TECHNICAL APPENDICES

for the paper

"Forward Aggregation in Electronic B2B Markets: Model and Experimental Findings"

Technical Appendix A: Mathematical Proofs

Technical Appendix B: Experimental Design & Data

Technical Appendix A: Mathematical Proofs

Proof of Proposition 1: For any price choice $P \in [a,b]$, the probability of a sale is $F(P)$. Hence the profits are $S_D(P) = P \cdot (1 - F(P)), \forall P \in [a,b]$.

Any price $P \geq b$ yields zero profits (since $F(b) = 1$); further, any price $P < a$ is dominated by setting $P = a$, and $S_D(a) = a$. Hence, the optimal price $P^* \in [a,b)$.

Differentiating the profit expression yields:

$$\Delta(P) = \frac{\partial S_D(P)}{\partial P} = 1 - F(P) - P \cdot f(P). \qquad \textit{(A.1)}$$

Expression *(A.1)* can be rewritten as

$\Delta(P) = \left[1 - F(P)\right]\left(1 - P \cdot \frac{f(P)}{1 - F(P)}\right) = \left[1 - F(P)\right]\left(1 - P \cdot H(P)\right)$. Since the hazard rate is increasing, each of the factors of $\Delta(P)$ is decreasing. Hence, $\Delta(P)$ is decreasing, and so the profit function $S_D(P)$ is concave in P (second order conditions are satisfied).

Further, we see that $\left.\frac{\partial S_D(P)}{\partial P}\right|_{P \to b} = -b \cdot f(b) < 0,$ and $\left.\frac{\partial S_D(P)}{\partial P}\right|_{P \to a} = 1 - a \cdot f(a).$ This yields two cases. If $a \cdot f(a) \geq 1$, $\Delta(a) \leq 0$, and hence, $\Delta(P) \leq 0, \forall P \in [a,b]$. In this case, $S_D(P)$ is maximized at $P = a$. If $a \cdot f(a) < 1$, $\Delta(a) > 0$, and hence an interior maximum exists, obtained by setting expression *(A.1)* to 0. This yields the profit maximizing price $P^* = \frac{1 - F(P^*)}{f(P^*)} = \frac{1}{H(P^*)}$. The corresponding supplier profits are derived by setting $P = P^*$ in the expression for $S_D(P)$.

An arbitrary buyer's surplus is $(v - P^*)$ when his valuation v exceeds the supplier's price P^*, and 0 otherwise, since he buys the good only when his valuation exceeds the price. Thus his surplus is $(v - P^*)^+ = \max\left\{v - P^*, 0\right\}$. His *expected* pre-aggregation surplus is given by $B_D^* = E\left[\max\left\{v - P^*, 0\right\}\right]$, which simplifies to $\left\{E\left[v \mid v > P^*\right] - P^*\right\} \cdot \left(1 - F(P^*)\right)$.

♦

Proof of Proposition 2: The expected price in a Vickrey auction is equal to the highest losing bid. Since the dominant strategy for any buyer under the Vickrey auction is to bid truthfully, the expected price when k units are offered for sale is the expected $(k+1)^{th}$ highest valuation. Thus, $P^*(k) = E[v_{(k+1)}]$. Further, by the Revenue Equivalence Theorem, this result applies to most commonly used types of auctions, and not just to the Vickrey auction. Now, the density of the $(k+1)^{th}$ order statistic is given by

$$f_{v_{(k+1)}}(x) = \frac{n!}{(n-k-1)!k!}\left[F(x)\right]^{n-k-1}\left[1 - F(x)\right]^k f(x). \qquad \textit{(A.2)}$$

This result is derived in most intermediate-level statistic texts (see, for instance, Theorem 12 on page 254 of Mood et al (1974));[1] the coefficient of $f(x)$ in the RHS of *(A.2)* is obtained from the Binomial distribution. Taking expectations throughout *(A.2)* yields Result *(i)* of the Proposition. Results *(ii)* and *(iii)* are straight-forward to derive from Result *(i)* and the supplier's revenue expression under auctions, given by $\pi_A(k) = k \cdot P^*(k)$ (Since production costs are normalized to zero, revenues equal profits). Result *(iv)* is derived from the auction-clearing price of Result *(i)*. ♦

Proof of Proposition 3: These results are obtained by specializing the results of *Proposition 1* (proved above) to the parameters of this distribution. ♦

Proof of Proposition 4: These results are obtained by specializing the results of *Proposition 2* (proved above) to the parameters of this distribution. To derive the optimal quantity and corresponding profits, note that, in deciding on how many items should be produced for the auction, the aggregator solves the following maximization problem:

$$\max_{1\le k\le n-1} \pi_A(k) = P^*(k)\cdot k = \left[(R-s)+2s\cdot\frac{(n-k)}{(n+1)}\right]\cdot k$$

Differentiating the objective function, it is easy to see that $\frac{d\pi_A(k)}{dk}$ is greater than 0 at $k=1$ (for $n\ge 2$), and strictly decreasing in k. The boundary condition ($k^*=n-1$) is obtained when either (a) $\left.\frac{d\pi_A}{dk}\right|_{k=n-1}\ge 0$, *or* (b) $\left.\frac{d\pi_A}{dk}\right|_{k=n-1}<0$, but $\pi_A(n-2)\le\pi_A(n-1)$. ♦

Proof of Proposition 5:

The proof builds on the results of Propositions 3 and 4. Based on the profit expressions provided in these Lemmas, we will analyze three cases separately: *(i)* $\frac{R}{s}\le\frac{3n-5}{n+1}$, *(ii)* $\frac{3n-5}{n+1}<\frac{R}{s}<3$, and *(iii)* $\frac{R}{s}\ge 3$.

__*Case (i)*:__ $\frac{R}{s}\le\frac{3n-5}{n+1}$ (<3). In this range, $\pi_D^* = n\cdot S_D^* = \frac{n\cdot(R+s)^2}{8s}$ (across the n markets) and $\pi_A^* = \frac{\left[R(n+1)+s(n-1)\right]^2}{8(n+1)s}$.

Thus, $\pi_D^*>\pi_A^*$ if and only if

$$\frac{n\cdot(R+s)^2}{8s} > \frac{\left[R(n+1)+s(n-1)\right]^2}{8(n+1)s}$$

[1] Mood, A.M., F.A.Graybill and D.C.Boes, *Introduction to the Theory of Statistics*, Third Edition, McGraw-Hill Series in Probability and Statistics, 1974.

iff $$n(n+1)\cdot(R+s)^2 > \left[R(n+1)+s(n-1)\right]^2.$$

Expanding and rearranging terms, $\pi_D^* > \pi_A^*$ if and only if

$$(n+1)\cdot R^2 - 2(n+1)R\cdot s - (3n-1)\cdot s^2 < 0$$

iff $$(n+1)\cdot\left(\frac{R}{s}\right)^2 - 2(n+1)\left(\frac{R}{s}\right) - (3n-1) < 0 \quad (\because s > 0)$$

iff $$(n+1)\cdot\left(\frac{R}{s} - 1 - 2\sqrt{\frac{n}{n+1}}\right)\cdot\left(\frac{R}{s} - 1 + 2\sqrt{\frac{n}{n+1}}\right) < 0.$$

One of the roots, $1-2\sqrt{\frac{n}{n+1}}$, is a decreasing function of *n*; further, this root is negative even at $n=1$. Therefore, $\frac{R}{s}$ is always greater than this root. Hence, $\pi_D^* > \pi_A^*$ if and only if $\left(\frac{R}{s} - 1 - 2\sqrt{\frac{n}{n+1}}\right) < 0$. Observe that this is always true under Case *1*, since

$$\frac{R}{s} \le \frac{3n-5}{n+1} = 1 + 2\cdot\left(\frac{n-3}{n+1}\right)$$
$$< 1 + 2\cdot\left(\frac{n}{n+1}\right)$$
$$< 1 + 2\sqrt{\frac{n}{n+1}}.$$

This proves Case *(i)*.

Case *(ii)*: $\frac{3n-5}{n+1} < \frac{R}{s} < 3$. Since $\frac{R}{s} \ge 1$ always, this case is feasible only for $n \ge 3$. In this range, $\pi_D{}^* = \frac{n\cdot(R+s)^2}{8s}$ and $\pi_A{}^* = (n-1)\cdot\left(R - \frac{n-1}{n+1}\cdot s\right)$.

Thus, $\pi_D^* > \pi_A^*$ if and only if

$$\frac{n\cdot(R+s)^2}{8s} > (n-1)\cdot\left(R - \frac{n-1}{n+1}\cdot s\right)$$

Which translates to:

$$\frac{n}{(n-1)} > \frac{\left(R - \frac{n-1}{n+1}\cdot s\right)}{\frac{(R+s)^2}{8s}}$$

Let $\frac{R}{s} = t \ : 1 \le t < 3$. Then the condition simplifies to

$$\frac{n}{(n-1)} > \frac{8 \cdot \left(t - \frac{n-1}{n+1}\right)}{(t+1)^2}.$$

Rewriting the above as a quadratic in t, the condition becomes $f(t) > 0,$ where
$f(t) = n(t+1)^2 - 8 \cdot \left(t - \frac{n-1}{n+1}\right) \cdot (n-1)$.
Since the co-efficient of the squared term t is positive – we can take the usual value of roots method. Solving for t in $f(t) = 0$, the two roots are:

$$t = \frac{3n^2 - n - 4 + 4\sqrt{1-n^2}}{n \cdot (1+n)} \quad \text{OR } t = \frac{3n^2 - n - 4 - 4\sqrt{1-n^2}}{n \cdot (1+n)}$$

Both roots are complex for $n > 1$ (has to be both or none in any case since it is quadratic), which means that $f(t)$ is either always greater than 0 or always less than 0 (since there are no Zeros for the polynomial). It is easily checked (by plugging in any value for t in the feasible range; e.g. $t = 2$) that $f(t) > 0$.
This proves Case *(ii)*.

Case *(iii)*: $\frac{R}{s} \ge 3$. In this range, $\pi_D^* = n \cdot (R - s)$ and $\pi_A^* = (n-1) \cdot \left(R - \frac{n-1}{n+1} \cdot s\right)$.
Thus, $\pi_D^* > \pi_A^*$ if and only if

$$n \cdot (R - s) > (n-1) \cdot \left(R - \frac{n-1}{n+1} \cdot s\right)$$

iff

$$\frac{R}{s} > n - \frac{(n-1)^2}{(n+1)} = \frac{3n-1}{n+1},$$

which is always true for this Case.
This proves Case *(iii)*, and hence concludes the proof of Proposition 5. ♦

Proof of Proposition 6: $\Delta\pi = \pi_D^* - \pi_A^* = \frac{n \cdot (R+s)^2}{8s} - \frac{[R(n+1) + s(n-1)]^2}{8(n+1)s}$
$= \frac{s}{8}\left[\frac{3n-1}{n+1} + 2 \cdot \frac{R}{s} - \left(\frac{R}{s}\right)^2\right] = \frac{s}{8}\left[3 - \frac{4}{n+1} + 2 \cdot \frac{R}{s} - \left(\frac{R}{s}\right)^2\right]$, which is increasing in n. ♦

Technical Appendix B: Experimental Design & Data

In this online Technical Appendix we provide a detailed description of the design of the experiments that test the predictions of our model, by placing experimental subjects in conditions similar to those in the model and providing them with incentives as described in the model.

1 Experimental Platform

An electronic auction platform – Virtual Auction Server Engine (VASE) - developed at a north eastern university that supports web-based bidders and suppliers was used to conduct the auction experiment. The platform supports several types of auctions including traditional auctions and reverse auctions. Further, it provides a control panel for the auctioneer to be able to set auction parameters (start and end time, bid increments, minimum bid, quantity under auction etc.). The platform allows a distribution of values to be created that can subsequently be assigned to bidders randomly. It is a very versatile platform that is rich in the feature set that it supports and was developed over a 2 year period to support auction-based experiments.

2 Experimental Design

We introduced the game to the participants as follows:

In this experiment you will play a game. You are the owner of a factory that produces a product to sell to the end consumer market. You need to buy a component from suppliers that goes into the manufacture of your product. Your total cost of manufacturing consists of two sets of costs: *(i)* the cost of labor and value addition and *(ii)* the price that you will pay to suppliers for this component. Your manufacturing costs, including all costs <u>other than</u> the price that you pay for this component, will be supplied to you via a message generated by the VASE system. The revenue (the unit price) that you expect to receive from selling the product will be made available to you when you log on to the VASE system.

There are other buyers that also want to buy this component for manufacturing their goods (sold to end-consumers not in your market). They will compete with you to buy the component via a procurement auction. Each buyer – like you – demands one unit of this good. The number of such buyers in the market will be supplied to you when you sign on to the VASE system. The amount that each such buyer is willing to pay for this product is drawn from a uniform distribution with a lower and upper bound end points. The characteristics of the distribution will also be supplied to you when you sign on to the VASE system (if you are not sure about what this means, read the explanatory document “Understanding The Experiment & Terminology” that was given to you). When you enter the electronic auction market, you will be informed of the number of units of the item on sale. You will then bid according to the rules of the auction (refer to “Section 2: Vickrey Auctions” in the document “Understanding The Experiment &

Terminology” that was given to you). You will be informed of the results of the auction when the auction is over.

3 Experimental Subjects

We conducted experiments for a total of 38 sessions over 2002 – 2004. The experimental subjects were drawn from the following:

1. MBA students that were enrolled in OM / IS electives at a north eastern university in the United States.
2. MBA students enrolled in an IS / OM elective at a leading business management school in India.
3. Undergraduate students that were enrolled in OM / IS electives at a north eastern university in the United States.
4. Middle managers that were enrolled in Executive Education programs at a north eastern university in the United States.
5. Senior executives enrolled in Executive Education programs at a north eastern university in the United States.
6. Senior executives enrolled in Executive Education programs at a leading business management school in India.
7. Senior executives from two corporations in Thailand enrolled in Executive Education programs run by a north eastern university.

4 Incentives

The incentives used in the experiments across the three different types of experimental subjects were very similar. Predominantly cash-based incentives were used for all three groups of subjects. The incentives used in the experiments are described below.

4.1 Executives:

1. For participants in executive education programs both direct cash and lottery-based incentives were used. The predominant incentive used was a cash-based system. In the case of cash incentives, the actual surplus of each of the participants was calculated. This was scaled to result in a cash award using a scale factor that converted experimental surplus to cash. For instance, a surplus of 100 experimental dollars (see the above statement for how the surplus would be calculated) would result in a cash award of USD[2] $10.00 (using a scale factor of 10 to 1).

2. In the case of lottery-based awards, participants’ final surplus was calculated and the points were placed in a lottery. The chances of a participant being selected for an award was directly proportional to her final surplus. For instance, a participants

[2] Or appropriate amounts in equivalent currencies where applicable.

with 100 units of experimental surplus would 10 times more likely to be selected as a lottery winner as a participant with 10 units of surplus.

4.2 MBA Students:

1. Both cash-based and non-cash based incentives were used. The predominant incentive used was a cash-based system. Cash-based incentives were as described above.
2. Non-Cash based incentives: some of the experimental subjects participated in the experiment as a part of an assignment[3]. Their experimental surplus was tallied and grade points were awarded in proportion to their surplus.

4.3 Undergraduate Students:

1. Both cash-based and non-cash based incentives were used. The predominant incentive used was a cash-based system. Cash-based incentives were as described above.
2. Non cash-based incentives were either lottery-based incentives as described above or were part of an assignment for which students received grade points on completion of the exercise (also described above).

5 Parameterization

Subjects were allowed to participate in a 'dummy auction' (dry run) to familiarize themselves with the VASE system and the user interface.

The auction parameters that defined each auction were the following:

1. Number of bidders in each auction. This corresponds to the number of buyers given by N in the model.
2. The mid point of the interval from which bidder's valuations were drawn[4]. This corresponds to R in the model.
3. The variance of the distribution of buyer valuations which in our model is given by $\frac{s^2}{3}$. Thus determining s and R results in determining the distribution of buyer valuations given by $U[R-s,R+s]$ and the variance of the distribution of buyer valuations given by $\frac{s^2}{3}$.
4. The number of units of the good available for sale. Bidders were informed when they signed on to the auction interface as to how many units of the good were on sale.

[3] The experiment was one out of the three or four auction-based exercises that constituted the assignment.

[4] Note that $F(\cdot)$ of buyer valuations are specified to the uniform distribution $U[R-s,R+s]$, where $R \geq s > 0$, which has a mean of *R*.

These experimental parameters can be found in columns 2 through 5 of Table 2 for each of the 38 experiments.

6 Experimental Procedure

The typical experimental procedure is described as below.

1. Subject signs consent form.
2. Subject is randomly assigned to an experimental auction.
3. Subject views an experimental charter describing the rules of the auction and the bidding procedures.
4. Subjects do a dry run to familiarize themselves with the auction interface.
5. Subject reads a set of instructions (cost of production, price in the end market, etc. as described above).
6. Subject signs on to the electronic auction platform.
7. Subject participates in the experiment until it ends (auction terminates).
8. Subject is informed of the outcome of the experiments (results of the auction).
9. Subject is informed of the experimental surplus.
10. Subject is paid (cash incentives) or is entered in a lottery (lottery-based incentives) or is informed of the grade points that she earned (where the subject participates in the auction as a part of an assignment).

7 Experimental Data

Table 2 (below) lists the parameter settings for each experiment as well as the raw data (output) collected: Columns 2 through 4 of Table 2 provide the number of buyers in each market (*n*), the mid point of the support (*R*) and the span of the support (*s*). Column 5 shows the pre-aggregation revenues (i.e., the revenues under the Decentralized regime). Column 6 is the optimal quantity released in the auction market under Aggregation. Column 7 is the theoretical predicted revenue under aggregation, column 8 the actual revenue realized under aggregation in the experiment, column 9 the difference between the theoretical and actual (experimental) revenues under aggregation, and column 10 the difference between revenues under the Decentralized Regime and the actual revenue under aggregation.

Table 2: Observed Revenue Outcomes in B2B Markets: Experimental Data

Expt. # (1)	N (2)	R (3)	S (4)	Decentralized Revenue: Benchmark (5)	Aggregation: Optimal Auction Quantity (6)	Aggregate Revenue: Theoretical (7)	Aggregate Revenue: Actual (8)	Diff Rev: Theoretical – Actual (9)= (7) – (8)	Diff Rev: Benchmark - Actual (10) = (5) – (8)
1	25	100	80	$1,266	14	$1,217	$1,144	$73	$122
2	12	200	80	$1,470	11	$1,472	$1,448	$25	$22
3	40	50	50	$1,000	20	$976	$902	$73	$98
4	16	150	125	$1,210	9	$1,171	$1,127	$44	$83
5	10	500	225	$2,920	8	$2,736	$2,736	$0	$184
6	19	30	25	$287	11	$289	$275	$14	$12
7	20	1000	400	$12,250	18	$12,257	$11,644	$613	$606
8	5	2000	1600	$5,063	3	$4,600	$5,060	-$460	$3
9	21	40	36	$421	11	$400	$379	$21	$42
10	10	100	50	$563	8	$564	$564	$0	-$1
11	24	120	50	$1,734	21	$1,743	$1,641	$102	$93
12	25	300	150	$4,219	19	$4,165	$3,915	$250	$303
13	30	700	300	$12,500	25	$12,258	$11,441	$817	$1,059
14	26	1400	1400	$18,200	13	$17,526	$16,447	$1,079	$1,753
15	27	1600	700	$25,505	23	$25,875	$24,246	$1,629	$1,260
16	28	2000	800	$34,300	25	$34,310	$32,105	$2,206	$2,195
17	29	3600	2000	$56,840	21	$57,400	$53,639	$3,761	$3,201
18	5	3500	1800	$9,753	4	$9,400	$10,340	-$940	-$587
19	30	4000	1600	$73,500	27	$74,206	$69,259	$4,947	$4,241
20	10	3200	1400	$18,893	9	$19,555	$19,555	$0	-$662
21	33	3000	2900	$49,514	17	$48,700	$45,306	$3,394	$4,208
22	34	5000	1775	$109,903	33	$110,114	$102,341	$7,773	$7,562
23	20	5000	4000	$50,625	11	$47,405	$45,035	$2,370	$5,590
24	37	10000	3600	$237,622	35	$234,684	$217,559	$17,126	$20,064
25	15	10000	10000	$75,000	8	$75,000	$72,500	$2,500	$2,500
26	40	10000	5000	$225,000	30	$221,341	$204,741	$16,601	$20,259
27	14	9000	3500	$78,125	13	$78,217	$75,982	$2,235	$2,143
28	41	7000	2500	$185,013	39	$182,929	$169,097	$13,831	$15,915
29	13	7000	7000	$45,500	7	$45,500	$44,450	$1,050	$1,050
30	42	7000	3000	$175,000	35	$172,558	$159,411	$13,147	$15,589
31	20	6000	6000	$60,000	10	$57,143	$54,286	$2,857	$5,714
32	45	6000	2500	$162,563	39	$163,630	$150,904	$12,727	$11,659
33	5	5000	5000	$12,500	3	$12,500	$13,750	-$1,250	-$1,250
34	46	6000	2100	$179,646	45	$180,239	$166,134	$14,106	$13,513
35	47	5000	1750	$152,960	46	$153,573	$141,483	$12,090	$11,477
36	8	2000	2000	$8,000	4	$7,111	$7,289	-$178	$711
37	49	400	400	$9,800	25	$9,800	$9,020	$780	$780
38	9	350	150	$1,875	8	$1,880	$1,901	-$21	-$26