \frac{a_i^2}{8b_i}$. Thus the supplier can ensure a minimum profit of $2 \cdot \frac{a_i^2}{8b_i}$. ∎

Proof of Theorem 3:

The proof is by contradiction. Suppose that there exists an equilibrium in which the buyer does not carry inventories across the first and second periods. Let the equilibrium wholesale prices be w_1^* and w_2^*, and the purchase quantities be q_1^* and q_2^* (Since inventories are not carried, purchase and sales quantities in each period are identical.). Since the second period begins without any inventory, the optimal wholesale price is $w_2^* = \frac{a_i}{2}$ for some $i \in \{1, 2, ..., N\}$, with the corresponding induced purchase quantity being $q_2^* = \frac{a_i}{4b_i}$, by Proposition 5. The supplier makes a second-period profit of $\pi_2^S = \frac{a_i^2}{8b_i}$. Suppose that the supplier offers w_1 in the first period. In order to ensure that the buyer does not carry inventory into the second period, it must be the case that $w_1 \neq \frac{a_i}{2}$ (by Proposition 6). But by Proposition 5, $w_1 \neq \frac{a_i}{2}$ leads to supplier profits $\pi_1^S < \frac{a_i^2}{8b_i}$. Thus in the proposed equilibrium without inventories, $\pi^S = \pi_1^S + \pi_2^S < 2 \cdot \frac{a_i^2}{8b_i}$. This cannot be an equilibrium, since the supplier can make profits of at least $2 \cdot \frac{a_i^2}{8b_i}$ by Proposition 6. Since we picked an arbitrary equilibrium under which the buyer does not carry inventories, no such equilibrium exists: the supplier can always do strictly better by inducing an equilibrium in which the buyer carries inventories. ∎

Proof of Theorem 4:

We focus on the finite (n-period) horizon problem, with the per-period multiplicative discount factor of δ, where $0 < \delta \leq 1$. The proof extends in a straightforward way to the *discounted* infinite horizon ($0 < \delta < 1$).

We prove this result by contradiction, using an outcomes-based argument. Suppose that such a contract exists, and the supplier can make first-best profits which are $\frac{1-\delta^n}{1-\delta} \cdot R(q_{fb})$ for the discounted, finite horizon ($\delta < 1$), $n \cdot R(q_{fb})$ for the undiscounted, finite horizon, and $\frac{1}{1-\delta} \cdot R(q_{fb})$ for the discounted, infinite horizon. Then the *outcome* of the contract will need to satisfy the following conditions:

1. *Sales and Purchase Quantities, and Inventories:* The quantities sold by the buyer in the market in each period *must* be $q_1 = q_2 = \ldots = q_n = q_{fb}$. These are the *unique* set of sales quantities that implement the first-best solution in each period, and generate per-period, channel-revenue maximizing sales of $R(q_{fb})$. Further, since inventories will lead to channel losses via holding costs, we must have $I_1 = I_2 = \ldots = I_{n-1} = 0$, where I_j is the inventory carried by the buyer from period j to $j+1$. Thus, the buyer's *purchase* quantities in each period (induced by the dynamic contract) must equal the sales quantities for that period; i.e., $Q_1 = Q_2 = \ldots = Q_n = q_{fb}$, ensuring that inventories are not carried.

2. *Transfer Payments from Buyer to Supplier:* The payment from buyer to supplier can be via fixed fees, unit prices, a combination of the two, or any other non-linear device. In making our argument here, we are only concerned with the *total* transfer payments. The buyer can and will choose not to participate in any period if he expects to make a loss by accepting the supplier's terms. This is because, under dynamic contracting, future periods are not contractible in the current

period, and past-period contracts have expired. To ensure the buyer's participation, his total per-period transfer payment to the supplier, say H_i in period i, must be bounded from above by his sales revenues of $R(q_{fb})$ per period. Thus the supplier's total profits are bounded from above by $\frac{1-\delta^n}{1-\delta} \cdot R(q_{fb})$, the maximum total payment he can get from the buyer. By assumption, the supplier's profits must attain this bound under the contract. Thus, the total payment to the supplier by the buyer, H_i, in each period must be exactly $R(q_{fb})$ – the single-period, first-best profit.

The *outcome* of the optimal dynamic contract that *both* implements the first-best solution *and* extracts away all of the buyer's residual profits over the n periods must be as follows: Total emoluments transferred from buyer to supplier are $H_1 = H_2 = \ldots = H_n = R(q_{fb})$, purchase quantities are $Q_1 = Q_2 = \ldots = Q_n = q_{fb}$, and Inventories are $I_1 = I_2 = \ldots = I_{n-1} = 0$, where I_j is the inventory carried by the buyer from period j to $j+1$.

Now we demonstrate that such an outcome cannot arise from any (sub-game perfect) equilibrium, since the buyer can do better by unilateral deviation. Suppose in the first period, the buyer has purchased the quantity $Q_1 = q_{fb}$, and paid a sum of $H_1 = R(q_{fb})$ to the supplier. If the buyer sells this entire quantity in the first period, he makes zero residual profits in the first period, and then, from the second period onwards, the supplier can implement the rest of the optimal contract. Knowing that this will be the outcome *if* he sells all his purchased quantities in the first period, the buyer can try to do better by selling some of his purchased quantity in the first period, and carrying the rest as inventory. Observe that, once the buyer has purchased the quantity q_{fb} in the first period, he is free to sell that quantity or carry it forward to future periods: The supplier has no credible enforcing mechanism to ensure sales of the entire purchased quantity within the buyer's period of purchase. Punishments via future contracts are neither credible (history-dependency in the finite horizon fails to meet the subgame-perfection criterion) nor feasible (since the buyer's residual profits in each period are already driven to his participation constraint). As an example, the buyer could sell whatever he carries forward from the first period in the second period. We analyze the result of such an unilateral deviation.[20] Under this strategy, the buyer's optimization

[20] Other deviations are possible, such as selling the purchased quantity gradually, over multiple periods. Some of these

is given by:

$$\max_{q_1} \Pi(q_1) = R(q_1) + \delta \cdot R(q_{fb} - q_1) - h(q_{fb} - q_1) - H_1,$$

subject to the constraint $0 \leq q_1 \leq q_{fb}$, where q_1 is the quantity sold in the first period, $(q_{fb} - q_1)$ is the inventory carried and sold in the second period, and $H_1 = R(q_{fb})$ is the first-period payment to the supplier.[21] Observe that H_1 was paid by the buyer in the first period to procure the quantity q_{fb}, and is a sunk cost when the buyer decides on how much to sell. Setting aside the constraint on the range of q_1 for now, the First Order Conditions with respect to q_1 are:

$$\Gamma(q_1) = R'(q_1) - \delta \cdot R'(q_{fb} - q_1) + h = 0 \qquad \text{(EC.18)}$$

Since $R'(q_{fb}) = 0$ (which is the first-best solution in the static case) and $\delta \cdot R'(0) > h$ (by assumption), $\Gamma(0) > 0 > \Gamma(q_{fb})$. Thus there exists at least one $q_1 \in (0, q_{fb})$ such that $\Gamma(q_1) = 0$. Let q_1^* be the largest such q_1.

Since $\Gamma(q_1^*) = 0$ and $\Gamma(q_{fb}) < 0$, it is clear that the buyer's profits at this maximizing solution are $\Pi(q_1^*) > \Pi(q_{fb}) = 0$ (the latter equality holds by construction of H_1). Hence the buyer will carry inventories (given by $I = q_{fb} - q_1^*$), and sell only a part of his first period purchase q_{fb} in the first period. Faced with the threat of residual profits of $\Pi(q_1^*)$ (with the buyer carrying inventories), the supplier can only implement those contracts in the second period that guarantee the buyer at least $\Pi(q_1^*) = R(q_1^*) + \delta \cdot R(q_{fb} - q_1^*) - h(q_{fb} - q_1^*) - H_1$ in profits.

Thus, *the contractual outcomes specified by conditions (1) and (2) above, are inconsistent with the requirement of subgame perfection. But we showed that conditions (1) and (2) must be satisfied by any contract that generates first-best profits to the supplier. Hence, by contradiction, no such contract that generates first-best profits to the supplier is feasible.* ∎

Proof of Theorem 5:

may yield even higher residual profits to the buyer than the deviation we analyze. However, positive residual buyer profits under the simple deviation we consider will be sufficient to demonstrate that the posited first-best dynamic contract is an infeasible equilibrium under subgame-perfection.

[21] We fix the strategies and outcomes from periods 3 to n as in the posited contract, so the buyer's profits are zero from period 3 onwards.

This is similar to the proof of Theorem 4 except that the per period transfer payment is of the form $H_i = K_i + w_i q_{fb}$. ∎

Proof of Theorem 6: The simplest proof of this result is to construct a commitment contract that *(i)* ensures buyer-participation, *(ii)* implements the first-best solution and *(iii)* extracts away all of the buyer's residual profits. Variants of "selling the firm" to the buyer, at a fee equal to the total discounted first-best profits, will satisfy all three conditions.

We know that the first-best profits in each period are $R(q_{fb})$. Thus, the total discounted first-best profits over the n period horizon are $\Sigma_{i=1}^{n} \delta^{i-1} \cdot R(q_{fb})$, which simplifies to $\frac{1-\delta^n}{1-\delta} \cdot R(q_{fb})$ for $0 < \delta < 1$, and $n \cdot R(q_{fb})$ when $\delta = 1$. Over the infinite horizon, the total discounted first-best profits are $\Sigma_{i=1}^{\infty} \delta^{i-1} \cdot R(q_{fb})$, which simplifies to $\frac{R(q_{fb})}{1-\delta}$.

A commitment contract with upfront fees equal to the total discounted first-best profits over the horizon (as derived above) and marginal unit-cost pricing (i.e., providing any quantity the buyer desires at zero incremental cost), will accomplish the supplier's objectives. The buyer will optimize and buy the quantity q_{fb} every period, to make profits of $R(q_{fb})$ every period. The buyer's total optimal discounted profits over the horizon will be equal to the upfront fee paid to the supplier, and so his residual profits will be driven to zero. The supplier makes first-best channel profits. ∎