

On the Effects of Consumer Search and Firm Entry in a Multiproduct Competitive Market

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This paper studies a model in which consumers search among multiple competing firms for products that match their preferences at a reasonable price. We focus on how easier search, possibly due to the adoption of search-facilitating technologies such as the Internet, influences equilibrium prices, assortments, firm profits, and consumer welfare. Conventional wisdom suggests that easier search creates a competition-intensifying effect that puts pressure on firms to lower their prices and reduce assortments. However, in our model we demonstrate that search also exhibits a market-expansion effect that encourages firms to expand their assortment—easier search means that each firm is searched by more consumers. Because of broader assortments, consumers are more likely to find products that better match their ideal preferences, improving the efficiency of the market. In fact, we demonstrate that the market-expansion effect can even dominate the competition-intensifying effect potentially leading to higher prices, broader assortments, more profits, and expanded welfare.

Key words: search; Internet; price competition; assortment; product variety; long-tail phenomenon; game theory; differentiated competition

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1. Introduction

It was widely believed that the emergence of the Internet would lead to brutal price competition and an erosion of retail profit. The argument is simple and compelling: the Internet lowers the costs consumers incur to search for goods and services, so consumers search more, resulting in more opportunities to compare prices and ultimately more severe price competition among sellers. There is indeed some evidence that the Internet has reduced prices in several markets (e.g., Brynjolfsson and Smith 2000, Brown and Goolsbee 2002, Sorensen 2000). Nevertheless, there is also evidence that firms are able to support prices above marginal cost (e.g., Brynjolfsson et al. 2004, Clemons et al. 2002, Hortacsu and Syverson 2004).

This paper provides a simple explanation for why firms have been able to maintain at least some pricing power and hence profits. Suppose consumers care about the price they pay but also want to find a product or service that best matches their preferences. For example, a shopper may know she wishes to purchase a pair of shoes but is not exactly sure what pair she wants. Different retailers offer different prices and different selections of shoes, so our shopper searches among a sample of retailers to find a suitable pair at an acceptable price. All else being equal,

the wider a store's assortment, the more likely the shopper will find a pair of shoes that closely matches her ideal. Although a wider assortment increases a retailer's demand, a wider assortment is costly (e.g., additional inventory costs, space requirements, etc.). Lower search costs then have two effects in this market. With a lower cost to search, a consumer is likely to search more stores, so each retailer is put into greater competition with other retailers. This *competition-intensifying effect* is precisely the intuition behind the conventional wisdom that easier search reduces market prices. However, if consumers search more, then each retailer effectively gains access to a broader pool of potential customers. A larger pool of potential customers allows the retailer to justify a broader assortment. Because a larger assortment provides value to consumers, the retailer can then support a *higher price* and potentially even make *more profit*. In other words, when retailers choose their assortments in addition to their prices, lower search costs have a *market expansion effect* that dampens competition. In fact, we demonstrate that the market-expansion effect can even dominate; i.e., lower search costs in a competitive market can lead to higher prices, more industry profits, and increased consumer and social welfare. At the very least, the reduction in market prices and

profits is not as severe as would be predicted by considering only the competition-intensifying effect.

We present a model of multiple firms competing on price and assortment in which consumers search for a product that best fits their needs. This novel model setup lets us study the impact of search costs on prices, assortments, profits, and welfare. To further highlight the distinctiveness of search's market-expansion effect, we compare the influence of search with the influence of firm entry.

2. Literature Review

Bakos (1997) and Anderson and Renault (1999) study search among differentiated single-product firms with exogenous differentiation. They find that easier search leads to lower equilibrium prices, corresponding, again, to the intuition that cheaper search intensifies price competition. Anderson and Renault (1999) find that equilibrium prices increase with the diversity of consumers' tastes when consumers have a sufficiently strong preference for variety. Despite this relationship, we note that easier search in their model continues to lower prices. In our model firms have a second strategic decision (their assortment breadth) that yields our qualitatively different conclusions. Klemperer (1992) studies a duopoly model where each firm offers a line of products, but the number of products carried by each firm is exogenous, whereas assortment breadth is endogenous in our model.

There is an extensive empirical literature that measures the impact of search on market dynamics. (See Baye et al. 2006 for a review.) For example, Brynjolfsson et al. (2003) estimate the economic impact of increased product variety in online bookstores and show that consumer welfare has increased significantly more by broader assortments than by increased competition or lower prices. Our theoretical conclusions are consistent with their findings: easier search in a competitive equilibrium can lead to broader assortments and reasonably stable prices.

Several recent papers develop theoretical models to explain why easier search may not lead to excessive price competition. Lal and Sarvary (1999) argue that the Internet reduces consumer search costs for digital attributes of a product (such as its price and dimension) but not for nondigital attributes (such as how well it fits, in the case of clothing).¹ This effectively increases a consumer's cost to search, so prices can rise. In an experimental study, Lynch and Ariely (2000) consider separately the ease of searching for price or quality information. They find that as quality information becomes easier to search, then consumers

are less price sensitive. We assume that consumers incur a single cost to learn each firm's price and product attributes. Kuksov (2004) is closely related to this paper. Similar to this paper, Kuksov establishes the counterintuitive result that average prices can increase if search costs decrease. Despite this similar result, because our model setup is quite different, we obtain different results and empirical predications concerning profits, assortments, and welfare. We summarize these differences at the end of §5.

3. Model

There are n risk-neutral multiproduct firms who each offers a set of differentiated products within the same category to a market of risk-neutral consumers. Firms simultaneously choose their prices and assortments. (Our qualitative results continue to hold if firms choose assortments first and then prices.²) Consumers then shop among firms searching for a product that matches their taste at a reasonable price (they do not know *a priori* whether such a product exists or which firm carries it).³ Consumers either choose to purchase one unit of a product from one firm or choose to abstain from purchasing (the no-purchase option). We normalize the consumer population to one (w.l.o.g.).

We study two variants of consumer search in our model, parallel and sequential search. (These are the common two variants in the search literature; see Baye et al. 2006.) Both variants reflect the presence of search costs, yet they differ in the process with which consumers search. In the case of parallel search, consumers sample a fixed number of firms and then purchase their most preferred product among the firms searched, which might be the no-purchase option. With sequential search, consumers visit firms sequentially and decide after each visit whether to purchase an item, to continue their search at another firm, or to purchase nothing. We suspect that other search models are qualitatively a mixture of these two extremes, and they are analytically more cumbersome.

In both models firms set their prices and assortments to maximize their expected profit given consumer shopping behavior and consumers choose a search strategy to maximize their utility given the price and assortment decisions of the firms. We focus on symmetric equilibria throughout the paper.

Our consumer choice process is based on the multinomial logit (MNL) model (see Anderson et al. 1992). The basic form of the model works as follows. A consumer considers whether to purchase one unit from a

² See Technical Appendix at <http://mktsci.pubs.informs.org> for details.

³ See Iyer et al. (2005) for a model in which firms use target advertising to inform consumers about which firm offers their desired product.

¹ See He and Chen (2006) for a model of how the Internet influences consumer search through the presence of an e-marketplace.

set of products, S , or to purchase nothing, product 0, which we also call the no-purchase option. Let U_j be the consumer's utility from product j , $j \in S \cup \{0\}$: $U_j = (u_j - p_j) + \zeta_j$, where u_j is a constant, p_j is the price of product j ($p_0 = 0$), and ζ_j is a random variable with a zero-mean Gumbel distribution and variance $\pi^2 \mu^2 / 6$, where μ is a scale parameter. Let $F(x)$ and $f(x)$ be the distribution and density functions of ζ_j and γ is Euler's constant,

$$F(x) = \exp[-\exp(-(x/\mu + \gamma))].$$

A consumer chooses the product with the highest realized utility.⁴ Let $q_j(S \cup \{0\})$ be the probability that a consumer chooses product j given the choice set. It is well known that

$$\begin{aligned} q_j(S \cup \{0\}) &= \Pr(U_j = \max\{U_i, i \in S \cup \{0\}\}) \\ &= \frac{\exp((u_j - p_j)/\mu)}{\sum_{i \in S} \exp((u_i - p_i)/\mu) + \exp(u_0/\mu)}. \end{aligned} \quad (1)$$

It follows from (1) that demand for product j decreases as more products are added to the consumer's choice set: $q_j(S) > q_j(S^+)$, $\forall j \in S \subset S^+$.

Let U_{ik} be a consumer's utility associated with firm k 's i th product, $i \geq 0$: $U_{ik} = (u_{ik} - p_{ik}) + \zeta_{ik}$. We assume that all actual products are equally likely to be preferred, adjusting for any price differences: for all $i \geq 1$ and k , $\zeta_{ik} = \zeta$ (i.e., the same random variable, ζ , represents the utility shock to all products) and $u_{ik} = 0$ (normalized to zero w.l.o.g.). For the no-purchase product we choose $u_{0k} = u_0$ and $p_{0k} = p_0 = 0$ (normalized to zero w.l.o.g.). MNL-based models have been widely used in empirical research on product variety/assortment-related issues in both economics (e.g., Watson 2004) and marketing (e.g., Draganska and Jain 2005, 2006).

In our model, each firm offers a unique set of products (i.e., each firm is the exclusive provider of its assortment). This is appropriate in a market with substantial variety and diffuse preferences, such as women's shoes, eyeglasses, apparel, furniture, ceramic tiles, antiques, paintings, bicycles, or paperback romance novels, among others. We feel that our qualitative results should continue to hold even if firms have overlapping assortments. But overlapping assortments require modeling consumer expectations for how assortments overlap and how those expectations influence consumer search. Intuitively, the greater the overlap in assortments across firms,

⁴ We assume that there is no residual uncertainty with respect to the utility of a product once the consumer has the opportunity to inspect the product at some firm. See Guo (2006) for a model in which consumers are uncertain about their future utility when they make a purchase.

the lower the value of consumer search to find a suitable product. Therefore, in our model the value of search to consumers is maximized.

Because all products have the same probability of appealing to a consumer, it can be shown that it is optimal for each firm k to choose the same price for all of its products,⁵ p_k (e.g., Shugan 1989 indicates that the majority of flavors within a product line are sold at the same price in the ice cream industry), and the firm's assortment decision amounts to choosing the number of products to offer, x_k .⁶ For analytical simplicity, throughout this paper we treat x_k as a continuous variable. We assume that each firm purchases each product for the same marginal cost (normalized to zero), but variety is costly from an operational perspective. In particular, let $c(x)$ be a firm's operational costs when it carries x products, $c(0) = 0$, $c'(x) \geq 0$, and $c''(x) \geq 0$.

In summary, we present an oligopoly model with n firms that provide an assortment of differentiated product variants to a set of consumers that may or may not purchase a unit (i.e., consumers have a no-purchase option). This model setup is distinct from the duopoly model presented by Kuksov (2004). Kuksov studies two firms that sell a single product variant to a set of consumers that purchase one unit with certainty (i.e., they do not have a no-purchase option).

4. Competition with Parallel Search

In this section consumers sample firms in parallel: each consumer *a priori* commits to search m firms. The consumer observes the product offerings of m randomly selected firms and then purchases the product that maximizes utility net price, which may be the no-purchase option. The consumer incurs a search cost of τ per firm included in the search, for a total search cost of $m\tau$.⁷ (Note that search can be costly even in electronic markets: e.g., Hann and Terwiesch 2003 show that consumers value the dis-utility of an online search activity at about \$5 per search.) Consumers choose m to maximize their expected utility conditional on the expectation that firms choose a symmetric equilibrium; i.e., each consumer expects

⁵ See the Technical Appendix at <http://mktsci.pubs.informs.org> for details on the optimality of identical pricing in an assortment.

⁶ In our model there is no personalized pricing (e.g., Iyer et al. 2005 and Liu and Zhang 2006) or pricing based on purchase history (e.g., Acquisti and Varian 2005).

⁷ This model can be extended to the case that the cost of searching m firms is a convex and increasing function of m , $S(m)$ with $S'(m) \geq 0$ and $S''(m) \geq 0$. In our model consumers incur a cost to search between firms, but once a consumer incurs the cost to visit the firm, the consumer does not incur a cost to evaluate the firm's set of products. See Randall et al. (2007) for a discussion of how firms can influence consumers' product evaluation cost.

each of the n firms to choose price p^* and assortment x^* . Naturally, each consumer makes an optimal purchase decision given the actual prices and assortments observed in the m chosen firms.

A parallel search strategy is appropriate if there is some lag to receive information from firms, such as requesting catalogs from several mail-order firms, or if consumers use a simple search heuristic that commits them to a fixed number of searches (e.g., “obtain price quotes from three insurance companies before choosing one”). Our parallel search model is analogous to the consideration set theory of consumer choice in the consumer behavior literature (e.g., Howard and Sheth 1969, Roberts and Lattin 1991). Consideration set theory posits that consumers first decide which brands/options to include in a consideration set and then they choose the best alternative among their pre-specified set.

Firms choose their prices and assortments optimally given their expectation of m . A symmetric equilibrium in this setting is a 3-tuple, $\{p^*, x^*, m^*\}$, such that consumers have no incentive to deviate from their search strategy and no firm has an incentive to deviate from its price and assortment strategy.

From the properties of the Gumbel distribution and the MNL model (p. 60, Anderson et al. 1992), a consumer’s expected utility of searching m firms is

$$\widehat{U}(m) = \mu \ln[mx^* \exp(-p^*/\mu) + \exp(u_0/\mu)].$$

For ease of exposition, we approximate $\widehat{U}(m)$ with $U(m)$, where

$$U(m) = \mu \ln[mx^* \exp(-p^*/\mu)] \\ = -p^* + \mu \ln m + \mu \ln x^*.$$

The approximation is most accurate when the no-purchase utility is low. In the Technical Appendix at <http://mktsci.pubs.informs.org>, we establish that our qualitative results based on our approximation can be established with the exact formulation of the consumer’s expected utility function, $\widehat{U}(m)$. However, the analysis of the exact model is more cumbersome.

A consumer’s search strategy maximizes $U(m)$, a consumer’s expected net utility of searching m firms,

$$\max_{1 \leq m \leq n} U(m) = -p^* + \mu \ln m + \mu \ln x^* - m\tau.$$

The solution to the consumer’s problem is

$$m^* = \min \left[n, \max \left(1, \frac{\mu}{\tau} \right) \right], \quad (2)$$

where we treat m as a continuous variable. The consumer’s optimal search sample size, m^* , is decreasing in the search cost τ and is independent of p^* and x^* .⁸

⁸ This convenient result is obtained because the consumer’s maximum utility is the log of a multiplicative term that includes the

Now consider the firms’ pricing and assortment decisions. Let $\{p_i, x_i\}$ be firm i ’s choices and let $\{p, x\}$ be firm i ’s expectation for the choices of the other firms, where, as the reader will recall, firm i expects the other firms to follow a symmetric equilibrium. Conditional that firm i is included in a consumer’s search, that consumer purchases from firm i with probability

$$q_i(p_i, x_i | p, x, m^*) \\ = \frac{x_i \exp(-p_i/\mu)}{x_i \exp(-p_i/\mu) + (m^* - 1)x \exp(-p/\mu) + \exp(u_0/\mu)}. \quad (3)$$

Although a consumer expects all firms to choose p^* and x^* , (3) implies that a consumer makes an optimal choice given the actual prices and assortments observed.

Our model of each firm’s market share, $q_i(\cdot)$, is similar to the share equations in the literature on market share models, but there are some important differences (Basuroy and Nguyen 1998, Bell et al. 1975, Gruca et al. 1992, Gruca and Sudharshan 1991, Karnani 1985, Monahan 1987). Those models express firm i ’s sales as

$$SALES_i = \left(\frac{A_i}{A_i + \sum_{j \neq i} A_j} \right) \times SIZE(\sum A_i),$$

where A_i is the attractiveness of firm i and $SIZE(A)$ is the total sales in the market, which is possibly an increasing function of the firms’ total attraction. Our model fixes the total potential sales of the market, and each firm’s share of that total potential depends on the sum of the firms’ attractions plus the attraction of the no-purchase option (i.e., the adjustment for market size in our model is in the denominator rather than in the numerator). Existence of equilibrium can be an issue in market share models based on the MNL (Basuroy and Nguyen 1998, Gruca and Sudharshan 1991), but existence is not an issue in our model even though our model is also based on the MNL.⁹

number of products and price. Hence, m separates from both x^* and p^* . It allows us to simplify the exposition significantly. In the exact version of the model, m depends on both x^* and p^* . See the Technical Appendix at <http://mktsci.pubs.informs.org> for a detailed analysis of that case.

⁹ Previous research derives each firm’s attractiveness score to be of the form $a_i e^{\alpha_i p_i} e^{\beta_i x_i}$, where a_i , α_i , and β_i are constants, p_i is the firm’s price, and m_i is its expenditure on a marketing variable (such as advertising or service). Price enters into our attractiveness in the same way, but our additional strategic variable, the number of products offered, is in linear form, x_i , rather than exponential form, $e^{\beta_i x_i}$, which is the key difference. Equilibrium existence does not appear to be an issue with multiplicative competitive interaction (MCI) models. Those models have attractiveness of the form $a_i p_i^{-\alpha} x_i^{\beta}$, which is similar to our form with respect to x_i but different with respect to price.

Firm i 's expected profit is

$$\begin{aligned} \Pi_i(p_i, x_i | p, x, m^*) \\ = p_i \left(\frac{m^*}{n} \right) q_i(p_i, x_i | p, x, m^*) - c(x_i) \end{aligned} \quad (4)$$

because m^*/n is the probability that firm i is included in a consumer's random sample of m^* firms. The next result establishes the existence and uniqueness of an equilibrium. All proofs are in the appendix.

PROPOSITION 1. *In the parallel search model there exists a unique symmetric equilibrium, $\{p^*, x^*, m^*\}$. If $c'(0) < \mu^2/(n\tau \exp(1 + u_0/\mu))$, then*

$$m^* = \min \left[n, \max \left(1, \frac{\mu}{\tau} \right) \right]$$

and p^* and x^* are implicitly defined by the following:

$$p^* = \frac{\mu^2}{\mu - (n/m^*)x^*c'(x^*)} \quad (5)$$

$$x^* = \frac{\mu}{nc'(x^*)} - \frac{\exp((u_0 + p^*)/\mu)}{m^*} \quad (6)$$

Otherwise, $p^* = \mu$, $m^* = 0$, $x^* = 0$.

PROPOSITION 2. *In the parallel search model (assuming an equilibrium with positive assortments), while firm entry reduces both product variety, x^* , and price, p^* , easier search increases product variety, x^* , and does not necessarily reduce price, p^* . Specifically, if $c'(x) = c$ for some constant c , then there exists some threshold $\bar{\tau}$ such that easier search (decreasing τ) increases the equilibrium price when $\tau > \bar{\tau}$, otherwise easier search decreases the equilibrium price. Furthermore, while firm entry may not always increase the total market variety nx^* , easier search always does.*

The effects of firm entry are consistent with the existing literature: firm entry lowers both equilibrium price and product variety and may not necessarily increase the total variety in the market.¹⁰ This is due to the competition-intensifying effect of firm entry: a more crowded market results in more competition, which shrinks market share (m^*/n), which reduces each firm's incentive to invest in variety. Furthermore, with less product variety and more competitors, firms must reduce prices.

¹⁰ Watson (2007) provides an exception. He too has consumers who search over price and products and firms that compete on price and assortment. However, his model does not include a no-purchase option. He finds that entry reduces assortment, as we do. Narrower assortments cause consumers to search less, which dampens price competition. The price competition-intensifying effect of entry exists in his model as well, but that effect may be dominated by the effect of narrower assortments; i.e., prices can rise with entry in his model.

The effects of search, on the other hand, are quite different from those of firm entry. For example, easier search raises the equilibrium product variety x^* and the total product variety in the market nx^* . As the search cost, τ , decreases, consumers are more likely to expand their search of firms before making a final purchase decision. Consequently, easier search pressures firms to compete for the patronage of a consumer with a greater number of competitors. This is similar to firm entry: easier search leads to more competition in the market, which induces firms to reduce their variety. This competition-intensifying effect of easier search is identified in the choice probability (3): everything else equal, a higher value of m^* (which is caused by easier search) results in lower $q_i(p_i, x_i | p, x, m^*)$. But easier search has an additional effect on market dynamics: when consumers search more actively, a firm gains access to (in the sense of "is being visited by") more consumers; i.e., an increase in m^* also increases the m^*/n term in the objective function (4). That is the market expansion effect. The overall effect of easier search on the equilibrium product offering depends on the relative strength of these two effects. According to Proposition 2, in the case of parallel search, the market expansion effect of easier search always dominates the competition intensifying effect; i.e., $(m^*/n)q_i(p_i, x_i | p, x, m^*)$ is increasing in m^* . Hence, overall, easier search raises the equilibrium assortment of each firm and the total assortment in the market.

Proposition 2 also indicates that the market price may actually increase as search becomes easier (i.e., as τ decreases). This result is in sharp contrast to the conventional economic view that increased competition leads to a lower market price (e.g., Bakos 1997, Anderson and Renault 1999). Prices may rise because firms offer more variety, thereby generating greater social welfare that allows the firms to support higher prices. In Anderson and Renault (1999), prices may rise only if consumers' preferences for variety increases; i.e., they test the relationship between prices and μ . As μ increases in their model, there are two effects: consumers search more because they care more about variety, and consumer welfare (net of prices and search costs) increases conditional on a given number of firms visited (each firm carries a single product in their model). The first effect in Anderson and Renault (1999) unambiguously decreases prices, but in our model additional consumer search can increase prices because firms also choose the breadth of their assortment. Thus, although the intuition in the two models is somewhat similar—equilibrium prices may rise when welfare increases—the mechanism to achieve this outcome is different (in our model it can occur via easier search;

in their model it occurs only via more intense consumer preferences).¹¹

It can be shown that when the variety cost function is linear, there exists a key threshold, \bar{m} , that determines the influence of search on prices. When the search cost is sufficiently high that $m^* < \bar{m}$, then a decrease in the search cost increases equilibrium prices: with a high search cost the firms offer minimal variety, so a marginal expansion in variety creates a significant improvement in consumer welfare (again, net of prices and search costs). When the search cost is already low, such that $m^* > \bar{m}$, then further expansion in variety does not increase welfare enough to increase price. This result implies that poorly served markets (because of limited variety and high search costs) are likely to support price increases when technology is implemented to reduce search costs.

PROPOSITION 3. *In the parallel search model, while firm entry reduces the equilibrium firm profit, easier search does not necessarily reduce the equilibrium firm profit. If the equilibrium price increases as the search cost decreases, then the firms' equilibrium profit increases.*

So firm entry indeed always reduces the firms' equilibrium profit, because the operating cost savings of less product variety are not enough to compensate for the revenue loss due to a lower price and market share. However, easier search may not necessarily lead to a profit decline: when easier search increases the equilibrium price, the revenue increase driven by a higher price offsets the cost increase associated with higher product variety, and equilibrium profit increases. (Note that this is a sufficient but not necessary condition: it is possible for equilibrium profits to increase even if equilibrium prices decrease. In that case profits increase because of a sufficiently large increase in sales driven by the decline in the popularity of the no-purchase option.)

PROPOSITION 4. *In the parallel search model, while firm entry does not necessarily increase consumer and social welfare, easier search always increases consumer welfare and can increase social welfare.*

Proposition 4 indicates that the social welfare (consumer welfare and industry profit) can increase as search becomes easier. More striking, Proposition 4 reveals that consumers always benefit from lower search costs even if prices rise because broader assortments generate sufficiently higher utilities to outweigh any welfare loss due to higher prices. Furthermore, we find that firm entry may benefit neither

consumers nor society as a whole. These results are consistent with Brynjolfsson et al. (2003), who find that the consumer welfare gain from increased product variety available through electronic markets is significant and between 7 and 10 times as large as the gain from the increased number of competitors.

5. Competition with Sequential Search

With sequential search, upon observing a firm's offerings a consumer decides among three options: (1) purchase a product, (2) incur a search cost τ and engage in further search, or (3) discontinue the search process. As with parallel search, we assume that consumers anticipate a symmetric equilibrium (each firm chooses price p^* and assortment x^*).

Suppose a consumer is visiting firm i with price p_i and her utility of the best product offered by firm i is y . Given her expectation of $\{p, x\}$, it is worthwhile for the consumer with search cost τ to search another firm who offers x products with price p instead of purchasing the current best product if the expected value of search is greater than the search cost τ ; i.e.,

$$\int_{y-p_i+p}^{\infty} (\phi - (y - p_i + p)) dH(\phi, x) \geq \tau, \quad (7)$$

where $H(\phi, x)$ is the Cumulative Distribution Function (CDF) of the maximum utility of x products and

$$H(\phi, x) = \exp \left[-x \exp \left(- \left(\frac{\phi}{\mu} + \gamma \right) \right) \right].$$

As the left-hand side of (7) is decreasing in the term $y - p_i + p$ there exists a threshold $\bar{U}(x, \tau)$ for any given x and τ such that the consumer will choose to search another firm if $y - p_i + p \leq \bar{U}(x, \tau)$; i.e., $\bar{U}(x, \tau)$ is implicitly defined by

$$\int_{\bar{U}(x, \tau)}^{\infty} (\phi - \bar{U}(x, \tau)) dH(\phi, x) = \tau. \quad (8)$$

From (8), $\bar{U}(x, \tau)$ is increasing in x (because the left-hand side of (8) is increasing in x) and decreasing in τ (because the left-hand side of (8) is decreasing in $\bar{U}(x, \tau)$). Weitzman (1979) shows that this threshold search strategy is the optimal sequential search strategy.

The following lemma establishes a useful property of the search threshold.

LEMMA 5. *$H(\bar{U}(x, \tau), x)$ is independent of x , where $\bar{U}(x, \tau)$ is implicitly defined by (8).*

Based on Lemma 5, we write $H(\bar{U}(x, \tau), x)$ as $H(\tau)$ whenever it is convenient. Lemma 5 indicates that the probability that a consumer searches

¹¹ In our model, increasing μ increases m^* ; i.e., consumers search more. Also, the direct effect of μ on p is positive in equilibrium, holding search constant. Hence, our model exhibits the same effects as in Anderson and Renault (1999) with respect to the parameter that measures consumers' preference for variety.

another firm is independent of the firms' assortments if they all choose the same assortment breadth (the probability of search then depends only on the search cost). However, the probability of search does depend on the assortment choices when assortment breadths vary across firms, which is relevant when a firm is contemplating a deviation from equilibrium. Define $\rho(x, \tau) = \exp(-(\bar{U}(x, \tau)/\mu + \gamma))$. Because $H(\bar{U}(x, \tau), x) = \exp[-x\rho(x, \tau)]$, Lemma 5 implies that $x\rho(x, \tau)$ is also independent of x . Furthermore, $H(\tau)$ is decreasing in τ and $\rho(x, \tau)$ is increasing in τ .

We assume that consumers observe their no-purchase utility first. As a result, a consumer visits at least one firm only if her no-purchase utility is lower than $\bar{U}(x, \tau) - p$. Therefore, $H_0(\bar{U}(x, \tau) - p)$ is the fraction of consumers that actually enter the market, where H_0 is the CDF of the no-purchase utility and

$$H_0(\phi) = \exp\left[-\exp\left(-\left(\frac{\phi - u_0}{\mu} + \gamma\right)\right)\right].$$

Note that the probability that a consumer searches at least one firm does depend on the firms' assortment choices.

A symmetric equilibrium in this setting is a 3-tuple, $\{p^*, x^*, \bar{U}(x^*, \tau)\}$, such that consumers have no incentive to deviate from their search strategy and no firm has an incentive to deviate from its price and assortment strategy given their expectations. Suppose firm i 's choices are $\{p_i, x_i\}$ and all other firms' choices are $\{p, x\}$. If a consumer visits firm i , the probability she purchases at firm i is $1 - H(\bar{U}(x, \tau) + (p_i - p), x_i)$; otherwise she is equally likely to choose among the firms that have not been visited. The probability that firm i is visited first is $1/n$, and, obviously, the probability that it is not visited first is $(n - 1)/n$. If firm i is not visited first, then with probability $H(\tau)$ the consumer does not purchase at the first firm and visits firm i in her next search with probability $1/(n - 1)$. Thus, the probability that firm i is the second firm in the consumer's search is

$$\left(\frac{n-1}{n}\right)H(\tau)\left(\frac{1}{n-1}\right) = \frac{H(\tau)}{n}.$$

In general, the probability a consumer eventually visits firm i is

$$\sum_{j=1}^n \frac{H(\tau)^{j-1}}{j} = \frac{1}{n} \left(\frac{1 - H(\tau)^n}{1 - H(\tau)} \right). \quad (9)$$

Hence, with probability $H(\bar{U}(x, \tau) + (p_i - p), x_i)H(\tau)^{n-1}$ a consumer will visit all n firms in the market and still cannot find a product (including the no-purchase option) that has a higher utility than her search threshold $\bar{U}(x, \tau)$. In our analysis we assume that this probability is small, i.e., $H(\bar{U}(x, \tau) +$

$(p_i - p), x_i)H(\tau)^{n-1} \approx 0$, which also implies that the probability (9) is approximately equal to

$$\frac{1}{n(1 - H(\tau))}.$$

This is a mild assumption as long as the total assortment in the market is reasonably large and the number of firms is not too small.

Thus, the probability that a consumer visits firm i and also makes a purchase from firm i is

$$q_i(p_i, x_i | p, x, \tau) = \frac{H_0(\bar{U}(x, \tau) - p)[1 - H(\bar{U}(x, \tau) + (p_i - p), x_i)]}{n[1 - H(\tau)]}. \quad (10)$$

Firm i 's expected profit is

$$\Pi_i(p_i, x_i | p, x, \tau) = p_i q_i(p_i, x_i | p, x, \tau) - c(x_i). \quad (11)$$

The following proposition establishes, under a relatively mild condition (e.g., which holds for linear and quadratic $c(x)$), the existence and uniqueness of an equilibrium for our sequential search model.

PROPOSITION 6. Define

$$g(x) = \frac{c'(x) + xc''(x)}{x^2}.$$

If $g(x)$ is decreasing in x , there exists a unique symmetric equilibrium in the sequential search model, $\{p^*, x^*, \bar{U}(x^*, \tau)\}$, where $\bar{U}(x^*, \tau)$, p^* , and x^* are implicitly defined by the following:

$$\int_{\bar{U}(x^*, \tau)}^{\infty} (\phi - \bar{U}(x^*, \tau)) dH(\phi, x^*) = \tau, \quad (12)$$

$$p^* = \frac{\mu[1 - H(\tau)]}{x^* \rho(x^*, \tau) H(\tau)}, \quad (13)$$

and

$$x^* = \frac{\mu H_0(\bar{U}(x^*, \tau) - p^*)}{nc'(x^*)}. \quad (14)$$

PROPOSITION 7. In the sequential search model, while firm entry reduces product variety, x^* , and leaves price, p^* , unchanged, easier search raises x^* and reduces p^* . Furthermore, firm entry may not always increase the total variety available in the market, nx^* , while easier search always does.

The qualitative findings of the sequential search model are consistent with those of the parallel search model. For example, while firm entry reduces the equilibrium product variety, easier search increases the equilibrium product variety and the total market variety. Again, the difference between easier search and firm entry is driven by the market expansion effect of easier search. However, in the sequential search model, the market expansion effect of easier search is not strong enough to elicit an increase in the equilibrium price. (But price competition is not as severe as it would be without the market-expansion effect.)

PROPOSITION 8. *In the sequential search model, while firm entry reduces the equilibrium firm profit, easier search does not necessarily reduce the equilibrium firm profit. If the consumers' expected no-purchase utility u_0 is sufficiently high, easier search increases equilibrium profit.*

As Proposition 8 shows, firm entry still reduces the firms' profits in equilibrium. This occurs for two reasons. First, because firms respond to new entry by reducing product variety and maintaining prices, the market as a whole is less attractive to consumers. As a result, fewer consumers enter the market. Second, each firm's market share falls post-entry. Although prices stay unchanged, the firms' demand post-entry decreases enough to lead to lower revenues and profits.

In contrast, Proposition 8 indicates that, although easier search cannot elicit an increase in the equilibrium price, it can still lead to a profit increase, especially when the consumers' expected no-purchase utility u_0 is large. When u_0 is large, consumers are less likely to enter the market to search firms at all. In this scenario, the market expansion effect of easier search on the whole market (the term $H_0(\bar{U}(x, \tau) - p)$ in (10)) becomes stronger and more important to the firms' profitability. If u_0 is sufficiently large, this expansion of the total market size caused by easier search outweighs the price decline and the increase in product variety costs.

PROPOSITION 9. *In the sequential search model, while firm entry reduces consumer welfare and does not necessarily increase social welfare, easier search always increases both consumer and social welfare.*

The qualitative findings of the sequential search model concerning welfare are consistent with those of the parallel search model. Thus, the market expansion effect and its impact on assortments, profits, and welfare are robust to the consumer search model. Because the effect of easier search on price differs in the parallel and sequential models, we conclude that the effect of easier search on assortments, profits, and welfare is stronger than the effect on prices.

There are subtle differences between the parallel and sequential search models, but the complexity of the MNL functional forms make a detailed comparison of the two models difficult. However, for a given search cost, we can show that consumers search more in the sequential model than in the parallel model conditional that they choose to search; i.e., $E[N | N > 0] > m^*$, where N is the number of firms searched in the sequential model. Furthermore, $\partial m^* / \partial \tau \leq \partial E[N | N > 0] / \partial \tau < 0$; i.e., the amount of search in the parallel model is more sensitive to a change in the search cost than in the sequential model.

Thus, a reduction in the search cost τ generates a greater increase in search with parallel search than with sequential search, thereby leading to a greater expansion in assortment. If initially τ is sufficiently high, then the expansion of the assortment in the parallel model is sufficiently great to support a price increase, whereas in the sequential search model the expansion of the assortment is never enough to lead to an increase in the equilibrium price. Independent of search mode, as search costs decrease, consumers search more in equilibrium.

In summary, we find that lower search costs (a) lead to an increase in industry sales, (b) lead to broader assortments, (c) can increase social welfare, (d) can make consumers better off, even if prices increase, (e) increase search activity in equilibrium, (f) increase industry profit even if prices decrease, and (g) can increase profits for all firms. Our results are obtained because lower search costs lead consumers to search more, which expands each firm's potential demand, which leads firms to expand their assortment, which expands total industry demand and provides additional value to consumers that either mitigates the consequence of additional price competition due to increased search or even allows firms to raise their prices. Even if lower search costs reduce equilibrium prices, firm profits may rise because the market expansion effect allows them to sell more (i.e., the firms "make it up in volume"). In our model it is the no-purchase option that is the big loser because of lower search costs—consumer choice expands with lower search costs, so consumers are more likely to purchase something from some firm.

Our findings complement the work by Kuksov (2004), who, in a different model setup, finds that lower search costs (a) do not change overall industry sales, (b) do not lead to a change in assortment (as there is only one product variant in the market), (c) do not increase social welfare, (d) make consumers worse off if prices are increased, (e) do not lead to any search activity in equilibrium (i.e., every consumer visits exactly one firm), (f) increase industry profit only if prices increase, and (g) never increase both firm's profits; i.e., easier search benefits one firm at the expense of the other. Our results contrast with Kuksov (2004) in particular because in his model there is no market expansion effect—every consumer buys one unit because the no-purchase option is not available to them. In other words, consumers in his model search only over price whereas in our model they search over price and product attributes.¹² As a result,

¹² Put another way, in our model a consumer can be better off searching another firm even if the other firm charges the same price whereas in Kuksov (2004) a consumer can be better off from search only if the consumer can obtain a better price.

firms in his model set prices to prevent all search; i.e., consumers are indifferent between paying the price at the first firm visited and searching for a potentially lower price. Furthermore, in his model social welfare cannot increase and lower search costs cannot increase the profits of all firms because only one firm invests in socially wasteful differentiation merely to mitigate price competition.¹³

6. Conclusion

Our work explores multiproduct firms' decision making and profitability in the presence of changing search costs or firm entry. In our model firm entry intensifies competition, resulting in less variety, lower prices, and lower profits. Easier search (modeled as a lower search cost) also intensifies competition, as suggested by conventional wisdom (e.g., Bakos 1997). But we demonstrate that easier search also has a market expansion effect: if consumers search more, then a firm has access to more consumers. Because of this market expansion effect, which is not present with firm entry, firms in equilibrium invest in broader assortments. With more products to choose from, consumers are more likely to find products that match their ideal preferences, increasing the efficiency of the market, which may manifest itself in higher prices, more profits, and increased welfare.

In our model we treat search costs as exogenous, but in practice it is possible that firms may be able to influence consumer search costs via their actions; i.e., search costs may be partially endogenous. For example, retailers have some control over their location, either in physical space or in e-commerce space, and

¹³ The Kuksov (2004) result is actually similar to the Lal and Sarvary (1999) result. In Lal and Sarvary (1999) the introduction of lower search cost technology (the Internet) makes it more costly for consumers to shop a second firm. In Kuksov (2004) there are two cases, undifferentiated firms and differentiated firms. With undifferentiated firms one consumer segment has a choice of two retailers, neither of which the segment prefers over the other. As a result, the consumers in this segment experience no reduction in utility (net of price paid and search costs) by searching the other firm. In contrast, in the differentiated case consumers have a strict preference for the firms and they know which firm they prefer, again, net of prices paid and search costs. Naturally, they visit their preferred firm first. But now search is relatively more costly because visiting the other firm means a reduction in the consumers' utility (all else being equal, the consumer strictly prefers the first firm). In Lal and Sarvary (1999) it is an exogenous event that makes search relatively more expensive whereas in Kuksov (2004) it is an endogenous decision by one firm to make search relatively more expensive (the firm invests in differentiation). In our model, lower search costs indeed lower search costs—because firms expand their assortments, lower search costs make search relatively more attractive all else being equal.

our results suggest that locating more closely to competitors may be the prudent action. Doing so reduces consumer search costs, which allows firms to expand their assortments and possibly generate higher profits despite the more intense price competition.¹⁴

The advent of the Internet has clearly motivated the recent interest in the influence of search on market dynamics. Our results, as we have already stated, are consistent with an expansion of variety due to (in part) easier search associated with Internet-enabled markets. But our results are not specific to any one technology—our model suggests that any technology that reduces search costs should expand the available assortment in a market. For example, according to our model, the development of better roads (e.g., pavement), telecommunications (e.g., telephone), and transportation (e.g., automobiles) in the early part of the last century should have contributed to larger retail formats (broader assortments). The Internet represents only the latest technological advancement in this trend.

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Appendix

Outlines of proofs are provided in this appendix. See the Technical Appendix at <http://mktsci.pubs.informs.org> for more details.

PROOF OF PROPOSITION 1. To simplify notation, we use the following abbreviations: $\Pi_i = \Pi_i(p_i, x_i | p, x, m^*)$ and $q_i = q_i(p_i, x_i | p, x, m^*)$ and

$$q_i = \frac{x_i}{x_i + a(p_i)};$$

$$a(p_i) = \frac{(m^* - 1)x \exp(-p/\mu) + \exp(u_0/\mu)}{\exp(-p_i/\mu)}.$$

¹⁴ See Dudey (1990) and Wolinsky (1983) for other models of retail clustering. In these models firms choose only their location and not their assortments. As a result, consumers experience greater assortment in a retail cluster only because there are more firms in the cluster. Our model suggests that consumers would experience more assortment in a cluster because there are more firms and each firm carries a broader assortment.

Differentiate (4) to obtain the following first-order conditions:

$$(m^*/n)q_i - (m^*/n)p_i(1 - q_i)q_i/\mu = 0,$$

$$\frac{(m^*/n)p_i(1 - q_i)q_i}{x_i} - c'(x_i) = 0.$$

Note that firm i optimizes p_i and x_i given an expectation that consumers will search m^* firms and the other firms will choose $\{p, x\}$. A symmetric equilibrium then requires that the firms and consumers have correct expectations regarding everyone's behavior and that behavior is optimal given these expectations. Simplification of the first-order conditions yield (5) and (6). Evaluation of the Hessian determines that this solution is indeed a maximum. The symmetric equilibrium is unique because the first-order conditions have a unique solution (and hence the only candidate equilibrium). The condition on $c'(0)$ ensures an interior equilibrium. \square

PROOF OF PROPOSITION 2. The right-hand side of (6) is decreasing in n , so x^* is decreasing in n . From (5), p^* is increasing in x^* , so p^* is decreasing in n as well. It is straightforward to show that nx^* can be decreasing in n when $c(x) = cx$. It is not difficult to verify that the right-hand side of (6) is increasing in m^* , so x^* is increasing in m^* . Because m^* is decreasing in the consumer search cost τ , easier search increases x^* .

To illustrate that easier search can increase p^* , assume $c(x) = cx$. The equilibrium price is then implicitly defined by

$$p^* - \frac{\mu^2 m^2}{\mu m^2 - \mu m + nce^{(u_0 + p^*)/\mu}} = 0.$$

Define \hat{p} such that

$$2nce^{(u_0 + \hat{p})/\mu} = \mu m.$$

Note that $\partial \hat{p} / \partial m > 0$. From the implicit function theorem, the sign of $\partial p^* / \partial m$ is positive if $p^* > \hat{p}$; otherwise it is negative. It can be shown that there is a unique \bar{m} such that $p^* = \hat{p} = \mu \bar{m}(\bar{m} - 1/2)$. It follows that $\partial p^*(\bar{m}) / \partial m = 0$. Hence, for all $m < \bar{m}$, $\hat{p} < p^*$, which implies $\partial p^* / \partial m > 0$. Furthermore, for all $m > \bar{m}$, $\hat{p} > p^*$, which implies $\partial p^* / \partial m < 0$. Thus, for sufficiently high search costs, $m^* < \bar{m}$, and the equilibrium price is increasing as search becomes easier (as m^* increases), and for sufficiently low search costs, $m^* > \bar{m}$, and the equilibrium price is decreasing as search becomes easier (as m^* continues to increase). \square

PROOF OF PROPOSITION 3: Differentiate the firm's equilibrium profit with respect to n :

$$\frac{\partial \Pi^*}{\partial n} = \left((\mu m^* - nx^*c'(x^*)) \frac{\partial x^*}{\partial n} [\mu m^* x c''(x) + nx^*c'(x)^2] \right. \\ \left. + \mu m^* x^* c'(x^*) \frac{\partial (nx^*c'(x^*))}{\partial n} \right) \\ \cdot (\mu m^* - nx^*c'(x^*))^{-2} < 0.$$

To ensure $p^* \geq 0$, we must have $\mu m^* - nx^*c'(x^*) \geq 0$. Because p^* is decreasing in n , $nx^*c'(x^*)$ must be decreasing in n . Together with the convexity of $c(x^*)$ and $\partial x^* / \partial n < 0$, the inequality follows.

Differentiating the firm's equilibrium profit with respect to m^* :

$$\frac{\partial \Pi^*}{\partial m^*} = \left(\mu nx^*c'(x^*) \left[m^*(c'(x^*) + x^*c''(x^*)) \frac{\partial x^*}{\partial m^*} - x^*c'(x^*) \right. \right. \\ \left. \left. + \frac{c'(x^*)(\mu m^* - nx^*c'(x^*))}{\mu} \frac{\partial x^*}{\partial m^*} \right] \right) \\ \cdot (\mu m^* - nx^*c'(x^*))^{-2}. \quad (15)$$

The above derivative cannot be signed unambiguously. Differentiating (5) with respect to m^* and collecting terms, we have

$$\frac{\partial p^*}{\partial m^*} = \frac{\mu^2 n [m^*(c'(x^*) + x^*c''(x^*)) (\partial x^* / \partial m^*) - x^*c'(x^*)]}{(\mu m^* - nx^*c'(x^*))^2}.$$

The sign of $\partial p^* / \partial m^*$ is determined by the sign of the term in the square brackets in the numerator, which is the same as the first term in the square brackets in the numerator of (15). As we discussed above, the second term in the square brackets in the numerator of (15) is always positive. Therefore, if $\partial p^* / \partial m^* \geq 0$, it is sufficient to ensure $\partial \Pi^* / \partial m^* \geq 0$. This implies that if easier search leads to a price increase in the market, it will surely increase the equilibrium firm profit as well. \square

PROOF OF PROPOSITION 4. In the parallel search model, equilibrium consumer welfare is

$$CW = \mu \ln [m^* x^* \exp(-p^*/\mu) + \exp(u_0/\mu)] - m^* \tau,$$

where the first term is the expected utility of searching m^* firms, which is the expected value of the maximum of the utilities of the no-purchase option and m^*x^* products with price p^* offered by m^* firms, and the second term is the cost of searching m^* firms. Because $m^* = \min[n, \max(1, \mu/\tau)]$, $m^*\tau$ is nonincreasing in τ . Thus, the sign of $\partial CW / \partial m^*$ is the same as the sign of $\partial (\mu \ln m^* x^* - p^*) / \partial m^*$. Differentiating, we find that to show $\partial (\mu \ln m^* x^* - p^*) / \partial m^* \geq 0$, it is sufficient to show $\partial p^* / \partial m^* \leq \mu / m^*$. Differentiating (6) and using $\partial x^* / \partial m^* \geq 0$, we find that $\partial p^* / \partial m^* \leq \mu / m^*$ is true. Thus, that consumer welfare always increases as search costs decrease. The social welfare in the parallel search model is the sum of consumer welfare and the total profit for all n firms. Because we have shown in Proposition 3 that equilibrium firm profit can increase as search costs decrease, the social welfare also can increase. Numerical studies show that both consumer and social welfare is not monotone in n ; i.e., firm entry may reduce or increase consumer and social welfare. \square

PROOF OF LEMMA 5. To show that $H(\bar{U}(x, \tau), x) = \exp[-x \exp(-(\bar{U}(x, \tau)/\mu + \gamma))]$ is independent of x , it is sufficient to show that the term in the square bracket is inde-

pendent of x . Differentiating the term with respect to x , we have

$$\frac{\partial [x \exp(-(\bar{U}(x, \tau)/\mu + \gamma))]}{\partial x} = \exp(-(\bar{U}(x, \tau)/\mu + \gamma)) \left[1 - \frac{x}{\mu} \frac{\partial \bar{U}(x, \tau)}{\partial x} \right].$$

Thus, we just need to show that $\partial \bar{U}(x, \tau)/\partial x = \mu/x$. Note that $\bar{U}(x, \tau)$ is the solution of the following equation

$$\Psi(x) = \int_{\bar{U}(x, \tau)}^{\infty} (\phi - \bar{U}(x, \tau)) dH(\phi, x) = \tau.$$

Differentiating by using Leibnitz's rule and implicit function theorem, we have

$$\frac{\partial \bar{U}(x, \tau)}{\partial x} = - \left(\frac{\mu}{x} \right) \frac{\int_{\bar{U}(x, \tau)}^{\infty} \frac{\partial[(\phi - \bar{U}(x, \tau))h(\phi, x)]}{\partial \phi} d\phi - \int_{\bar{U}(x, \tau)}^{\infty} h(\phi, x) d\phi}{\int_{\bar{U}(x, \tau)}^{\infty} h(\phi, x) d\phi} = \frac{\mu}{x},$$

because

$$\int_{\bar{U}(x, \tau)}^{\infty} \frac{\partial[(\phi - \bar{U}(x, \tau))h(\phi, x)]}{\partial \phi} d\phi = [\phi - \bar{U}(x, \tau)]h(\phi, x)|_{\bar{U}(x, \tau)}^{\infty} = 0. \quad \square$$

PROOF OF PROPOSITION 6. Let $\rho(p_i - p, x, \tau) = \exp(-((\bar{U}(x, \tau) + (p_i - p))/\mu + \gamma))$. To simplify notation, we use the following abbreviations: $\rho(p_i) = \rho(p_i - p, x, \tau)$, $H(x) = H(\bar{U}(x, \tau), x)$, and $H(p_i, x_i) = H(\bar{U}(x, \tau) + (p_i - p), x_i)$. Furthermore, let

$$A = \frac{H_0(\bar{U}(x, \tau) + p)}{n(1 - H(\bar{U}(x, \tau), x))} \quad \text{and} \quad Q_i = [1 - H(p_i, x_i)].$$

Differentiating (11), we obtain the following first-order conditions:

$$AQ_i + p_i A \frac{\partial Q_i}{\partial p_i} = 0, \\ p_i A \frac{\partial Q_i}{\partial x_i} - c'(x_i) = 0.$$

Because for any symmetric equilibrium $\{p^*, x^*\}$ there exists a unique $\bar{U}(x^*, \tau)$ satisfying (12), it is sufficient to show the existence and uniqueness of a $\{p^*, x^*\}$ equilibrium for a given $\bar{U}(\cdot)$. To prove the existence of the equilibrium we show that firm i 's expected profit function (11) is jointly quasi-concave in $\{p_i, x_i\}$. To establish uniqueness, we show that if function $g(x) = (c'(x) + xc''(x))/x^2$ is decreasing in x , there exists a unique positive $\{p^*, x^*\}$ that solves the first-order conditions of all firms simultaneously. \square

We need the following lemma for the proof of Proposition 7.

LEMMA 10. For a given x , $\rho(x, \tau)H(\tau)/(1 - H(\tau))$ is decreasing in τ .

PROOF. Define functions $K(\omega) = \exp(-x \exp(-\omega))$, where x is a constant and $T(\omega) = \exp(-\omega)K(\omega)/(1 - K(\omega))$. Thus, $K'(\omega) = x \exp(-\omega)K(\omega)$ and $T(\omega) = K'(\omega)/(x(1 - K(\omega)))$. We then have

$$T'(\omega) = \frac{x \exp(-\omega)K(\omega)[x \exp(-\omega) + \exp(-x \exp(-\omega)) - 1]}{x(1 - K(\omega))^2} \geq 0.$$

The inequality follows from the fact that function $f(\epsilon) = \epsilon + \exp(-\epsilon) - 1$ is convex and minimized at $\epsilon = 0$ with $f(0) = 0$. Therefore, $T(\omega)$ is an increasing function. The lemma follows from $\rho(x, \tau)H(\tau)/(1 - H(\tau)) = T(\bar{U}(x, \tau)/\mu + \gamma)$, and $\bar{U}(x, \tau)$ is decreasing in τ . \square

PROOF OF PROPOSITION 7. From (13), it is clear that p^* is independent of n . Condition (14) can be written as

$$w(x^*) = x^* c'(x^*) - \frac{\mu}{n} H_0(\bar{U}(x^*, \tau) - p^*) = 0. \quad (16)$$

Because $w(x^*)$ is increasing in n and quasi-convex with $w(0) = 0, x^*$, which is the unique positive solution of (16), is decreasing in n . To illustrate the possibility that nx^* can be decreasing in n , consider a linear cost function $c(x) = cx$ with $c'(x) = c$. For this linear cost function, defining $v^* = nx^*$ and multiplying both sides of equation (16) by n , it yields $cv^* - \mu H_0(\bar{U}(v^*/n, \tau) - p^*) = 0$. Because the right-hand side of the equation is quasi-convex in v^* and increasing in n , $v^* = nx^*$, which is the unique positive solution of the above equation, is decreasing in n . By Lemma 10, $\rho(x^*, \tau)H(\tau)/(1 - H(\tau))$ is decreasing in the search cost τ . Thus, from (13), p^* is increasing in τ . Because $w(x^*)$ in condition (16) is increasing in τ and quasi-convex with $w(0) = 0, x^*$ is decreasing in τ . \square

PROOF OF PROPOSITION 8. Differentiating the firm's equilibrium profit with respect to n and arranging terms, we have

$$\frac{\partial \Pi^*}{\partial n} = \left(\frac{1 - H(\tau) - x^* \rho(x^*, \tau)H(\tau)}{x^* \rho(x^*, \tau)H(\tau)} \right) c'(x^*) \frac{\partial x^*}{\partial n} + \frac{[1 - H(\tau)]x^* c''(x^*)}{x^* \rho(x^*, \tau)H(\tau)} \frac{\partial x^*}{\partial n} \leq 0.$$

The inequality follows from $1 - H(\tau) - x^* \rho(x^*, \tau)H(\tau) \geq 0$, $c(x^*)$ is convex, and $\partial x^*/\partial n < 0$.

Define $\theta(\bar{U}(x^*, \tau)) = (1 - H(\tau))/(\rho(x^*, \tau)H(\tau))$. Differentiating the firm's equilibrium profit with respect to $\bar{U}(x^*, \tau)$ and abbreviating notation, we have

$$\frac{\partial \Pi^*}{\partial \bar{U}} = \left(\frac{1 - x^* \rho(x^*, \tau) - H(\tau) + \rho(x^*, \tau)(1 - H(\tau))(\partial x^*/\partial \bar{U})}{\rho(x^*, \tau)H(\tau)} \right) \cdot c'(x^*) + \theta c''(x^*) \frac{\partial x^*}{\partial \bar{U}}. \quad (17)$$

The above derivative cannot be signed unambiguously. Let us consider the linear cost function $c(x) = cx$ with $c'(x) = c$. Then, (17) becomes

$$\frac{\partial \Pi^*}{\partial \bar{U}} = \left(\frac{1 - x^* \rho(x^*, \tau) - H(\tau) + x^* \rho(x^*, \tau)(1 - H(\tau))(\rho_0(x^*, \tau)/\mu)}{\rho(x^*, \tau)H(\tau)} \right) \cdot c'(x^*) + \theta c''(x^*) \frac{\partial x^*}{\partial \bar{U}}.$$

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For $\rho_0(x^*, \tau)/\mu = 1$, the numerator of the term inside the parentheses becomes $1 - H(\tau) - x^*\rho(x^*, \tau)H(\tau) \geq 0$, which makes $\partial\Pi^*/\partial\bar{U}(x^*, \tau) \geq 0$. All terms in the numerator are independent of x^* except the term $\rho_0(x^*, \tau)$. Because $w(x^*)$ in condition (34) is increasing in u_0 and quasi-convex with $w(0) = 0$, x^* is decreasing in u_0 . Hence, $\rho_0(x^*, \tau) = \exp(-((\bar{U}(x^*, \tau) - p^* - u_0)/\mu + \gamma))$ is increasing in u_0 . Thus, the numerator inside the parentheses is more likely to be positive, in which case $\partial\Pi^*/\partial\bar{U} \geq 0$ is more likely to hold, which implies that easier search does not necessarily reduce the equilibrium firm profit. \square

PROOF OF PROPOSITION 9. Using the definition of $\bar{U}(x^*, \tau)$, the conditional expected net utility of a consumer who enters the market and searches is

$$E[\text{consumer utility} \mid \text{search}] = \frac{\int_{\bar{U}(x^*, \tau)}^{\infty} \xi dH(\xi) - \tau}{(1 - H(\tau))} - p^* \\ = \bar{U}(x^*, \tau) - p^*.$$

Thus, the expected consumer welfare is

$$CW = H_0(\bar{U}(x^*, \tau) - p^*)(\bar{U}(x^*, \tau) - p^*) + \int_{\bar{U}(x^*, \tau) - p^*}^{\infty} \xi h_0(\xi) d\xi \\ = E[\max[\bar{U}(x^*, \tau) - p^*, U_0]],$$

where U_0 is the utility of consumer's no-purchase option, which is a random variable with CDF H_0 and pdf h_0 . It is clear that $E[\max[\bar{U}(x^*, \tau) - p^*, U_0]]$ is increasing in $\bar{U}(x^*, \tau) - p^*$. Therefore, consumer welfare CW increases as search cost τ decreases because $\bar{U}(x^*, \tau)$ increases and p^* decreases as τ decreases. As more firms enter the market, $\bar{U}(x^*, \tau)$ decreases because x^* decreases, and p^* is unchanged. As a result, $\bar{U}(x^*, \tau) - p^*$ decreases. Therefore, firm entry always reduces consumer welfare.

The social welfare is

$$SW = H_0(\bar{U}(x^*, \tau) - p^*)\bar{U}(x^*, \tau) + \int_{\bar{U}(x^*, \tau) - p^*}^{\infty} \xi h_0(\xi) d\xi - nc(x^*).$$

To simplify notation, we use the following abbreviations: $h_0 = h_0(\bar{U}(x^*, \tau) - p^*)$, $H_0 = H_0(\bar{U}(x^*, \tau) - p^*)$, and $\bar{U} = \bar{U}(x^*, \tau)$. Differentiating with respect to x^* and collecting terms, we have

$$\frac{\partial SW}{\partial x^*} = H_0 \frac{\partial \bar{U}}{\partial x^*} + p^* h_0 \frac{\partial \bar{U}}{\partial x^*} - nc'(x^*).$$

From (14), we know that $H_0 = nx^*c'(x^*)/\mu$, and in the proof of Lemma 5 we have shown that $\partial\bar{U}/\partial x^* = \mu/x^*$. Therefore, $\partial SW/\partial x^*$ simplifies to

$$\frac{\partial SW}{\partial x^*} = p^* h_0 \frac{\partial \bar{U}}{\partial x^*} \geq 0.$$

Because x^* increases as search costs decrease, it implies that social welfare also increases as search costs decrease. Numerical studies show that social welfare is not monotone in n ; i.e., firm entry may reduce or increase social welfare. \square

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