Online Appendix
for the paper
Group Buying On The Web: A Comparison Of Price Discovery Mechanisms

This online appendix contains the following items:

1. An extended list of academic and trade references.
2. An extended list of group-buying markets surveyed.
3. Mathematical proofs and derivations of all results in the main paper.

Extended List of References:

1. Anand, K.S., R. Anupindi, and Y. Bassok, “Strategic Inventories in Procurement Contracts”, Working Paper, The Wharton School of the University of Pennsylvania, 2002.
2. Anand, K.S., and H. Mendelson, “Information and Organization for Horizontal Multimarket Coordination”, *Management Science*, December 1997, Vol. 43, No: 12, pp. 1609-1627.
3. Anand, K.S., and H. Mendelson, “Postponement and Information in a Supply Chain”, Technical Report, Northwestern University, July 1998.
4. Appelman, H., "I Scream You Scream: Consumers Vent over the Net", *The New York Times*, March 4, 2001.
5. Cachon, G.P, “The Allocation of Inventory Risk and Advanced Purchase Discounts in a Supply Chain”, Working Paper, The Wharton School of the University of Pennsylvania, 2002.
6. Cremer, J., and R.P.McLean, “Optimal selling strategies under uncertainty for a discriminating monopolist when demands are interdependent”, *Econometrica*, Vol. 53, No: 2, 1985, pp. 345-362.
7. Dennehey, M., " MobShop Closes Consumer Division" http://www.auctionwatch.com/awdaily/dailynews/january01/1-011501.html
8. Dolan, R.J, “Quantity Discounts: Managerial Issues and Research Opportunities”, Marketing Science, Vol. 6, No: 1, 1987, pp. 1-22.

9. Kauffman, R.J., and Wang, B., “New Buyers’ arrival under Dynamic Pricing Market Microstructure: The case of Group-Buying discounts on the Internet”, *Journal of Management Information Systems*, Vol. 18, No: 2, 2001, pp. 157-188.
10. Kauffman, R.J., and Wang, B., “Bid together, buy together: On the efficacy of Group-Buying business models in Internet-based selling,” in *The E-Business Handbook*, P.B.Lowry, J.O.Cherrington and R.R.Watson (editors), Chapter 6, pp. 99-137, CRC Press, 2002.
11. Kohli, R. and Heungsoo P., “A Cooperative Game Theory Model Of Quantity Discounts”, *Management Science*, Vol. 35, No: 6, Jun 1989; pp 693-707.
12. Kreps, David, *A course in Microeconomic Theory*, Princeton University Press, 1990.
13. Lee, Hau L. and Rosenblatt, M., “A Generalized Quantity Discount Pricing Model to Increase Supplier's Profits”, *Management Science*, Vol. 32, No: 9, Sep 1986, pp 1177-1185.
14. Lissack, M., "Of Chaos And Complexity: Managerial Insights From A New Science", *Management Decision*, 35 (1997), 205 - 218.
15. Marschak, J. “Elements for a Theory of Teams”, *Management Science 1,2*, 1955, pp 127-137.
16. Marschak, J. and Radner, R., *Economic Theory of Teams*, Yale University Press, 1972.
17. Mas-Colell, A., M.D.Whinston and J.R.Green, *Microeconomic Theory*, Oxford University Press, 1995.

18. Maskin, Eric and J. Riley, “Monopoly with Incomplete Information”, The RAND Journal of Economics, Vol 15, No: 2, 1984, pp. 171-196.
19. Monahan, J.P., “A Quantity Discount pricing model to increase Vendor profits”, *Management Science*, Vol. 30, No:6, June 1984, pp. 720-726.
20. Myerson, R.B., *Game Theory: Analysis of Conflict*, Harvard University Press, 1991.
21. Myerson, R.B., “Optimal Auction Design”, *Mathematics of Operations Research*, Vol. 6, No: 1, 1981, pp. 58-73.
22. Nahmias, Steve, *Production and Operations Analysis*. Third Edition. Irwin Series in Production Operations Management. McGraw-Hill, 1997.
23. Notess, G.R., "Web Wanderings: Consumers' Revenge: Online Product Reviews and Ratings", *EContent*, April 2000.
24. Sandoval, G., Kawamoto, D., "Mercata Shuts Doors" http://news.cnet.com/news/0-1007-200-4372403.html
25. Shankland, S., "Sun Micosystems Tries Group-buying Strategy" http://news.cnet.com/news/0-1003-200-1851571.html
26. Simon, H.A., " Rational Decision Making in Business Organizations", *The American Economic Review*, Vol. 69, No. 4. (Sep., 1979), pp. 493-513.
27. Simon, H.A., " *The Journal of Economic Perspectives*, Vol. 5, No. 2. (Spring, 1991), pp. 25-44.
28. Solomon., K, " Customer Reviews: It's All a Matter of Opinion", *The Industry Standard*, October 11, 1999.

29. Swaminathan, J.M., and H.L.Lee, "Design for Postponement", in *Handbook of OR/MS on Supply Chain Management*, Graves and de Kok (editors), forthcoming.
30. Tang, C.S., K. Rajaram, A. Alptekinoglu and J. Ou, "The Benefits of Advance Booking Discount Programs: Model and Analysis", 2001, *Management Science*, forthcoming.
31. Tirole, Jean, *The Theory of Industrial Organization*. The MIT Press, 1988.
32. Value America Story: http://news.cnet.com/news/0-1007-201-1518733-0.html
33. Weng, Z. K., "Channel Coordination and Quantity Discounts", Management Science, Vol. 41, No: 9, September 1995, pp. 1509-1522.

Extended List of Group-Buying Markets Surveyed

Business-to-Consumer Markets in the US:

Online Choice: http://www.onlinechoice.com/home/default.asp

McNopoly.com: http://www.mcnopoly.com/

e.Conomy – PreiceWaterhouseCoopers: http://www.pwcglobal.com

The Happy Many: http://www.happymany.com/index_en.html

The Buying Group: http://www.the-buying-group.com/

LetsBuyit.Com (UK, Germany, France): http://www.letsbuyit.com/

Chennai Online Bazaar (India): http://www.chennaionline.com/services

Ciao.com – (A German group-buying directory):

http://www.ciao.com/kategorien/1,202643,218630,55164.html

From Egypt.Com [Egyptian group-buying Market]:

http://groupbuy.fromegypt.com/users/Default.asp

IP Circles: http://www.contractsxml.org/ecap2000/students/FINAL/f-marko-new/page3.html

Clust.Com [French Consumer Market]: http://www.clust.com/

Business-to-Business Group-Buying Markets

Independent Restaurant Purchasing Group (US): http://www.independentrestaurants.com/

MaxGroup (US): http://www.maxgroup.com/Services/b2b_fremont.htm

Group Buying Partnership (UK): http://www.groupbuying.co.uk/

StockBuzz.Com: (Thailand): www.stockbuz.com

HCIS Group Buying: http://www.hcis.org/groupbuying.htm (B2B Health care services).

The Printing Industries of New England: http://www.pine.org/MS/groupbuying.htm

Printing and Graphics Communications Association: http://www.pgca.org/pages/groupbuy.htm

Printing and Imaging Association of Mid-America: http://www.piamidam.org/groupbuy.php

BuildByte Construction Portal [Indian]: http://www.buildbyte.com/

ShopeMates [group-buying enabler]: http://www.shopmates.com/

BazaarE.com - Group buying services for businesses (and consumers): http://www.blonnet.com/businessline/2001/05/03/stories/150339j1.htm

Group-Buying Practices in Non-Profit Organizations

The Maryland Public Service Commission: http://www.md-electric-info.com/index.html

APPA: American Public Power and Environmental Agency

http://www.appanet.org/about/why/aggregation/f_buyingpower.pdf

The Center for Non-Profits: http://www.njnonprofits.org/groupbuy.html

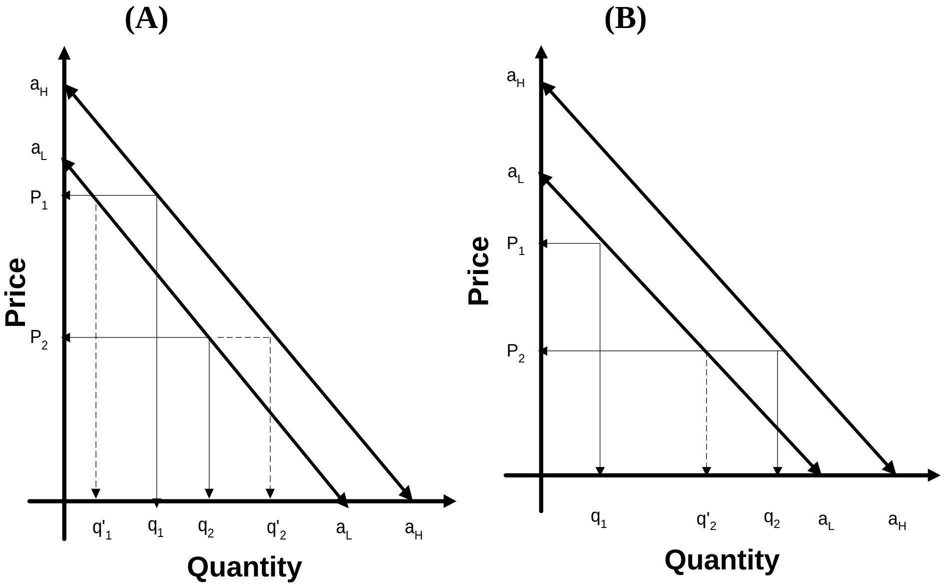
Master Source Corp. : http://www.mastersourcecorp.com/

Mathematical Proofs and Derivations

Generalization of High and Low Demand Regimes

The generic demand function is: Under the high and low demand regimes, the demands are given by: $q = \hat{a}_h - mp$ and $q = \hat{a}_l - mp.$ Without loss of generality, we normalize m to 1 and the resulting demand curves are given by $q = a_h - p$ and $q = a_l - p.$

Figure 1: (A) The monopolist attempts to set the higher of the two price points from the high demand regime. (**B)** The higher price point is captured from the low demand regime while the lower price point is captured from the high demand regime.



Lemma 1: The seller's total revenue in expectation when he chooses the higher price point from the higher demand regime is given by:

$$p^*\left[(q_1, p_1),(q_2, p_2)\right] = \frac{(a_h + a_l)^2}{16}$$

and the optimal price P^* is given by: (0.1)

$$P^* = \frac{a_h + a_l}{4}$$

Proof:

When the seller picks price-quantity schedules (p_1, q_1) and (p_2, q_2), the price that the buyers face in either demand regime is p_2 since the seller cannot stop the price from sliding down to p_2 when the higher demand regime is realized.

Revenue to the seller from this price level is given by:

$$p\left(p_2\right) = \frac{1}{2} p_2 \left(a_l - p_2\right) + \frac{1}{2} p_2 \left(a_h - p_2\right)$$

Taking First Order Conditions w.r.t. p_2

$$\frac{\partial p}{\partial p_2} = \frac{1}{2}\left(a_l + a_h\right) - 2 p_2 = 0$$

$$\Rightarrow p^*{}_2 = \frac{a_l + a_h}{4} \qquad [\because \frac{\partial^2 p}{\partial p^2{}_2} = -2 < 0]$$

$$\therefore p\left(p_2{}^*\right) = \frac{\left(a_l + a_h\right)^2}{8}$$

Which completes the proof.

Lemma 2: The optimal revenue to the seller under this pricing schedule is given by:

$$p^*\left[\left(q_1, p_1\right), \left(q_2, p_2\right)\right] = \frac{\left(a_h + a_l\right)^2}{16}$$

and the resulting prices are:

$$p_2 = \frac{a_h}{2} \text{ and } p_1 > \frac{a_l}{2}$$

Proof:

If $a_l < a_h\left(\sqrt{2} - 1\right)$ it can be shown easily that the seller will ignore the sales from the low demand regime (should it be realized) and pick a price that maximizes the total expected revenue from the high demand regime. This price and revenue in this case are given by: $p_1^* = p_2^* = \frac{a_h}{2}$ and $p_G\left[\left(p_1, q_1\right), \left(p_2, q_2\right)\right] = \frac{a_h^2}{8}$

When $a_l \geq a_h\left(\sqrt{2} - 1\right)$ the seller can pick the profit maximizing price in either demand regime and ensure that the price in the other demand regime is maximized subject to the constraints. This leads to two possibilities. Pick a price in the high demand regime and adjust the price in the low demand regime or vice versa. We will prove the result using the first option here. It can be shown easily that the same result is arrived at when using the second option.

Revenue in the high demand regime is given by:

$$p\left(p_2\right) = p_2\left(a_h - p_2\right) \Rightarrow p_2^* = \frac{a_h}{2}$$

$$\therefore p\left(p_2^*\right) = \frac{a_h^2}{2}$$

Let the price in the low demand regime be given by: $p_1 = p_2 + x$

The unconstrained optimum price in this demand regime is given by:

$p_1^* = \frac{a_l}{2}$ and the unconstrained maximum revenue is given by:

$$p\left(p_1^*\right) = \frac{a_l^2}{4}$$

Let the constrained optimum price be given by: $p_1^{'}$

It is necessary that $p_1^{'} \geq p_2$ for the group buying mechanism to work.

$\therefore$ Let $p_1^{'} = p_2 + x$

$$p\left(p_1^{'}\right) = \left(p_2 + x\right)\left(a_l - p_2 - x\right)$$

We consider two cases. Case 1: $\frac{a_l}{2} > a_h$

Taking First Order Conditions w.r.t. x: $\frac{\partial p}{\partial x} = a_l - a_h - 2x$

Substituting for $p_2 = \frac{a_h}{2}$

$$p\left(p_1^{'}\right) = \left(\frac{a_h}{2} + x\right)\left(a_l - \frac{a_h}{2} - x\right)$$

Since $\frac{\partial p}{\partial x} < 0$ in the above expression, the profits to the seller are decreasing in *x*. Since $x = 0$ is the limiting case of the feasible solution, the seller will pick a price given by $p_2 = \frac{a_h}{2}$ in the higher demand regime and a price $p_1 \geq \frac{a_l}{2}$ in the lower demand regime.

Case 2: $\frac{a_h}{2} \geq a_l$

In this case the seller will pick a price $p_2^* = \frac{a_h}{2}$ in the high demand regime and $p_1^* \geq \frac{a_h}{2}$ in the low demand regime. The market will not clear in the low demand regime, however the seller's expected revenue of $p\left(p_2^*\right) = \frac{a_h^2}{8}$ cannot be exceeded by any other pricing scheme (in the interest of expositional brevity we omit the proof of this claim here, but we can be make the proof available on request).

In either case the maximum possible revenue to the seller is given by:

$$p^*\left[(q_1, p_1),(q_2, p_2)\right] = \frac{a_h.a_l}{4}$$

and the resulting prices are:

$$p_2 = \frac{a_h}{2} \text{ and } p_1 > \frac{a_h}{2}$$

Finally, $\frac{(a_h + a_l)^2}{16} > \frac{a_h.a_l}{4} \ \forall a_h > a_l$ the seller will always pick a price-quantity schedule given by: $p_2 = \frac{a_h}{2}$ and $p_1 > \frac{a_l}{2}$ with resulting expected revenues of:

$$p^*\left[(q_1, p_1),(q_2, p_2)\right] = \frac{(a_h + a_l)^2}{16}$$

Which completes the proof.

Proposition 1: *Under demand uncertainty, the revenue to the seller that operates Group-Buy schemes is either less than the revenue from posted pricing or is exactly equal to it. The Group-Buy mechanisms does not generate superior revenues to the seller under any conditions.*

Proof: It suffices if we prove that revenue from the Group-Buy scheme in case 2 is dominated by the revenues from posted pricing, since the revenues from the Group-Buy scheme is in case –1 are exactly the same as the revenue from posted pricing.

We first determine the maximum revenue to the monopolist that operates a posted price market. The monopolist picks a price *p* and the induced demand and revenue are given by:

$$p\,(p) = \frac{1}{2}\left[p(a_l - p) + p\,(a_h - p)\right]$$

$$\Rightarrow p\,(p) = \frac{1}{2}\left[p(a_l + a_h)\right] - p^2$$

Taking First Order Conditions *w.r.t.* to *p* we have:

$$\frac{\partial p}{\partial p}=\frac{\left(a_l+a_h\right)}{2}-2p$$

$$p^*=\frac{a_l+a_h}{4} \qquad [\because \frac{\partial^2 p}{\partial p^2}=-2<0]$$

$$\therefore p\left(p^*\right)=\frac{\left(a_l+a_h\right)^2}{16}$$

From Lemma 2 the revenue from the Group-Buy mechanism is given by:

$$p\left(g\right)=\frac{a_l.a_h}{4}$$

$$\therefore p\left(p^*\right)-p\left(g\right)=\frac{\left(a_l+a_h\right)^2}{16}-\frac{a_l.a_h}{4}=\frac{\left(a_l-a_h\right)^2}{16}>0$$

$$p\left(p^*\right)>p\left(g\right)$$

Which completes the proof.

Lemma 3: When the demand regimes are characterized by the following binding constraint, $\frac{a}{m}\le b<a$, the profit maximizing Group-Buy price-quantity schedule under demand uncertainty is given by:

$$\text{Price } P*$$

$$p_1^{\ *}=\frac{b}{2} \text{ if quantity } q_1\le b-\frac{a}{2m} \tag{0.2}$$

$$p_2^{\ *}=\frac{a}{2m}\text{if quantity } q_1>b-\frac{a}{2m}$$

The expected revenue to the seller is given by:

$$p\left(p_1^{\ *},p_2^{\ *}\right)=\frac{a^2+mb^2}{8m}$$

Proof:

Note that it is implied in the construction of the demand regimes, that $b>\frac{a}{m}$.

Now we consider the maximum revenue from Group-Buy schemes.

The seller picks prices as follows:

$$\text{Price } P*$$

$$p_1^{\ *}=\frac{b}{2} \text{ ; if quantity } q_1\le b-\frac{a}{2m}$$

$$p_2^{\ *}=\frac{a}{2m} \text{ ; if quantity } q_1>b-\frac{a}{2m}$$

From figure 2 (see below) it is clear that the quantity used to separate the two prices should be at least equal to $b-\dfrac{a}{2m}$ to prevent the price from sliding down to p_2 if the demand regime given by $q=b-p$ is realized. The reader can convince herself from inspection that the necessary conditions defined by the Group-Buy scheme, i.e. price falls with an increase in quantity demanded do hold.

The seller's revenues from the above Group-Buy price schedule is given by:

$$p\left(p_1,p_2\right)=\frac{1}{2}\left[p_1\left(b-p_1\right)+p_2\left(a-mp_2\right)\right]$$

such that: $p_1 \geq p_2$ and $q_1 < q_2$

$$\therefore p_1^{*}=\frac{b}{2} \text{ and } q_1^{*}=\frac{b}{2}$$

$$p_2^{*}=\frac{a}{2m} \text{ and } q_2^{*}=\frac{a}{2}$$

For the binding condition $q_1^{*} < q_2^{*}$ to hold, it is necessary that:

$b < a$

Substituting for the values of p_1^{*} and p_2^{*} in $p\left(p_1,p_2\right)$, we have:

$$\therefore p\left(p_1^{*},p_2^{*}\right)=\frac{a^2+mb^2}{8m} \quad \forall\ b<a$$

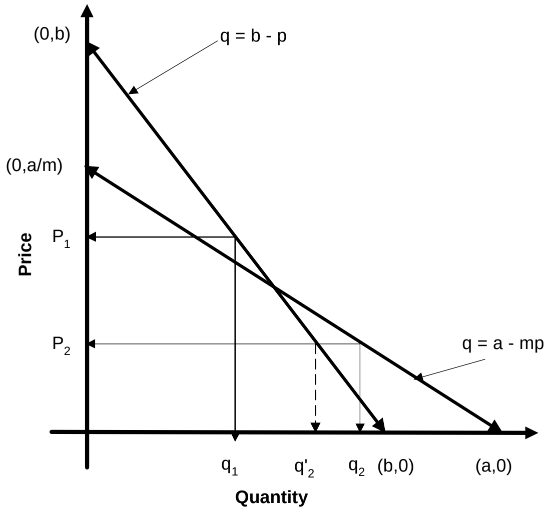
Which completes the proof.

Proposition 2: *When the demand regimes are characterized by the following binding constraint,* $\dfrac{a}{m}\leq b<a$ *,: The revenue to the seller from group buying strictly dominates the revenue from posted pricing. The profit maximizing price and revenue under posted pricing are as follows:*

$$p^{*}=\frac{a+b}{2\left(m+1\right)}$$

$$p\left(p^{*}\right)=\frac{\left(a+b\right)^2}{8\left(m+1\right)}$$

Figure 2: Intersecting Demand Curves: Neither Demand Regime Dominates The Other Universally.



Proof:

We first determine the optimal price and maximum revenue under posted pricing. If the monopolist seller picks a price given by *p* under both demand regimes, then his revenues are given by:

$$p\left(p\right)=\frac{1}{2}\left[p\left(b-p\right)+p\left(a-mp\right)\right]$$

Taking First Order Conditions w.r.t. *p*,

$$\frac{\partial p}{\partial p}=\frac{1}{2}\left[\left(a+b\right)-2p\left(m+1\right)\right]$$

$$\therefore p^{*}=\frac{a+b}{2\left(m+1\right)}\qquad [\because \frac{\partial^{2}p}{\partial p^{2}}=-\left(m+1\right)<0]$$

$$\therefore p\left(p^{*}\right)=\frac{\left(a+b\right)^{2}}{8\left(m+1\right)}$$

From Lemma 3 the revenue from the Group-Buy mechanism is given by:

$$p\left(p_{1}^{*},p_{2}^{*}\right)=\frac{a^{2}+mb^{2}}{8m}\ \ \forall a>b$$

Comparing the two revenues we have:

$$p\left(p_{1}^{*},p_{2}^{*}\right)-p\left(p^{*}\right)=\frac{a^{2}+mb^{2}}{8m}-\frac{\left(a+b\right)^{2}}{8\left(m+1\right)}\quad\forall\ b<a$$

$$p\left(p_{1}^{*},p_{2}^{*}\right)-p\left(p^{*}\right)=\frac{\left(a-mb\right)^{2}}{8m\left(m+1\right)}>0\quad[\forall\ b<a]$$

$$p\left(p_{1}^{*},p_{2}^{*}\right)>p\left(p^{*}\right)\ \ \forall\ b<a$$

Since $\frac{a}{m} < b$ by defintion, we have: $p\left(p_1^{*}, p_2^{*}\right) > p\left(p^{*}\right) \;\; \forall \; \frac{a}{m} \le b < a$

Which completes the proof.

Proposition 3: *The gains from using group-buying mechanism increase as m increases.*

Proof: From proposition 2, corollary 1, the difference in revenues between group buying and posted pricing is given by:

$$p_g - p_p = \frac{(a - bm)^2}{8m(m+1)}$$

Let $D = p_g - p_p = \frac{(a - bm)^2}{8m(m+1)}$

$$\frac{\partial D}{\partial m} = \frac{2abm^2 + b^2m^2 - a^2(1+2m)}{8m^2(1+m)^2}$$

$$\frac{\partial D}{\partial m} > 0 \text{ iff } \; 2abm^2 + b^2m^2 - a^2(1+2m) > 0$$

The above expression can be re-written as follows:

$$m^2\left(2ab + b^2\right) - 2a^2m - a^2$$

Solving for m in the above expression yeilds the following result:

$$m^2\left(2ab + b^2\right) - 2a^2m - a^2 \;\; iff \;\; m > \frac{a}{b} \text{ or } m < \frac{-a}{2a+b}$$

Clealry $m < \frac{-a}{2a+b}$ is infeasible. However, $m > \frac{a}{b}$ is both feasible and binding (by our defintion of the two demand curves).

$$\therefore m^2\left(2ab + b^2\right) - 2a^2m - a^2 \;\; > 0$$

This in turn implies that $\frac{\partial D}{\partial m} > 0 \;\; \therefore D$ is increasing in *m.*

Which completes the proof.

Proposition 4: *Under production precommitment, the equilibrium solution as well as the supplier's profits are identical under group buying and posted pricing.*

Proof:

Remark: The seller can produce 0,1, or 2 units. Clearly when he produces 0 or 1 unit(s) for sale, it results in price and revenue outcomes that are identical under group buying and posted pricing. The remaining case to be examined is when he produces 2 units. Note however, that producing two units does not imply that the actual quantity sold (under either mechanism) is exactly two units. Under either mechanism, the actual quantity sold can be 0,1 or 2 units depending on the seller's optimal price schedule and the buyers' valuations.

Posted Prices: Customer values are distributed as $U[0,1]$, and the production cost is normalized to 0. Let $p \in [0,1]$ be the posted price set by the firm. The firm will sell 0, 1 or 2 units. The probability of selling two units (for revenues of $2*p$) is the probability that both customers assign a value to the good that is greater than the price; this is equal to $(1-p)^2$. The probability of selling one unit (for revenues of p) is $\binom{2}{1} * p*(1-p) = 2p*(1-p)$. Thus, supplier's expected profits are given by

$p_p = (1-p)^2 * 2*p + 2p*(1-p)*p = 2p*(1-p)$, a simple quadratic. The first order conditions yield $p_p^* = 0.5$ and $p_p^* = 0.5$.

Group Buying : The two distinguishing features of customers' bidding are that (i) they bid without knowing the final price, but only based on the current price and their *expected* final price, and (ii) in so far as each bid lowers the final price, each customer must factor other customers' behavior into their strategies. Observe that, in fact, customers have an incentive to bid even if the price slightly exceeds their value. This is because their bidding might induce other customers to bid as well, which would lead to lower prices. If the prices are sufficiently lowered by the cascade induced by their bidding, they would get the good at a price lower than their value, generating a positive surplus. On the other hand, if they don't succeed in inducing a cascade of bids by bidding at a price higher than their value, they will lose surplus. Customers' optimal bidding price point is to trade off these two possibilities. It is clear that their optimal bidding price point will typically be *greater* than their value. Directly attempting to solve for each customer's bidding strategy as a function of her value and the other customer's strategy is difficult. Hence we solve for the equilibrium by employing the *revelation principle*, widely used in the economics/game theory literature on adverse selection and mechanism design. The revelation principle is found in most advanced textbooks in microeconomics (cf. Kreps [1990], Mas-Collell *et. al.* [1995]); the following statement of the revelation principle is modified from Proposition 14.C.2, page 493 of Mas-Collell *et. al.* [1995] to apply to our problem.

Revelation Principle applied to our problem:

In essence, the supplier has to figure out the optimal pricing mechanism to offer to customers, while allowing for customers to lie, if it were in their interest to do so. Recall that customer values are distributed as U[0,1], and not known to the supplier. The revelation principle states that in searching for the optimal (profit-maximizing) group-buying mechanism (the price-points as a function of the total quantities demanded), the supplier can without loss restrict himself to schemes of the following form:

Each customer reveals his true ‘value’ to the supplier.
The pricing mechanism specifies an outcome (whether each customer gets a unit of the good or not), and [Incentive Compatibility] Whatever her value, each customer finds it optimal to report the state truthfully.

To meet these conditions, we propose the following mechanism: The supplier publicly announces the pricing schedule $[(p_1,q_1), p_2]$, where the variables are interpreted as discussed in the text. Then the customers will reveal their true values publicly, and the supplier will pick the lowest price point possible (corresponding to the maximum number of customers who can be served), *consistent* with the pricing schedule. The incentive compatibility constraint is easily seen to be met: if feasible, the customer will get a unit of the good at the lowest feasible price point consistent with the total quantity allotted. The argument for why the customer’s dominant strategy is to announce her true value is almost identical to the well-known argument for truth-revelation in Vickery (second-price sealed bid) auctions, and is summarized below:

Announcing a value higher or lower than her true value will only make a difference when the price point is in between the announced value and the true value. If the announced value > final price > true value, the customer gets the good but at a negative surplus. If the announced value < final price < true value, the customer fails to get the good even though getting the good at the final price would have generated positive surplus for her. Under all other scenarios, the outcome is the same for the announced value as well as if she had announced her true value.

So now, the supplier’s objective function can be set up assuming that customers announce their true values, and that the supplier credibly commits to and implements the lowest price point consistent with the announced price schedule. In our current model, with two units being sold and two customers, we can set $q_1 = 1$ without loss of generality. The firm will sell *0, 1* or *2* units. The probability of selling two units (for revenues of $2 * p_2$) is the probability that both customers assign a value to the good that is greater than the price; this is equal to $(1-p_2)^2$. The probability of selling exactly one unit (for revenues of p_1) is equal to the probability that *(a)* one customer has a value for the good that is $\geq p_1$ (so that at least one unit will be sold) and *(b)* the other customer has a value $\leq p_2$ (so that two units are not sold), which is given by $\binom{2}{1} * p_2 * (1-p_1) = 2p_2 * (1-p_1)$. Thus, the supplier’s expected profits are given by

$$p_{\boldsymbol{g}} = (1-p_2)^2 * 2 * p_2 + 2p_2 * (1-p_1) * p_1,$$

and he solves for the price schedule (p_1, p_2) that maximizes this, subject to the constraint that $p_1 \geq p_2$. The Lagrangian is

$$\Lambda \quad (p_1, p_2, l) = (1-p_2)^2 * 2 * \mathrm{p}_2 + 2p_2 * (1-p_1) * p_1 + l * (p_1 - p_2).$$

The first order conditions (foc), obtained by differentiating with respect to p_1 and p_2, and the complementary slackness condition (csc) give the following three equations:

$2\text{-}8^* p_2 + 6^* p_2^{\,2} + 2^* p_1 \text{-} 2^* p_1^{\,2} - l = 0,$ (foc1)
$p_2(2\text{-}4^* p_1) + l = 0$, and (foc2)
$l^*(p_1 - p_2) = 0$ (csc)

There are four solutions to the above equations. These are given by *(p_1, p_2, l) = (0.5,0.8333,0); (0.5,0.5,0);(1.6180,0,0) and lastly, (-0.6180,0,0).* Of these four solutions, three are infeasible: the first has $p_1 < p_2$ *which violates the constraint, and the last two has* p_1 outside the range of *[0,1]*. We now check the second order conditions for the only feasible solution: *(p_1, p_2) = (0.5,0.5).* The Hessian for the objective function is:

$$\mathrm{H}(p_{\mathrm{g}}) = \begin{pmatrix} \dfrac{\partial^2 p_{\mathrm{g}}}{\partial p_1^{\,2}} & \dfrac{\partial^2 p_{\mathrm{g}}}{\partial p_1 \partial p_2} \\ \dfrac{\partial^2 p_{\mathrm{g}}}{\partial p_1 \partial p_2} & \dfrac{\partial^2 p_{\mathrm{g}}}{\partial p_2^{\,2}} \end{pmatrix}$$

$$= \begin{pmatrix} -4 * p_2 & 2*(1-2*p_1) \\ 2*(1-2*p_1) & -8+12*p_2 \end{pmatrix},$$

which at (0.5,0.5) is $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$. A cursory examination of the principal determinants reveals that the Hessian is negative definite. Hence the point *(p_1, p_2) = (0.5,0.5)* maximizes the objective function (supplier profits). This solution is identical to the posted price solution, and yields the same profits.

Which completes the proof.

Note: The following Lemma is not a part of the main paper. We offer it as a lead up to Proposition 5 – to demonstrate the derivation of results that have been subsumed in the discussion that leads up to and follows Proposition 5.

Lemma 4: *Under production postponement with linear production costs of c per unit, the unique subgame-perfect equilibrium and supplier profits under posted pricing and under group buying are as given below.*

Posted Pricing: $P_p^* = \dfrac{1+c}{2}$ *and* $p_p^* = \dfrac{(1-c)^2}{2}$.

Group Buying: $P_{1g}^* = P_{2g}^* = \dfrac{1+c}{2}$ *and* $p_g^* = \dfrac{(1-c)^2}{2}$.

The production quantities under each pricing scheme are determined by the realized orders. Thus the equilibrium solution as well as the supplier's profit are identical under group buying and posted pricing.

Proof:

Posted Prices: Customer values are distributed as *U[0,1]*. Let $p \in [0,1]$ be the posted price set by the firm. The firm will sell *0, 1* or *2* units, and the corresponding production costs would be *0, c* and *2c* respectively. The probability of selling two units (for revenues of *2*p*) is the probability that both customers assign a value to the good that is greater than the price; this is equal to $(1-p)^2$. The probability of selling one unit (for revenues of *p*) is $\binom{2}{1} * p*(1-p) = 2p*(1-p)$. Thus, supplier's expected profits are given by

$p_{\mathbf{p}} = (1-p)^2$*(2*p-2*c)+2p*(1-p)*(p-c) = 2*(p-c)*(1-p), a simple quadratic. The first order conditions yield $P_p^* = \frac{1+c}{2}$ and $p_p^* = \frac{(1-c)^2}{2}$.

Group Buying : As before, the supplier announces a price schedule, and the customers then bid based on their values. Once again, customers have an incentive to shade their behavior above their actual valuation of the good, since their bid might induce a cascade of bids. As in the proof of Proposition 4, we can apply the revelation principle, and assume *(i)* truthful revelation of their values by customers, in return for *(ii)* a credible commitment from the supplier to pick the lowest price point possible (corresponding to the maximum number of customers who can be served), *consistent* with the announced pricing schedule. In the current scenario, the supplier incurs production costs only after the exact demand is revealed via pricing, and this should be reflected in the supplier's profit function. The supplier will sell *0, 1* or *2* units. The probability of selling two units (for a surplus of $(2*p_2 - 2*c)$ is the probability that both customers assign a value to the good that is greater than the price; this is equal to $(1-p_2)^2$. The probability of selling exactly one unit (for a surplus of $p_1 - c$) is equal to $2p_2*(1-p_1)$, as derived previously in the proof of Proposition 4. Thus, the supplier's expected profits are given by

$p_{\boldsymbol{g}} = (1-p_2)^2*(2*p_2-2*c) + 2p_2*(1-p_1)*(p_1-c)$, and he solves for the price schedule (p_1, p_2) that maximizes this, subject to the constraint that $p_1 \ge p_2$. The Lagrangian is

$$\Lambda(p_1, p_2, l) = (1-p_2)^2*(2*p_2-2*c)+2p_2*(1-p_1)*(p_1-c)+l*(p_1-p_2).$$

The first order conditions (foc), obtained by differentiating with respect to p_1 and p_2, and the complementary slackness condition (csc) give the following three equations:

$2-8*p_2+6*p_2^2+2*p_1-2*p_1^2+c*(2-4*p_2+2*p_1)-l = 0,$ (foc1)

$2 * p_2(1 - p_1) - 2 * p_2(-c + p_1) + l = 0$, and (foc2)
$l * (p_1 - p_2) = 0.$ (csc)

The only feasible solution to the above equations (that satisfies the constraint $p_1 \geq p_2$ and the range restriction $p_1, p_2 \in [0,1]$) is the point $(p_1, p_2, l) = (\frac{1+c}{2}, \frac{1+c}{2}, 0)$. We now check whether the second order conditions for the maximum are satisfied at the point $(p_1, p_2) = (\frac{1+c}{2}, \frac{1+c}{2})$. The Hessian for the objective function is:

$$\mathrm{H}(p_g) = \begin{pmatrix} \frac{\partial^2 p_g}{\partial p_1^2} & \frac{\partial^2 p_g}{\partial p_1 \partial p_2} \\ \frac{\partial^2 p_g}{\partial p_1 \partial p_2} & \frac{\partial^2 p_g}{\partial p_2^2} \end{pmatrix}$$

$$= \begin{pmatrix} -4 * p_2 & 2 * (1 + c - 2 * p_1) \\ 2 * (1 + c - 2 * p_1) & -4 * (2 + c - 3 * p_2) \end{pmatrix},$$

which at $(\frac{1+c}{2}, \frac{1+c}{2})$ reduces to $\begin{pmatrix} -2(1+c) & 0 \\ 0 & -2(1-c) \end{pmatrix}$. A cursory examination of the principal diagonal reveals that the Hessian is negative definite. Hence the point $(p_1, p_2) = (\frac{1+c}{2}, \frac{1+c}{2})$ maximizes the objective function (supplier profits). This solution is identical to the posted price solution, and hence yields the same profits.

Which completes the proof.

Proposition 5: *Under production scale economies, when pricing precedes production, the unique subgame-perfect equilibrium and supplier profits under posted pricing and under group buying are as given below.*

Posted Pricing: $p^* = \frac{1 - c_1 + c_2}{2 - 2c_1 + c_2}$ *and* $p_p^* = \frac{(1 - c_1)^2}{2 - 2c_1 + 2c_2}$.

Group Buying: $p_1^* = \frac{1 + c_1}{2}$ *and*

$$p_2^* = \frac{1}{6}\left(4 + c_2 - \frac{6(1 - c_1 + c_2)}{2 - 2c_1 + c_2} - \sqrt{1 + 3(2 - c_1)c_1 + (c_2 - 4)c_2}\right) \text{and}$$

$$p_G = \frac{1}{108}\left(52 - 72c_1 + 36c_1^2 - 15c_2 - 18c_1c_2 + 9c_1^2c_2 + 12c_2^2 - 2c_2^3 + 2\left(1 + 3(2 - c_1)c_1 + (c_2 - 4)c_2\right)^{3/2}\right)$$

Proof: The proof of this claim is based on reasoning that is similar to the analysis employed in the proof of Lemma 4 (above). A more detailed proof will be made available on request by the authors.