Capacity Investment by Competitive Start-ups*

Robert Swinney, Gérard Cachon and Serguei Netessine†

The Wharton School, University of Pennsylvania

Philadelphia, PA, 19104

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Abstract

Start-up firms can have objectives that differ greatly from more established companies because of the threat of bankruptcy. In order to understand the effect that such risk imposes on the decisions of a start-up firm, we analyze games in which a start-up maximizes the probability of survival while an established firm maximizes expected profits. Competition is modeled as a capacity investment duopoly game with endogenous pricing, and the capacity game is embedded in a higher level entry order game in which firms must strategically choose to commit to capacity "early" or "late." To determine how competition involving start-ups differs from competition between established firms, we consider three potential combinations of players: two established firms, an established firm and a start-up, and two start-ups. In each case, we analyze the resulting equilibria and compare capacity investments, profits, and survival probabilities. We find that when two established firms with similar investment costs compete, the dominant strategy is for both firms to commit to capacity early. If a start-up competes against an established firm or if two start-ups compete with one another, the only equilibria are sequential; one firm leads, and the other firm follows, and both cases are possible. This finding helps explain why new markets/technologies are often pioneered by start-up firms. Surprisingly, a start-up firm may invest in higher capacity, achieve higher expected profit and enjoy lower variance in payoffs than an established firm would under similar conditions. We conclude that a start-up's

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†rswinney@wharton.upenn.edu, cachon@wharton.upenn.edu, netessine@wharton.upenn.edu

preference to maximize survival probability significantly influences the nature of competition in these markets.

1 Introduction

A darling of the Internet revolution, on-line grocer Webvan raised $375 million in its initial public offering. Expansion of the firm during its early years was rapid. In 1999, Webvan signed a $1 billion contract with Bechtel to build automated distribution and delivery warehouses across the country at a cost of $30 million each. In 2000, Webvan acquired Homegrocer, its chief rival, in an all-stock deal valued at $1.2 billion. When demand did not meet the massive capacity that Webvan had invested in through construction and mergers, Wall Street began to sour on the company. In July 2001, after months of plummeting stock prices, Webvan declared bankruptcy and never recovered. Though the dizzying rise and fall of Webvan was a part of the larger dot com bubble of the late 1990s, the story of the grocery delivery service provides a poignant example of how mismanagement of capacity and inventory can have tragic consequences for a start-up company. The company over-invested in capacity and failed to recognize the competitive interactions that it faced with brick-and-mortar groceries. When demand for on-line grocery services turned out to be lower than expected, Webvan was essentially ruined, whereas the existing market players (the established groceries) lost very little, and in fact currently dominate the market.

Despite the failure of Webvan, it is often the case that an aggressive start-up taking a leadership role in a new market encounters great success over established rivals. Christensen and Bower (1996), in their comprehensive study of the disk drive industry, provide numerous examples of start-ups that triumph over established competitors in new markets. By committing to early entry in these markets, often with technology pioneered but ignored by established firms, start-ups in the disk drive industry take a leadership role and capture a dominant market share before the more established competitors can mount an effective defense. The work of Christensen and Bower (1996), however, has drawn some criticism suggesting that, in many cases, start-ups pioneering new markets do not succeed, and a late-coming established firm performs better (see Daneels (2004)). One technology that fits such an example is the Iridium global satellite phone system. Moreover, the work of Lieberman (2005) shows empirically, using Internet start-up companies as an example,

that there are limited benefits to being the first-mover. Thus, it appears that there is no single dominant entry timing strategy for new markets, a fact supported both by anecdotal evidence and empirical work in the marketing literature such as Green et al. (1995). These phenomena leave us with many intriguing questions. How does competition between a start-up and an established firm differ from that between two established firms or, for that matter, between two start-ups? What determines the order of entry into new markets? Do capacity investment decisions made by start-up firms and established firms differ? These are the questions we investigate in this paper using a stylized, game theoretic model that seeks to capture the behavior and environment of start-up firms.

In a competitive world, the operational problems faced by a start-up can be complex. Over-investment in capacity can lead to the exhaustion of capital streams, with disastrous results to the firm if demand is too low, whereas established competitors may face less dire consequences. Conversely, under-investment in capacity may place the firm at a strategic disadvantage with its competitors or yield unacceptably poor financial performance. A delicate balance needs to be struck between possibly conflicting goals to behave strategically in a competitive marketplace and to avoid bankruptcy. It is precisely this balance that we seek to achieve by proposing a model consisting of three primary characteristics: one, start-up firms maximize the probability of survival, as opposed to expected profit; two, the market is uncertain and price-sensitive, and three, firms are capable of credible capacity commitment and enter the market strategically. Although it may appear that the threat of bankruptcy is, perhaps, of secondary importance to most firms, there is empirical evidence suggesting otherwise. Dunne et al. (1989) find that 39.7% of U.S. manufacturing plants fail within the first 5 years of existence. Bartelsman et al. (2005) use data for 10 OECD countries to show that about 20% of all firms exit most markets and about 20-40% of entering firms fail within the first 2 years of life. Hillegeist et al. (2004) find that, although the overall rate of bankruptcy for the U.S. economy between 1980 and 2000 was low (i.e., the rate of bankruptcy for all firms, including start-ups and established firms), it was quite high for certain industries including coal (19.23%), retail (11.48%) and textiles (15.83%). Romanelli (1989) cites a study by Dun and Bradstreet showing that 53% of all failures and bankruptcies posted in 1980 occurred less than five years from a firm's founding; fully 80% of businesses failed in less than ten years. In the same paper, 59.3% of firms manufacturing minicomputers between 1957 and 1981 failed. Given

	Firm i Early	**Firm i Late**
Firm j Early	*Simultaneous competition*	*Firm j as leader*
Firm j Late	*Firm i as leader*	*Simultaneous competition*

Table 1: Normal form representation of the entry order game.

these significant numbers, it is somewhat surprising that there have been relatively few attempts to model the behavior of start-ups or, more generally, the behavior of companies facing the risk of bankruptcy.

To determine how competitive interactions for a start-up differ from those of an established firm, we analyze three distinct games: competition between two established firms that maximize expected profits, competition between a profit maximizing firm and a survival maximizing start-up, and competition between two survival maximizing start-ups. In each case, we allow firms to commit strategically to capacity in either an "early" or "late" period. The normal form representation of this game is depicted in Table 1.

We find that a start-up is averse to simultaneous competition: in a game with one or two start-ups the only equilibria are sequential. This finding is consistent with observation that new markets are often pioneered by start-up firms. An established firm, however, may or may not be averse to simultaneous competition. In particular, if two established firms compete and their capacity costs are close enough, they prefer to compete simultaneously. On the other hand, if two established firms have dissimilar costs, contrary to our intuition, the firm with the higher capacity cost leads in the game. Thus, the equilibria of the entry order game depicted above are fundamentally altered by the presence of a start-up. Furthermore, we find that an established firm with capacity costs that far exceed the costs of its rival is often forced out of the market altogether, while a start-up is never forced out *ex ante*. Finally, through counterexamples we show that, contrary to intuition, a start-up firm may invest in higher capacity, achieve higher expected profit and enjoy lower variance in payoffs than an established firm would under similar conditions.

The details of our model and a survey of related literature follow in §2. In §§3 − 5, we analyze competition between two expected profit maximizing firms, a profit maximizing firm and a survival maximizing start-up, and two survival maximizing start-ups, respectively. In §6 we provide illustrative numerical examples and in §7 we summarize our main findings.

	Firm i Early	**Firm i Late**
Firm j Early	$U_j\left(K_j^n, K_i^n\right), U_i\left(K_j^n, K_i^n\right)$	$U_j\left(K_j^l, K_i^f\right), U_i\left(K_j^l, K_i^f\right)$
Firm j Late	$U_j\left(K_j^f, K_i^l\right), U_i\left(K_j^f, K_i^l\right)$	$U_j\left(K_j^n, K_i^n\right), U_i\left(K_j^n, K_i^n\right)$

Table 2: Normal form representation of the entry order game with payoffs.

2 Model and Related Literature

When analyzing behavior of a start-up company, we emphasize the joint importance of demand and competitive conditions for young firms' survival likelihood (see citations and extensive discussion in Romanelli (1989) about the importance of these aspects as reflected in many strategic management studies). To keep the model parsimonious, we allow up to two firms in the market and either firm may commit to capacity investment in one of two periods, which we label "early" and "late" (the decision not to invest is also considered). This choice is purely strategic in nature; there is no explicit penalty, cost, or discount factor assigned to committing in either period. The commitment to enter is credible and irreversible which is plausible since the firm entering early invests in capacity K. Thus, there are three subgames to the entry order game, each representing a different order of moves. Whether both firms commit early or both firms commit late, the outcome is identical: the firms invest in capacity simultaneously and we seek the Nash equilibrium of this capacity investment game. When firm i commits early and firm j commits late, firm i is the leader in a sequential game; thus, this scenario corresponds to a Stackelberg game with firm i as the first mover and firm j as the follower, and vice-versa if firm j leads. Call K_i the capacity of firm i, and K_i^g the equilibrium capacity of firm i in game g, $g \in \{n, l, f\}$ corresponding to firm i competing simultaneously in a Nash game, leading, and following, respectively. Then the entry order game is represented in Table 2 (each entry in the table corresponds to the utility functions of both players).

Each of the two firms is either a start-up (hereafter denoted by "she" and the subscript s, for "survival maximizing") or an established firm ("he" and the subscript p, for "profit maximizing"). We assume that firms produce identical, fully substitutable products. When entering the market, firm $i \in \{s, p\}$ invests into capacity K_i at a linear unit cost c_i which may include financing costs. We assume throughout that both firms are capable of financing any level of capacity installation through financial markets. Conventional wisdom suggests that start-ups should be resource constrained. We do not consider this possibility because, if such a constraint is binding, it becomes trivial to

incorporate into the game. Furthermore, we believe that the intuition that start-ups enter "small" is made in comparison to incumbent firms that already have a presence in the industry. In this paper, we compare the scale of entry employed by start-ups relative to the diversifying entry of established firms into new markets. This idea is explored empirically by Hariharan and Brush (1999), who find that start-ups enter with larger scale plants than diversifying established firms.

To be able to capture a variety of practical situations, we make no assumption on the ordering of c_p and c_s; the start-up may have higher or lower capacity costs than the established firm. After both firms enter, uncertain demand is realized and the firms choose to costlessly produce quantities Q_i to maximize their utility functions. At the conclusion of the selling season, the firm must cover a fixed, exogenous amount of operating expenses α_i, which may include various costs such as overhead, pre-existing loans, or the development costs for a new technology. Both firms are fully aware of one another's cost structures so that there is no private information in the model.

The market is price sensitive, and uncertain demand is modeled as a stochastic inverse demand curve. In particular, we use an isoelastic inverse demand curve with the market price $P(Q_i, Q_j) = A(Q_i + Q_j)^{-\gamma}$, $\gamma \in (0, 1)$, where the intercept A is a random variable with mean μ_A. If we define $Q = Q_i + Q_j$ to be the total quantity of the good in the market, this demand function corresponds to a demand curve $Q = A^{1/\gamma} P^{-1/\gamma}$, hence the price elasticity of demand is $\frac{\partial Q}{\partial P} \frac{P}{Q} = -1/\gamma \in (-\infty, -1)$. Higher values of γ imply less elastic demand and less price sensitivity. Such demand curves have long been customary in economics, but are only now fueling a growing stream of research in operations, particularly with stochastic elements and in competitive settings. Dada and Petruzzi (1999) include a general discussion of modeling price as a function of supply in operational problems. Goyal and Netessine (2004), Plambeck and Taylor (2005), and Anand and Girotra (2005) all examine competitive models wherein the underlying market mechanism is a downward sloping linear inverse demand curve with a random intercept. Isoelastic demand functions like those utilized in this paper result from consumers having quasi-linear preferences. Monahan et al. (2004) introduce a newsvendor model with an isoelastic inverse demand curve in a multiperiod setting in the absence of competition. Chod and Rudi (2005) model two firms that face isoelastic demand, invest into capacity competitively and thereafter cooperate by trading excess inventories. Xu and Hopp (2004) discuss isoelastic demand function in great detail and use it to model a game among firms that make procurement decisions once and then compete dynamically while demand uncertainty evolves

according to the geometric Brownian motion process. None of these papers considers bankruptcy-averse behavior.

In the last (production) stage, the profit function for firm $i \in \{s, p\}$ facing competitor $j \neq i$ is:

$$\pi_i = \max_{K_i \geq Q_i \geq 0} A \left(Q_i + Q_j\right)^{-\gamma} Q_i$$

With isoelastic demand and no marginal production cost, production clearing is the optimal strategy; the optimal solution for the start-up is to produce $Q_s^* = K_s$ while the established firm produces $Q_p^* = K_p$, see Chod and Rudi (2005). The absence of production costs in the model is a standard assumption in the literature which is justified in industries where the greatest expense is in the installation of capacity. We refer to Monahan et al. (2004) and Xu and Hopp (2004) for detailed discussions of isoelastic demand curves, their advantages and limitations.

To motivate the concept of survival probability maximization, consider the following model. If a firm cannot cover its end of period costs, it defaults on any outstanding loans and incurs a penalty D_i. The objective function of firm $i \in \{s, p\}$ in the capacity game is then:

$$\Pi_i = \max_{K_i \geq 0} \mathbb{E}\left[A \left(K_i + K_j\right)^{-\gamma} K_i - c_i K_i\right] - D_i \Pr\left(A \left(K_i + K_j\right)^{-\gamma} K_i < \alpha_i + c_i K_i\right). \tag{1}$$

This model is first introduced in the seminal paper by Greenwald and Stiglitz (1990) and is shown to produce behavior consistent with that of a risk-averse firm by Walls and Dyer (1996). As the latter paper points out, "in a homogeneous industry where investment projects have equivalent properties ... competition for scarce capital requires the firm to obtain an appropriate trade-off between maximization of expected profits and the probability of bankruptcy[1] (or financial distress)..."

If D_i is small compared to the assets of the firm or if the probability of default is very low, then the firm may safely ignore the last term and simply maximize expected profits. We therefore assume that the established firm p solves the following problem in the capacity investment game:

$$\Pi_p = \max_{K_p \geq 0} \mu_A \left(K_p + K_s\right)^{-\gamma} K_p - c_p K_p. \tag{2}$$

[1] From the accounting/financial point of view the meaning of the word "bankruptcy" is often complex and does not necessarily imply that the company fails. When using this term we simply imply that the company becomes insolvent and ceases to exist due to the negative cash-flow.

If the default penalty D_i is large compared to the assets of the firm and would result in financial ruin, or if the probability of default is high (either of which is likely to be the case for a start-up), the penalty term dominates the expression; the maximization problem may then be thought of as approximately equal to minimizing the probability of default or, equivalently, maximizing the probability of survival. We therefore assume that the start-up s solves the following problem in the capacity investment game:

$$\Psi_s = \max_{K_s \geq 0} \Pr\left(A\left(K_s + K_p\right)^{-\gamma} K_s \geq \alpha_s + c_s K_s\right). \tag{3}$$

Effectively, the start-up maximizes the chance of ending the game with a profit level (after accounting for capacity costs) of at least α_s; consequently, we frequently refer to α_s as the target capital or profit level. This dichotomization of the objective function allows us to obtain sharper results and agrees in spirit with the results of Walls and Dyer (1996) who empirically estimate the parameters of a risk-averse utility function and find that firm size is negatively correlated with the degree of risk aversion in petroleum exploration firms. The result that large firms are less risk averse than small firms implies that for smaller firms, the bankruptcy portion of the objective function carries greater weight.

An alternative assumption that can be made regarding start-up's objective function is the maximization of a risk averse utility function (Eeckhoudt et al. (1995), Agrawal and Seshadri (2000), Chen and Federgruen (2000), Van Mieghem (2003a) and Van Mieghem (2003b)). The drawback of this assumption is that a risk-averse decision maker dislikes gains (upside risk) as well as losses (downside risk) which is unlikely for a start-up firm. Chen et al. (2003) compare and contrast different criteria for modeling risk-aversion in inventory and pricing management, including risk averse utility and Conditional Value at Risk (CVaR), an alternative to risk aversion that essentially penalizes only for downside risk. We feel that, since starting a company is by definition a risky proposition, it is unlikely that owners or managers are strictly risk-averse. It might even be the case that they are risk seeking, at least in the domain of gains (see, for example, Daneels (2004)). Such behavior could lead to any form of risk-averse utility performing rather poorly as a descriptive model of a start-up's behavior, including CVaR. Consequently, several recent works examine alternative optimization criteria for start-up firms which are in the spirit of

our assumption. Babich and Sobel (2004) examine a model wherein a bankruptcy-averse start-up maximizes expected IPO value by adopting a monotone threshold rule in revenue, assets, and profit. Li et al. (2004) propose a model wherein a firm maximizes the expected present value of dividends net of capital subscriptions by controlling inventory levels and dividend payouts. Archibald et al. (2002) propose survival probability maximization criterion in an infinite horizon newsvendor model. The same approach is used in Swinney (2004) in a finite-horizon newsvendor model. None of these papers considers the effect of competition, which is the key focus of our paper.

In economics, Spence (1977) pioneered the stream of research on the competitive capacity investment and market entry. The most relevant paper to our work in this stream is Gal-Or (1985) and several follow-ups that endogenize the sequence of moves in the game and discuss first and second mover advantages. Another stream of work analyzes the effects of risk sensitivity on traditional competitive models. For example, Bhatt and Sobel (2004) explore the effects of risk-averse (exponential) utility in sequential, dynamic oligopoly games. However, to the best of our knowledge, none of these papers considers bankruptcy-averse firms.

Our model is stylized and, for the most part, applies to firms that need to make significant capacity/inventory investments to operate. Examples of such firms are Amazon.com, Netflix, Webvan, Iridium and others. Often, however, a start-up company is founded based on a technological advancement. Our model is less applicable to such situations. Furthermore, our model is static while the inherently dynamic nature of competition may be interesting. For example, after it becomes clear that the new market is of significant size, new firms (both established firms and start-ups) may enter. Another possibility we do not model is that the first entrant might want to build a high-enough capacity to prevent further entry and enjoy monopoly profits in subsequent time periods (see the stream of work pioneered by Spence (1977)). Finally, the strategic management literature cites many reasons for first-mover advantages that we do not model. These include technological leadership, preemption of scarce assets and switching costs (see Lieberman and Montgomery (1988)).

3 Competition Between Established Firms

In this section we examine a model with two established firm maximizing their respective expected profits. This analysis provides a basis for comparison with duopolies involving one or two start-ups, and allows us to infer how competition involving a start-up is fundamentally different from that involving only established firms. Since both firms are expected profit maximizers in this section, we drop the subscripts p and s in favor of the subscripts i and j. Both firms solve the optimization problems described in (2). As a preliminary observation, note that, as firm j's capacity investment increases, the optimal expected profit of firm i decreases and this result holds regardless of the objective function of firm j. This observation will become useful shortly.

3.1 Simultaneous Competition

First, we consider a simultaneous move game between established firms. Our first result shows that an equilibrium to this game exists and is unique.

Lemma 1 *When two established firms compete simultaneously, a Nash equilibrium in the capacity game exists and is unique. If $c_j/c_i > 1-\gamma$, equilibrium capacities are:*

$$K_i^n = \left(\frac{\mu_A (2-\gamma)}{c_j + c_i}\right)^{1/\gamma} \frac{(c_j - (1-\gamma) c_i)}{\gamma (c_j + c_i)}. \tag{4}$$

Otherwise, firm i does not invest in capacity while firm j invests in the monopoly quantity $K_j^n = (\mu_A (1-\gamma)/c_j)^{1/\gamma}$.

Proof. Note that

$$\frac{\partial^2 \Pi_j}{\partial K_j^2} = \frac{\gamma \mu_A}{(K_j + K_i)^{\gamma+1}} \left((\gamma+1)\frac{K_j}{K_j + K_i} - 2\right) < 0,$$

so that each firm's profit function is concave implying existence of an equilibrium. Optimality conditions are obtained from the first-order conditions and it is straightforward to see from these expressions that equilibrium capacities are unique. The condition for the interior equilibrium results from requiring the capacities in (4) to be positive. If one of the capacities is negative, then the best response curves intersect on the boundary of the first quadrant, i.e., at a monopoly point. ■

For equilibrium to be interior, capacity costs should not be "too different," where the magnitude of this restriction is a function of the elasticity parameter γ. A smaller γ implies greater price sensitivity of demand and a more restrictive condition on the relationship between firm costs: a firm with expensive capacity in a highly price-sensitive market cannot compete against a more efficient firm and decides not to enter (builds zero capacity).

3.2 Sequential Competition

Since the objective functions in a sequential game are symmetric, we need only to perform the following analysis once as it holds in the event that either firm is the leader. To this end, whenever we describe a sequential game between firms with symmetric objectives, we assume firm i is the leader and firm j is the follower. Existence of an equilibrium in sequential games follows immediately from the continuity of the profit functions in (2). In the following lemma, we find equilibrium capacities in the sequential game.

Lemma 2 *When two established firms compete sequentially, the solution is obtained as follows. Let $\hat{T} = K_i^* + K_j^*$. Then*

$$K_j^*(\hat{T}) = \left(\frac{\hat{T}}{\gamma}\right)\left(1 - \frac{c_j \hat{T}^\gamma}{\mu_A}\right), \tag{5}$$

$$\hat{T}^\gamma = \mu_A \left(\frac{(c_i(1-\gamma) + c_j) + \sqrt{(c_i(1-\gamma) + c_j)^2 - 4c_i c_j (1+\gamma)(1-\gamma)^2}}{2c_i c_j (1+\gamma)}\right). \tag{6}$$

Proof. Let $T = K_i + K_j^*(K_i)$, the total capacity, be the decision variable of the leader. T is strictly increasing in K_i since the implicit function theorem yields:

$$\frac{\partial K_j^*(K_i)}{\partial K_i} = (\gamma K_j - K_i) / ((1-\gamma) K_j + 2K_i) > -0.5. \tag{7}$$

It follows that the minimum total capacity in the sequential game is achieved when $K_i = 0$ such that $T_m = ((1-\gamma)(\mu_A/c_j))^{1/\gamma}$ and for every $T \geq T_m$ there is a unique K_i and $K_j^*(K_i)$ pair such that $K_i + K_j^*(K_i) = T$. Define $K_i(T)$ and $K_j(T)$ such that $K_i(T) + K_j(T) = T$. From the follower's first-order optimality condition, we calculate the follower's capacity for a given total capacity T as

in (5) so that the profit function for the leader is:

$$\Pi_i(T) = (T/\gamma)\left(\mu_A T^{-\gamma} - c_i\right)\left((c_j/\mu_A)\,T^{\gamma} - (1-\gamma)\right).$$

For profit to be positive, we must have $\mu_A/c_i > T^{\gamma} > (1-\gamma)\,\mu_A/c_j$. Differentiating, we obtain

$$\begin{aligned}
\frac{\partial \Pi_i}{\partial T} &= \left(\frac{T^{-\gamma}}{\gamma\mu_A}\right)\left(-(1-\gamma)^2\,\mu_A^2 + (c_i\,(1-\gamma) + c_j)\,\mu_A T^{\gamma} - c_i c_j\,(1+\gamma)\,T^{2\gamma}\right), \\
\frac{\partial^2 \Pi_i}{\partial T^2} &= \frac{T^{-\gamma-1}}{\mu_A}\left((1-\gamma)^2\,\mu_A - c_i c_j\,(1+\gamma)\,T^{2\gamma}\right).
\end{aligned}$$

Note that the first derivative is first increasing and then decreasing, i.e., the profit function is convex and then concave. Hence, there are two roots and the higher root is the optimal decision for the leader. The first-order condition is quadratic in T^{γ}, yielding a solution (6). Because Π_i is convex-concave, we need to demonstrate that $\Pi_i(\hat{T}, K_j(\hat{T})) > 0$ or else the interior $\hat{T}$ is not a global optimum; however, the simultaneous-move game is always profitable for each firm, so the leader's profit in the sequential game must be positive as well. ■

3.3 The Entry Order Game

We begin by comparing the capacity investment decisions of players in the simultaneous move and sequential games. This comparison subsequently allows us to compare expected profits.

Lemma 3 *Four situations arise:*

1. If $c_i/c_j > 1/(1-\gamma)$, then in the simultaneous game, firm i is forced out of the market, while in the sequential game firm i may be in the market.

2. If $1/(1-\gamma) > c_i/c_j > 1+\gamma-\gamma^2$, then the sequential game equilibrium capacity of firm i is less than the simultaneous equilibrium capacity of firm i.

3. If $1+\gamma-\gamma^2 > c_i/c_j > 1-\gamma$, then the sequential game equilibrium capacity of firm i is greater than the simultaneous equilibrium capacity of firm i.

4. If $1-\gamma > c_i/c_j$, then firm j is forced out of the market and firm i produces the monopoly capacity in both the simultaneous and sequential games.

Proof. First note that the total capacity in the sequential game equilibrium is higher than in the simultaneous game if $\left(\hat{T}\right)^{\gamma} > (T^{*})^{\gamma}$, i.e., if

$$\frac{(c_i(1-\gamma)+c_j)+\sqrt{(c_i(1-\gamma)+c_j)^2-4c_ic_j(1+\gamma)(1-\gamma)^2}}{2c_ic_j(1+\gamma)} > \frac{(2-\gamma)}{c_i+c_j}.$$

The above expression depends only on γ, c_i and c_j. Let $c_i + c_j = c$ and let $c_i = \phi c$, $\phi \in (0,1)$, resulting in the following equivalent condition:

$$\phi^2\left(\gamma^3-3\gamma^2+4\right)+\phi\left(4\gamma^2-2\gamma-\gamma^3-4\right)+\left(1+\gamma-\gamma^2\right)>0,$$

which clearly holds for $\phi = 0$ and for $\phi = 1$. The function on the left-hand side is convex in ϕ and hence possesses a minimum. It is straightforward to verify that the desired condition never holds at the minimum with respect to ϕ implying that the condition fails for all $\phi \in \left(\underline{\phi}, \overline{\phi}\right)$, where $\underline{\phi} < \overline{\phi}$ are the zeroes of the above equation and can be found as follows:

$$\overline{\phi} = 1/(2-\gamma), \ \underline{\phi} = 1-(2-\gamma)/\left(\gamma^3-3\gamma^2+4\right).$$

Thus, if ϕ lies anywhere between these two points, the sequential game equilibrium total capacity is *less* than the simultaneous equilibrium total capacity. Recall that, in order for an interior equilibrium to exist in the simultaneous game, we required $(1-\gamma) < c_i/c_j < 1/(1-\gamma)$ which is equivalent to $(1-\gamma)/2-\gamma < \phi < 1/(2-\gamma)$. Note that the upper bound on ϕ for existence of an interior equilibrium for the simultaneous game is equal to $\overline{\phi}$. Since T is strictly increasing in K_i, a greater total capacity implies a greater K_i. Define $\overline{\phi} = \left(1-\overline{\phi}\right)\left(1+\gamma-\gamma^2\right)$ to be the critical cost ratio and the result follows after combining these thresholds with the result of Lemmas 1 and 2. ■

Figure 1, which plots the various regions of the parameter space with respect to c_i/c_j and γ, illustrates the lemma. In the upper and lower left regions, the simultaneous equilibria are boundary: the firm with higher capacity cost is forced out of the market because high demand elasticity causes a low market price, which in turn allows only the more efficient firm to make a positive profit. These two regions are larger for small γ (highly price-elastic demand), because in this case the ability to price low is most important. In the upper-middle region, the leader's cost

is higher than the follower's, and hence the leader's sequential game equilibrium capacity is lower than his corresponding simultaneous equilibrium capacity. In the lower-middle region, the leader's cost is comparable to or is lower than the follower's cost. Hence, the leader builds more capacity in the sequential game than he would in the simultaneous game.

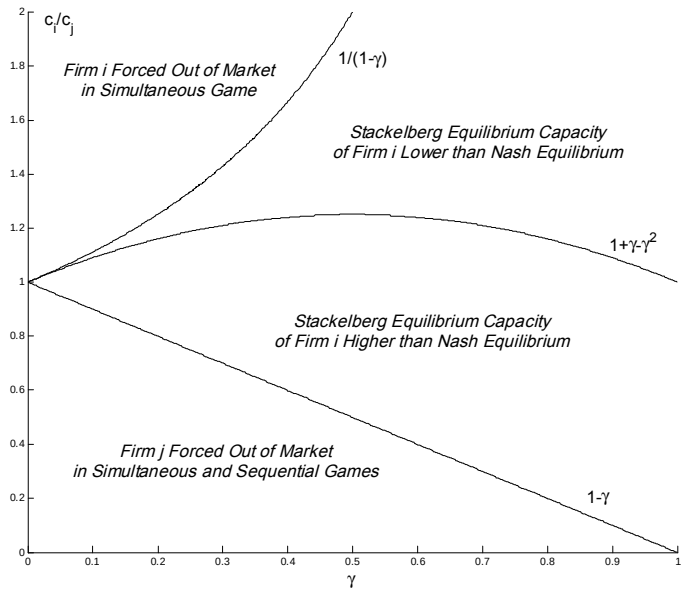

Figure 1: Comparison of the Nash equilibrium capacity (in the simultaneous game) with the Stackelberg equilibrium capacity (in the sequential game), when firm i leads and both firms are established.

To understand the intuition of why a high-cost sequential leader invests less in capacity than in the simultaneous game, consider Figure 2, which plots best responses of two players in the simultaneous game. Note that, unlike in Gal-Or (1985), best response functions are neither increasing nor decreasing, and therefore equilibria in the entry order game depend on problem parameters. First, observe that with the higher capacity cost ($c_i = 1.9$ in the figure) of firm i, the best response of firm j is increasing at the simultaneous game equilibrium: higher capacity of firm i leads to higher capacity investment of firm j. Thus, if firm i were to lead in the sequential game (recalling that firm i's profit is decreasing in firm j's capacity, implying firm i wants firm j to build less capacity), firm i would decrease its capacity investment. This is why a high-cost firm builds lower capacity when it leads in the sequential game than when it competes simultaneously. Alternatively, a low capacity cost ($c_i = 1.2$ in Figure 2) places the simultaneous equilibrium on the decreasing part of the best response of firm j. Therefore, in order to decrease capacity investment by the follower (firm

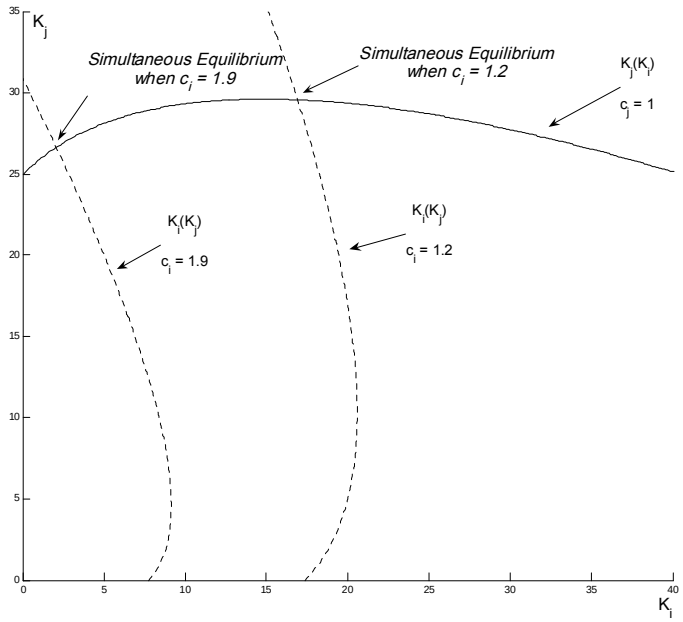

Figure 2: Best response functions for two established, profit maximizing firms, with $c_j = 1$, and $c_i = 1.2$ or $c_i = 1.9$.

j) in the sequential game, firm i prefers to build higher capacity than it would in the simultaneous game.

Now that we have ordered the capacities in the various games, we are ready to state the main result of this Section which allows us to compare profits in the simultaneous and sequential games and hence determine when firm j prefers simultaneous competition to being a follower.

Theorem 1 *In the entry-order game, three outcomes arise:*

1. If $c_i/c_j > 1/(1-\gamma)$ or if $c_i/c_j < 1-\gamma$, then the high-cost firm is forced out of the market.

2. If $1+\gamma-\gamma^2 < c_i/c_j < 1/(1-\gamma)$ or if $1-\gamma < c_i/c_j < 1-\gamma+\gamma^2$, then the high-cost firm leads and the low-cost firm follows.

3. If $1-\gamma+\gamma^2 < c_i/c_j < 1+\gamma-\gamma^2$, then firms compete simultaneously.

Proof. From Lemma 2 and Lemma 3 observe that, $\Pi_j\left(K_i^n, K_j^n\right) > \Pi_j\left(K_i^l, K_j^f\right)$ iff $c_i/c_j \notin \left(1+\gamma-\gamma^2, 1/(1-\gamma)\right)$. Thus, if $c_i/c_j < 1+\gamma-\gamma^2$, both firms prefer simultaneous competition. Since both firms clearly prefer leading to simultaneous play, entry is the dominant action. Conversely, if the condition in Part 2 fails, $c_i/c_j > 1+\gamma-\gamma^2 > 1$, $\forall\gamma$ so c_j/c_i must be less than 1, and therefore, $c_j/c_i < 1 < 1+\gamma-\gamma^2$, $\forall\gamma$. Part 1 then implies that firm j prefers following to

simultaneous play, but firm i still prefers to commit early, so the equilibrium to entry order game is for firm i to enter early and firm j to enter late. ■

Figure 3 provides a graphical representation of Theorem 1. Intuitively, if one firm is much less efficient than the other, then the high-cost firm is forced out of the market leaving the low-cost firm a monopolist. Furthermore, if the relative capacity costs of the two firms are close, then both firms prefer to compete simultaneously. What is most surprising is that when costs of two firms are sufficiently different, the high-cost firm leads. Intuitively, we would expect a firm that is more cost-efficient to pioneer the market but this does not happen. The intuition behind this result follows from the observation already discussed that a high cost firm invests less in the sequential game than in the simultaneous game. This unexpected result drives high-cost firms to lead rather than compete simultaneously which results in lower capacity and higher prices. In effect, the firm prefers to lead because this allows the high cost firm to commit to an unaggressive strategy. Our results are closely related to the findings in Gal-Or (1985): when two identical players move sequentially in a game, the player that moves first earns lower profits than the player that moves second if the reaction functions of the players are upward sloping. In our game the reaction functions are neither downward nor upward sloping which explains why both the sequential and the simultaneous equilibria are possible.

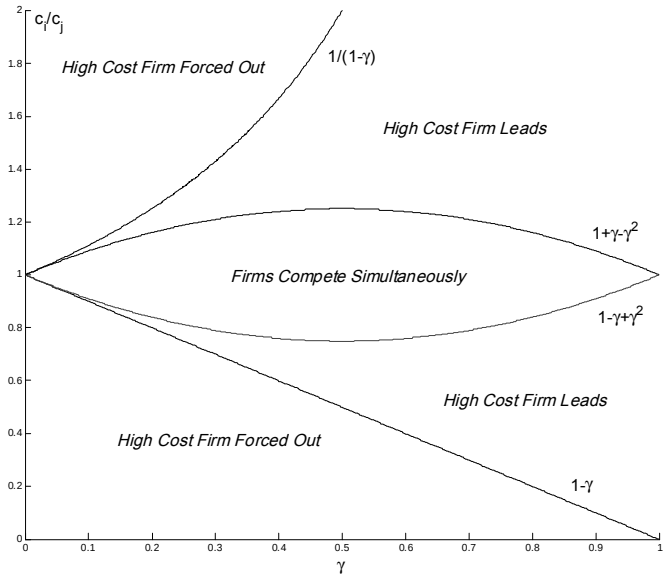

Figure 3: Equilibria of the entry-order game between two established, profit maximizing firms.

4 Competition Between a Start-up and an Established Firm

We now shift our focus to a failure-averse start-up who seeks to maximize the probability of survival. It is useful to first explore properties of the reaction function of a survival-maximizing firm. The objective function of the start-up in the capacity game is given by (3) which can be equivalently written as follows:

$$\psi_s = \max_{K_s \geq 0} -\alpha_s \left(K_p + K_s\right)^{\gamma} / K_s - c_s \left(K_p + K_s\right)^{\gamma}.$$

The following Lemma summarizes properties of the best response function of a start-up firm.

Lemma 4 *1. ψ_s is quasi-concave and hence yields a unique best-response function*

$$K_s(K_p) = \frac{(1-\gamma)\,\alpha_s + \sqrt{(1-\gamma)^2\,\alpha_s^2 + 4c_s\gamma\alpha_s K_p}}{2c_s\gamma}. \tag{8}$$

2. The equilibrium survival probability of a start-up is decreasing in the capacity of the opposing player.

Proof. 1. The first derivative of ψ_s is

$$\frac{\partial \psi_s}{\partial K_s} = -\gamma\alpha_s \left(K_p + K_s\right)^{\gamma-1} / K_s + \alpha_s \left(K_p + K_s\right)^{\gamma} / K_s^2 - c_s\gamma \left(K_p + K_s\right)^{\gamma-1}.$$

Note that $\lim_{K_s \to 0} \partial\psi_s / \partial K_s > 0$ and $\lim_{K_s \to \infty} \partial\psi_s / \partial K_s < 0$ so there is at least one point at which the first derivative crosses zero. The second derivative evaluated at a point at which the first derivative is zero is found as follows

$$\frac{\partial^2 \psi_s}{\partial K_s^2} = \alpha_s \left(K_p + K_s\right)^{\gamma-1} \left((\gamma - 1)\, K_s - 2K_p\right) / K_s^3 < 0,$$

implying that the objective function of the start-up is quasi-concave. The first-order condition is thus sufficient for optimality, and it yields two best response functions

$$K_s(K_p) = \frac{(1-\gamma)\,\alpha_s \pm \sqrt{(1-\gamma)^2\,\alpha_s^2 + 4c_s\gamma\alpha_s K_p}}{2c_s\gamma},$$

of which one can be eliminated since $(1-\gamma)\,\alpha - \sqrt{(1-\gamma)^2\,\alpha^2 + 4c_s\gamma\alpha K_p} < 0$.

2. From the Envelope Theorem, $d\psi_s/dK_p = -\alpha_s\gamma\left(K_p+K_s\right)^{\gamma-1}/K_s - c_s\gamma\left(K_p+K_s\right)^{\gamma-1} < 0$ for all K_s and K_p. ■

Note that $K_s(K_p)$ is increasing in K_p and is unique for any given capacity of the other firm. Evaluating the best response at the point $K_p = 0$ yields the monopoly solution to the start-up's problem $K_s(0) = (1-\gamma)\,\alpha_s/\left(c_s\gamma\right)$. It is immediately apparent that the equilibrium capacity of a start-up in any game is greater than the start-up's monopoly capacity and that the reaction function of a start-up is increasing and concave.

4.1 The Simultaneous Game

We first demonstrate existence and uniqueness of an equilibrium in the simultaneous game. As in Lemma 1, there is a restriction on the costs that is necessary for the equilibrium to be interior. Unlike the restriction in Lemma 1, that restriction does not depend solely on the costs and γ.

Lemma 5 *In a simultaneous game between a start-up and an established firm, a Nash equilibrium to the capacity game exists and is unique. The equilibrium is interior ($K_p^n > 0, K_s^n > 0$) if $\mu_A/c_p > \left((1-\gamma)\,\alpha_s/\left(c_s\gamma\right)\right)^{\gamma}$. Otherwise, $K_s^n = (1-\gamma)\,\alpha_s/\left(c_s\gamma\right)$ and $K_p^n = 0$.*

Proof. From the proof of Lemma 1, the established firm's objective function is concave. Since ψ_s is quasi-concave, an equilibrium exists. To demonstrate uniqueness, note that the optimal capacity of the established firm is found from the solution to the following first-order optimality condition:

$$-\gamma\mu_A\left(K_p+K_s(K_p)\right)^{-\gamma-1}K_p + \mu_A\left(K_p+K_s(K_p)\right)^{-\gamma} - c_p = 0.$$

To show uniqueness, it suffices to demonstrate that the function on the left-hand side is monotone in K_p. It is convenient to re-arrange this expression as follows:

$$z(K_p) = \left(1-\gamma K_p/\left(K_p+K_s(K_p)\right)\right) - c_p\left(K_p+K_s(K_p)\right)^{\gamma}/\mu_A = 0. \tag{9}$$

To see that the first term of $z(K_p)$ is decreasing in K_p, define $\theta = \sqrt{(1-\gamma)^2 \alpha_s^2 + 4c_s\gamma\alpha_s K_p} > 0$ so that

$$\begin{aligned}
\frac{d}{dK_p}\left(1-\gamma\frac{K_p}{K_p+K_s(K_p)}\right) &= \frac{-\gamma}{K_p+K_s(K_p)}\left(1-\frac{K_p}{K_p+K_s(K_p)}\left(1+\frac{\partial K_s(K_p)}{\partial K_p}\right)\right) \\
&= \frac{-\gamma}{(K_p+K_s(K_p))^2}\left(\frac{(1-\gamma)\,\alpha_s\theta+(1-\gamma)^2\,\alpha_s^2+2c_s\gamma\alpha_s K_p}{2c_s\gamma\theta}\right) < 0.
\end{aligned}$$

To see that the second term of $z(K_p)$ is increasing in K_p, note that

$$\frac{d}{dK_p}\left(-\frac{c_p}{\mu_A}\left(K_p+K_s(K_p)\right)^{\gamma}\right) = \frac{c_p}{\mu_A}\gamma\left(K_p+K_s(K_p)\right)^{\gamma-1}\left(1+\frac{\partial K_s(K_p)}{\partial K_p}\right) > 0.$$

Thus, $z(K_p)$ is a decreasing function of K_p, and the equilibrium is unique. To ensure that the equilibrium is interior (i.e. that $K_p > 0$), $z(0)$ must be positive implying that the following condition must hold $(1-\gamma K_p/\left(K_p+K_s(K_p)\right)) > c_p\left(K_p+K_s(K_p)\right)^{\gamma}/\mu_A$ at $K_p = 0$. We obtain a simplified condition by plugging in the best response function and evaluating the above expression at $K_p = 0$.

■

The interior equilibrium condition ensures that capacity costs are not too great for either player and that the disparity between capacity costs is not too large when the costs have been normalized by the objective of each firm. The established firm is an expected profit maximizer, so its cost is normalized by the mean demand μ_A; the start-up is maximizing the probability of achieving a target level of capital so its cost is normalized by that target level, α_s. The condition implies, for instance, that the equilibrium is not interior if α_s is very large (the start-up has a very high goal), μ_A is small (mean demand is very low), or if either c_p is very high or c_s is very low. In either of these cases, the start-up is either so efficient or so desperate relative to the established firm that the established firm is forced completely out of the market, and the start-up enjoys a monopoly. Interestingly, the start-up is never forced out of the market in our model, which is consistent with the notion in Christensen and Bower (1996) that start-ups often pursue markets that are considered to be unprofitable by established firms. Within the context of our model, the intuition behind this result is that, conditional on the start-up going into business and incurring a fixed cost $\alpha_s > 0$, the start-up cannot be forced to invest zero in capacity, as this would result in a probability of survival equal to zero.

4.2 The Sequential Game: Established Firm Leads

In the sequential game with the established firm as the leader, an equilibrium exists due to the continuity of the payoff functions. The equilibrium solution is obtained by solving simultaneously the optimality condition for the established firm

$$\frac{d\Pi_p}{dK_p} = \mu_A \left(K_p + K_s(K_p)\right)^{-\gamma} - \gamma\mu_A \left(K_p + K_s(K_p)\right)^{-\gamma-1} \left(1 + \frac{dK_s(K_p)}{dK_p}\right) K_p - c_p = 0 \qquad (10)$$

and the best response $K_s(K_p)$ of the start-up (8). Analysis of this implicit function leads to the following result.

Lemma 6 *$K_p^n \geq K_p^l$ for all parameters.*

Proof. Recall that, in the simultaneous-move game, the optimal capacity of the established firm is the solution to $z(K_p) = 0$ where $z(K_p)$ is defined in (9). Furthermore, we re-arrange (10) to obtain

$$w(K_p) = \left(1 - \gamma\frac{K_p}{K_p + K_s(K_p)}\right) - \frac{c_p}{\mu_A}\left(K_p + K_s(K_p)\right)^{\gamma} - \gamma\frac{K_p}{K_p + K_s(K_p)}\frac{dK_s(K_p)}{dK_p}.$$

The difference between $z(K_p)$ and $w(K_p)$ is

$$z(K_p) - w(K_p) = \left(dK_s(K_p)/dK_p\right)\gamma K_p/\left(K_p + K_s(K_p)\right).$$

Since $dK_s(K_p)/dK_p > 0$, $z(K_p) - w(K_p) > 0$ for all K_p. In particular, when $w(K_p^l) = 0$, $z(K_p^l) > 0$. That is, if K_p^l is an optimal solution to the sequential game, the optimality condition for the simultaneous game is not satisfied because the first derivative is positive. From the proof of Lemma 5, $z(K_p)$ is decreasing so the equilibrium capacity in the simultaneous game must be greater than the equilibrium capacity in the sequential game. ■

Intuitively, we see that the start-up is more aggressive than the established firm: even though the established firm leads in the game, the start-up still invests in more capacity to maximize survival probability, hence the established firm cannot leverage the leadership position to increase capacity investment. Consequently, the established firm reduces its capacity investment when leading, a finding consistent with the empirical results of Hariharan and Brush (1999) indicating that established firms enter on a smaller scale than start-ups do.

4.3 The Sequential Game: Start-up Leads

In the sequential game in which the start-up leads, the equilibrium is unique, which allows us to compare equilibrium capacities later on.

Lemma 7 *In the sequential game in which the start-up leads, the equilibrium exists and is unique.*

Proof. Existence follows from continuity of the payoff functions. In order to demonstrate uniqueness, we once again employ the technique of allowing the leader to select the total capacity T. Since the follower is a profit maximizer, the best response to a given T remains $K_p(T) = (T/\gamma)\left(1 - c_p T^\gamma/\mu_A\right)$. Note that

$$K_s = T\left(1 - \frac{1}{\gamma} + \frac{c_p T^\gamma}{\gamma \mu_A}\right)$$

must be positive in order for the solution to be feasible. The leader chooses T to maximize

$$\psi_s = \max_{T \geq T_m} -\frac{\alpha T^{\gamma-1}}{\left(1 - \frac{1}{\gamma} + \frac{c_p T^\gamma}{\gamma \mu_A}\right)} - c_s T^\gamma.$$

The first derivative is

$$\frac{\partial \psi_s}{\partial T} = T^{\gamma-1}\left[\frac{\alpha_s(1-\gamma)}{T\left(1 - \frac{1}{\gamma} + \frac{c_p T^\gamma}{\gamma \mu_A}\right)} + \frac{\alpha_s c_p}{\mu_A T^{1-\gamma}\left(1 - \frac{1}{\gamma} + \frac{c_p T^\gamma}{\gamma \mu_A}\right)^2} - c_s \gamma\right].$$

Note that the first two terms inside the bracket are always positive and strictly decreasing in T for $T \geq T_m$, approaching zero as T goes to infinity, while the third term is constant and negative. Thus, ψ_s is quasi-concave; the first derivative changes sign from positive to negative and crosses zero precisely once. It follows that the equilibrium is unique. ■

4.4 The Entry Order Game

Before analyzing the entry order game, we compare capacity investments by the start-up in the simultaneous move game and in the sequential move game when the start-up leads. The comparison can go either way, and there is a simple condition that dictates precisely when one capacity is greater than the other.

Lemma 8 *Define* $\eta = \left(1+\gamma-\gamma^2\right)(1+\gamma)^{1+1/\gamma}/\gamma^3$. *Then,* $K_s^l > K_s^n$ *if and only if* $(\alpha_s\eta/c_s)^\gamma > \mu_A/c_p$.

Proof. In the simultaneous equilibrium, the following optimality condition must hold,

$$y(K_s) = \alpha_s\left(K_p + K_s\right) - \gamma\alpha_s K_s - \gamma c_s K_s^2 = 0. \tag{11}$$

Clearly, $y(K_s)$ is decreasing at all equilibria because the equilibrium is unique (from §4.2). In a sequential game equilibrium, the following optimality condition must hold:

$$x(K_s) = \alpha_s\left(K_p(K_s) + K_s\right) - \gamma\alpha_s K_s - \gamma c_s K_s^2 - \left(\gamma c_s K_s^2 + \gamma\alpha_s K_s\right)\partial K_p(K_s)/\partial K_s = 0. \tag{12}$$

Thus,

$$y(K_s) - x(K_s) = \left(\gamma c_s K_s^2 + \gamma\alpha_s K_s\right)\partial K_p(K_s)/\partial K_s.$$

Note that

$$\frac{\partial^2 K_p^*(K_s)}{\partial K_s^2} = -\frac{1}{(1-\gamma)K_p + 2K_s}\left(1 + (1-\gamma)\frac{\partial K_p^*(K_s)}{\partial K_s}\right)\left(1 + \frac{\partial K_p^*(K_s)}{\partial K_s}\right) < 0,$$

and furthermore, $\lim_{K_s\to 0}\partial K_p/\partial K_s = \gamma/(1-\gamma) > 0$, while $\lim_{K_s\to\infty}\partial K_p/\partial K_s = -0.5 < 0$. Together, these observations imply that there is a unique $K_s > 0$ such that $\partial K_p(K_s)/\partial K_s = 0$, and by extension, there exists a unique $K_s > 0$ such that $y(K_s) = x(K_s)$. From (7), it is straightforward to see that $K_s = \gamma K_p$ is the unique solution to $\partial K_p(K_s)/\partial K_s = 0$. It remains to demonstrate that at this value, $y(K_s) > 0$ (if a certain condition holds), and since the sequential game equilibrium is unique, this implies that the start-up's sequential game equilibrium capacity is greater than the simultaneous equilibrium capacity. Using $K_s = \gamma K_p$, we solve for the capacity of the established firm:

$$K_p = \left(\mu_A/\left(c_p(1+\gamma)\right)\right)^{1/\gamma}/(1+\gamma)\,.$$

The function $y(K_s)$ evaluated at this point is:

$$y(K_s) = \frac{\gamma c_s}{(1+\gamma)^2}\left(\frac{\mu_A}{c_p(1+\gamma)}\right)^{1/\gamma}\left(\alpha_s\frac{\left(1+\gamma-\gamma^2\right)(1+\gamma)}{\gamma^3 c_s} - \left(\frac{\mu_A}{c_p(1+\gamma)}\right)^{1/\gamma}\right),$$

and the result follows. ■

The condition $(\alpha_s \eta / c_s)^\gamma > \mu_A / c_p$ is essentially a reflection of how "desperate" the start-up is. If α_s is high relative to other problem parameters, then the start-up builds more capacity in the sequential game than in the simultaneous game. If α_s is low, then the start-up builds more capacity in the simultaneous game. By examining the above condition, we see that $\eta > (1-\gamma)/\gamma$, the constant from Theorem 3, for all $\gamma > 0$, so that both conditions may be satisfied simultaneously; that is, it is possible for the equilibrium in the simultaneous game to be interior and for the capacity of the sequential game with a leading start-up to be greater than the capacity of the game in which the start-up competes simultaneously.

We are now ready to solve the entry order game between an established firm and a start-up. The following theorem demonstrates when firms prefer following to simultaneous competition:

Theorem 2 *When a start-up competes with an established firm, three situations arise:*

1. If $\mu_A / c_p > (\alpha_s \eta / c_s)^\gamma$, then there are two equilibria with one firm leading and the other firm following.

2. If $(\alpha_s (1-\gamma) / (c_s \gamma))^\gamma < \mu_A / c_p < (\alpha_s \eta / c_s)^\gamma$, then the established firm leads and the start-up follows.

3. If $\mu_A / c_p < (\alpha_s (1-\gamma) / c_s \gamma)^\gamma$, then the established firm is forced out of the market.

Proof. First, note (from Lemmas 5 and 6) that $\Psi_s \left(K_s^f, K_p^l\right) > \Psi_s \left(K_s^n, K_p^n\right)$ for all parameters. Second, observe (from Lemmas 7 and 8) that $\Pi_p \left(K_s^n, K_p^n\right) > \Pi_p \left(K_s^l, K_p^f\right)$ if $(\alpha_s \eta / c_s)^\gamma > \mu_A / c_p > (\alpha_s (1-\gamma) / (c_s \gamma))^\gamma$, that $\Pi_p \left(K_s^n, K_p^n\right) < \Pi_p \left(K_s^l, K_p^f\right)$ if $\mu_A / c_p > (\alpha_s \eta / c_s)^\gamma$ and $\Pi_p \left(K_s^n, K_p^n\right) = \Pi_p \left(K_s^l, K_p^f\right) = 0$ otherwise. The results follow. ■

The two last results are illustrated in Figures 4 and 5. The lower region (in which the established firm is forced out of the market) is consistent with several situations described in Christensen and Bower (1996), in which the established firm has high production costs and perceives the market as being of limited size. As a result, the established firm chooses not to enter the market at all, while a start-up company rushes in. In the other two regions, either the start-up rushes to beat the established firm to the market and dictate capacity and price levels (again, similar to some situations in the work of Christensen and Bower (1996)) or the start-up waits to see the capacity levels of the established firm before committing to an output level. Moving simultaneously is detrimental to

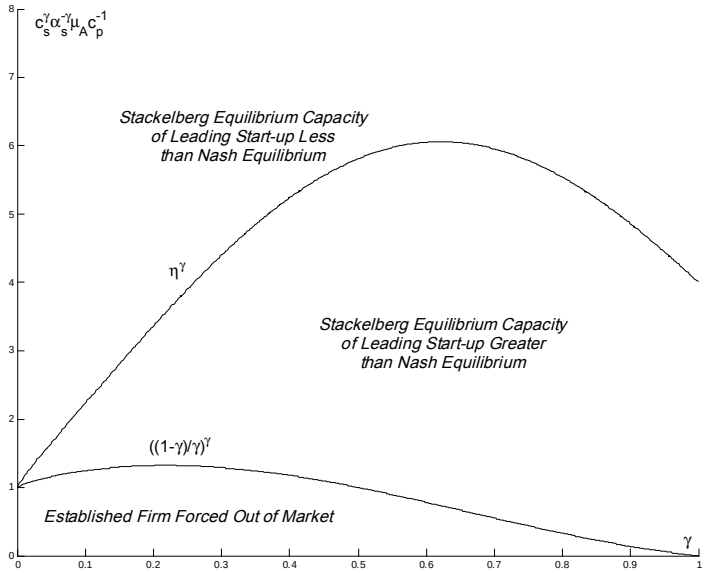

Figure 4: Comparison of the Nash equilibrium capacity (in the simultaneous game) with the Stackelberg equilibrium capacity (in the sequential game), when one firm is a start-up and the other an established firm.

both firms: the start-up always avoids simultaneous competition while the established firm avoids it because the start-up tends to invest in large capacity. This finding is in sharp contrast with the result in the previous section that established firms often compete simultaneously, especially if their costs are approximately equal. In this case, think of c_p/μ_A as the adjusted cost of the established firm and $(c_s/\alpha_s\eta)^\gamma$ as the adjusted cost of the start-up. Then, Figure 4 demonstrates that, if an established firm and a start-up have approximately equal adjusted costs (corresponding to 1 on the vertical axis), the established firm is very likely to be forced out of the market, especially if price sensitivity is high.

5 Competition Between Start-ups

Finally, we turn our attention to the situation in which two start-ups compete in the market. We drop the subscripts s and p and replace them with i and j since both firms maximize survival. Next, we demonstrate that the simultaneous move equilibrium is unique.

Lemma 9 *When two start-ups compete simultaneously, the game is supermodular, and an equilib-*

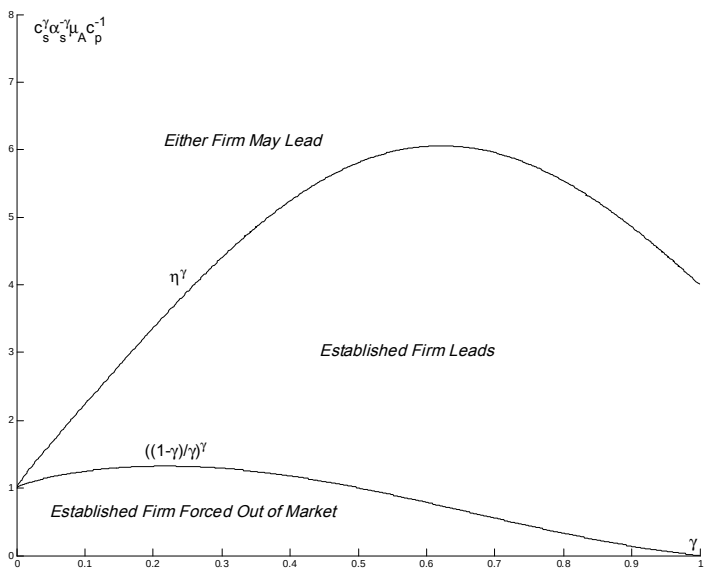

Figure 5: Equilibria of the entry-order game between a start-up and an established firm.

rium exists and is unique. The equilibrium is found by solving the system of two equations:

$$K_i(K_j) = \frac{\alpha_i (1-\gamma) + \sqrt{\alpha_i^2 (1-\gamma)^2 + 4c_i\gamma\alpha_i K_j}}{2c_i\gamma},\ i,j = 1,2,\ i \neq j. \tag{13}$$

Proof. The cross-partial derivative of the objective function is:

$$\frac{\partial^2 \psi_i}{\partial K_i \partial K_j} = \frac{\gamma(1-\gamma)\alpha_i (K_j + K_i)^{\gamma-2}}{K_i} + \frac{\gamma\alpha_i (K_j + K_i)^{\gamma-1}}{K_i^2} + c_i\gamma(1-\gamma)(K_j + K_i)^{\gamma-2} \geq 0,$$

and it immediately follows that the game is supermodular and hence an equilibrium exists. The expression for the equilibrium capacities is straightforward to obtain from the first-order conditions. To see that the equilibrium is unique, note that $K_i(0) > 0$ and $K_j(0) > 0$, and both best responses grow as the square root of the strategy of the other player. Thus, best responses intersect only once since one curve starts below the other and both are strictly increasing and concave. ■

Supermodularity of the game is an intuitive result: in the highly competitive world of start-up competition in which the very existence of a firm is at stake, the only reaction to an aggressive strategy (high capacity) of a competitor is to act aggressively (invest more in capacity) in return. Since the game is supermodular and $K_j(0)$ is the monopoly capacity, it is clear that the capacity investment in the simultaneous equilibrium is greater than the monopoly capacity, and further that

there is no boundary equilibrium: a start-up is never forced out of the market. We also note that, in the game between two start-ups, the equilibrium capacities are independent of expected market size, a consequence of (13). This is in contrast with the previous cases we have analyzed, where even if a start-up's best response is independent of μ_A, in general the equilibrium capacity level is not. Though seemingly counterintuitive, this result implies that competition between start-ups may yield capacity levels that are out of proportion with expected demand, which agrees with the trend of over-investment observed in certain start-up industries.

When the start-ups compete sequentially, existence of an equilibrium follows immediately from the continuity of the payoff functions, but uniqueness is difficult to demonstrate. Nevertheless, we can order the equilibrium capacities in the simultaneous move and sequential games, and the following lemma shows that the ordering is unambiguous in this case.

Lemma 10 *When two start-ups compete, $K_i^n > K_i^l$.*

Proof. Suppose firm i is the leader. Recall that, in the simultaneous-move game, $y(K_i)$ defined in (11) is decreasing in K_i at any equilibrium point. Furthermore, in a sequential game equilibrium the condition $x(K_i) = 0$ (where $x(K_i)$ is defined in (12)) must hold. Thus,

$$y(K_i) - x(K_i) = \left(\gamma c_i K_i^2 + \gamma \alpha_i K_i\right) \partial K_j(K_i)/\partial K_i > 0,$$

which implies that all solutions to $x(K_i) = 0$ are smaller than the solution to $y(K_i) = 0$. Hence all equilibria of the sequential game have lower capacity investments by the leader than the equilibrium investments in the simultaneous game. ■

An implication of the above lemma is that a start-up in the simultaneous game builds higher capacity than either the leader or the follower in both sequential games, thus depressing prices. Therefore, start-up firms competing against other start-ups are averse to simultaneous competition.

Theorem 3 *When two start-ups compete, there are two equilibria to the entry order game: one firm commits early and the other firm commits late.*

Proof. Follows from Lemmas 9 and 10. ■

Similar to the previous section, we see that competition involving start-ups never results in a simultaneous entry. The difference is that with two start-ups, it is never the case that one firm

	Established Firm 1	**Start-up Firm 1**
Order	Simultaneous	Firm 2 Leads
$(\boldsymbol{K}_1^*, \boldsymbol{K}_2^*)$	$(703, 703)$	$(707, 480)$
$(\mathbb{E}\boldsymbol{\Pi}_1^*, \mathbb{E}\boldsymbol{\Pi}_2^*)$	$(234, 234)$	$(319, 217)$

Table 3: Equilibrium values when firm 2 is established and firm 1's target capital level is 300.

is forced out of the market and moreover, cost advantage/disadvantage does not affect the firm's decision to lead or follow: both situations remain an equilibrium of the game.

6 Discussion

In addition to the analytical results discussed above, we have encountered a number of other observations that raise interesting questions regarding the understanding of the nature of start-up firms. For example, one might expect a start-up to invest less in capacity than an established firm would. Indeed, the operations literature points out that risk aversion (which, at first sight, appears to be synonymous with bankruptcy minimization) usually leads to lower investments (see, e.g., Van Mieghem (2003b)). Furthermore, we expect a start-up firm to generate lower profits than an established firm would under the same circumstances: after all, an established firm maximizes expected profits. As our first example demonstrates, neither intuition is necessarily true.

In all of the examples, $c_1 = c_2 = 1$, $\gamma = 0.5$, and A is normally distributed with mean 50 and standard deviation 50. Note that, for $c_1 = c_2 = 1$, our results have a particularly simple form (for all γ): when both firms are established, they compete simultaneously, but when there is at least one start-up, the firms compete sequentially. In the first example (Table 3), we compare the equilibrium capacities and profits when two established firms compete with the results when a start-up competes with an established firm. For these parameters, when two established firms compete, they do so simultaneously; when a start-up competes with an established firm, the established firm leads.

Not only does the start-up invest more (even though it enters second), it also enjoys a higher expected profit. Note, the start-up's target capital level is $\alpha_1 = 300$, while her expected profit is 319. Thus, the start-up is not particularly aggressive with her choice of target capital level, and still the equilibrium investment is greater than that of a *simultaneous* game between established firms. The aggressiveness of the start-up gives it an upper hand against the established firm in

	Two Established Firms	**Two Start-ups**
Order	Simultaneous	Firm 1 Leads
$(\boldsymbol{K}_1^*, \boldsymbol{K}_2^*)$	$(703, 703)$	$(206, 259)$
$(\mathbb{E}\boldsymbol{\Pi}_1^*, \mathbb{E}\boldsymbol{\Pi}_2^*)$	$(234, 234)$	$(271, 342)$
$(\mathbb{E}\boldsymbol{\Psi}_1^*, \mathbb{E}\boldsymbol{\Psi}_2^*)$	$(-, -)$	$(54\%, 58\%)$

Table 4: Equilibrium values when both firms have a target capital level of 100.

this example. This observation is consistent with examples in Christensen and Bower (1996). Additionally, Hariharan and Brush (1999) find empirically that established firms enter at a smaller scale than start-ups.

Even when we compare competition between two start-ups and two established firms, it is not true that start-ups always experience lower profits than established firms. Furthermore, we often expect first-mover advantages from the start-up leading into the new market. Yet, it very well might be that the second firm invests in more capacity and enjoys higher profits. Our next example (Table 4) illustrates both of these points by comparing the equilibrium between two profit maximizing firms with the equilibrium between two survival maximizing firms.

The performance of the start-ups dominates the performance of the established firms in this example; both start-ups have greater expected profit than the established firms. Here, the start-ups are cautious, and that caution helps mitigate the competitive effect of increased capacity. Note the follower in the game of start-ups enjoys higher profitability and higher survival probability, which is consistent with some observations during the Internet era (e.g., Amazon.com was preceded by BookStacksUnlimited, yet it invested into higher capacity and performed better).

This example also addresses the issue of first and second mover advantages discussed in Gal-Or (1985). The follower in the game of start-ups has a higher survival probability than the leader; this second mover advantage is a general result when players are identical and reaction functions are upward sloping. The survival maximizing case contrasts with the case of profit maximizing firms. When the firms are identical, an implication of Theorem 1 is that the simultaneous equilibrium is on the downward sloping portion of both reaction functions. Since players are identical and reaction functions are downward sloping in this regime, there is a first mover advantage among profit maximizing firms.

Our third example (Table 5) concerns the variance of profits. One might expect start-ups to have a lower variance of profits compared to established firms because start-ups maximize survival

	Two Established Firms	**Two Start-ups**
Order	Simultaneous	Firm 1 Leads
$(\boldsymbol{K}_1^*, \boldsymbol{K}_2^*)$	$(703, 703)$	$(616, 776)$
$(\mathbb{E}\boldsymbol{\Pi}_1^*, \mathbb{E}\boldsymbol{\Pi}_2^*)$	$(234, 234)$	$(210, 264)$
$(\boldsymbol{Var}(\Pi_1^*), \boldsymbol{Var}(\Pi_2^*))$	$(937^2, 937^2)$	$(1040^2, 825^2)$

Table 5: Equilibrium values when both firms have a target capital level of 300.

probability. This is, again, not necessarily the case. We see that the start-up entering first has a *greater* variance of profit than either of the established firms or the start-up that follows. This is consistent with the notion that the leader among start-ups may face greater uncertainty in revenue as a company breaks new ground.

7 Conclusion

In this work, we assume that, consistent with practical observations, start-up firms maximize the probability of survival. We argue that this is a better representation of a start-up's objective than expected profit maximization or risk aversion. Within this context, we analyze the difference in competitive interactions introduced by a start-up firm. The insights that we derive from these stylized models of start-up competition are compelling. We find that profit-maximizing firms either compete simultaneously (if their costs are similar) or the firm with the high cost leads. However, when one of the competitors is a start-up firm, the simultaneous entry is never an equilibrium: one firm always leads and the other firm follows. Thus, we find support for multiple observed phenomena, such as start-ups leading established firms in a sequential setting, simultaneous competition for profit maximizing firms with similar costs, and an overall preference among survival maximizing firms to avoid simultaneous competition. In our model, start-ups are never forced out of the market while the established firm often prefers not to enter at all if it is at a cost disadvantage. This outcome is, again, consistent with the observation that start-ups often venture into new markets that may be considered unprofitable by established firms (see Christensen and Bower (1996)). The market in which only start-up firms compete is characterized by complementarities: the more one firm invests into capacity the more others do, which is consistent with overinvestments we observe in some start-up dominated markets (e.g., the Webvan example and Internet retailing). Finally, our results are consistent with the empirical finding in Atuahene-Gima and Ko (2001) that entrepre-

neurial firms generally enter markets earlier than established firms. We conclude that competition involving start-ups is fundamentally different in nature from the competition between established firms, and our model offers a plausible explanation of some observed phenomena.

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