TECHNICAL APPENDIX TO: STRATEGIC INFORMATION MANAGEMENT UNDER LEAKAGE IN A SUPPLY CHAIN

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$\tilde{A}$	random demand intercept.
w	wholesale price.
A_H	a 'high' realization of $\tilde{A}$.
A_L	a 'low' realization of $\tilde{A}$.
p	Probability of realizing A_H.
μ	Mean demand $= pA_H + (1-p)\,A_L$
θ	$(A_H - w)\,/\,(A_L - w)$
q_{iH}^*	High type incumbent's order quantity under the separating equilibrium.
q_{iL}^*	Low type incumbent's order quantity under the separating equilibrium.
q_{eH}^*	Entrant's order quantity under the separating equilibrium when demand is high.
q_{eL}^*	Entrant's order quantity under the separating equilibrium when demand is low.
q_{ip}^*	Incumbent's order quantity under a Pareto-dominant pooling equilibrium.
q_{ep}^*	Entrant's order quantity under a Pareto-dominant pooling equilibrium.
$\bar{\Theta}\,(p)$	Threshold on θ below which a pooling equilibrium exists.
$\bar{\bar{\Theta}}\,(p)$	Threshold on θ, under the composite belief structure, below which pooling is an equilibrium, and above which separation is an equilibrium.
$\Theta^*\,(p)$	Threshold on θ above which information is acquired (under IAG)

The Technical Appendix is divided into five Sections. Section 1 is the 'preliminaries': In specific, it details the extensive form of a signaling game. In Section 2, we prove that the supplier leaks. This establishes that the information exchange, or the signaling, is in effect between the incumbent and the entrant. In Section 3 we prove Propositions 1,2 and 3 of the Information Dissemination Game (IDG). We also show that the composite equilibrium of Proposition 3 satisfies the Intuitive Criterion of Cho and Kreps (1987). In Section 4 we prove Proposition 4 of the Information Acquisition Game (IAG). Finally in Section 5, we delineate the mathematical details of Section 6 of the paper ('Exclusive sourcing strategies to counter leakage').

Unless otherwise stated, all references to sections, equations and figures correspond to this document.

1 Preliminaries

As a first step, for future reference, it is useful to summarize the outcomes of a sequential move (Stackelberg) game and simultaneous move (Cournot) game with complete information for the demand system used in the paper: $P(Q) = (A - Q)$.

Lemma 1 *(i) In a sequential move (Stackelberg) game with complete information with a wholesale price of w, the subgame-perfect Nash equilibrium outcomes in terms of the order quantities (q) and the profits (π) of the Leader (incumbent) and the Follower (entrant) are (the subscript i refers to the incumbent and e to the entrant, while the subscript S refers to Stackelberg):*

$$\begin{aligned} q_{iS}^{*} &= \frac{(A-w)}{2}; q_{eS}^{*} = \frac{(A-w)}{4} \\ \pi_{iS} &= \frac{(A-w)^2}{8}; \pi_{eS} = \frac{(A-w)^2}{16} \end{aligned}$$

(ii) In a simultaneous move (Cournot) game with complete information with a wholesale price of w, the subgame-perfect Nash equilibrium outcomes in terms of the order quantities (q) and the profits (π) of the incumbent and the entrant are (the subscript i refers to the incumbent and e to the entrant, while the subscript C refers to Cournot):

$$\begin{aligned} q_{iC}^{*} &= q_{eC}^{*} = \frac{(A-w)}{3} \\ \pi_{iC} &= \pi_{eC} = \frac{(A-w)^2}{9} \end{aligned}$$

Proof. (i) If the incumbent puts in q_i, the entrant solves

$$\begin{aligned} q_{eS}^{*}(q_{iS}) &= \arg\max_{q_{eS}} \left((A - q_{iS} - q_{eS}) q_{eS} - w q_{eS} \right) \\ \Rightarrow \quad q_{eS}^{*}(q_{iS}) &= \frac{(A-w) - q_{iS}}{2} \end{aligned} \tag{1}$$

The incumbent in turn solves for

$$\begin{aligned} q_{iS}^{*} &= \arg\max_{q_{iS}} \left((A - q_{iS} - q_{eS}^{*}(q_{iS})) q_{iS} - w q_{iS} \right) \\ \Rightarrow \quad q_{iS}^{*} &= \frac{(A-w)}{2}, \end{aligned}$$

and the results follow.

(ii) The incumbent and the entrant correspondingly solve the following simultaneous move game of complete information.

$$\begin{aligned} \pi_{iC} &= \max_{q_{iC}} (A - q_{iC} - q_{eC}) q_{iC} - w q_{iC} \\ \pi_{eC} &= \max_{q_{eC}} (A - q_{iC} - q_{eC}) q_{eC} - w q_{eC} \end{aligned}$$

The solution to the first-order conditions (these are also sufficient) gives the order quantities and profits as noted. ■

1.1 Extensive form representation of a signaling game

In a signaling game, let T represent the *type* space for the 'Sender', with $t \in T$. The Sender sends a message m from a set $M(t)$ and the set of types that have message m available is $T(m)$. The 'Receiver', having seen the message, chooses a response r from a set $R(m)$. The notation translates to our case as follows: the Sender is our incumbent with type space, $T = \{High, Low\}$, $m = q_i \in [0, \infty) = M(t)$. The Receiver is our entrant with $r = q_e \in [0, \infty) = R(m)$.

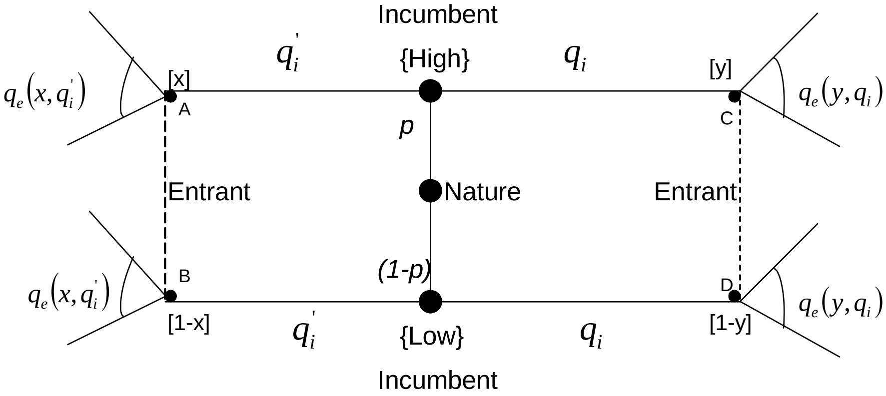

Figure A.1: Extensive form representation of the signaling game

Figure A.1 shows an extensive form of the signaling game between the incumbent and the entrant. For brevity, the final payoffs and the role of the supplier are suppressed. Moreover, the figure is drawn assuming that the supplier always leaks the incumbent's order quantity to the entrant, as proved in the next section. Consistent with such games of incomplete information, 'Nature' moves first and draws a 'type' for the incumbent: In our context Nature draws a high type incumbent with probability p and low type with probability $(1 - p)$. The incumbent's strategy space, contingent on the move by Nature, is to order a quantity $q_i \in [0, \infty)$. For expositional simplicity, we show only two possible quantities, q_i and $q_i^{'}$, for the incumbent in Figure A.1. The entrant's strategy space is his order quantity $q_e \in [0, \infty)$, which is a function of both the incumbent's order quantity and the entrant's belief about the incumbent's type based on the incumbent's observed order quantity. These beliefs are simplistically represented as node probabilities [x], [1-x], [y] and [1-y] in Figure A.1. For instance, when the entrant learns that the incumbent's order quantity is $q_i^{'}$, he is unsure whether is he is on node A or node B. He thus assigns a probability of [x] to being on node A (i.e., to demand being high) and a probability of [1-x] to being on node B (i.e., to demand being low).

Unless otherwise stated, without loss of generality, we normalize $w = 0$ in the proofs below. For non-zero w, we can interpret θ as $(A_H - w) / (A_L - w)$ without changing the proofs or the results.

2 Establishing that the supplier leaks.

We accomplish this with the help of Lemmas 2 and 3, and Theorems 1 and 2.

Lemma 2 *If the supplier could credibly commit not to leak before the incumbent places his order with the supplier, the order quantities of the high type incumbent, the low type incumbent and the entrant correspondingly are,*

$$\begin{aligned} q_{iH}^{N} &= \frac{(3(A_H - w) - (\mu - w))}{6} \\ q_{iL}^{N} &= \frac{(3(A_L - w) - (\mu - w))}{6} \\ q_{e}^{N} &= \frac{(\mu - w)}{3} \end{aligned} \tag{2}$$

Proof. If, before the incumbent places his order quantity with the supplier, the supplier credibly commits not to leak, then the game is *akin* to a simultaneous move game, even though the incumbent and the entrant move sequentially (*cf* Gibbons (1992), Page 4). Hence, the high type incumbent, the low type incumbent and the entrant correspondingly solve the following simultaneous move (static) game of incomplete information.

$$\begin{aligned} &\pi_{iH} = \max_{q_{iH}} (A_H - q_{iH} - q_e) q_{iH} - w q_{iH} \\ &\pi_{iL} = \max_{q_{iL}} (A_L - q_{iL} - q_e) q_{iL} - w q_{iL} \\ &\pi_e = \max_{q_e} (p (A_H - q_{iH} - q_e) q_e + (1-p)(A_L - q_{iL} - q_e) q_e) - w q_e \end{aligned}$$

where q_{iH} is the order quantity of the high type incumbent, q_{iL} is the order quantity of the low type incumbent and q_e is the order quantity of the entrant.

The solution to the first-order conditions (these are also sufficient) gives the order quantities as noted. ■

The next Lemma derives the necessary and sufficient condition on the incumbent's order quantity for the supplier to leak (or equivalently, to not leak). It then shows that the incumbent will never order so as to meet the conditions for non-leakage. Thus for any feasible order quantity by the incumbent, the supplier leaks.

Lemma 3 *(i) If the high type orders $\tilde{q}_{iH}$ and the low type orders $\tilde{q}_{iL}$, the supplier does not leak <u>if and only if</u>*

$$\begin{aligned} \tilde{q}_{iH} &< \frac{(\mu - w)}{3}, \text{ and} \\ \tilde{q}_{iL} &< \frac{(\mu - w)}{3} \end{aligned} \tag{3}$$

(ii) The incumbent will never order $\tilde{q}_{iH} < \dfrac{(\mu - w)}{3}$.

Proof. (*i*) Suppose the incumbent orders *distinct* order quantities, $\tilde{q}_{iH}$ and $\tilde{q}_{iL}$, correspondingly in the high and low demand states (quantities in (2) would be a special case of this). For the supplier to not leak, his *ex post* profits in *both* demand states with no leakage ought to be higher than with leakage. Suppose not. Then the supplier leaks in one demand state (say, high) and not in the other (say, low). But then no leakage by the supplier is correctly inferred as realization of a low demand state by the entrant.

If the supplier leaks, the entrant infers the demand state and the standard Stackelberg game comes into play via supplier leakage: the entrant orders $((A_H - w) - \tilde{q}_{iH})/2$ when demand is high and $((A_L - w) - \tilde{q}_{iL})/2$ when demand is low (from equation (1)). Hence for the supplier to not leak, the following inequalities should hold *simultaneously*,

$$\begin{aligned} w\left(\tilde{q}_{iH} + \frac{(A_H - w) - \tilde{q}_{iH}}{2}\right) &< w\left(\frac{(3(A_H - w) - (\mu - w))}{6} + \frac{(\mu - w)}{3}\right), \text{ and} \\ w\left(\tilde{q}_{iL} + \frac{(A_L - w) - \tilde{q}_{iL}}{2}\right) &< w\left(\frac{(3(A_L - w) - (\mu - w))}{6} + \frac{(\mu - w)}{3}\right), \end{aligned}$$

where the RHS of the inequalities is the supplier profit under no leakage – the sum of the order quantities of the incumbent and the entrant from (2) multiplied by the wholesale price, and the LHS is the supplier profit under the Stackelberg game with leakage. This simplifies to

$$\begin{aligned} \left(\frac{(A_H - w) + \tilde{q}_{iH}}{2}\right) &< \left(\frac{3(A_H - w) + (\mu - w)}{6}\right), \text{ and} \\ \left(\frac{(A_L - w) + \tilde{q}_{iL}}{2}\right) &< \left(\frac{3(A_L - w) + (\mu - w)}{6}\right) \end{aligned}$$

which further simplifies to

$$\begin{aligned} \tilde{q}_{iH} &< \frac{(\mu - w)}{3}, \text{ and} \\ \tilde{q}_{iL} &< \frac{(\mu - w)}{3} \end{aligned}$$

(*ii*) To prevent leakage, the incumbent must order a quantity lower than $(\mu - w)/3$ in the high demand state. However, if the supplier leaks, the incumbent orders $((A_H - w)/2) > (\mu - w)/3$ (from Lemma 1(i)), which is more profitable than ordering $\tilde{q}_{iH} < (\mu - w)/3$ and preventing leakage. ■

The Theorems below integrate the previous results to show that the supplier leaks in equilibrium.

Theorem 1 *Any ex-ante commitment by the supplier to not leak does not hold in equilibrium.*

Proof. If the incumbent, believing the supplier's commitment to not leak, orders as per equations (2) in Lemma 2, then the supplier leaks. This follows from Lemmas 2, 3 and from the fact that $(3(A_H - w) + (\mu - w))/6 > (\mu - w)/3$. ■

Theorem 2 *The supplier always leaks in equilibrium.*

Proof. Since the supplier's commitment to not leak as per Theorem 1 is not credible, the incumbent assumes that the supplier leaks and orders $((A_H - w)/2)$ when demand is high and $((A_L - w)/2)$ when demand is low (Lemma 1(i)). Given these order quantities, the supplier indeed leaks as per Lemma 3(i), which provides the necessary and sufficient conditions for no-leakage. ■

3 The information dissemination game (IDG).

3.1 The Separating Equilibrium

Proposition 1 *A separating PBNE exists for all θ, and is as follows:*

(i) The incumbent orders:

$$\begin{aligned} q_{iH}^{*} &= \left(A_H/2\right), \text{ if demand is high, and} \\ q_{iL}^{*} &= \begin{cases} \left(A_L/2\right), & \text{if demand is low and } \theta \geq 3; \\ \dfrac{2A_H - A_L - \sqrt{3A_H^2 - 4A_H A_L + A_L^2}}{2}, & \text{if demand is low and } \theta < 3. \end{cases} \end{aligned}$$

(ii) The supplier always leaks.

(iii) The entrant orders

$$\begin{aligned} q_{eH}^{*} &= \left(A_H/4\right), \text{ if } Pr_e\left(\tilde{A} = A_H\right) = 1, \text{ and} \\ q_{eL}^{*} &= \begin{cases} A_L/4 & \text{if } \theta \geq 3 \text{ and } Pr_e\left(\tilde{A} = A_H\right) = 0; \\ \dfrac{3A_L - 2A_H + \sqrt{\left(A_H - A_L\right)\left(3A_H - A_L\right)}}{4} & \text{if } \theta < 3 \text{ and } Pr_e\left(\tilde{A} = A_H\right) = 0, \end{cases} \end{aligned}$$

consistent with his beliefs that:

$$Pr_e\left(\tilde{A} = A_H\right) = \begin{cases} 0 \text{ if the supplier leaks } \underline{\text{and}} \; q_i \leq q_{iL}^{*}, \text{ OR the supplier does not leak;} \\ 1, \text{ otherwise.} \end{cases} \tag{4}$$

(iv) The profits of the incumbent and the entrant are as follows:

(a) The incumbent's earns a profit of

$$\begin{aligned} \pi_{iH} &= A_H^2/8, \text{ if demand is high, and} \\ \pi_{iL} &= \begin{cases} \left(A_L^2/8\right), & \text{if demand is low and } \theta \geq 3; \\ \frac{\left(-4A_L^2 + 12A_H A_L - 7A_H^2 + 4(A_H - A_L)\sqrt{(A_H - A_L)(3A_H - A_L)}\right)}{8}, & \text{if demand is low and } \theta < 3. \end{cases} \end{aligned}$$

(b) The entrant earns a profit of

$$\begin{aligned} \pi_{eH} &= \left(A_H^2/16\right), \text{ if } Pr_e\left(\tilde{A} = A_H\right) = 1, \text{ and} \\ \pi_{eL} &= \begin{cases} A_L^2/16 & \text{if } \theta \geq 3 \text{ and } Pr_e\left(\tilde{A} = A_H\right) = 0; \\ \left(\dfrac{3A_L - 2A_H + \sqrt{\left(A_H - A_L\right)\left(3A_H - A_L\right)}}{4}\right)^2 & \text{if } \theta < 3 \text{ and } Pr_e\left(\tilde{A} = A_H\right) = 0. \end{cases} \end{aligned}$$

3.1.1 Extensive form representation of the separating equilibrium

The separating equilibrium, on the equilibrium path, is shown in Figure A.2. Recall the discussion in Section 1.1 on node probabilities. In the separating equilibrium, the entrant assigns a probability of [0] to being on node A upon observing the incumbent's quantity of q_{iL}^{*} (and consequently a probability of

[1] to being on node B). Similarly, he assigns a probability of [1] to being on node C and a probability of [0] to being on node D upon observing the incumbent's order quantity of q^*_{iH}. Hence, in the separating equilibrium, the entrant (like the incumbent) has perfect demand information when he decides his own order quantities: he knows that demand is low when he sees q^*_{iL}, and similarly he knows that demand is high when he sees q^*_{iH}.

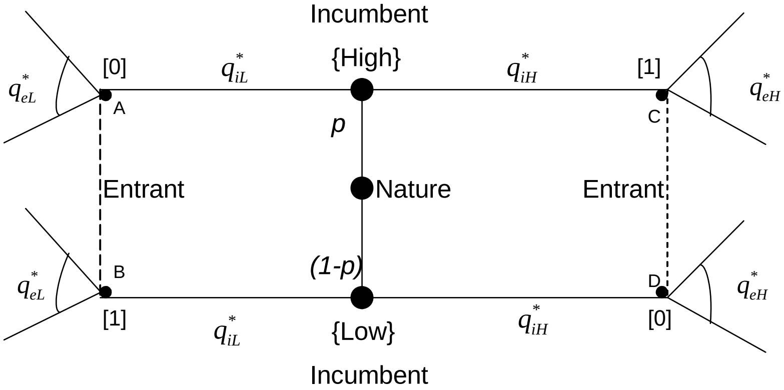

Figure A.2. The Separating Equilibrium of Proposition 1.

3.1.2 Proof of the separating equilibrium

For a pure strategy separating equilibrium to exist, q^*_{iL} and q^*_{iH} must emerge as a simultaneous solution to the following constrained optimization program:

$$\pi_{iL} = \max_{q_{iL}} \left(A_L - q_{iL} - q^*_{eL}\left(q_{iL}\right)\right) q_{iL} \tag{5}$$

$$\pi_{iH} = \max_{q_{iH}} \left(A_H - q_{iH} - q^*_{eH}\left(q_{iH}\right)\right) q_{iH} \tag{6}$$

subject to :

$$\left(A_L - q_{iH} - q^*_{eH}\left(q_{iH}\right)\right) q_{iH} \leq \left(A_L - q^*_{iL} - q^*_{eL}\left(q^*_{iL}\right)\right) q^*_{iL} \tag{7}$$

$$\left(A_H - q_{iL} - q^*_{eL}\left(q_{iL}\right)\right) q_{iL} \leq \left(A_H - q^*_{iH} - q^*_{eH}\left(q^*_{iH}\right)\right) q^*_{iH} \tag{8}$$

where

$$q^*_{eL}\left(q_{iL}\right) = \arg\max_{q_{eL}} \left(A_L - q_{iL} - q_{eL}\right) q_{eL} = \left(\frac{A_L - q_{iL}}{2}\right) \tag{9}$$

$$q^*_{eH}\left(q_{iH}\right) = \arg\max_{q_{eH}} \left(A_H - q_{iH} - q_{eH}\right) q_{eH} = \left(\frac{A_H - q_{iH}}{2}\right) \tag{10}$$

Inequalities (7) and (8) are the incentive compatibility constraints for a separating equilibrium that ensure that each type of incumbent prefers not to mimic the other; the RHS of these inequalities are the equilibrium profits which are greater then the LHS off-equilibrium profits.

The above formulation embeds the notion of a Perfect Bayesian Nash Equilibrium in the following sense: *(i)* The incumbent's actions (the optimal order quantities derived from (5) and (6)) are a best response to what he knows at that point (the realized demand state), what the entrant optimizes and to his own conjecture on the entrant's beliefs. *(ii)* The entrant's optimization in (9) and (10) is in turn a best response to what he knows at that stage (the incumbent's order quantity) and his beliefs on the actual demand state given by (4). *(iii)* The entrant's actual beliefs and the incumbent's conjectures on the entrant's beliefs coincide.

Constraint (7) does not bind, and since only the high type incumbent has incentives to mimic the low type, the optimization program for the low type reduces to

$$\pi_{iL} = \max_{q_{iL}} \left(\frac{A_L - q_{iL}}{2} \right) q_{iL} \tag{11}$$

$$s.t$$

$$\left(A_H - \frac{A_L}{2} - \frac{q_{iL}}{2} \right) q_{iL} \leq \frac{A_H^2}{8} \tag{12}$$

The Lagrangian for the above formulation is

$$\pounds\left(q_{iL}, u\right) = \max_{q_{iL}} \left(\left(\frac{A_L - q_{iL}}{2} \right) q_{iL} - u \left(\left(A_H - \frac{A_L}{2} - \frac{q_{iL}}{2} \right) q_{iL} - \frac{A_H^2}{8} \right) \right)$$

The first-order Karush-Kuhn-Tucker (KKT) conditions for the Lagrangian are:

$$\begin{aligned}
&(1a) : \frac{\partial \pounds\left(q_{iL}, u\right)}{\partial q_{iL}} \leq 0 \Rightarrow \frac{A_L - 2q_{iL}}{2} - u \left(A_H - \frac{A_L}{2} - q_{iL} \right) + v_1 = 0, \text{ and} \\
&(1b) : q_{iL} \frac{\partial \pounds\left(q_{iL}, u\right)}{\partial q_{iL}} = 0 \text{ by complementary slackness.} \\
&(2a) : \frac{\partial \pounds\left(q_{iL}, u\right)}{\partial u} \geq 0 \Rightarrow \left(A_H - \frac{A_L}{2} - \frac{q_{iL}}{2} \right) q_{iL} + v_2 = \frac{A_H^2}{8}, \text{ and} \\
&(2b) : u \frac{\partial \pounds\left(q_{iL}, u\right)}{\partial u} = 0 \text{ by complementary slackness.}
\end{aligned} \tag{13}$$

Solving the above system we get: For $u = 0$, (unconstrained optimal)

$$q_{iL}^* = \left(A_L/2\right),$$

$$v_2 \geq 0 \Rightarrow A_H \geq 3A_L \tag{14}$$

For $u > 0$, we have

$$q_{iL}^* = \frac{2A_H - A_L \pm \sqrt{3A_H^2 - 4A_H A_L + A_L^2}}{2} \tag{15}$$

Call the lower root q_{Li}^1 and the upper root q_{Li}^2. We know that the unconstrained maximum of equation (11) is at $\left(A_L/2\right)$, while the maxima for the LHS of the constraint (12) is at $\left(A_H - A_L/2\right)$, where $\left(A_H - A_L/2\right) > \left(A_L/2\right)$. Secondly, it is a matter of straightforward algebra to show that $q_{Li}^1 < \left(A_L/2\right) <$

$(A_H - A_L/2) < q_{Li}^2$. If the constraint (12) binds at $q_{iL}^* = (A_L/2)$, then any $q_{iL} \in [A_L/2, (A_H - A_L/2)]$ does not permit a separating equilibrium since the LHS of (12) is increasing in this interval. Hence, in order to separate out, the low type must order a quantity lower than $(A_L/2)$ to prevent the high type from mimicking him, and we discard the upper root. Therefore as per (14), if $\theta \geq 3$, $q_{iL}^* = (A_L/2)$ and if $\theta < 3$, q_{iL}^* is given by the lower root of (15). Algebraic manipulation using equation (9) results in the entrant's order quantity as shown in Proposition 1.

The last thing that we need to check is whether the low type's strategy is susceptible to a 'one-shot-deviation'. If it is too costly for the low type to separate and order q_{iL}^* given by the lower root of (15), then the low type incumbent puts a quantity $\tilde{q}_{iL}^* > q_{iL}^*$ such that

$$\begin{aligned}
\tilde{q}_{iL}^* &= \arg\max_{\tilde{q}_{iL} > q_{iL}^*} \left(A_L - \tilde{q}_{iL} - \tilde{q}_{eL}^*\left(\tilde{q}_{iL}\right)\right)\tilde{q}_{iL} = \left(A_L - \frac{A_H}{2}\right)^+ \text{ where} \\
\tilde{q}_{eL}^*\left(\tilde{q}_{iL}\right) &= \arg\max_{\tilde{q}_{eL}} \left(A_H - \tilde{q}_{iL} - \tilde{q}_{eL}\right)\tilde{q}_{eL} = \frac{A_H - \tilde{q}_{iL}}{2},
\end{aligned}$$

the entrant's response being determined by the belief structure in (4). Hence, for the low type, the maximum off-equilibrium profit is given by

$$\tilde{\pi}_{iL} = \begin{cases} 0 \text{ if } \theta \geq 2 \\ \frac{1}{2}\left(A_L - \frac{A_H}{2}\right)^2 \text{ otherwise} \end{cases}$$

For $\theta \geq 2$, the off-equilibrium profit is zero, hence the low type always separates. For $\theta < 2$, the profit off the equilibrium path ought to be lower than the profit on the equilibrium path for the equilibrium to hold. This implies that

$$\pi_{iL} \geq \tilde{\pi}_{iL} \text{ for } \theta < 2$$

This leads us to the following inequality (the LHS below is the profit of the low type for $\theta < 2$, which can be obtained from q_{iL}^* and q_{eL}^* by straightforward algebraic manipulations, and which is summarized Proposition 1, (iv)):

$$\begin{aligned}
\frac{A_L^2\left(-4 + 12\theta - 7\theta^2 + 4\left(\theta - 1\right)\sqrt{\left(\theta - 1\right)\left(3\theta - 1\right)}\right)}{8} &\geq \frac{A_L^2}{2}\left(1 - \frac{\theta}{2}\right)^2 \\
\left(\theta - 1\right)\left(2 - 2\theta + \sqrt{\left(\theta - 1\right)\left(3\theta - 1\right)}\right) &\geq 0
\end{aligned}$$

Since $\theta > 1$, it is sufficient to verify that

$$\left(2 - 2\theta + \sqrt{\left(\theta - 1\right)\left(3\theta - 1\right)}\right) \geq 0$$

The above inequality holds for all $\theta < 3$. Hence, the low type always separates out and no one-shot-deviation is profitable.

The high type incumbent always orders $A_H/2$ $\forall\theta$ since the low type successfully separates out. The supplier always leaks as per Theorem 2.

The profit expressions for part (iv) can be obtained from the equilibrium order quantities after some tedious algebra.

3.2 The Pooling Equilibrium

Proposition 2 *I. When* $\theta \leq \bar{\Theta}(p) = \frac{3+2p-p^2}{1+4p-p^2}$, *a* Pareto-dominant *pooling equilibrium exists and is as follows:*

(i) The incumbent orders $q_{ip}^{*} = A_L - \dfrac{\mu}{2}$.

(ii) The supplier always leaks.

(iii) The entrant orders $q_{ep}^{*} = \dfrac{3\mu - 2A_L}{4}$,
consistent with his beliefs that:

$$Pr_e(\tilde{A} = A_H) = \begin{cases} 0, & \text{if the supplier leaks } \underline{\text{and}}\ q_i < \underline{q}_p \text{, OR the supplier does not leak;} \\ p, & \text{if the supplier leaks } \underline{\text{and}}\ \underline{q}_p \leq q_i \leq q_{ip}^{*}\text{;} \\ 1, & \text{otherwise.} \end{cases} \tag{16}$$

where $\underline{q}_p = \left(A_H - \frac{\mu}{2} - \frac{1}{2}\sqrt{(A_H - \mu)(3A_H - \mu)}\right)$.

(iv) *The profits of the incumbent and the entrant are summarized below:*

(a) *The incumbent earns a profit of*

$$\begin{aligned} \pi_{ipH} &= \left(A_H - \frac{\mu}{4} - \frac{A_L}{2}\right)\left(A_L - \frac{\mu}{2}\right) \text{ when demand is high, and} \\ \pi_{ipL} &= \frac{1}{2}\left(A_L - \frac{\mu}{2}\right)^2 \text{ when demand is low.} \end{aligned}$$

(b) *The entrant earns an expected profit of*

$$\pi_{ep} = \left(\frac{3\mu - 2A_L}{4}\right)^2$$

II. When $\theta > \bar{\Theta}(p)$, *there can be no pooling equilibrium.*

3.2.1 Extensive form representation of the pooling equilibrium

The pooling equilibrium is shown in extensive form in Figure A.3. For brevity, only two possible pooling quantities, q_{ip}^{*} and $\underline{q}_p$, are shown (these are the extreme points in the range of the incumbent's order quantities for which the entrant sticks to his priors). For instance, having observed the incumbent's order quantity of $\underline{q}_p$, the entrant is unable to derive any inference as to which node he is on, A or B. Hence he sticks to his priors, thereby assigning a probability of p to being on node A (i.e., to demand being high) and a probability of $(1-p)$ to being on node B (i.e., to demand being low). Similarly reasoning holds when the entrant observes q_{ip}^{*}, the uncertainty now pertains to nodes C and D.

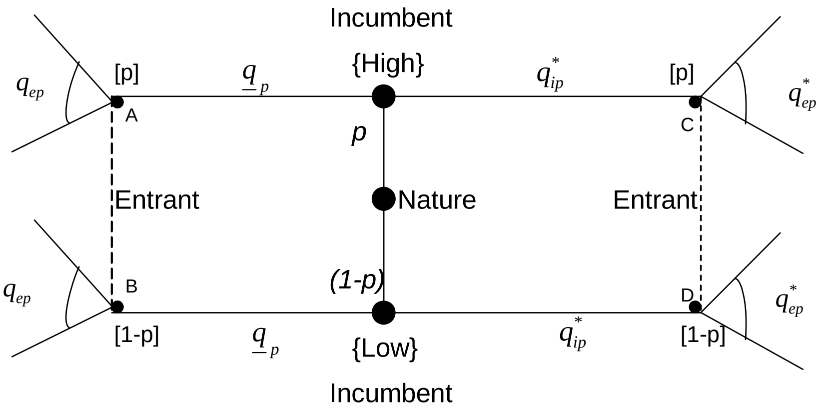

Figure A.3. The Pooling Equilibrium.

3.2.2 Proof of the pooling equilibrium

The proof is through a series of Lemmas (In the proofs, q_{ip} denotes a candidate pooling quantity for the incumbent).

Lemma 4 *The maximum quantity where a pooling equilibrium can be sustained is*

$$q^*_{ip} = \left(A_L - \frac{\mu}{2}\right)^+ \tag{17}$$

Proof. We first establish the optimal preferred pooling quantities, q^{*H}_{ip} and q^{*L}_{ip}, for the high and low types respectively. Suppose $\exists\ q_{ip}$ such that on observing this quantity, the entrant is unable to tease the demand state and sticks to his priors. Hence, the entrant releases q^*_{ep} where

$$q^*_{ep}\left(q_{ip}\right) = \arg\max_{q_{ep}} \left(p\left(A_H - q_{ip} - q_{ep}\right)q_{ep} + (1-p)\left(A_L - q_{ip} - q_{ep}\right)q_{ep}\right) = \frac{\mu - q_{ip}}{2} \tag{18}$$

Now the high type incumbent's maximizes

$$\pi^p_{iH} = \max_{q_{ip}} \Big(A_H - q_{ip} - q^*_{ep}\left(q_{ip}\right)\Big)\, q_{ip} \tag{19}$$

while the low type incumbent maximizes

$$\pi^p_{iL} = \max_{q_{ip}} \Big(A_L - q_{ip} - q^*_{ep}\left(q_{ip}\right)\Big)\, q_{ip} \tag{20}$$

The required optimal quantities for the low type and the high type are thus obtained from (18), (19) and (20), and are as follows:

$$q^{*H}_{ip} = \left(A_H - \frac{\mu}{2}\right); q^{*L}_{ip} = \left(A_L - \frac{\mu}{2}\right)^+$$

We now show that any candidate quantity q_{ip} for a pooling equilibrium must satisfy:

$q_{ip} \leq \min\left(q_{ip}^{*L}, q_{ip}^{*H}\right) = q_{ip}^{*L} = \left(A_L - \frac{\mu}{2}\right)^+ = q_{ip}^*$. The low type would never agree to pool at a quantity greater than q_{ip}^{*L} for any reasonable belief structure (where the probability the entrant ascribes to high demand is non-decreasing in the incumbent's order quantity, footnote 10 in the main paper). The proof is by contradiction. Suppose $\exists q_{ip}$ such that $q_{ip} > q_{ip}^{*L}$. Then, the low type incumbent orders q_{ip}^{*L} and does strictly better as long as the entrant assigns a probability of atmost p to demand being high. ■

Lemma 5 *The lowest quantity that the high type prefers to pool on is*

$$q_{(ip)\min}^H = \left(A_H - \frac{\mu}{2} - \frac{1}{2}\sqrt{(A_H - \mu)(3A_H - \mu)}\right)$$

while the lowest quantity the low type prefers to pool on is

$$q_{(ip)\min}^L = \left(A_L - \frac{\mu}{2} - \frac{1}{2}\sqrt{(A_H - \mu)(4A_L - A_H - \mu)}\right)$$

Proof. The high type incumbent prefers to pool as long as his profits while pooling dominate the profits he would make by revealing his type by ordering a high enough quantity (by equation 16). That is,

$$\left(A_H - q_{ip} - q_{ep}^*(q_{ip})\right) q_{ip} \geq \max_{q_{iH} > q_{ip}^*} (A_H - q_{iH} - q_{eH}^*(q_{iH}))q_{iH}, \tag{21}$$

where

$$q_{eH}^*(q_{iH}) = \arg\max_{q_{eH}} (A_H - q_{iH} - q_{eH})\, q_{eH} = \frac{A_H - q_{iH}}{2}$$

is derived from the entrant's beliefs in (16) and $q_{ep}^*(q_{ip})$ is given by equation (18). Consequently, inequality (21) simplifies to:

$$\left(A_H - \frac{\mu}{2} - \frac{q_{ip}}{2}\right) q_{ip} \geq \frac{A_H^2}{8}$$

Solving we get,

$q_{ip} \in \left[\left(A_H - \frac{\mu}{2} - \frac{1}{2}\sqrt{(A_H - \mu)(3A_H - \mu)}\right), \left(A_H - \frac{\mu}{2} + \frac{1}{2}\sqrt{(A_H - \mu)(3A_H - \mu)}\right)\right]$. Hence, the lowest pooling quantity for the high type is $q_{(ip)\min}^H = \left(A_H - \frac{\mu}{2} - \frac{1}{2}\sqrt{(A_H - \mu)(3A_H - \mu)}\right)$.

Similarly, for the low type, we verify

$$\left(A_L - q_{ip} - q_{ep}^*(q_{ip})\right) q_{ip} \geq \max_{q_{iL} > q_{ip}^*} (A_L - q_{iL} - q_{eL}^*(q_{iL}))q_{iL}, \tag{22}$$

where

$$q_{eL}^*(q_{iL}) = \arg\max_{q_{eL}} (A_H - q_{iL} - q_{eL})\, q_{eL} = \frac{A_H - q_{iL}}{2}$$

is derived from the entrant's beliefs in (16) and q_{ep}^* is given by equation (18). Consequently, inequality (22) simplifies to

$$\left(A_L - \frac{\mu}{2} - \frac{q_{ip}}{2}\right) q_{ip} \geq \begin{cases} \frac{1}{2}\left(A_L - \frac{A_H}{2}\right)^2 \text{ if } \theta < 2 \\ 0 \text{ otherwise} \end{cases}$$

Hence, we need to check for $\theta < 2$ only. Solving we get

$q_{ip} \in \left[\left(A_L - \frac{\mu}{2} - \frac{1}{2}\sqrt{(A_H-\mu)(4A_L - A_H - \mu)}\right), \left(A_L - \frac{\mu}{2} + \frac{1}{2}\sqrt{(A_H-\mu)(4A_L - A_H - \mu)}\right)\right]$. Hence, the lowest pooling quantity for the low type is

$q^L_{(ip)\min} = \left(A_L - \frac{\mu}{2} - \frac{1}{2}\sqrt{(A_H-\mu)(4A_L - A_H - \mu)}\right)$. ■

Corollary 1 *There is no pooling equilibria below* $\max\left(q^H_{(ip)\min}, q^L_{(ip)\min}\right) = q^H_{(ip)\min} = \underline{q}_p$.

Proof. This is the lowest quantity where *both* types are willing to pool at. ■

Lemma 6 *The pooling equilibrium exists for* $\theta \leq \bar{\Theta}(p) = \dfrac{3+2p-p^2}{1+4p-p^2}$

Proof. From Lemma 4 and Corollary 1, the pooling equilibrium exists if $\underline{q}_p < q^*_{ip}$, i.e., $q_{ip} \in \left[\underline{q}_p, q^*_{ip}\right]$. For this to happen,

$$\underline{q}_p = \left(A_H - \frac{\mu}{2} - \frac{1}{2}\sqrt{(A_H-\mu)(3A_H-\mu)}\right) < q^*_{ip} = \left(A_L - \frac{\mu}{2}\right)^+ \tag{23}$$

Now $(A_L - \mu/2) \geq 0$ if $\theta \leq \grave{\theta}(p) = ((1+p)/p)$. Assuming this holds and using $(A_H/A_L) = \theta$, inequality (23) simplifies to

$$(\theta - (p\theta + (1-p)))(3\theta - (p\theta + (1-p))) - 4(\theta-1)^2 > 0$$

The above inequality is satisfied for $\theta \in \left(1, \bar{\Theta}(p)\right)$ where

$$\bar{\Theta}(p) = \frac{3+2p-p^2}{1+4p-p^2} \in [1,3]$$

Moreover, $\bar{\Theta}(p) \leq \grave{\theta}(p)\ \forall p$ (since $\left(\grave{\theta}(p) - \bar{\Theta}(p)\right) = (1+p)^2 / (p + 4p^2 - p^3) > 0$). Hence, $\theta \leq \bar{\Theta}(p)$ is sufficient to ensure the existence of a pooling equilibrium. ■

Lemma 7 *Both types prefer to pool at the Pareto-dominant pooling quantity of* q^*_{ip}.

Proof. Recall by Lemma 4 that since $q_{ip} \leq \min\left(q^{*L}_{ip}, q^{*H}_{ip}\right) = q^{*L}_{ip} = q^*_{ip}$, the pooling profit for the low type is maximized at $q^{*L}_{ip} = q^*_{ip}$. The high type's pooling profit is concave in the pooling quantity with a maximum at $q^{*H}_{ip} > q^*_{ip}$. Hence, the high type prefers pooling at a (feasible) quantity closest to q^{*H}_{ip}, which is q^*_{ip}. Hence, both types prefer to pool on $q^*_{ip} = (A_L - \mu/2)$ which Pareto-dominates all other possible pooling quantities. ■

The following Lemma summarizes the complete proof for part I of Proposition 2.

Lemma 8 (*i*) *The incumbent orders* $q^*_{ip} = A_L - \dfrac{\mu}{2}$.

(*ii*) *The supplier always leaks.*

(*iii*) *The entrant orders* $q^*_{ep} = \dfrac{3\mu - 2A_L}{4}$

(*iv*) *The profits of the entrant and the incumbent are as in part* (*iv*) *of Proposition 2.*

Proof. (*i*) As per Lemma 7.

(*ii*) As per Theorem 2.

(*iii*) The entrant's pooling quantity in response to $q_{ip}^* = (A_L - \mu/2)$ can be obtained from (18).

(*iv*) The associated profits for the players can be obtained from straightforward algebra using the quantities in parts (*i*) and (*ii*). ■

Lemma 9 below proves part II of Proposition 2.

Lemma 9 *When* $\theta > \bar{\Theta}(p)$, *there can be no pooling equilibrium.*

Proof. The proof is by contradiction. Suppose $\exists\ q_{ip} > \underline{q}_p$ such that pooling exists (recall Corollary 1). Since $\theta > \bar{\Theta}(p)$, $\underline{q}_p > q_{ip}^*$ (follows from the proof of Lemma 6) $\Rightarrow q_{ip} > q_{ip}^*$. But this contradicts Lemma 4. ■

3.3 The Composite Equilibrium

Proposition 3 *(I) There exists a pure strategy PBNE, composite of Propositions 1 and 2, which parses to the separating equilibrium of Proposition 1 when* $\theta > \bar{\bar{\Theta}}(p)$ *and to the pooling equilibrium of Proposition 2 otherwise. The incumbent's and the entrant's order quantities, the supplier's strategy and the entrant's beliefs in equilibrium are as specified in Propositions 1 (for* $\theta > \bar{\bar{\Theta}}(p)$*) and 2 (for* $\theta \leq \bar{\bar{\Theta}}(p)$*). The entrant's composite threshold belief structure is given by:*

(*a*) *For* $\theta > \bar{\bar{\Theta}}(p)$

$$Pr_e(\tilde{A} = A_H) = \begin{cases} 0 \text{ if the supplier leaks } \underline{\text{and}}\ q_i \leq q_{iL}^*, \text{ OR the supplier does not leak;} \\ 1, \text{ otherwise.} \end{cases} \tag{24}$$

(*b*) *For* $\theta \leq \bar{\bar{\Theta}}(p)$

$$Pr_e(\tilde{A} = A_H) = \begin{cases} 0, \text{ if the supplier leaks } \underline{\text{and}}\ q_i < \underline{q}_p, \text{ OR the supplier does not leak;} \\ p, \text{ if the supplier leaks } \underline{\text{and}}\ \underline{q}_p \leq q_i \leq q_{ip}^*; \\ 1, \text{ otherwise.} \end{cases} \tag{25}$$

(II) The composite equilibrium satisfies the Intuitive Criterion of Cho and Kreps (1987).

We prove part I of this proposition in Section 3.3.1. Thereafter in Section 3.3.2, we detail the Intuitive Criterion of Cho and Kreps (1987) and prove that the proposed composite equilibrium satisfies the Intuitive Criterion.

3.3.1 Proof of part I of the composite equilibrium

We prove part I of the proposition through a series of Lemmas.

Lemma 10 *While pooling at the Pareto-dominant pooling quantity of* q_{ip}^**, the high type incumbent prefers the pooling equilibrium to the separating equilibrium for* $\theta \leq \bar{\Theta}(p)$*. However, the low type prefers the pooling equilibrium (over the separating equilibrium) only when* $\theta \leq \bar{\bar{\Theta}}(p)$*, where*

$$\bar{\bar{\Theta}}(p) = \frac{3 - 8\sqrt{p} + 14p - 8p^{3/2} + 12p^2 + 2p^3 + p^4}{1 + 14p^2 + p^4} \leq \bar{\Theta}(p)$$

Proof. Separation is the only pure strategy equilibrium (and hence unique) for $\theta > \bar{\Theta}(p)$. Now, for $\theta \leq \bar{\Theta}(p)$, by comparing the separating and the pooling profits of Proposition 1(*iv*) and Proposition 2, I(*iv*), it can be shown that the high type incumbent prefers pooling for $\theta \leq \bar{\Theta}(p)$ (i.e., wherever pooling exists). The low type incumbent prefers to pool for $\theta \leq \bar{\bar{\Theta}}(p)$ and separate otherwise, where

$$\bar{\bar{\Theta}}(p) = \frac{3 - 8\sqrt{p} + 14p - 8p^{3/2} + 12p^2 + 2p^3 + p^4}{1 + 14p^2 + p^4}$$

We prove that $\bar{\bar{\Theta}}(p) \leq \bar{\Theta}(p)$ by contradiction. Suppose not, i.e., $\bar{\bar{\Theta}}(p) > \bar{\Theta}(p)$. Then we get a pooling equilibrium (since both types of incumbent prefer the pooling equilibrium over the separating equilibrium) for $\theta = \bar{\bar{\Theta}}(p) > \bar{\Theta}(p)$ which contradicts Lemma 4. ■

Lemma 11 *The strategies of the incumbent, the supplier and the entrant reduce to those under the separating equilibrium of Proposition 1 when* $\theta > \bar{\bar{\Theta}}(p)$ *and to the pooling equilibrium of Proposition 2 otherwise.*

Proof. We first show that where ever pooling exists, $q_{iL}^* \leq \underline{q}_p \leq q_{ip}^* < q_{iH}^*$. We start with $q_{iL}^* \leq \underline{q}_p$. It can be shown that $\underline{q}_p$ is convex increasing in μ. At $(\mu)_{\text{min}} = A_L$, $\underline{q}_p = q_{iL}^*$. At $(\mu)_{\text{max}} = A_H$, $\underline{q}_p = (A_H/2) > q_{iL}^*$. And since for $\theta \leq \bar{\Theta}(p)$, $\underline{q}_p \leq q_{ip}^*$ from equation (23) in Lemma 6, $q_{iL}^* \leq \underline{q}_p \leq q_{ip}^*$. Lastly, it is straightforward to show that $q_{ip}^* = \left(A_L - \frac{\mu}{2}\right) < \frac{A_H}{2} = q_{iH}^*$.

Consider the case where $\theta > \bar{\bar{\Theta}}(p)$. As per Lemma 10, the low type separates by ordering q_{iL}^*. Hence, the entrant must assign a probability of 1 to demand being high for any quantity greater than q_{iL}^*, i.e., $Pr_e(\tilde{A} = A_H \,|q_i > q_{iL}^*) = 1$. Since the high type does not find it profitable to order q_{iL}^* or lower as per Proposition 1, and any quantity greater than q_{iL}^* reveals him to be of high type (including the zone where pooling exists), he orders q_{iH}^* and the separating equilibrium of Proposition 1 ensues.

Whenever $\theta \leq \bar{\bar{\Theta}}(p)$, both the high and low types prefer the pooling equilibrium as per Lemma 10. Consequently, both order q_{ip}^* and the pooling equilibrium of Proposition 2 ensues. ■

3.3.2 The Intuitive Criterion of Cho and Kreps (1987) and the proof of part II of the composite equilibrium

As before, in a signaling game, let T represent the type space for the 'Sender', with $t \in T$. The Sender sends a message m from a set $M(t)$ and the set of types that have message m available is $T(m)$. The 'Receiver', having seen the message, chooses a response r from a set $R(m)$. The payoff to the Sender is

$u(t,m,r)$ while the payoff for the Receiver is $v(t,m,r)$. We denote the equilibrium payoff of the sender of type t as $u^*(t)$. Let μ be the probability assessment, assigned by the Receiver under the equilibrium, over the set $T(m)$ of Sender types that might have sent message m. Then the Best Response of the Receiver to a message m is the set

$$BR(m;T(m)) = \arg\max_{r \in R(m)} \sum_{t \in T(m)} v(t,m,r)\,\mu(t).$$

The Intuitive Criterion stipulates the following:

(i) For each off-equilibrium message m, form the set $S(m)$ consisting of all types t such that

$$u^*(t) > \max_{r \in BR(m;T(m))} u(t,m,r)$$

(ii) If for each and every (off-equilibrium) message m, and for every type $t^{'} \in T$

$$u^*\left(t^{'}\right) \geq \min_{r \in BR(m;T(m)\backslash S(m))} u\left(t^{'},m,r\right)$$

then the equilibrium satisfies the Intuitive Criterion.

Requirement (i) identifies the set of types $S(m)$ that can never send an off-equilibrium message m, no matter what inference the Receiver draws from m (note that since this is an off-equilibrium message, the beliefs are no longer determined by Bayes' Rule). Then the receiver must (rationally) place a belief of zero on each type $t \in S(m)$ on observing an off-equilibrium message m. Requirement (ii) stipulates that if every one of the remaining types in $T(m)$, which are not in $S(m)$, does at least as well under the equilibrium as by deviating to any off-equilibrium message m, then the equilibrium satisfies the test of Intuitive Criterion.

We now prove that the composite equilibrium satisfies the Intuitive Criterion of Cho and Kreps (1987).

Lemma 12 *The composite equilibrium of Proposition 3 satisfies the Intuitive Criterion of Cho and Kreps* (1987).

Proof. The equilibrium parses into the separating equilibrium of Proposition 1 for $\theta > \overline{\overline{\Theta}}(p)$, and the pooling equilibrium of Proposition 2 for $\theta \leq \overline{\overline{\Theta}}(p)$. We prove that each of these equilibria satisfy the Intuitive Criterion.

Part 1: $\theta > \overline{\overline{\Theta}}(p)$. In this range, the separating equilibrium of Proposition 1 is played (Recall Section 3.1.1 and Figure A.2.). On the equilibrium path, the incumbent orders q_{iH}^* when demand is high and q_{iL}^* when demand is low. In the proof below, we show that the profit under each off-equilibrium outcome (as determined by requirements (i) and (ii)) is dominated by the on-equilibrium separating profit for both types of incumbent. The following off-equilibrium outcomes are possible:

(a) $m \in \{q_i : q_i > q_{iH}^*\}$. In this case, the set $S(m) = \{Low\}$ by requirement (i), since no matter what beliefs the entrant places on observing $q_i > q_{iH}^*$, the equilibrium payoff for the low type incumbent is greater than the off-equilibrium payoff from any $q_i > q_{iH}^*$. Hence, the set $T(m)\backslash S(m) = \{High\}$, i.e., the entrant must assign a belief that the message $q_i > q_{iH}^*$ is sent by the high type. However, the high type does worse by ordering any off-equilibrium quantity in excess of q_{iH}^* – the profit maximizing quantity – given the entrant's belief that demand is high. Hence, by requirement (ii) the Intuitive Criterion is satisfied for $m \in \{q_i : q_i > q_{iH}^*\}$.

(b) $m \in \{q_i : q_{iL}^* < q_i < q_{iH}^*\}$. Consider two cases. **Case I**, $\theta \geq 3$, $q_{iL}^* = (A_L/2)$. In this case the set $S(m) = \{Low\}$ (requirement (i)). The low type does better by ordering q_{iL}^* if $\theta \geq 3$ since this is the unconstrained maximum. Hence, $T(m) \backslash S(m) = \{High\}$, i.e., any quantity q_i such that $q_{iL}^* < q_i < q_{iH}^*$ reveals the incumbent to be of the high type, who in turn is better off ordering q_{iH}^*. Hence, by requirement (ii) the Intuitive Criterion is satisfied. **Case II**, $\theta < 3$, $q_{iL}^* < (A_L/2)$. A profitable off-equilibrium deviation for the low type is to order $q_{iL}^* < q_i \leq q_{SL} = \frac{A_L}{2} + \frac{\sqrt{\left((A_H-1)\left(7A_H-5-4\sqrt{(A_H-1)(3A_H-1)}\right)\right)^+}}{2}$, if the entrant believes that demand is low. Similarly, the high type prefers deviating to $q_{iL}^* < q_i \leq q_S$ if the entrant believes that demand is low. Hence, $S(m) = \{\}$ since there exists a profitable deviation for both types. Consequently, the set $T(m) \backslash S(m) = \{High, Low\}$. However, the Intuitive Criterion is satisfied as per requirement (ii) – if the entrant believes that demand is high, no type has an incentive to deviate. (Recall that the entrant is allowed to have any beliefs as long he places a belief of zero on all types in the set $S(m)$ by requirement (i). Since the set $S(m)$ is Null in this case, no beliefs are ruled out while applying requirement (ii)). For $q_{iL}^* < q_{SL} < q_i < q_{iH}^*$, the set $S(m) = \{Low\}$ as the low type's equilibrium profits are higher no matter what beliefs the entrant holds (including the belief that demand is low). Hence the set $T(m) \backslash S(m) = \{High\}$ which means that the entrant must believe that the deviation came from the high type. The high type in turn does better by ordering q_{iH}^* satisfying requirement (ii).

(c) $m \in \{q_i : q_i < q_{iL}^*\}$. In this case the set $S(m) = \{High, Low\}$. Clearly the high type does better in equilibrium given the incentive constraint (8) in the proof of Proposition 1 which is satisfied for $q_i < q_{iL}^*$. Moreover, the low type incumbent does better by ordering q_{iL}^* as this is the profit maximizing quantity $(A_L/2)$ for $\theta \geq 3$, or it is the quantity closest to $(A_L/2)$ that allows for separation when $\theta < 3$ (Recall that by Lemma 10, the low type prefers separation for $\theta > \overline{\overline{\Theta}}(p)$). Hence, requirement (ii) is trivially satisfied since the set $T(m) \backslash S(m) = \{\}$.

Part 2: $\theta \leq \overline{\overline{\Theta}}(p)$. In this range, the pooling equilibrium of Proposition 2 is played (Recall Section 3.2.1 and Figure A.3.). On the equilibrium path, the incumbent orders q_{ip}^*, in which case the entrant sticks to his priors and assigns a probability of p to high demand and a probability of $(1-p)$ to low demand. In the proof below, we show that the profit under each off-equilibrium outcome (as determined by requirements (i) and (ii)) is dominated by the on-equilibrium pooling profit for both types of incumbent. The following off-equilibrium outcomes are possible:

(a) $m \in \left\{q_i : q_i > q_{ip}^*\right\}$. **Case I:** $q_{ip}^* < q_i \leq \overline{q} = (1/2)\left(A_L + \sqrt{((\mu - A_L)(3A_L - \mu))^+}\right)$. There exists a profitable deviation for both types of incumbent if the entrant thinks that demand is low, as follows: The high type incumbent is better off ordering a quantity greater than q_{ip}^* when the entrant thinks that demand is low, as long as this quantity is less than

$\overline{q}_H = (1/2)\left(2A_H - A_L + \sqrt{\left(4A_H^2 + 5A_L^2 - \mu^2 + 4A_H\left(\mu - 3A_L\right)\right)^+}\right)$. Similarly, when the entrant thinks that demand is low, the maximum quantity the low type is willing to order is

$\overline{q}_L = (1/2)\left(A_L + \sqrt{\left(\left(\mu - A_L\right)\left(3A_L - \mu\right)\right)^+}\right)$. Note that $\overline{q} = \min\left(\overline{q}_H, \overline{q}_L\right) = \overline{q}_L$. Then $S(m) = \{\}$, by requirement (i). Hence, the set $T(m)/S(m)$ is $\{High, Low\}$. However, if the entrant sticks to his priors, then both types are better off ordering the Pareto-dominant q_{ip}^* and receiving their equilibrium payoffs. Hence requirement (ii) is satisfied. **Case II:** $\overline{q} < q_i \leq \overline{q}_H$. The low type's equilibrium payoffs are higher no matter what beliefs the entrant holds (including a belief that demand is low), i.e., the set $S(m) = \{Low\}$. Consequently, the set $T(m)/S(m)$ is $\{High\}$, which implies that the entrant must believe that the deviation came from the high type, who in turn is better off ordering q_{ip}^* (as per Lemma 10). Hence, requirement (ii) is satisfied. **Case III:** $q_i > \overline{q}_H$. The equilibrium payoffs for both types of incumbent are higher no matter what beliefs the entrant holds (including a belief that demand is low), i.e., the set $S(m) = \{High, Low\}$. Hence, requirement (ii) is trivially satisfied.

(b) $m \in \left\{q_i : \underline{q}_p \leq q_i < q_{ip}^*\right\}$. If the entrant believes that demand is low, both types are better off deviating. Hence, the set $T(m)/S(m)$ is $\{High, Low\}$. However, requirement (ii) is again satisfied if the entrant sticks to his priors, in which case both types are better off ordering Pareto-dominant q_{ip}^* and receiving their equilibrium payoffs.

(c) $m \in \left\{q_i : q_{iL}^* < q_i < \underline{q}_p\right\}$. Again the set $S(m) = \{\}$: If the entrant believes that demand is low, both types are better off deviating. Hence, the set $T(m)/S(m)$ is $\{High, Low\}$. However, requirement (ii) is again satisfied if the entrant sticks to his priors, in which case both types are better off ordering the Pareto-dominant q_{ip}^* and receiving their equilibrium payoffs.

d) $m \in \{q_i : q_i \leq q_{iL}^*\}$. The set $S(m) = \{High, Low\}$ as there is no profitable deviation for either type of incumbent: As per Lemma 10, both types of incumbent prefer the pooling equilibrium for $\theta \leq \overline{\overline{\Theta}}(p)$. Hence, the Intuitive Criterion is satisfied.

In summary, the composite equilibrium of Proposition 3 satisfies the Intuitive Criterion of Cho and Kreps (1987). ■

4 Proof of Proposition 4 of IAG.

Proposition 4 *There exists a pure strategy PBNE for the Information Acquisition Game, which depends on the value of* θ *relative to the threshold* $\Theta^*(p) = \frac{3-12p+p^2-8\sqrt{p}}{1-14p+p^2}$.

Case (i) $\theta < \Theta^*(p)$*: No information is acquired by the incumbent even when such information acquisition is costless. The supplier always leaks the incumbent's order quantity to the entrant. The order quantities of the incumbent and the entrant are* $q_i^* = \frac{\mu}{2}$ *and* $q_e^* = \frac{\mu}{4}$.

Case (ii) $\theta \geq \Theta^*(p)$*: The incumbent acquires information. Once information is acquired, the equilibrium of the continuation game (in terms of the incumbent's and the entrant's order quantities, the supplier's strategy and the entrant's beliefs) is given by the separating equilibrium of Proposition 3.*

We prove the Proposition in three stages: We first show that the supplier leaks even when no information is acquired by the incumbent (Lemma 13). Thereafter we show that not acquiring information dominates playing the pooling equilibrium for the incumbent, i.e., once information is acquired, only the separating equilibrium is played (Lemma 14). Finally we establish the information acquisition threshold $\Theta^*(p)$ for the incumbent in Lemma 15. These lemmas thus prove Cases (*i*) and (*ii*) of Proposition 4.

Lemma 13 *The supplier leaks even when no information is acquired by the incumbent.*

Proof. The game is sequential with the entrant moving after the incumbent (Figure 1 of the main paper). When the supplier leaks, the incumbent orders $((\mu - w)/2)$ and the entrant orders $((\mu - w)/4)$ as per Lemma 1, part (*i*). When the supplier does not leak, the incumbent and the entrant each order $(\mu - w)\,3$ as per part (*ii*) of Lemma 1 (the game is akin to a simultaneous move game; see Gibbons (1992), Page 4). The supplier earns a profit of $w(3(\mu - w)/4)$ with leakage, which is higher than his profit of $w(2(\mu - w)/3)$ with no leakage. Hence, the supplier leaks. ■

Lemma 14 *Once information is acquired, the continuation game is the same as the separating equilibrium of Proposition 3. Hence*

$$\Theta^*(p) \geq \bar{\bar{\Theta}}(p)$$

Proof. For the incumbent, 'no information acquisition' (NI) dominates pooling whenever

$$\max_{q_{NI}} \left(p\left(A_H - q_{NI} - q^*_{eNI}(q_{NI})\right) q_{NI} + (1-p)\left(A_L - q_{NI} - q^*_{eNI}(q_{NI})\right) q_{NI}\right) \geq$$
$$\left(p\left(A_H - q_{ip} - q^*_{ep}(q_{ip})\right) q_{ip} + (1-p)\left(A_L - q_{ip} - q^*_{ep}(q_{ip})\right) q_{ip}\right)$$

where,

$$\begin{aligned} q^*_{eNI}(q_{NI}) &= \arg\max_{q_{eNI}} \left(p\left(A_L - q_{NI} - q_{eNI}\right) q_{eNI} + (1-p)\left(A_H - q_{eNI} - q_{eNI}\right) q_{eNI}\right) = \frac{\mu - q_{NI}}{2} \\ q^*_{ep}(q_{ip}) &= \arg\max_{q_{ep}} \left(p\left(A_L - q_{ip} - q_{ep}\right) q_{ep} + (1-p)\left(A_H - q_{ip} - q_{ep}\right) q_{ep}\right) = \frac{\mu - q_{ip}}{2} \end{aligned}$$

Note that the maximand of the LHS in the inequality is the same as the RHS, with the RHS fixed at q_{ip}. However, q_{NI} is chosen so as to maximize the LHS, with q_{ip} being a point in the feasible set of q_{NI}. Hence, the result follows. ■

Lemma 15 *The incumbent acquires information whenever* $\theta \geq \Theta^*(p)$ *where* $\Theta^*(p) = \frac{3-12p+p^2-8\sqrt{p}}{1-14p+p^2}$

Proof. (*i*) $\theta \geq 3$. The profit with information acquisition is greater if

$$p\frac{A_H^2}{8} + (1-p)\frac{A_L^2}{8} \geq \frac{\mu^2}{8} = \frac{\left(pA_H + (1-p)A_L\right)^2}{8}, \tag{26}$$

which is true from Jensen's inequality. The RHS of the inequality is the profit that the incumbent makes when he does not acquire information (follows directly from Lemma 1, part (*i*)). The LHS is the expected profit for the separating equilibrium from part (*iv*) of Proposition 1 for $\theta \geq 3$ (since pooling is never played as per Lemma 14).

(ii) $\overset{=}{\Theta}(p) < \theta < 3$. The incumbent acquires information if

$$\begin{aligned} & p\frac{A_H^2}{8} + (1-p)\frac{-4A_L^2 + 12A_H A_L - 7A_H^2 + 4(A_H - A_L)\sqrt{(A_H - A_L)(3A_H - A_L)}}{8} \\ \geq \ & \frac{(pA_H + (1-p)A_L)^2}{8} \end{aligned}$$

The LHS is the expected profit for the separating equilibrium from part (iv) of Proposition 1 for $\theta < 3$. The RHS, as above, is the profit that the incumbent makes when he does not acquire information. Simplifying and writing in terms of the parameter θ, we get,

$$\frac{1}{8}(1-p)(\theta - 1)\left(5 + p(\theta - 1) - 7\theta + 4\sqrt{(\theta - 1)(3\theta - 1)}\right) \geq 0$$

Since, $p < 1$ and $\theta > 1$, this simplifies to

$$16(\theta - 1)(3\theta - 1) - (7\theta - 5 - p(\theta - 1))^2 \geq 0 \tag{27}$$

Define

$$f(\theta) \equiv 16(\theta - 1)(3\theta - 1) - (7\theta - 5 - p(\theta - 1))^2$$

The roots of $f(\theta)$ are

$$\begin{aligned} \theta_1 &= \frac{3 - 12p + p^2 + 8\sqrt{p}}{1 - 14p + p^2} \\ \theta_2 &= \frac{3 - 12p + p^2 - 8\sqrt{p}}{1 - 14p + p^2} \end{aligned} \tag{28}$$

Now,

$$\frac{\partial^2 f(\theta)}{\partial \theta^2} = 96 - 2(7 - p)^2$$

Hence, the function f is convex for $p > .07$ and concave below it. Hence, we take two cases.

Case I: $p \geq 0.07$. Since, f is convex, information is acquired for $\theta \geq \theta_2$ or for $\theta \leq \theta_1$. It can be seen that $\theta_1 \leq 0 \ \forall p \in [0, 1]$. Hence, information is acquired for $\theta \geq \theta_2 = \Theta^*(p)$.

Case II: $p < 0.07$. In this range f is concave and $\theta_2 < \theta_1$. Hence, information acquisition dominates no information acquisition for $\theta \in [\theta_2, \theta_1]$. However, $\theta_1 > 3$ when $p < 0.07$. Hence, information is acquired for $\theta \geq \theta_2 = \Theta^*(p)$. ■

5 Exclusive sourcing strategies to counter leakage

Section 6 of the main paper analyzes the following two-stage game numerically: In stage 1, the incumbent chooses an $\alpha \in [0, 1]$, which is the proportion of his supplier base that is common with the entrant. This uniquely determines both the incumbent's *unit* production cost (given by $k_0(1 - \alpha)$) and the *precision* of the entrant's signal on the demand (given by $\Pr\left\{s_e = High/\tilde{A} = A_H\right\} = \Pr\left\{s_e = Low/\tilde{A} = A_L\right\} = (1 + \alpha)/2$). Then in stage 2, the incumbent and the entrant choose their order quantities as a function of their demand information and other model parameters. Consequently, the profits of all players are determined as a function of α.

We solve backwards to first obtain the incumbent's order quantities and his profits in stage 2 as a function of α. We then numerically optimize the incumbent's expected profits over the range of possible α, in stage 1.

5.1 Stage 2: Derivation of the incumbent's order quantities and profits as a function of α.

Lemma 16 *Let the proportion of common suppliers in the incumbent's sourcing base be represented by* $\alpha \in [0,1]$; *i.e., the incumbent's unit production cost is* $k_0(1-\alpha)$, *and the entrant receives a signal* s_e *through leakage such that* $\Pr\left\{s_e = High/\tilde{A} = A_H\right\} = \Pr\left\{s_e = Low/\tilde{A} = A_L\right\} = (1+\alpha)/2$. *Then:*

(*a*) *The order quantity and the profits for the incumbent when demand is high are*

$$q_{iH}^{e}(\alpha) = \frac{\left(\begin{array}{c} A_H \left(\begin{array}{c} 4+p-p^2-2\left(4+11\left(-1+p\right)p\right)\alpha^2 \\ +\left(1+\left(-1+p\right)p\left(23+32\left(-1+p\right)p\right)\right)\alpha^4 \end{array}\right) \\ +\left(\alpha-1\right)\left(\begin{array}{c} A_L\left(-1+p\right)\left(1+\alpha\right)\left(-1-p+\left(1+p\left(-3+4p\right)\right)\alpha^2\right) \\ +2k_0\left(\begin{array}{c} 3+p-2\left(3+p\left(-9+2p\left(4+p\right)\right)\right)\alpha^2 \\ +\left(1-2p\right)^2\left(3+p\left(-7+8p\right)\right)\alpha^4 \end{array}\right) \end{array}\right) \end{array}\right)}{\left(8+3p-3p^2-2\left(8+21\left(-1+p\right)p\right)\alpha^2+\left(8+\left(-1+p\right)p\left(45+64\left(-1+p\right)p\right)\right)\alpha^4\right)},$$

$$\pi_{iH}^{e}(\alpha) = \frac{\left(\begin{array}{c} \left(-2+p+\left(2+p\left(-5+4p\right)\right)\alpha^2\right) \\ \left(A_H\left(\begin{array}{c} 4+p-p^2-2\left(4+11\left(-1+p\right)p\right)\alpha^2 \\ +\left(4+\left(-1+p\right)p\left(23+32\left(-1+p\right)p\right)\alpha^4\right) \\ +\left(-1+\alpha\right)\left(A_L\left(-1+p\right)\left(1+\alpha\right)\left(-1-p+\left(-3+4p\right)\right)\alpha^2\right) \\ +2k_0\left(\begin{array}{c} 3+p-2\left(3+p\left(-9+2p\left(4+p\right)\right)\right)\alpha^2 \\ +\left(1-2p\right)^2\left(3+p\left(-7+8p\right)\right)\alpha^4 \end{array}\right) \end{array}\right)\right)^2 \end{array}\right)}{\left(\begin{array}{c} 2\left(-1+\left(1-2p\right)^2\alpha^2\right) \\ \left(8+3p-3p^2-2\left(8+21\left(-1+p\right)p\right)\alpha^2+\left(8+\left(-1+p\right)p\left(45+64\left(-1+p\right)p\right)\right)\alpha^4\right)^2 \end{array}\right)}.$$

(*b*) *The order quantity and the profits for the incumbent when demand is low are*

$$q_{iL}^{e}(\alpha) = \frac{\left(\begin{array}{c} A_L \left(\begin{array}{c} 4+p-p^2-2\left(4+11\left(-1+p\right)p\right)\alpha^2 \\ +\left(4+\left(-1+p\right)p\left(23+32\left(-1+p\right)p\right)\right)\alpha^4 \end{array}\right) \\ +\left(\alpha-1\right)\left(\begin{array}{c} A_H\left(-1\right)p\left(1+\alpha\right)\left(-2+p+\left(2+p\left(-5+4p\right)\right)\alpha^2\right) \\ +2k_0\left(-1-\alpha+2p\alpha\right)\left(1-\alpha+2p\alpha\right)\left(-4+p+\left(4+p\left(-9+8p\right)\right)\alpha^4\right) \end{array}\right) \end{array}\right)}{\left(8+3p-3p^2-2\left(8+21\left(-1+p\right)p\right)\alpha^2+\left(8+\left(-1+p\right)p\left(45+64\left(-1+p\right)p\right)\right)\alpha^4\right)},$$

$$\pi_{iL}^{e}(\alpha)=\frac{\begin{pmatrix}\left(-1-p+\left(1+p\left(-3+4p\right)\right)\alpha^{2}\right)\\ \left(A_L\begin{pmatrix}4+p-p^{2}-2\left(4+11\left(-1+p\right)p\right)\alpha^{2}\\ +\left(4+\left(-1+p\right)p\left(23+32\left(-1+p\right)p\right)\alpha^{4}\right)\\ +\left(-1+\alpha\right)\left(A_H p\left(1+\alpha\right)\left(-2+p+\left(2+p\left(-5+4p\right)\right)\right)\alpha^{2}\right)\\ +2k_0\left(-1-\alpha+2p\alpha\right)\left(1-\alpha+2p\alpha\right)\left(-4+p+\left(4+p\left(-9+8p\right)\right)\alpha^{2}\right)\end{pmatrix}\right)^{2}\end{pmatrix}}{\begin{pmatrix}2\left(-1+\left(1-2p\right)^{2}\alpha^{2}\right)\\ \left(8+3p-3p^{2}-2\left(8+21\left(-1+p\right)p\right)\alpha^{2}+\left(8+\left(-1+p\right)p\left(45+64\left(-1+p\right)p\right)\right)\alpha^{4}\right)^{2}\end{pmatrix}}.$$

Proof. Given that the demand state is high, the incumbent solves

$$\pi_{iH}^{e}(\alpha)=\max_{q_{iH}^{e}}\begin{pmatrix}\Pr\left(s_e=\text{“}High\text{”}/\tilde{A}=A_H\right)\left(A_H-q_{iH}^{e}-q_{eH}^{e*}\left(q_{iH}^{e},q_{iL}^{e},\alpha\right)\right)q_{iH}^{e}\\ +\Pr\left(s_e=\text{“}Low\text{”}/\tilde{A}=A_H\right)\left(A_H-q_{iH}^{e}-q_{eL}^{e*}\left(q_{iH}^{e},q_{iL}^{e},\alpha\right)\right)q_{iH}^{e}\\ -k_0\left(1-\alpha\right)q_{iH}^{e}\end{pmatrix}$$

$$\Rightarrow\quad \pi_{iH}^{e}(\alpha)=\max_{q_{iH}^{e}}\begin{pmatrix}\left(\dfrac{1+\alpha}{2}\right)\left(A_H-q_{iH}^{e}-q_{eH}^{e*}\left(q_{iH}^{e},q_{iL}^{e},\alpha\right)\right)q_{iH}^{e}\\ +\left(\dfrac{1-\alpha}{2}\right)\left(A_H-q_{iH}^{e}-q_{eL}^{e*}\left(q_{iH}^{e},q_{iL}^{e},\alpha\right)\right)q_{iH}^{e}-k_0\left(1-\alpha\right)q_{iH}^{e}\end{pmatrix} \tag{29}$$

Similarly for the low demand state, the incumbent solves:

$$\pi_{iL}^{e}(\alpha)=\max_{q_{iL}^{e}}\begin{pmatrix}\left(\dfrac{1+\alpha}{2}\right)\left(A_L-q_{iL}^{e}-q_{eL}^{e*}\left(q_{iH}^{e},q_{iL}^{e},\alpha\right)\right)q_{iL}^{e}\\ +\left(\dfrac{1-\alpha}{2}\right)\left(A_L-q_{iL}^{e}-q_{eH}^{e*}\left(q_{iH}^{e},q_{iL}^{e},\alpha\right)\right)q_{iL}^{e}-k_0\left(1-\alpha\right)q_{iL}^{e}\end{pmatrix}. \tag{30}$$

The entrant's order quantities as a function of the signal s_e and the incumbent's order quantities are:

$$q_{eH}^{e*}\left(q_{iH}^{e},q_{iL}^{e},\alpha\right)=\arg\max_{q_{eH}^{e}}\begin{pmatrix}\Pr\left(\tilde{A}=A_H/s_e=\text{“}High\text{”}\right)\left(A_H-q_{iH}^{e}-q_{eH}^{e}\right)q_{eH}^{e}\\ +\Pr\left(\tilde{A}=A_L/s_e=\text{“}High\text{”}\right)\left(A_L-q_{iL}^{e}-q_{eH}^{e}\right)q_{eH}^{e}\end{pmatrix}$$

$$\Rightarrow\quad q_{eH}^{e*}\left(q_{iH}^{e},q_{iL}^{e},\alpha\right)=\arg\max_{q_{eH}^{e}}\begin{pmatrix}\left(\dfrac{\left(\dfrac{1+\alpha}{2}\right)p}{\left(\dfrac{1+\alpha}{2}\right)p+\left(1-\dfrac{1+\alpha}{2}\right)\left(1-p\right)}\right)\left(A_H-q_{iH}^{e}-q_{eH}^{e}\right)q_{eH}^{e}\\ +\left(\dfrac{\left(1-\dfrac{1+\alpha}{2}\right)\left(1-p\right)}{\left(\dfrac{1+\alpha}{2}\right)p+\left(1-\dfrac{1+\alpha}{2}\right)\left(1-p\right)}\right)\left(A_L-q_{iL}^{e}-q_{eH}^{e}\right)q_{eH}^{e}\end{pmatrix},$$

$$q_{eL}^{e*}\left(q_{iH}^{e}, q_{iL}^{e}, \alpha\right) = \arg\max_{q_{eL}^{e}} \begin{pmatrix} \Pr\left(\tilde{A} = A_L / s_e = \text{“}\mathit{Low}\text{”}\right)\left(A_L - q_{iL}^{e} - q_{eL}^{e}\right) q_{eL}^{e} \\ +\Pr\left(\tilde{A} = A_H / s_e = \text{“}\mathit{Low}\text{”}\right)\left(A_H - q_{iH}^{e} - q_{eL}^{e}\right) q_{eL}^{e} \end{pmatrix}$$

$$\Rightarrow \quad q_{eL}^{e*}\left(q_{iH}^{e}, q_{iL}^{e}, \alpha\right) = \arg\max_{q_{eL}^{e}} \begin{pmatrix} \left(\dfrac{\left(\dfrac{1+\alpha}{2}\right)(1-p)}{\left(\dfrac{1+\alpha}{2}\right)(1-p) + p\left(1 - \dfrac{1+\alpha}{2}\right)}\right)\left(A_L - q_{iL}^{e} - q_{eL}^{e}\right) q_{eL}^{e} \\ +\left(\dfrac{p\left(1 - \dfrac{1+\alpha}{2}\right)}{\left(\dfrac{1+\alpha}{2}\right)(1-p) + p\left(1 - \dfrac{1+\alpha}{2}\right)}\right)\left(A_H - q_{iH}^{e} - q_{eL}^{e}\right) q_{eL}^{e} \end{pmatrix}.$$

The entrant's order quantities are plugged back into the incumbent's profit functions in (29) and (30). These profit functions are maximized by simultaneously solving for the incumbent's optimal order quantities $q_{iH}^{e}(\alpha)$ and $q_{iL}^{e}(\alpha)$. The incumbent's corresponding profits are then calculated using these order quantities. Lemma 16 provides the results. ■

5.2 Stage 1: Solving for the optimal α

In this stage, the incumbent optimizes the following program:

$$\alpha^{*} = \arg\max_{\alpha}\left(p\pi_{iH}^{e}(\alpha) + (1-p)\,\pi_{iL}^{e}(\alpha)\right), \tag{31}$$

which was done numerically using Mathematica.

5.3 Alternate formulation: α as the probability of leakage

In stage 1, the incumbent chooses an $\alpha \in [0,1]$, which is the proportion of his supplier base that is common with the entrant. This uniquely determines both the incumbent's *unit* production cost (given by $k_0(1-\alpha)$) and the *probability* of leakage of demand information to the entrant (given by α). Then in stage 2, the incumbent and the entrant choose their order quantities as a function of their demand information and other model parameters. Consequently, the profits of all players are determined as a function of α.

As before, we solve backwards to first obtain the incumbent's order quantities and his profits in stage 2 as a function of α. We then numerically optimize the incumbent's expected profits over the range of possible α, in stage 1.

With the alternate formulation, where α is the probability of leakage, in stage 2 the high type incumbent solves:

$$\pi_{iH}^{e}(\alpha) = \max_{q_{iH}^{e}}\left(\alpha\left(A_H - q_{iH}^{e} - q_{eH}^{e*}(q_{iH})\right) q_{iH}^{e} + (1-\alpha)\left(A_H - q_{iH}^{e} - q_{e}^{e*}(q_{iH}, q_{iL})\right) q_{iH}^{e} - k_0(1-\alpha)\, q_{iH}^{e}\right) \tag{32}$$

The low type incumbent solves

$$\pi_{iL}^{e} = \max_{q_{iL}^{e}} \left(\alpha \left(A_L - q_{iL}^{e} - q_{eL}^{e*}\left(q_{iL}\right)\right) q_{iL}^{e} + \left(1-\alpha\right)\left(A_L - q_{iL}^{e} - q_{e}^{e*}\left(q_{iH}, q_{iL}\right)\right) q_{iL}^{e} - k_0 \left(1-\alpha\right) q_{iL}^{e}\right) \tag{33}$$

where

$$\begin{aligned} q_{eH}^{e*}\left(q_{iH}\right) &= \arg\max_{q_{eH}^{e}} \left(\left(A_H - q_{iH}^{e} - q_{eH}^{e}\right) q_{eH}^{e}\right), \\ q_{eL}^{e*}\left(q_{iL}\right) &= \arg\max_{q_{eL}^{e}} \left(\left(A_L - q_{iL}^{e} - q_{eL}^{e}\right) q_{eL}^{e}\right), \text{ and} \\ q_{e}^{e*}\left(q_{iH}, q_{iL}\right) &= \arg\max_{q_{e}^{e}} \left(p\left(A_L - q_{iH}^{e} - q_{e}^{e}\right) q_{e}^{e} + \left(1-p\right)\left(A_L - q_{iL}^{e} - q_{e}^{e}\right) q_{e}^{e}\right). \end{aligned}$$

After solving for the entrant's optimal order quantities, these are plugged back in the incumbent's profit functions in equations (32) and (33). Thereafter, by simultaneously solving for the incumbent's optimal order quantities, the optimal profits are obtained.

Finally in stage 1, the incumbent optimizes over the entire range of feasible α. This program is the same as in (31), but with the profit expressions for $\pi_{iH}^{e}\left(\alpha\right)$ and π_{iL}^{e} given by the expressions (32) and (33) respectively.

References

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