

Competing Manufacturers in a Retail Supply Chain: On Contractual Form and Coordination¹

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November 25, 2007

Abstract

It is common for a retailer to sell products from competing manufacturers. How then should the firms manage their contract negotiations? The supply chain coordination literature focuses either on a single manufacturer selling to a single retailer or one manufacturer selling to many (possibly competing) retailers. We find that some key conclusions from those market structures do not apply in our setting. We allow the manufacturers to compete for the retailer's business using one of three types of contracts, a wholesale-price contract, a quantity-discount contract or a two-part tariff. It is well known that there are two reasons why a monopolist manufacturer prefers either of the latter two, more sophisticated, contracts relative to the wholesale-price contract. First, they can be used to coordinate the supply chain, meaning that they induce the retailer to sell more because they reduce the double marginalization caused by wholesale-price contracts. Second, they can be used to extract rents from the retailer, in theory allowing the manufacturer to leave the retailer only with her reservation profit. However, we show that in our market structure these two sophisticated contracts force the manufacturers to compete more aggressively than when they only offer wholesale-price contracts, and this may leave them worse off and the retailer substantially better off. In other words, although in a serial supply chain a retailer may have just cause to fear quantity discounts and two-part tariffs, a retailer may actually prefer those contracts when offered by competing manufacturers. We conclude that the properties a contractual form exhibits in a one-manufacturer supply chain may not carry over to the realistic setting in which multiple manufacturers must compete to sell their goods through a single retailer.

¹The authors would like to thank Awi Federgruen and Richard Staelin for helpful discussions.

1. Introduction

The literature on supply chain coordination has studied several contractual forms in settings with a single manufacturer and one or more retailers. One of the key results from this literature is that wholesale-price contracts lead to suboptimal decisions for the supply chain (i.e., double marginalization) and more sophisticated contracts (like quantity discounts or two-part tariffs) can be employed to achieve both channel coordination (i.e., maximize the supply chain's profit) and rent extraction (i.e., the ability to allocate a high share of the profits to the manufacturer). Our objective in this paper is to test this conclusion in a setting in which multiple manufacturers compete to sell their products through a single retailer.

In our model, two manufacturers simultaneously offer to the retailer one of three types of contracts: a wholesale-price contract, a quantity-discount contract (i.e., a decreasing per-unit price in the quantity purchased) or a two-part tariff (i.e., a per unit price and a fixed fee). The retailer's prices determine the products' demand rates, and she sets her prices to maximize her total profit given the offered contracts and her inventory costs, which may exhibit economies of scale.

As with all supply chain structures, the firms are indirectly interested in the total profit in the supply chain, and more directly interested in their share of that profit. In supply chains with a monopolist manufacturer it has been shown that a properly designed quantity-discount or two-part tariff allows the manufacturer to maximize his product's total profit (i.e., the two firms' combined profit). Furthermore, the manufacturer can extract the largest possible share of that profit, thereby leaving the retailer with her reservation profit, which is the profit the retailer can earn if the retailer rejects the contract. In a serial supply chain the retailer's reservation profit is assumed to be an exogenous constant that reflects the retailer's bargaining power - an increase in the retailer's bargaining power is modeled by increasing the retailer's reservation profit. However, in our structure with two manufacturers, the retailer's reservation profit is endogenous - the profit the retailer can earn if it were to reject manufacturer A's offer depends on what manufacturer B offers, and vice-versa. This distinction is significant and, as we demonstrate, important for our findings.

We show that, holding the other manufacturer's contract offer fixed, a manufacturer can increase its profit by using a more sophisticated contract relative to a wholesale-price contract. Furthermore,

in equilibrium (i.e., a pair of contracts such that neither manufacturer has an incentive to offer a different contract given the other manufacturer's contract offer), we show that the more sophisticated contracts can increase the supply chain's total profit, again relative to the wholesale-price contract. These results are analogous to those found in models with a monopolist manufacturer. However, in sharp contrast to the results with one manufacturer, we also find that in equilibrium, if the products are close substitutes, the manufacturers may earn substantially less when sophisticated contracts are offered and the retailer may earn substantially more.

In a one-manufacturer setting sophisticated contracts are advantageous to the manufacturer because they allow the manufacturer to increase and extract rents. These abilities are present even with multiple manufacturers (holding the other manufacturer's contract fixed), but now there is an additional effect - the sophisticated contracts also yield more aggressive competition between the manufacturers. This additional effect may dominate the others - quantity discounts and two-part tariffs effectively increase the retailer's bargaining power relative to wholesale price contracts, so much so that the manufacturers can be worse off with them relative to an equilibrium in which they both offer wholesale-price contracts. To explain further, in our multiple-manufacturer model the retailer's reservation profit is endogenous and competition between the manufacturers serves to raise it, in particular when sophisticated contracts are used.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the model. Section 4 present our analysis of the retailer's problem, Section 5 the analysis of the wholesale and quantity-discount games, and Section 6 the analysis of two-part tariff games. Section 7 concludes the paper. All proofs are presented in Appendix A.

2. Literature Review

The present paper is foremost a commentary on the supply chain coordination literature (see Cachon 2003 for a review). As already discussed, this literature focuses on either relationships with bilateral monopoly or models with one manufacturer and multiple retailers. Wholesale-price contracts are nearly always found to be inefficient and more sophisticated contracts can be used to eliminate that inefficiency and reallocate rents arbitrarily between the parties in the supply

chain.² Choi (1991), Trivedi (1998), Lee and Staelin (1997), and Martinez de Albeniz and Roels (2007) do study systems with multiple manufacturers and a common retailer, but they only consider wholesale-price contracts.

There is an extensive literature on supply chain coordination with quantity-discount contracts (e.g., Jeuland and Shugan 1983, Moorthy 1987, Ingene and Perry 1995) but they consider only one manufacturer. There is also a considerable literature in operation management on lot-size coordination with fixed demand (e.g., Monahan 1984, Corbett and de Groote 2000). We also consider operational issues, but our retailer can adjust demand via her prices. Two-part tariffs are studied by Bernheim and Whinston (1998), O'Brien and Shaffer (1997), and Mathewson and Winter (2001) in models with multiple manufacturers and a single retailer. They explore whether or not the exclusion of a manufacturer occurs and the implications for anti-trust laws. They do not consider the wholesale or quantity-discount contracts and the profit level of individual firms. Related to two-part tariffs, there is a literature on slotting fees (which are essentially two-part tariffs with negative payments to the manufacturer, see, for example, Marx and Shaffer 2004). Kuksov and Pazgal (2007) show that slotting fees do not occur in a setting with simultaneous manufacturer competition and a single retailer, and the same result applies in our model.

McGuire and Staelin (1983) study two competing supply chains under two structural forms: in each supply chain either the manufacturer sells to a dedicated retailer via a wholesale-price contract or the manufacturer vertically integrates into retailing. In either structure the products of the two manufacturers are sold to consumers from different firms, whereas in our model the manufacturers' products are sold through a single independent retailer. Nevertheless, there are some similarities in our results. McGuire and Staelin (1983) find that the manufacturers may prefer to sell via wholesale-price contracts, despite the fact that they do not coordinate the channel nor allow the manufacturer to extract all rents, because they dampen retail competition between the two products relative to the vertically integrated structure. In our model, competition to consumers is held constant, because we have a single retailer, so what changes is the retailer's bargaining power relative to the manufacturers.

²A wholesale-price contract can maximize profits in a system with one manufacturer and multiple quantity competing retailers. However, it provides only one allocation of the system's rents and it isn't even the manufacturer's optimal wholesale price (see Cachon and Lariviere 2005). If retailers compete on price and quantity, then the wholesale price no longer guarantees coordination (see Bernstein and Federgruen 2003, 2005).

3. The Model

There are two products in the market supplied by two different manufacturers. The products are partial substitutes and are sold through a common retailer. In the first stage of the game, the manufacturers simultaneously announce the payment schemes for their products. In the second stage, the retailer chooses prices, which determine the products' demand rates, to maximize her profit. In addition to the payments to the manufacturer, the retailer incurs operating costs that depend on the average volume sold of each product. The manufacturers incur constant marginal production costs.

The retailer faces price sensitive customers. The revenue from product i is

$$R_i(d) = p_i(d)d_i,$$

where d_i is the demand rate of product i , d is the pair of demand rates, and the inverse demand function is

$$p_i(d) = \theta_i - \beta_i d_i - \gamma_i d_j, \text{ and } \beta_i > \gamma_j > 0 \text{ for all } i, j.$$

We elect to work with the inverse demand function for expositional simplicity. The formulation with the demand function is equivalent to the above.

Let $G_i(d_i)$ be the retailer's inventory related operational costs of product i ,

$$G_i(d_i) = K_i d_i^\lambda, \quad K_i \geq 0, \quad 0 < \lambda < 1,$$

where K_i and λ are exogenous constants. This functional form, which exhibits economies of scale, is a general representation of the inventory costs that arise in common inventory replenishment models such as a base-stock model³ or an economic order quantity model⁴.

Let π_i denote the retailer's profit from product i and $\pi = \pi_i + \pi_j$, the retailer's total profit. It follows that

$$\pi_i = R_i(d) - G_i(d_i) - T_i(d_i),$$

³In a periodic review model where demand follows a Normal distribution with mean d_i and standard deviation $\sigma_i d_i^\lambda$, the total inventory related costs with the optimal base-stock level is given by $(b + h) \phi(z^*) \sigma_i d_i^\lambda$, where b is the backlog penalty per unit, and h is the inventory holding cost per period. Defining $K_i = (b + h) \phi(z^*) \sigma_i$ leads to the G_i function.

⁴In the economic order quantity (EOQ) model, the retailer incurs a fixed cost k_i per order and a holding cost h_i per unit of inventory held for one period. The well known EOQ formula suggests ordering every $\sqrt{2k_i/h_i d_i}$ periods. The resulting optimal inventory and ordering costs is given by $\sqrt{2k_i d_i h_i}$. Defining $K_i = \sqrt{2k_i h_i}$ leads to the G_i function with $\lambda = 1/2$.

and $T_i(d_i)$ is the payment made to the manufacturer based on the retailer's demand rate and their agreed upon contract.⁵ Manufacturer i 's profit is

$$\Pi_i = T_i(d_i) - c_i d_i, \quad (1)$$

where c_i is the manufacturer's cost per unit.

We consider three different types of contracts. With a wholesale-price contract, the payment function is

$$T_i(d_i) = w_i d_i,$$

where w_i is the wholesale price chosen by manufacturer i .

We include the following set of quantity-discount contracts:

$$T_i(d_i) = \begin{cases} w_i d_i - v_i d_i^2 / 2, & \text{if } d_i \leq (w_i - c_i) / v_i \\ T_i((w_i - c_i) / v_i) + c_i (d_i - (w_i - c_i) / v_i), & \text{otherwise.} \end{cases} \quad (2)$$

where w_i is a flexible parameter chosen by the manufacturer, c_i is the manufacturer's marginal cost per unit, and v_i is an exogenous constant, where $v_i \in [0, \bar{v})$, $\bar{v} = 2\beta_i - (\gamma_i + \gamma_j)$. We have intentionally designed this set of quantity-discount contracts such that (1) they have a single parameter, just like the wholesale-price contract, w_i , and (2) wholesale-price contracts are a subset of our quantity-discount contracts - a quantity discount with $v_i = 0$ is a wholesale-price contract.

Although we only consider a subset of possible quantity discounts, this is not overly restrictive. Our quantity discounts are continuous, differentiable, concave, and the manufacturer does not sell even the marginal unit for less than its production cost. The bound on v_i implies that the quantity-discount is not too "aggressive" in the sense that the marginal price paid does not fall too rapidly as the purchase volume increases, i.e., $T_i''(d_i) \geq -v_i$. In fact, it can be shown that our quantity-discounts are optimal for the manufacturer (holding the other manufacturer's contract offer fixed) given the $T_i''(d_i) \geq -v_i$ constraint and assuming differentiability (see Proposition 2 in Appendix A). Furthermore, $T_i''(d_i) \geq -v_i$ ensures the retailer's profit function minus operational costs is strictly concave in (d_1, d_2) . This naturally raises the question of whether the manufacturer could do better by offering an even more aggressive quantity discount. The manufacturer can

⁵The retailer's payment $T_i(d_i)$ to a manufacturer can be interpreted as a yearly (average) payment based on yearly (average) volume. Many manufacturer-retailer purchasing contracts are based on the yearly volume rather than on the volume of individual shipments.

do better, but we have found that an equilibrium in the manufacturer’s contract offer game may not exist when we consider more aggressive quantity discounts. However, a two-part tariff can be interpreted as the most aggressive quantity discount, and we do have results for those contracts. Thus, although we cannot characterize the equilibrium dynamics for all quantity discounts, we have results for quantity-discounts that are more aggressive than wholesale-price contracts and results for the most aggressive contract, the two-part tariff.

As already mentioned, our third contract form is the two-part tariff, which is characterized by a fixed fee F_i and a marginal cost w_i :

$$T_i(d_i) = F_i 1_{\{d_i > 0\}} + w_i d_i, \tag{3}$$

where indicator function $1_{\{d_i > 0\}} = \{1, \text{ if } d_i > 0; 0, \text{ otherwise}\}$.

A symmetric game across manufacturers means that the data for the two products are identical, i.e., $c_i, \theta_i, \beta_i, \gamma_i, K_i, v_i$ are the same for any i . The subscript i will be dropped in those cases. In a symmetric solution, the decisions (d_i at the retail level, w_i or T_i at the manufacturer level) are identical across products.

In the following sections, we solve the problem using backward induction. We analyze the retailer’s decision first and then the game between the manufacturers.

4. The Retailer’s Problem

In the first stage of the game, the manufacturers simultaneously announce their contract offers. We assume that the particular contractual form offered is established before the game begins, but we later discuss what happens when the manufacturers choose their contractual form and parameters simultaneously. In the second stage, given functions T_1 and T_2 , the retailer chooses the demand rates to maximize her profit.

4.1 The retailer’s decision with quantity discounts

In this section we assume the manufacturers offer the retailer quantity discounts, which, as discussed earlier, are wholesale-price contracts when $v_i = 0$. We defer the discussion of two-part tariffs to Section 6.

Define

$$d_i(d_j) = \arg \max_{d_i} \{\pi_i(d_i, d_j)\}, \quad (4)$$

$$(\tilde{d}_i, \tilde{d}_j) = \arg \max_{d_i, d_j} \{\pi(d_i, d_j) : d_i d_j > 0\}, \quad (5)$$

$$\tilde{\pi} = \pi(\tilde{d}_i, \tilde{d}_j) \quad (6)$$

The retailer's optimization problem can now be written as

$$\max\{\tilde{\pi}, \pi_1(d_1(0), 0), \pi_2(0, d_2(0))\}. \quad (7)$$

Hence, the optimal solution to the retailer's problem is

$$(d_1^*, d_2^*) \in \{(\tilde{d}_1, \tilde{d}_2), (d_1(0), 0), (0, d_2(0))\}.$$

For expositional simplicity, we assume the retailer breaks ties in favor of carrying a full product line, if possible, over a single product.

Consider the problem of maximizing total supply chain profit. It is equivalent to the retailer's problem if the manufacturers charge only their production cost. Define $\hat{\pi}_1 = \pi_1(d_1(0), 0)$, $\hat{\pi}_2 = \pi_2(0, d_2(0))$, and $\hat{\pi}_{12} = \pi(\tilde{d}_1, \tilde{d}_2)$ when $T_i(d_i) = c_i d_i$ for $i = 1, 2$. These profit levels are respectively the maximum profit the system earns, if it were to carry only product 1, only product 2 and both products. We assume

$$\hat{\pi}_{12} > \hat{\pi}_i > 0, \text{ for } i = 1, 2, \quad (8)$$

which implies that it is always optimal for the system to carry both products.

Now return to the system with independent manufacturers and a retailer. Let H_i denote the first derivative of π with respect to d_i . We have

$$H_i = \partial\pi/\partial d_i = \theta_i - 2\beta_i d_i - (\gamma_i + \gamma_j) d_j - G'_i(d_i) - T'_i(d_i), \quad j \neq i. \quad (9)$$

Consider first the case with no economies of scale (i.e., $K_1 = K_2 = 0$). The retailer's profit function π is jointly concave in (d_1, d_2) , so the unique solution to $\{H_i = 0, i = 1, 2\}$ is the unique optimal solution. If an interior solution does not exist, then the optimal solution is either $(d_1(0), 0)$ or $(0, d_2(0))$, where $d_i(0)$ is the unique solution to $H_i = 0$ with $d_j = 0$. Because π is well behaved, the optimal solution (d_1^*, d_2^*) can be easily characterized and it is a continuous, differentiable function of the problem inputs such as the parameters of the manufacturer contracts.

The case with economies of scale, however, is rather complicated. Observe that $\partial^2\pi/\partial d_i^2 = -2\beta_i - G_i''(d_i) - T_i''(d_i)$ is positive at $d_i = 0$ and then decreasing in d_i . Thus, π is convex-concave in d_i for fixed d_j . There are up to two solutions to $H_i = 0$ and the larger of the two solutions is a local maximum. $d_i(0)$ is either the larger solution or zero. Evidently, π is not jointly concave in (d_i, d_j) . As a result, there may be multiple solutions to $\{H_i = 0, i = 1, 2\}$ and we do not know which one could be the interior optimal solution. Furthermore, the global optimal solution may be at one of the boundaries $d_i = 0$. A final technical note is that the profit function is not necessarily unimodal (in one or two dimensions).

The next theorem shows that there can be at most one interior local maximum and that the optimal solution is either that interior solution or at one of the boundary lines. That is, there are at most three candidate optimal solutions and each is characterized by a set of first order conditions. Furthermore, in a symmetric problem, the unique interior maximum is a symmetric solution.

Theorem 1 *The retailer's optimal solution $(d_1^*, d_2^*) \in \{(\tilde{d}_1, \tilde{d}_2), (d_1(0), 0), (0, d_2(0))\}$. $(\tilde{d}_1, \tilde{d}_2)$ is the unique interior optimal solution to $\{H_i = 0, i = 1, 2\}$ and $d_i(0)$ is given by the larger of the two solutions to $\{H_i = 0, \text{ s.t. } d_j = 0\}$. In a symmetric problem, $(\tilde{d}_1, \tilde{d}_2) = (\tilde{d}, \tilde{d})$, where \tilde{d} is the larger of the two solutions to*

$$\theta - 2(\beta + \gamma)d - G'(d) - T'(d) = 0.$$

In summary, there are at most three local maxima for a retailer's problem: one interior solution in which the retailer carries both products, and two boundary solutions in which the retailer carries only one product.

While the retailer's profit function is generally complex in the presence of economies of scale, the following conditions ensure that it is jointly concave in (d_1, d_2) . We state and prove this result in Lemma 1 in Appendix A.

$$\beta_1 = \beta_2 = \beta, \quad \gamma_1 = \gamma_2 = \gamma, \tag{10}$$

$$T_i''(d_i) \geq -v_i, \quad \text{where } 0 \leq v_i < \beta - \gamma, \tag{11}$$

$$2R_i/G_i \geq |\varepsilon_{ii}|, \quad \text{for all } i, \quad \text{where } \varepsilon_{ii} = -\frac{\beta_j}{\beta_i\beta_j - \gamma_i\gamma_j} \frac{p_i}{d_i} \tag{12}$$

The condition (10) requires the own- and cross-price coefficients to be symmetric. This is not a very restrictive assumption, because we allow nonidentical θ_i , which implies different demand rates

and price elasticities for the products. The condition (11) is stricter than our earlier assumption: it requires the quantity discount to be less concave to guarantee the concavity of the retailer's profit function. The condition (12) stipulates that the own-price elasticity (ε_{ii}) is less than two times the revenues-to-average inventory costs ratio of the product. (It is similar to the conditions Bernstein and Federgruen (2003) developed for decentralized retailers.)

5. Manufacturers' Problem

In this section, we analyze the game between the manufacturers assuming they offer quantity discounts (possibly with $v_i = 0$). In the presence of economies of scale (i.e., $K_i > 0$ for any i), we assume that conditions (10)-(11) hold and we restrict our attention to a region defined by (12). Each manufacturer chooses its own best response $T_i(T_j)$ given the other manufacturer's contract T_j .

$$T_i(T_j) = \arg \max_{T_i(d)} \Pi_i(d^*) \text{ for all } i, \text{ where } d^* = \arg \max \pi. \quad (13)$$

An equilibrium of the game is a pair of contracts (T_i^*, T_j^*) such that neither manufacturer has an incentive to offer a different contract.

The following remark demonstrates how the contracting problem with multiple manufacturers is different from that with a single manufacturer.

Remark 1 *For any fixed contract offered by manufacturer 2 such that $\pi_2(0, d_2(0)) > 0$:*

1. *Consider the set of contracts such that the retailer's payment to manufacturer 1 is a non-decreasing function of d_1 . There does not exist a contract in this set such that the manufacturer can extract all of the profit from his product (i.e., it is not possible to have $\Pi_1 > 0$ and $\pi_1 = 0$).*
2. *The retailer accepts manufacturer 1's contract offer and stocks both products only if*

$$\pi_1(\tilde{d}_i, \tilde{d}_j) \geq \pi_2(0, d_2(0)) - \pi_2(\tilde{d}_i, \tilde{d}_j) \quad (14)$$

Unlike in a **serial supply chain**, the first statement implies that a manufacturer must leave the retailer with some profit to induce the retailer to carry the manufacturer's product (see Appendix A for a detailed proof). In other words, the retailer's reservation profit for carrying a product is

greater than zero. However, this does not mean that a single reservation profit exists. The second statement provides an intuitive condition, (14), for when the retailer is willing to carry manufacturer 1's product - the retailer must earn more with product 1 in the assortment than without product 1 in the assortment. The right hand side of (14) can be considered the retailer's reservation profit that the retailer must earn from product 1 for the retailer to be willing to carry product 1. The first term, $\pi_2(0, d_2(0))$, is fixed, given manufacturer 2's contract offer. However, the second term, $\pi_2(\tilde{d}_i, \tilde{d}_j)$, depends on manufacturer 1's contract offer (assuming the mild condition that the optimal interior demands, $(\tilde{d}_i, \tilde{d}_j)$, change smoothly in the contract parameters). Therefore, even if manufacturer 2's contract offer is held fixed, there does not exist a reservation profit for product 1 that is independent of manufacturer 1's contract offer, as is assumed in models with a single manufacturer. Put another way, a serial supply chain with a fixed reservation profit for the retailer cannot replicate the dynamics of our model.

It remains to characterize the equilibrium contract offers by the manufacturers. We are able to partially characterize the equilibrium of the contract offer game when the products are symmetric (i.e., identical $\theta_i, \beta_i, \gamma_i, c_i, K_i, v_i$). We start with the following observation that rules out equilibria in which the retailer carries only one product.

Remark 2 *In a symmetric game, there does not exist a manufacturer equilibrium where $d_i = 0$ for some i .*

The result is due to (8), which guarantees that the inclusion of a manufacturer strictly increases system profit. Suppose there were an equilibrium in which manufacturer i is excluded. Regardless of the fraction of $\hat{\pi}_j$ the retailer earns, manufacturer i can offer to sell to the retailer at $c_i + \varepsilon$ for an arbitrarily small ε and then the retailer's profit increases if it carries product i . If j is excluded as a result, it will react similarly and be included.

Define $\underline{w}_i(w_j) = \max\{w_i : d_j^*(w_i, w_j) = 0\}$, the maximum w_i that makes the retailer exclude product j , $\bar{w}_i(w_j) = \min\{w_i : d_i^*(w_i, w_j) = 0\}$, the minimum wholesale price of i that makes the retailer exclude product i . Note that $\underline{w}_i(w_j)$ may not exist for every w_j . In that case, set $\underline{w}_i(w_j) = c_i$. Define $w_i(w_j)$ as the best response of manufacturer i , which can be found via a line-search between $[\underline{w}_i, \bar{w}_i]$.

The following theorem characterizes the unique symmetric equilibrium of the contract offer

game.

Theorem 2 *Consider a symmetric game in which the manufacturers offer quantity discounts.*

There exists a unique solution to

$$d_i^*(w_1, w_2) + (w_i - v_i - c_i) \frac{\partial d_i^*}{\partial w_i} = 0, \text{ for all } i,$$

denoted (w_1^, w_2^*) , which is the unique candidate to be a symmetric equilibrium.*

As can be seen in the proof of the above theorem, showing the unimodality of a manufacturer's profit in w_i requires the use of the second-order properties of the retailer's optimal solution (d_1^*, d_2^*) . The analysis of a manufacturer game that depends on the solution of a complex problem at the retailer presents technical difficulties in the presence of economies of scale, that have not been present in any other competition paper in the literature.

We cannot guarantee that the candidate point described in Theorem 2 is an equilibrium. We show that at (w_1^*, w_2^*) , w_i^* is a local optimum for manufacturer i , and Π_i is concave for $w_i > w_i^*$. However, the optimal solution $w_i(w_j^*)$ may be different than w_i^* in the range $[w_i, w_i^*]$. If $w_i(w_j^*) = w_i^*$, then (w_1^*, w_2^*) is indeed an equilibrium point. If not, there there exists no symmetric equilibrium.

Now consider the situation in which there are no economies of scale. This substantially simplifies the analysis of both the retailer's demand decisions and the manufacturers' contract offer problem. For any (v_1, v_2) and asymmetric products, we can now guarantee joint concavity of the retailer's profit and the existence and the uniqueness of the equilibrium without the symmetry assumptions. The next theorem provides the closed form solutions for the demand rates and the contract parameters by solving the first-order conditions given in Theorem 2 for the general case.

Theorem 3 *With no economies of scale (i.e., $K_i = 0$), there exists a unique equilibrium of the game in which the manufacturers offer quantity discounts. It is characterized by the following reaction functions and the optimal demand rates:*

$$d_i^* = \frac{(2\beta_j - v_j)(\theta_i - w_i) - (\gamma_i + \gamma_j)(\theta_j - w_j)}{(2\beta_i - v_i)(2\beta_j - v_j) - (\gamma_i + \gamma_j)^2},$$

$$w_i(w_j) = \frac{[(2\beta_j - v_j)(\theta_i) - (\gamma_i + \gamma_j)(\theta_j - w_j)](\delta + v_i(2\beta_j - v_j)) + c_i(2\beta_j - v_j)\delta}{(2\beta_j - v_j)(\delta + \delta + v_i(2\beta_j - v_j))},$$

where $\delta \equiv (2\beta_i - v_i)(2\beta_j - v_j) - (\gamma_i + \gamma_j)^2$.

In this section we have assumed that the manufacturers offer quantity discounts with an exogenously specified v_i . Suppose the manufacturers now simultaneously offer a (w_i, v_i) where they are free to choose any $v_i \in [0, \bar{v})$. In other words, they can choose to offer a wholesale-price contract ($v_i = 0$) or a quantity discount ($v_i > 0$). The next proposition indicates, as in supply chains with a single manufacturer, that a manufacturer prefers to offer a quantity-discount contract and, in particular, prefers more aggressive quantity discounts. Quantity discounts allow the manufacturer to increase supply chain rents (i.e., reduce double marginalization) and to extract rents, so they are the preferred contract when the other manufacturer's contract is held fixed even when the retailer can adjust its demand allocations between the two products in response.

Proposition 1 *Manufacturer i 's profit strictly increases with v_i at the optimal w_i . That is, if a manufacturer is given the option to choose between three contractual forms with $v_i \in \{0, a, b\}$ such that $0 < a < b$, then $\Pi_i(w_i^*|v_i = 0) < \Pi_i(w_i^*|v_i = a) < \Pi_i(w_i^*|v_i = b)$.*

5.1 Comparison of the Equilibrium under Wholesale Price and Quantity-Discount Contracts

This section presents numerical examples to compare the equilibrium solution when the manufacturers offer wholesale price and quantity-discount contracts. We start with the following example: $\theta_i = 20$, $\beta_i = 2$, $\gamma_i = 1$, $c_i = 1$, $K_i = 0$ for all i , and $v_1 = v_2 = v = 1.6$ when quantity discounts are offered. Table 1 provides the equilibrium results.

Contracts offered	v_1	v_2	w_1^*	w_2^*	d_1^*	d_2^*	π	Π_1	Π_2
wholesale price	0	0	7.33	7.33	2.11	2.11	26.74	13.37	13.37
quantity discounts	1.6	1.6	7.59	7.59	2.82	2.82	35.02	12.22	12.22

Table 1: Equilibrium contract parameters, demand rate, and profits under wholesale and quantity discount contracts.

Recall that a manufacturer prefers to offer a quantity discount for any fixed contract offer by the other manufacturer. However, this preference does not carry over to the equilibrium analysis. Competition between the manufacturers is different when they both offer quantity discounts than when they both offer wholesale-price contracts. Apparently, it can be a more aggressive type of competition. Note, the supply chain is better off with quantity-discounts - as in the serial supply chain, quantity discounts improve supply chain efficiency. However, in this example, the retailer

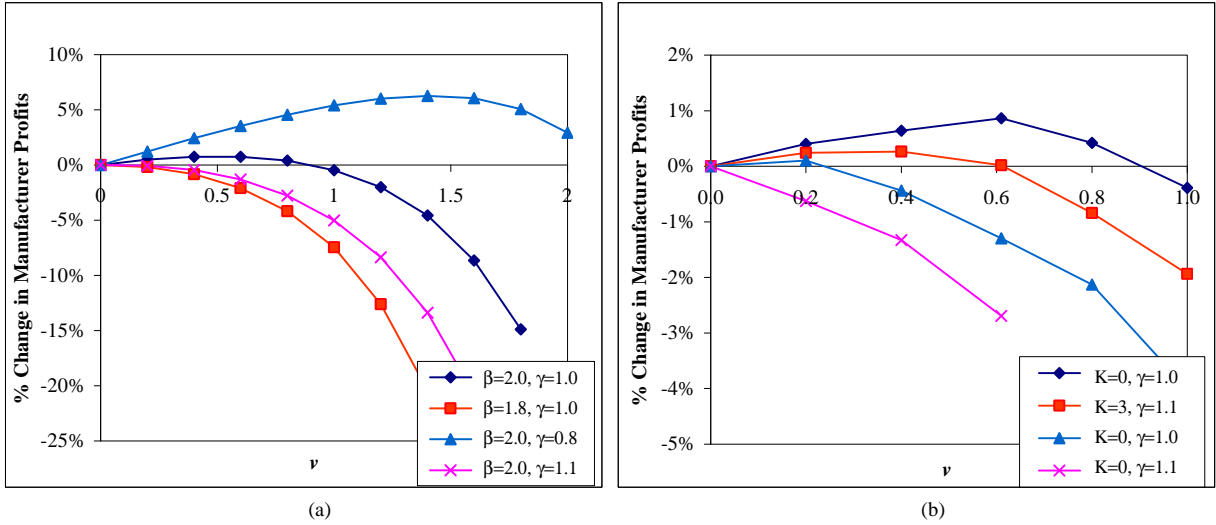


Figure 1: The effect of the concavity of quantity discount contracts on manufacturer profits: % change in Π_1 at equilibrium as v increases. Base case: $\theta = 20, \beta = 2, \gamma = 1, c = 1, K = 0$.

captures all of the improved efficiency and even more, leaving the manufacturers worse off. This need not always be true, as we next demonstrate.

Figure 1(a) plots the change in the manufacturers' profit as v is modified. In our example, if we reduce v from 1.6 to 0.4, then the manufacturers are somewhat better off offering quantity discounts than wholesale-price contracts - in this case the manufacturer competition has not been intensified substantially, allowing them to increase their profit. However, if the products become more substitutable (lower β or higher γ), then the manufacturers can be worse off with quantity-discounts no matter how weak the quantity discounts are (i.e., no matter how low v is). On the other hand, with less substitutable products (lower γ) the manufacturers benefit from quantity discounts at equilibrium. Figure 1(b) shows that including inventory costs can make the negative effects of competition with quantity discounts on the manufacturers' profits more pronounced.

We constructed a numerical study to better understand the extent of these observations. We chose parameters from the following sets, leading to 108 scenarios: $\theta = \{20, 40\}$, $\beta = \{1, 2, 4\}$, $\gamma = \{0.25, 0.5, 0.75\} \times \beta$, $c = \{1, 3\}$, $K = \{0, 1, 3\}$. Evaluating each scenario with $v = \{0, 0.5, 0.95\} \times (\beta - \gamma)$, i.e., under wholesale-price and two different quantity discount contracts, leads to a total 324 instances. In nine of the 108 scenarios, we haven't been able to find an equilibrium for at least one of the contracts (including the wholesale-price contracts in five scenarios). Investigating

the best response functions in those cases reveals that the effect of the economies of scale is very strong at the retailer and the manufacturers cycle between undercutting prices to get the retailer to exclude the other manufacturer and being undercut. In those cases, there does not exist an asymmetric equilibrium either. We report the results for the 297 instances that produced a symmetric equilibrium of the game. As a validity check, we compare the average cost per unit that the retailer pays to the manufacturers, $T^*(d)/d$, with the two types of contracts: the average cost is 5% lower in the quantity discounts equilibrium than the wholesale prices when $v = 0.5(\beta - \gamma)$ and 11% lower when $v = 0.95(\beta - \gamma)$, which indicates that the quantity discounts in equilibrium in these examples are modest.

Table 2 provides a summary of the comparison of profits under the quantity-discount equilibrium relative to the wholesale-price equilibrium. In all cases, the supply chain profit and the retailer's profit increase when quantity discounts are offered. For low substitutability levels (i.e., $\gamma = 0.25\beta$), the manufacturers' profits increase with the quantity-discount contract. Furthermore, the manufacturers' profit increase is greater than the supply chain's profit increase, which means that the manufacturers are able to extract a higher share of the supply chain's profit. With medium and high substitutability levels, however, the manufacturers' equilibrium profits are generally lower with quantity discounts than with wholesale-price contracts, and the magnitude of the profit loss can be substantial, especially with a more aggressive quantity discount (i.e., higher v). In those cases the effect of more aggressive competition among the manufacturers dominates the effect of increased supply chain profits. Finally, the negative effect of quantity discounts on the manufacturers' profits becomes more pronounced with increased economies of scale. For instance, with $\gamma = 0.5\beta$ and $v = 0.95(\beta - \gamma)$, both the percentage of cases in which the manufacturers are worse off and the average magnitude of the manufacturers' loss increase with k . In summary, the decisions for the two products are more tightly linked if they are close substitutes or the level of economies of scale is high. This increases the retailer's endogenous reservation profit (the value of the option to carry more of the competitor's product) and effectively the retailer's power in its relation with each manufacturer. Our results suggest the manufacturers are better off employing the simple wholesale-price contracts when dealing with a powerful retailer rather than the more sophisticated quantity discounts.

$\gamma = 0.25 \times \beta$						
$v = 0.5 \times (\beta - \gamma)$			$v = 0.95 \times (\beta - \gamma)$			
	$k = 0$	$k = 1$	$k = 3$	$k = 0$	$k = 1$	$k = 3$
Average % gain in total profit	5.7	6.0	6.0	11.2	11.5	11.7
Average % gain in the retailer's profit	3.3	3.2	3.2	6.0	5.9	6.0
% of cases manufacturers are better off	100	100	100	100	100	100
Average % gain in manufacturer profits	7.3	7.8	7.7	14.6	15.2	15.2
% of cases manufacturers are worse off	0	0	0	0	0	0
Average % loss in manufacturer profits	n/a	n/a	n/a	n/a	n/a	n/a
$\gamma = 0.5 \times \beta$						
Average % gain in total profit	3.1	3.6	3.7	6.8	6.9	7.1
Average % gain in the retailer's profit	6.0	6.6	7.1	13.8	14.7	15.8
% of cases manufacturers are better off	100	100	100	9	0	0
Average % gain in manufacturer profits	0.8	0.7	0.5	0.2	n/a	n/a
% of cases manufacturers are worse off	0	0	0	91	100	100
Average % loss in manufacturer profits	n/a	n/a	n/a	-0.2	-0.6	-1.2
$\gamma = 0.75 \times \beta$						
Average % gain in total profit	1.4	1.5	1.4	2.6	2.7	2.6
Average % gain in the retailer's profit	5.9	6.1	6.3	12.4	12.9	13.1
% of cases manufacturers are better off	0	0	0	0	0	0
Average % gain in manufacturer profits	n/a	n/a	n/a	n/a	n/a	n/a
% of cases manufacturers are worse off	100	100	100	100	100	100
Average % loss in manufacturer profits	-7.5	-8.1	-8.6	-16.9	-18.5	-19.3

Table 2: Summary statistics on supply chain, retailer and manufacturer profits in the quantity-discount equilibrium relative to the wholesale-price equilibrium.

6. Two-part Tariffs

In this section, we consider the game between manufacturers that offer two-part tariff contracts and compare it with the outcome of the wholesale price game. Several papers in the industrial organization literature (e.g., Bernheim and Whinston, 1998) provide a characterization of the equilibrium with two-part tariffs. The next theorem generalizes this characterization to the case with economies of scale at the retailer.

Theorem 4 *Given any contract by manufacturer j , the optimal contract of manufacturer i is a two-part tariff contract of the type (F_i, c_i) . The equilibrium of the two-part tariff game is given by $F_i^* = \hat{\pi}_{12} - \hat{\pi}_j$ for $i = 1, 2, j = 3 - i$. Hence, $\Pi_i = \hat{\pi}_{12} - \hat{\pi}_j$ and $\pi = \hat{\pi}_1 + \hat{\pi}_2 - \hat{\pi}_{12}$.*

As can be seen in the proof of the theorem, showing the optimality of a two-part tariff in the presence of economies of scale requires checking the boundary solutions of the multi-modal retail profit function so that its product does not get dropped when a manufacturer changes its contract. The first result states that the optimal contract for a manufacturer is to set the variable cost equal to the marginal production cost, thereby making a profit exclusively from the fixed fee, $\Pi_i = F_i$. The second part of the theorem states that in equilibrium each manufacturer charges a fixed fee that equals exactly the incremental benefit it brings to the whole system. Overall, just as we have shown that a manufacturer prefers quantity discounts over wholesale-price contracts for a fixed contract offer from the other manufacturer (Proposition 1), Theorem 4 indicates that two-part tariffs are a dominant strategy for the manufacturer - a manufacturer prefers the more aggressive two-part tariffs over a wholesale-price contract.

When the manufacturers offer two-part tariffs the retailer chooses system optimal pricing and ordering decisions (because the manufacturers' per unit prices equal their marginal costs). Hence, the system's profit is maximized and the allocation of that profit is determined by the fixed fees. System profits are not maximized when the manufacturers offer wholesale price or quantity discounts.

Although the manufacturers are able to extract rent more effectively with two-part tariffs than with quantity-discounts, they cannot extract all of the profit from their product. The retailer gains considerable power when she retains stocking and pricing decision rights and the manufacturers

compete. Especially when the products are close substitutes, $\hat{\pi}_{12}$ is not much higher than $\hat{\pi}_1$ or $\hat{\pi}_2$. Thus, in that case, the retailer captures a large part of the total profit and each manufacturer gets only $\hat{\pi}_{12} - \hat{\pi}_i$, which is relatively small.

The next theorem indicates that, despite being the dominant strategy, the manufacturers may be worse off in equilibrium with two-part tariffs relative to wholesale-price contracts.

Theorem 5 *Consider a symmetric system with no economies of scale (i.e., $K = 0$). The manufacturers' profits when they offer two-part tariffs is higher than when they offer wholesale-price contracts if and only if $\gamma < \beta(2 - \sqrt{2}) \cong 0.585\beta$. The retailer's profit is lower under two-part tariffs than under wholesale-price contracts if and only if $\beta^3 > 2\gamma(2\beta - \gamma)$.*

Although a two-part tariff allows a manufacturer to extract rents more effectively, if the products are highly substitutable, the manufacturers' profits are lower with two-part tariffs because they compete more aggressively. Appendix B shows that the same result could hold in cases with asymmetric market sizes and costs.

The second part of the theorem states that the retailer's profit is lower under two-part tariff contracts only if the product substitutability is very low, e.g., if $\gamma \leq 0.15$ when $\beta = 1$. In the extreme case of independent products, i.e., $\gamma = 0$, the retailer's profit is zero, because there is no competition and each manufacturer is able to extract all the profits with a two-part tariff.

Next we repeat the numerical study in Section 5.1 to compare the equilibrium profits with two-part tariffs to those under wholesale-price contracts for various levels of economies of scale, namely $K \in \{0, 1, 3, 5, 7\}$.

We present a summary of the results in Table 3, which illustrates several interesting insights on the effect of quantity discounts. First, when product substitutability is low, because the manufacturers can extract rents well with these contracts, the retailer's profit is lower and the manufacturers' profits are higher. As product substitutability increases, similar to our conclusion on the effect of quantity discounts, the manufacturers' profits are lower when they offer two-part tariffs, and the magnitude of the loss can be as high as 39%. Second, as the level of economies of scale increases, the retailer benefits more from the switch to two-part tariffs and the manufacturers benefit less. With a higher level of economies of scale, because the retailer has more incentive to consolidate its demand to one product, the manufacturer competition becomes more intense

and the retailer gains more power. In other words, the positive effect of two-part tariffs on the manufacturers' profits in the case of low substitutability diminishes with higher degree of economies of scale and the negative effect in the case of high substitutability becomes more pronounced. Third, total supply chain profit always increases when the manufacturers offer two-part tariffs, because two-part tariffs achieve system coordination. The percentage increase in total profit decreases as product substitutability increases, because wholesale-price competition is more intense with higher γ . In summary, when two-part tariffs are not very useful to the supply chain, because products are highly substitutable, they are particularly destructive for the manufacturers. If two-part tariffs benefit the supply chain substantially, then they are good for the manufacturers too.

	$\gamma = 0.25 \times \beta$				
	$k = 0$	$k = 1$	$k = 3$	$k = 5$	$k = 7$
Average % gain in total profit	22.5	22.6	22.9	23.2	23.3
Average % gain in the retailer's profit	-23.4	-20.1	-11.2	3.8	7.0
% of cases manufacturers are better off	100	100	100	100	100
Average % gain in manufacturer profits	53.1	50.2	43.6	35.9	34.0
% of cases manufacturers are worse off	0	0	0	0	0
Average % loss in manufacturer profits	n/a	n/a	n/a	n/a	n/a
	$\gamma = 0.5 \times \beta$				
Average % gain in total profit	12.5	12.4	12.1	12.0	12.0
Average % gain in the retailer's profit	12.5	16.3	25.6	28.0	29.7
Average % change in manufacturer profits	12.5	8.6	-0.3	-2.6	-4.3
% of cases manufacturers are better off	100	100	58	44	29
Average % gain in manufacturer profits	12.5	8.6	5.9	4.5	3.1
% of cases manufacturers are worse off	0	0	42	56	71
Average % loss in manufacturer profits	n/a	n/a	-9.1	-8.4	-7.2
	$\gamma = 0.75 \times \beta$				
Average % gain in total profit	4.2	3.9	3.7	3.6	3.5
Average % gain in the retailer's profit	17.3	19.5	21.7	22.6	23.5
% of cases manufacturers are better off	0	0	0	0	0
Average % gain in manufacturer profits	n/a	n/a	n/a	n/a	n/a
% of cases manufacturers are worse off	100	100	100	100	100
Average % loss in manufacturer profits	-21.9	-27.9	-33.8	-36.3	-38.8

Table 3: Summary statistics on supply chain, retailer and manufacturer profits in the two-part tariff equilibrium relative to the wholesale-price equilibrium.

7. Discussion

This paper is a first attempt to consider contracts other than wholesale-price contracts in systems with multiple competing manufacturers and a single common retailer. We demonstrate that, holding manufacturer B's contract offer fixed, manufacturer A can use a quantity discount or two-part tariff to improve supply chain performance and extract rents, as is known with contracting in supply chains with a monopolist manufacturer. However, when the manufacturers compete in the contract offers, then we show that the manufacturers may be worse off with quantity discounts or two-part tariffs relative to wholesale-price contracts even though the supply chain can be better off. When downstream competition between the manufacturers is high, i.e., when the products are close substitutes, the retailer benefits considerably when the manufacturers compete with the more aggressive contracts, and manufacturers are worse off. If the products are not close substitutes, then the retailer may be harmed by the more aggressive contracts and the manufacturers are better off, because the manufacturers are able to extract rent more effectively. We find these effects to be more significant when the level of economies of scale at the retailer is high. In summary, the decisions on the two products are more tightly linked if they are close substitutes or the level of economies of scale at the retailer is high. This increases the retailer's endogenous reservation profit (the value of the option to carry more of the competitor's product) and effectively the retailer's power in its relation with each manufacturer. Our results suggest the manufacturers are better off employing the simple wholesale-price contracts when dealing with a powerful retailer rather than the more sophisticated quantity discounts or two-part tariffs. We conclude that it is important to study the properties of a contractual form in the presence of competition between manufacturers.

The prominence of wholesale-price contracts in practice and their widespread use in many industries, despite the fact that the manufacturers' optimal contract is usually a more sophisticated one, has been an open empirical question for researchers. One possible explanation has to do with the simplicity and lower implementation costs associated with wholesale-price contracts. Our results may provide another hypothesis to include in this discussion. The manufacturers in our model essentially face a prisoners' dilemma game - their best myopic action is always to offer the most aggressive contract, but this then leads to an equilibrium in which they are both worse off. In a repeated game setting, it is well known that the players may be able to coordinate on the

good outcome (offering wholesale-price contracts in our setting with highly substitutable products) by utilizing trigger strategies (defections are penalized by the other firm for a limited number of periods even though the bad outcome, offering quantity-discount or two-part tariff contracts, is the Nash equilibrium in a single-shot game). It may be argued that the manufacturers have learned not to engage in actions that will trigger punishment, and hence choose wholesale-price contracts over more sophisticated contracts that are myopically optimal, but harmful at equilibrium. It will be interesting to test this hypothesis empirically by investigating which contract types are commonly used for various product groups with different levels of substitutability.

A quantity discount essentially reduces a retailer's marginal cost. Other contracts, such as buy-back and revenue-sharing contracts, have a similar effect on the retailer. Thus, we suspect that similar results can be found for manufacturers competing with these contracts. That may require, however, retailer models that are different than ours, such as a model with inventory competition (i.e., substitution between products based on availability). While the particular retailer model we have worked with presents many technical difficulties in the presence of economies of scale, alternative models may be no less challenging. For the inventory competition model, the characterization of the best response of a manufacturer even in the simpler wholesale price case can only be achieved under very restrictive assumptions (see Kk 2003). The derivation of the optimal wholesale-price contract as in Lariviere and Porteus (2001) is not possible because the retailer changes the quantities of both products in response to a change in the wholesale price of one manufacturer. Finally, extension of this discussion to a setting with multiple common retailers will lead to richer and more complicated dynamics.

References

- Bernstein, F. and A. Federgruen. 2003. Pricing and replenishment strategies in a distribution system with competing retailers. *Oper. Res.* **51** 409-426.
- Bernstein, F., A. Federgruen. 2005. Decentralized supply chains with competing retailers under demand uncertainty. *Management Sci.* **51**(1) 18-29.
- Bernheim, B. D., and M. D. Whinston. 1998. Exclusive Dealing. *Journal of Political Economy.* **106** 64-103.

- Cachon, G. P. 2003. Supply chain coordination with contracts. *Handbooks in Operations Research and Management Science: Supply Chain Management*. Eds. S. Graves and T. de Kok. North Holland.
- Cachon, G. P., M. Lariviere. 2005. Supply chain coordination with revenue sharing: strengths and limitations. *Management Sci.* **51**(1): 30-44.
- Corbett, C. and X. de Groot. 2000. A supplier's optimal quantity discount policy under asymmetric information. *Management Sci.* **46**(3) 444-50.
- Choi, S. C. 1991. Price competition in a channel structure with a common retailer. *Marketing Sci.* **10** 271-296.
- Ingene, C. A., M. E. Perry, 1995. Channel coordination when retailers compete. *Marketing Sci.* **14**(4) 360-377.
- Jeuland A. P., S. P. Shugan 1983. Managing channel profits. *Marketing Sci.* **2** 239-272.
- Kök, A. G. 2003. Management of product variety in retail operations. Ph.D. Dissertation. The Wharton School, University of Pennsylvania, Philadelphia, PA.
- Kuksov, D., A. Pazgal. 2007. The effects of costs and competition on slotting allowances. *Marketing Sci.* **26** (2) 259-267.
- Lariviere, M., E. Porteus. 2001. Selling to the newsvendor: an analysis of price-only contracts. *Manufacturing Service Oper. Management.* **3** 293-305.
- Lee, E., R. Staelin. 1997. Vertical strategic interaction: Implications for channel pricing strategy. *Marketing Sci.* **16**(3) 185-207.
- Martinez de Albeniz, V., G. Roels. 2007. Competing for shelf space. Working paper. University of California, Los Angeles, CA.
- Mathewson, G. F., R. A. Winter. 2001. The competitive effects of vertical agreements: Comment. *American Economic Review.* **77** 1057-1062.
- Marx, L., G. Shaffer. 2004. Slotting allowances and scarce shelf space. Working paper, Duke University, Fuqua School of Business, Durham, NC.
- McGuire T. W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical inte-

- gration. *Marketing Sci.* **2** 161-191.
- Monahan, J. P., 1984. A quantity discount pricing model to increase vendor profits. *Management Sci.* **30** 720-726.
- Moorthy, K. S. 1987. Managing channel profits: Comment. *Marketing Sci.* **6**(4) 375-379.
- O'Brien, D., G. Shaffer. 1997. Nonlinear supply contracts, exclusive dealing, and equilibrium market foreclosure. *Journal of Economics and Management Strategy.* **6**(4) 755-785.
- Tellis, G. J. 1988. The price sensitivity of competitive demand: A meta analysis of sales response models. *J Marketing Res.* **15** (3) 331-341.
- Trivedi, M. 1998. Distribution channels: An extension of exclusive retailership. *Management Sci.* **44**(7) 896-909.

Appendix A: Proofs

Theorem 1

Proof. The proof holds for general quantity discount contracts that satisfy

$$T_i(0) = 0, \quad T_i'(d) \geq c_i, \quad -v_i \leq T_i''(d) \leq 0, \quad T_i'''(d) \geq 0, \quad T_i''''(d) \leq 0. \quad (15)$$

The quantity discount contract given by (2) satisfies these conditions. The proof consists of two steps. The first step proves that the retailer's problem would not admit more than one local maximum in $\{(d_i, d_j) : d_i > 0 \text{ for } i = 1, 2\}$. The second step characterizes each of the solutions $(\tilde{d}_1, \tilde{d}_2), (\tilde{d}_1(0), 0), (0, \tilde{d}_2(0))$. The proof of the first step is by contradiction. Suppose that there are two interior local maxima: (x', y') and (x'', y'') . The line that connects (x', y') and (x'', y'') can be characterized by $(x' + \alpha t, y' + \delta t)$, where $\alpha = x'' - x', \delta = y'' - y', \alpha, \delta \in \Re, t \in [0, 1]$ represents the line segment between the two points, and $t \in \Re$ represents the whole line. Define $\Delta(t)$ as the value of π on that line,

$$\Delta(t) = \pi(x' + \alpha t, y' + \delta t)$$

We have,

$$\Delta'(t) = \frac{\partial \pi}{\partial d_i} \frac{\partial d_i}{\partial t} + \frac{\partial \pi}{\partial d_j} \frac{\partial d_j}{\partial t} = \alpha \frac{\partial \pi}{\partial d_i} + \delta \frac{\partial \pi}{\partial d_j}.$$

Because $\Delta(t)$ achieves local maxima at the points (x', y') and (y', x') , we have $\Delta'(t) = 0$ and $\Delta''(t) < 0$ at these points. We now derive higher order derivatives:

$$\begin{aligned} \Delta'''(t) &= -\alpha^3 G_i'''(x' + \alpha t) - \delta^3 G_j'''(y' + \delta t) - \alpha^3 T_i'''(x' + \alpha t) - \delta^3 T_j'''(y' + \delta t) \\ \Delta''''(t) &= -\alpha^4 G_i''''(x' + \alpha t) - \delta^4 G_j''''(y' + \delta t) - \alpha^4 T_i''''(x' + \alpha t) - \delta^4 T_j''''(y' + \delta t) \end{aligned}$$

It follows that $\Delta''''(t) > 0$ because $G_i''''(d_i) = -\lambda(1-\lambda)(2-\lambda)(3-\lambda)K_i d_i^{\lambda-3} < 0$ and $T_i'''' \leq 0$ by equation (2). Therefore, $\Delta'''(t)$ is increasing.

Recall that $\Delta'(t) = 0$ and $\Delta''(t) < 0$ at $t \in \{0, 1\}$. Given that $\Delta(t)$ is continuous in t , this can only occur if there is at least one segment in $t \in [0, 1]$ such that $\Delta'(t)$ is convex-concave, which requires that $\Delta'''(t)$ is decreasing along some segment. However, we have established that $\Delta'''(t)$ is increasing. Hence, a contradiction.

For the second step, it suffices to say that the interior optimal solution $(\tilde{d}_1, \tilde{d}_2)$ satisfies the first order conditions $\{H_i = 0, i = 1, 2\}$. The solution on the boundary $(\tilde{d}_1(0), 0)$ satisfies the first order condition $\{H_1 = 0, \text{ s.t. } d_2 = 0\}$. Similarly for $(0, \tilde{d}_2(0))$.

It is easy to see that the profit function is convex-concave along the $d_i = 0$ lines.

$$\begin{aligned}\partial\pi(d_i, 0)/\partial d_i &= \theta_i - 2\beta_i d_i - G'_i(d_i) - T'_i(d_i), \\ \partial^2\pi(d_i, 0)/\partial d_i^2 &= -2\beta_i - G''_i(d_i) - T''_i(d_i).\end{aligned}$$

The second derivative is (i) decreasing in d_i , (ii) positive at $d_i = 0$ (because $G''_i(d_i) = -\lambda(1-\lambda)K_i d_i^{\lambda-2} \rightarrow -\infty$ as $d_i \rightarrow 0^+$), and (iii) negative for sufficiently large d_i . Thus, there can be at most one local maximum for each problem.

For a symmetric problem, it follows from the above argument that the unique interior optimal solution is on the $d_i = d_j$ line. (If (d_i, d_j) with $d_i \neq d_j$ is an interior optimal solution, then there are at least two local maxima, because (d_j, d_i) is also an optimal solution by the symmetry of the profit function. This contradicts with the result above.) Similar to the boundary lines, the profit function is convex-concave on the $d_i = d_j$ line and the solution is characterized by the first order condition

$$\partial\pi(d, d)/\partial d = \theta - 2(\beta + \gamma)d - G'(d) - T'(d) = 0.$$

■

Lemma 1 *The the retailer function $\pi(d_1, d_2)$ is jointly concave in the region where (d_1, d_2) satisfies (12) and $T_i(d_i)$ satisfies (11).*

Proof. First consider the retailer's problem under wholesale price contracts from both manufacturers. We show that the Hessian is a negative semi-definite matrix, which guarantees joint concavity of the profit function. Note that (12) implies that $2R_i/G_i = 2p_i d_i/G_i > (\beta/(\beta^2 - \gamma^2))(p_i/d_i)$. First,

$$\partial^2\pi/\partial d_i^2 = -2\beta + (G_i/4)d_i^{-2} \leq 0$$

if and only if $(1/\beta_i)p_i/d_i \leq 8p_i d_i/G_i$, which is implied by (12) and $\beta/(\beta^2 - \gamma) > (1/\beta)$. Second, we show that the Hessian is a diagonally dominant matrix.

$$|\partial^2\pi/\partial d_i^2| = 2\beta - (G_i/4)d_i^{-2} \geq |\partial^2\pi/\partial d_j \partial d_i| = 2\gamma$$

if and only if $p_i/d_i \leq (2\beta - 2\gamma)4p_i d_i/G_i$, which holds under (12) if $\beta/(\beta^2 - \gamma^2) > 1/(2\beta - 2\gamma)$, equivalently $2\beta > \beta + \gamma$.

The proof of the case with quantity discount contracts is similar. To show diagonal dominance, we split the right-hand-side of the second derivatives to show $(\beta - \gamma) > -G_i''$ and $(\beta - \gamma) > -T_i''$. The former inequality holds by the condition (12) and the latter by (11). ■

Remark 1

Proof. Suppose manufacturer 1 offers $T_1(d_1)$ and there exists a demand pair (d'_1, d'_2) such that $\pi_1 = 0$. By definition, $\pi_2(d'_1, d'_2) = R_2(d'_1, d'_2) - T_2(d'_2) - G_2(d'_2)$. We have $\pi_2(d'_1, d'_2) < \pi_2(0, d'_2) \leq \pi_2(0, d_2(0))$. The first inequality is due to $\partial R'_2(d_1, d_2)/\partial d_1 < 0$ for all d_1, d_2 and $T_1(d_1)$ is non-decreasing: the retailer can always increase her revenue from product 2 by dropping product 1, and her payment to manufacturer 1 cannot increase. The second inequality follows by optimality of $d_2(0)$. Therefore $\pi(0, \tilde{d}_2(0)) > \pi(d'_1, d'_2)$.

Without loss of generality, let $j = 2$, and $i = 1$. Consider any $T_2(d_2)$ - a quantity discount that satisfies (15) or a two-part tariff of the form (3). Suppose that M1 offers a contract $T_1(d_1)$ and that the retailer chooses (d'_1, d'_2) and that $\pi_1 = 0$. By definition, $\pi_2(d'_1, d'_2) = R_2(d'_1, d'_2) - T_2(d'_2) - G_2(d'_2)$. We have $\pi_2(d'_1, d'_2) < \pi_2(0, d'_2) \leq \pi_2(0, \tilde{d}_2(0))$. The first inequality is due to $\partial R'_2(d_1, d_2)/\partial d_1 < 0$ for all d_1, d_2 and the second by optimality of $\tilde{d}_2(0)$. Therefore $\pi(0, \tilde{d}_2(0)) > \pi(d'_1, d'_2)$. ■

Proposition 1

The proof follows from the proof of Proposition 2.

Proposition 2 *Given v_i , the dominant strategy for manufacturer i has the following functional form.*

$$T_i(d_i, w_i, v_i) = \begin{cases} w_i d - v_i d_i^2 / 2, & \text{if } d_i \leq (w_i - c_i) / v_i \\ T_i((w_i - c_i) / v_i) + c_i (d_i - (w_i - c_i) / v_i), & \text{otherwise.} \end{cases} \quad (16)$$

Thus, for given $T_j(d_j, w_j, v_j)$ and fixed v_i , the best response of manufacturer i is given by $T_i(d_i, w_i^*, v_i)$ for some $w_i^* \geq [c_i, \theta_i (v_i / \beta_i + 1)]$.

Proof. Without loss of generality, let $j = 2$, and $i = 1$. Take any $T_1(d_1)$ and $T_2(d_2)$. Suppose that the retailer's optimal solution is an internal point $\tilde{d} = (\tilde{d}_1, \tilde{d}_2)$. (The proof is simpler if one of \tilde{d}_i is zero.) At the optimal solution the first order conditions are satisfied: $\{H_1 = 0, H_2 = 0\}$.

Define τ such that

$$\tau = H_1(\tilde{d}) + T'_1(\tilde{d}_1) = \frac{\partial (R_1(\tilde{d}) + R_2(\tilde{d}) - G_1(\tilde{d}_1))}{\partial d_1}$$

We have $T_1'(\tilde{d}_1) = \tau$. The optimal discount scheme for M1 among those that generate \tilde{d} is the solution to

$$\begin{aligned} & \max T_1(\tilde{d}_1) \\ & \text{subject to } H_i(\tilde{d}) = 0, \text{ for } i = 1, 2. \end{aligned}$$

The constraints guarantee that both first order conditions are satisfied at $(\tilde{d}_1, \tilde{d}_2)$. The the retailer function is jointly concave everywhere in the absence of economies of scale at the retailer as long as T_i satisfies (2). Thus $(\tilde{d}_1, \tilde{d}_2)$ remains the optimal solution (and the unique local maximum) for the retailer. In the presence of economies of scale, $(\tilde{d}_1, \tilde{d}_2)$ remains the unique interior optimal solution for the retailer by Theorem 1 as long as T_i satisfies (2) and (11). Because, the retailer profit is jointly concave when (10)-(12) are satisfied, $(\tilde{d}_1, \tilde{d}_2)$ remains the optimal solution in the region defined by (12).

The manufacturer's problem can be written as $\{\max T_1(\tilde{d}_1) : T_1'(\tilde{d}_1) = \tau\}$ because the condition implies $H_1(\tilde{d}) = 0$ and we already have $H_2(\tilde{d}) = 0$. Therefore, the objective is to increase $T_1(\tilde{d}_1)$ while keeping the marginal cost at \tilde{d}_1 the same. This can be achieved by reducing $T_1'(d_1)$ as little as possible for all $d_1 < \tilde{d}_1$, which can be achieved by setting $T_1''(d_1) = v_1$. Together with $T_1'(\tilde{d}_1) = \tau$, this implies that

$$T_1'(d_1) = -v_1 (d_1 - \tilde{d}_1) + \tau, \text{ for all } d_1 \in [0, \tilde{d}_1]$$

That is, the marginal cost to the retailer is decreasing as slowly as possible and equals τ at \tilde{d} . We have $T_1''(\tilde{d}_1) = -v_1$. $T_1'(d_1)$ can be specified in any way for $d_1 > \tilde{d}_1$ as long as it satisfies (15). We use the same functional form to specify $T_1'(d_1)$ (which implies that $T_1''(d_1) = -v$) for all $d_1 > \tilde{d}_1$ to make sure that the argument applies to all \tilde{d}_1 . Another condition in (15) requires $T_1'(d_1) \geq c_1$. T_1' is decreasing linearly in d_1 and it will reach c_1 at some finite value. Rewriting T_1' , we replace $v_1\tilde{d}_1 + \tau$ with w_1 to obtain,

$$T_1'(d_1) = \begin{cases} w_1 - v_1 d_1, & \text{for } d_1 \leq (w_1 - c_1)/v_1 \\ c_1, & \text{for } d_1 > (w_1 - c_1)/v_1 \end{cases}$$

Integrating T' and recalling the boundary value $T(0) = 0$ by (15), we obtain the quantity discount schedule

$$T_1(d_1) = \begin{cases} w_1 d_1 - v_1 d_1^2/2, & \text{if } d_1 \leq (w_1 - c_1)/v_1 \\ T_1((w_1 - c_1)/v_1) + c_1 (d_1 - (w_1 - c_1)/v_1), & \text{otherwise.} \end{cases}$$

Note that this is not the optimal discount scheme over all possible discount schemes. It is the best scheme among the ones that produce \tilde{d} . In other words, the functional form dominates other functional forms of $T_1(d_1)$. Thus, the optimal quantity discount scheme can be found by considering the functions of this type only. We will see in Theorem 2 that the constant part of $T_1'(d_1)$ is not relevant. A technical note is needed here that (15) requires continuous and differentiable payment functions, but the suggested solution has a non-differentiable point. This can be circumvented by replacing the two-piece marginal cost function with a differentiable convex decreasing function that approximates it very closely.

The manufacturer profit is zero if $w_1 \leq c_1$. If $w_1 \geq \theta_1 (v_1/\beta_1 + 1)$, then the marginal cost to the retailer $w_1 - v_1 d_1 > w_1 - v_1 \theta_1/\beta_1 > \theta_1 (v_1/\beta_1 + 1) - v_1 \theta_1/\beta_1 = \theta_1$. The first inequality is due to $d_1 \leq \theta_1/\beta_1$ to keep p_1 nonnegative. If the retailer charges θ_1 for product 1, the demand cannot be positive, which implies that $\Pi_i = 0$. ■

Theorem 2

Proof. Recall that $v_i = 0$ when the manufacturers employ wholesale-price contracts. Furthermore, while evaluating the equilibrium, we need to only consider the quadratic part of the payment functions. Suppose by contradiction, that the retailer chose d_i such that $d_i > (w_i - c_i)/v_i$. The manufacturer can increase w_i such that $d_i = (w_i - c_i)/v_i$ and increase its profit without affecting the retailer's or the other manufacturer's decisions.

We show in the proof that Π_i is concave in w_i at the symmetric solution to the first order condition stated in the theorem. Hence, the solution is (at least) a local maximum.

Using the quadratic part of T_i , we obtain the first order condition for a manufacturer as follows.

$$\frac{\partial \Pi_i}{\partial w_i} \text{ at } w_i^* = d_i^* + (w_i - c_i - v_i d_i^*) \frac{\partial d_i^*}{\partial w_i} = 0$$

Applying the Implicit Function Theorem on the retailer's first order conditions $\{Y_i = 0, \text{ for all } i\}$, we can derive the impact of the wholesale prices on the optimal demand rates. Define $A_i = (2\beta_i + G_i'' + T_i'')$, $A_i' = \partial A_i / \partial d_i = G_i''' = \lambda(1 - \lambda)(2 - \lambda)d_i^{\lambda-3} = -(2 - \lambda)d_i^{-1}G_i'' > 0$, $B = (\gamma_i + \gamma_j)$, and $\Delta = A_i A_j - B^2$. Note that $A > B > 0$ because $\partial^2 \pi(d, d) / \partial d^2 = A_i - B < 0$ at $(\tilde{d}_1, \tilde{d}_2)$ and $A_i' > 0$.

$$\begin{bmatrix} \partial d_i^* / \partial w_i \\ \partial d_j^* / \partial w_i \end{bmatrix} = \frac{1}{A_i A_j - B^2} \begin{bmatrix} -A_j \\ (\gamma_i + \gamma_j) \end{bmatrix}$$

As expected, d_i decreases with w_i and increases with w_j . The second derivative of Π_i yields

$$2 \frac{\partial d_i^*}{\partial w_i} - v \left(\frac{\partial d_i^*}{\partial w_i} \right)^2 + (w_i - c_i - v_i d_i^*) \frac{\partial^2 d_i^*}{\partial w_i^2},$$

which is negative when $d_1^* = d_2^*$, because we have $A'_1 = A'_2$ and

$$\begin{aligned} \frac{\partial^2 d_i^*}{\partial w_i^2} &= \Delta^{-2} [-A'_j B \Delta^{-1} \Delta + A_j (A_i A'_j B \Delta^{-1} + A'_i A_j (-A_j) \Delta^{-1})] \\ &= \Delta^{-3} [-A'_j B \Delta + (A_i A_j A'_j B - A'_i A_j^3)] \\ &= \Delta^{-3} [-A'_j B A_i A_j + A'_j B^3 + (A_i A_j A'_j B - A'_i A_j^3)] \\ &= \Delta^{-3} [-A'_i A_j^3 + A'_j B^3] < 0 \end{aligned}$$

Hence, the first order condition above has a unique solution when $d_1^* = d_2^*$. The solution (w_1^*, w_2^*) satisfies the first and second order conditions for both manufacturers. That is, w_i^* is a local maximum of Π_i for fixed w_j , and vice versa. If $w_i > w_i^*$, then d_1^* decreases and d_2^* increases, A'_i increases, A_j increases, and A'_j decreases, implying that $\partial^2 d_i^* / \partial w_i^2$ remains negative. Hence, Π_i is concave in w_i for $w_i > w_i^*$ for fixed w_j^* , therefore, there can be no other local maximum greater than w_i^* . ■

Theorem 3

Proof. The optimal demand rates can be solved as the unique solution to first order conditions $\{H_i = 0, i = 1, 2\}$ for given (w_1, w_2) . We have $\alpha \equiv \partial d_i / \partial w_i = -(2\beta_j - v_j) / \delta$, $\partial^2 d_i / \partial w_i^2 = 0$, and $\partial^2 d_i / \partial w_i \partial w_j = 0$, $\partial d_i / \partial w_j = (\gamma_i + \gamma_j) / \delta$. Substituting these in the first order conditions in Theorem 2, we verify that the profit function of manufacturer i is concave in w_i . This guarantees the existence of equilibria in the quantity discount game between the manufacturers. The best response $w_i(w_j)$ is the explicit solution to each first order condition. Differentiating, we obtain

$$\frac{\partial w_i}{\partial w_j} = \frac{(1 - v_i \alpha) (\gamma_i + \gamma_j) / \delta}{-2\alpha + v_i \alpha^2} = \frac{(1 - v_i \alpha) (\gamma_i + \gamma_j)}{(2\beta_j - v_j) (2 - v\alpha)} > 0 \text{ and } < 1.$$

The second inequality is due to $v_i < 2\beta_i - (\gamma_i + \gamma_j)$. Hence, we have increasing reaction functions with a slope less than 1. This implies that there is a unique equilibrium of the game between the manufacturers. ■

Theorem 4

Proof. The first part of the proof shows that for any set of contracts by the manufacturers, manufacturer i can do better by switching to a two-part tariff (F_i, c_i) . Let us focus without loss of generality on manufacturer 1. Let superscripts b and a denote the solutions before and after the contract change. Take any potential manufacturer equilibrium $(T_1(d_1), T_2(d_2))$, where the retailer chooses (d_1^b, d_2^b) with $d_1^b > 0$. Suppose that M1 switches to two-part tariff $(T_1(d_1^b) - c_1 d_1^b, c_1)$. If the retailer optimal solution (d_1^a, d_2^a) is interior or $d_2^a = 0$, M1's profit remains the same, i.e., equal to $T_1(d_1^b) - c_1 d_1^b$. It is not possible that $d_1^a = 0$, because $\pi^{r,a}(d_1^a, d_2^a) \geq \pi^{r,a}(d_1^b, d_2^b) = \pi^{r,b}(d_1^b, d_2^b) > \pi^{r,b}(0, d_2^b(0)) = \pi^{r,a}(0, d_2^a(0))$. The first inequality is due to optimization, the equality because the payment to M1 has not changed, the second inequality from the optimality of (d_1^b, d_2^b) before the contract change, and the last equality holds because the contract for product 2 has not been changed. If $d_1^b = 0$, then M1 profit was zero, switching the contract to two-part tariff $(0, c_1)$ yields zero profit for any (d_1^a, d_2^a) .

The second part characterizes the equilibrium fees (F_1^*, F_2^*) . For any $(F_1, c_1), (F_2, c_2)$, we have $\Pi_i = F_i 1_{\{d_i^* > 0\}}$, and $\pi = \max\{\hat{\pi}_1 - F_1, \hat{\pi}_2 - F_2, \hat{\pi}_{12} - F_1 - F_2\}$. Let us focus on M1. If $F_2 \leq \hat{\pi}_{12} - \hat{\pi}_1$, then $\hat{\pi}_1 - F_1 \leq \hat{\pi}_{12} - F_1 - F_2$, then $\pi = \max\{\hat{\pi}_2 - F_2, \hat{\pi}_{12} - F_1 - F_2\}$ and M1's best response to F_2 is to set F_1 as high as possible while ensuring that $\hat{\pi}_{12} - F_1 - F_2 \geq \hat{\pi}_2 - F_2$ hence $F_1^*(F_2) = \hat{\pi}_{12} - \hat{\pi}_2$. If $F_2 > \hat{\pi}_{12} - \hat{\pi}_1$, then $\hat{\pi}_1 - F_1 > \hat{\pi}_{12} - F_1 - F_2$, then $\pi = \max\{\hat{\pi}_1 - F_1, \hat{\pi}_2 - F_2\}$ and M1's best response to F_2 is to set F_1 as high as possible while ensuring that $\hat{\pi}_1 - F_1 \geq \hat{\pi}_2 - F_2$. Hence $F_1^*(F_2) = \hat{\pi}_1 - \hat{\pi}_2 + F_2 - \varepsilon$ for arbitrarily small ε . The best response function of M2 is similarly obtained: $F_2^*(F_1) = \{\hat{\pi}_{12} - \hat{\pi}_1, \text{ if } F_1 \leq \hat{\pi}_{12} - \hat{\pi}_2; \hat{\pi}_2 - \hat{\pi}_1 + F_1 - \varepsilon, \text{ otherwise}\}$. The unique equilibrium point is $(F_1^*, F_2^*) = (\hat{\pi}_{12} - \hat{\pi}_1, \hat{\pi}_{12} - \hat{\pi}_1)$. At equilibrium, the retailer's profit is $\pi = \hat{\pi}_{12} - (\hat{\pi}_{12} - \hat{\pi}_1) - (\hat{\pi}_{12} - \hat{\pi}_2) = \hat{\pi}_1 + \hat{\pi}_2 - \hat{\pi}_{12}$. The retailer's profit is nonnegative, because $\hat{\pi}_{12} \leq \hat{\pi}_1 + \hat{\pi}_2$ follows from the substitutability of the products. ■

Theorem 5

Proof. Appendix B presents the analysis and the results of the contract choice game with no economies of scale and asymmetric market sizes and production costs. Table 4 below is the simplified version of Table 5 in Appendix B for the symmetric manufacturers case. Let W and TP in the superscript denote wholesale price and two-part tariff contracts as the contract type of each

manufacturer.

		Manufacturer 2	
		Wholesale	Two-Part
Manufacturer 1	Wholesale	$\frac{\beta(\beta-\gamma)(\theta-c)^2}{2(\beta+\gamma)(2\beta-\gamma)^2}$, $\frac{\beta(\beta-\gamma)(\theta-c)^2}{2(\beta+\gamma)(2\beta-\gamma)^2}$	$\frac{(\beta-\gamma)(\theta-c)^2}{8\beta(\beta+\gamma)}$, Π_2^{W-TP}
	Two-Part	Π_1^{TP-W} , $\frac{(\beta-\gamma)(\theta-c)^2}{8\beta(\beta+\gamma)}$	$\frac{(\beta-\gamma)(\theta-c)^2}{4\beta(\beta+\gamma)}$, $\frac{(\beta-\gamma)(\theta-c)^2}{4\beta(\beta+\gamma)}$

Table 4: Equilibrium profits in the contract-choice game for symmetric products with no economies of scale.

Comparing the equilibrium profits, we make the following three observations that complete the proof.

- (i) If manufacturer i is offering two-part tariff, j should switch to a two-part tariff from wholesale-price contract to double its profit.
- (ii) If manufacturer i is offering a wholesale-price contract, j should switch to a two-part tariff from wholesale-price contract to increase its profit, as shown in (17) in Appendix B.
- (iii) $\Pi_i^{TP-TP} > \Pi_i^{W-W}$ if and only if

$$\frac{1}{4\beta} > \frac{\beta}{2(2\beta - \gamma)^2} \Leftrightarrow 2\beta - \gamma > \sqrt{2}\beta.$$

The retailer profit in the case of wholesale price contracts and under two-part tariff contracts are as follows.

$$\pi^{W-W} = \frac{\beta^2 (\theta - c)^2}{4(2\beta - \gamma)^2 (\beta + \gamma)}; \pi^{TP-TP} = \frac{\gamma (\theta - c)^2}{2\beta (\beta + \gamma)}$$

The condition $\beta^3 > 2\gamma(2\beta - \gamma)^2$ implies the result on the retailer's profit. ■

Appendix B: Analysis of Manufacturer Competition with Two-part Tariffs and No Economies of Scale

Assume β, γ are symmetric. Unless otherwise stated θ_i, c_i are asymmetric across manufacturers. The payoffs to the manufacturers in the case of asymmetric market sizes and costs are given in Table 5. Let W denote wholesale price contracts and TP denote two-part tariff contracts. The derivations are presented below.

M1	M2	Π_1, Π_2
W	W	$\frac{\beta((2\beta^2-\gamma^2)(\theta_i-c_i)-\beta\gamma(\theta_j-c_j))^2}{2(\beta^2-\gamma^2)(4\beta^2-\gamma^2)^2}, \frac{\beta((2\beta^2-\gamma^2)(\theta_j-c_j)-\beta\gamma(\theta_i-c_i))^2}{2(\beta^2-\gamma^2)(4\beta^2-\gamma^2)^2}$
TP	W	$\Pi_1^{TP-W}, \frac{(\beta(\theta_j-c_j)-\gamma(\theta_i-c_i))^2}{8\beta(\beta^2-\gamma^2)}$
W	TP	$\frac{(\beta(\theta_i-c_i)-\gamma(\theta_j-c_j))^2}{8\beta(\beta^2-\gamma^2)}, \Pi_2^{W-TP}$
TP	TP	$\frac{(\beta(\theta_i-c_i)-\gamma(\theta_j-c_j))^2}{4\beta(\beta^2-\gamma^2)}, \frac{(\beta(\theta_j-c_j)-\gamma(\theta_i-c_i))^2}{4\beta(\beta^2-\gamma^2)}$

Table 5: Equilibrium profits in the contract-choice game for asymmetric market sizes and costs

Total profit in the centralized system

The optimal demand rates and the profit are as follows.

$$\begin{aligned} \begin{bmatrix} d_i^* \\ d_j^* \end{bmatrix} &= \frac{1}{4(\beta^2-\gamma^2)} \begin{bmatrix} 2\beta & -2\gamma \\ -2\gamma & 2\beta \end{bmatrix} \begin{bmatrix} \theta_i + c_i \\ \theta_j + c_j \end{bmatrix}, \\ d_i^* &= \frac{\beta(\theta_i - c_i) - \gamma(\theta_j - c_j)}{2(\beta^2 - \gamma^2)}, \\ p_i^* - c_i &= \frac{\theta_i - c_i}{2}, \\ \hat{\pi}_{ij} &= \frac{\beta(\theta_i - c_i)^2 + \beta(\theta_j - c_j)^2 - 2\gamma(\theta_j - c_j)(\theta_i - c_i)}{4(\beta^2 - \gamma^2)} = 2\frac{(\theta - c)^2}{4(\beta + \gamma)}. \end{aligned}$$

If only product i were carried, then

$$\begin{aligned} p_i^* - c_i &= (\theta_i - c_i) / 2, \\ d_i^* &= \frac{(\theta_i - c_i)}{2\beta}, \\ \hat{\pi}_i &= \frac{(\theta_i - c_i)^2}{4\beta}. \end{aligned}$$

Equilibrium in TP-TP

By Theorem 4, manufacturer i 's optimal strategy is of a (F_i, c_i) type two-part tariff.

$$\begin{aligned}\Pi_i^{TP-TP} &= F_i^* = \pi_{ij} - \pi_j = \frac{\beta(\theta_i - c_i)^2 + \beta(\theta_j - c_j)^2 - 2\gamma(\theta_j - c_j)(\theta_i - c_i)}{4(\beta^2 - \gamma^2)} - \frac{(\theta_j - c_j)^2}{4\beta} \\ &= \frac{\beta^2(\theta_i - c_i)^2 + \gamma^2(\theta_j - c_j)^2 - 2\gamma\beta(\theta_j - c_j)(\theta_i - c_i)}{4\beta(\beta^2 - \gamma^2)},\end{aligned}$$

if symmetric : $\Pi_i^{TP-TP} = \frac{(\beta - \gamma)(\theta - c)^2}{4\beta(\beta + \gamma)}$.

Equilibrium in W-W

Optimal demand rates at the retailer are

$$d_i^* = \frac{\beta(\theta_i - w_i) - \gamma(\theta_j - w_j)}{2(\beta^2 - \gamma^2)}.$$

The first order conditions for the manufacturers are

$$(w_i - c_i)(-\beta) + \beta(\theta_i - w_i) - \gamma(\theta_j - w_j) = 0,$$

leading to the best response functions

$$w_i(w_j) = c_i/2 + \theta_i/2 - \gamma(\theta_j - w_j)/2\beta.$$

This is a supermodular game with linear increasing reaction functions. The unique wholesale price equilibrium is given by

$$\begin{aligned}\begin{bmatrix} w_i^* \\ w_j^* \end{bmatrix} &= \frac{1}{4\beta^2 - \gamma^2} \begin{bmatrix} 2\beta & \gamma \\ \gamma & 2\beta \end{bmatrix} \begin{bmatrix} \beta(\theta_i + c_i) - \gamma\theta_j \\ \beta(\theta_j + c_j) - \gamma\theta_i \end{bmatrix} \\ &= \frac{1}{4\beta^2 - \gamma^2} \begin{bmatrix} 2\beta^2(\theta_i + c_i) - \beta\gamma\theta_j + \beta\gamma c_j - \gamma^2\theta_i \\ 2\beta^2(\theta_j + c_j) - \beta\gamma\theta_i + \beta\gamma c_i - \gamma^2\theta_j \end{bmatrix} \\ w_i^* - c_i &= \frac{1}{4\beta^2 - \gamma^2} [(2\beta^2 - \gamma^2)(\theta_i - c_i) - \beta\gamma(\theta_j + c_j)], \\ (\theta_i - w_i^*) &= \frac{2\beta^2(\theta_i - c_i) + \beta\gamma(\theta_j - c_j)}{4\beta^2 - \gamma^2}, \\ \text{if symmetric: } w_i^* &= \frac{(\beta - \gamma)\theta + \beta c}{2\beta - \gamma}.\end{aligned}$$

The demand rates at the equilibrium are

$$\begin{aligned}
d_i^*(w_1^*, w_2^*) &= \frac{1}{2(\beta^2 - \gamma^2)} [\beta(\theta_i - w_i) - \gamma(\theta_j - w_j)] \\
&= \frac{1}{2(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)} \left[\begin{array}{l} \beta(2\beta^2(\theta_i - c_i) + \beta\gamma(\theta_j - c_j)) \\ -\gamma(2\beta^2(\theta_j - c_j) + \beta\gamma(\theta_i - c_i)) \end{array} \right] \\
&= \frac{\beta}{2(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)} [((2\beta^2 - \gamma^2)(\theta_i - c_i) - \beta\gamma(\theta_j - c_j))], \\
\text{if symmetric} &: d_i^*(w_1^*, w_2^*) = \frac{\beta(\theta - c)}{2(\beta + \gamma)(2\beta - \gamma)}. \\
\Pi_i^{W-W} &= \frac{\beta((2\beta^2 - \gamma^2)(\theta_i - c_i) - \beta\gamma(\theta_j - c_j))^2}{2(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)^2}, \\
\text{if symmetric} &: \Pi_i^{W-W} = (w_i - c_i) d_i = \frac{\beta(\beta - \gamma)(\theta - c)^2}{2(\beta + \gamma)(2\beta - \gamma)^2}.
\end{aligned}$$

We next compare the equilibrium profits under wholesale-price and two-part tariff contracts in the asymmetric case. The profits under two-part tariff contracts are higher if the following is satisfied.

$$\begin{aligned}
\Pi_i^{TP-TP} &> \Pi_i^{W-W} \\
\frac{(\beta(\theta_i - c_i) - \gamma(\theta_j - c_j))^2}{4\beta(\beta^2 - \gamma^2)} &> \frac{\beta((2\beta^2 - \gamma^2)(\theta_i - c_i) - \beta\gamma(\theta_j - c_j))^2}{2(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)^2} \\
(4\beta^2 - \gamma^2)(\beta(\theta_i - c_i) - \gamma(\theta_j - c_j)) &> \sqrt{2}\beta((2\beta^2 - \gamma^2)(\theta_i - c_i) - \beta\gamma(\theta_j - c_j)) \\
\left((4\beta^2 - \gamma^2) - \sqrt{2}(2\beta^2 - \gamma^2)\right)\beta(\theta_i - c_i) &> \left((4\beta^2 - \gamma^2) - \sqrt{2}\beta^2\right)\gamma(\theta_j - c_j) \\
\left(\beta^2(4 - 2\sqrt{2}) + \gamma^2(\sqrt{2} - 1)\right)\beta(\theta_i - c_i) &> \left(\beta^2(4 - \sqrt{2}) - \gamma^2\right)\gamma(\theta_j - c_j).
\end{aligned}$$

The manufacturers' profits are lower with two-part tariffs than wholesale-price contracts for high substitutability levels (i.e., high γ) and higher with low substitutability levels. Furthermore, if manufacturers are asymmetric with respect to $(\theta_i - c_i)$ values, the one with larger $(\theta_i - c_i)$ is more likely to benefit when they switch to two-part tariff contracts, while the reverse is true for the other manufacturer.

Equilibrium in TP-W

Manufacturer i offers contract (F_i, c_i) and manufacturer j offers wholesale-price contract

$$w_j^* = w_j(c_i) = (\theta_j + c_j)/2 - \gamma(\theta_i - c_i)/2\beta.$$

The demand rates are given by

$$\begin{aligned}
\begin{bmatrix} d_i^* \\ d_j^* \end{bmatrix} &= \frac{1}{2(\beta^2 - \gamma^2)} \begin{bmatrix} \beta(\theta_i - c_i) - \gamma(\theta_j - w_j) \\ \beta(\theta_j - w_j) - \gamma(\theta_i - c_i) \end{bmatrix} \\
&= \frac{1}{2(\beta^2 - \gamma^2)} \begin{bmatrix} (\beta - \gamma^2/2\beta)(\theta_i - c_i) - \gamma(\theta_j - c_j)/2 \\ \beta(\theta_j - c_j)/2 - \gamma(\theta_i - c_i)/2 \end{bmatrix} \\
\text{if symmetric} &: = \frac{(\theta - c)}{4(\beta^2 - \gamma^2)} \begin{bmatrix} (2\beta - \gamma^2/\beta) - \gamma \\ \beta - \gamma \end{bmatrix}
\end{aligned}$$

We derive the results of W-TP case only for symmetric θ_i, c_i .

$$\begin{aligned}
\begin{bmatrix} p_i - c_i \\ p_j - w_j^* \end{bmatrix} &= \begin{bmatrix} \theta - c - \beta d_i^* - \gamma d_j^* \\ \theta - \beta d_j^* - \gamma d_i^* - (c + (\theta - c)(1/2 - \gamma/2\beta)) \end{bmatrix} \\
&= \begin{bmatrix} \theta - c - \beta d_i - \gamma d_j \\ (\theta - c)(1 + (\gamma/\beta))/2 - \beta d_j - \gamma d_i \end{bmatrix} \\
&= \frac{\theta - c}{4(\beta^2 - \gamma^2)} \begin{bmatrix} 4(\beta^2 - \gamma^2) - \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \begin{bmatrix} (2\beta - \gamma^2/\beta) - \gamma \\ \beta - \gamma \end{bmatrix} \\ 2(\beta^2 - \gamma^2)(1 + (\gamma/\beta)) - \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \begin{bmatrix} \beta - \gamma \\ (2\beta - \gamma^2/\beta) - \gamma \end{bmatrix} \end{bmatrix} \\
&= \frac{\theta - c}{4(\beta^2 - \gamma^2)} \begin{bmatrix} 4(\beta^2 - \gamma^2) - [2\beta^2 - \gamma^2 - \gamma\beta + \gamma\beta - \gamma^2] \\ 2(\beta^2 - \gamma^2)(1 + (\gamma/\beta)) - [\beta^2 - \beta\gamma + 2\beta\gamma - \gamma^3/\beta - \gamma^2] \end{bmatrix} \\
&= \frac{\theta - c}{4(\beta^2 - \gamma^2)} \begin{bmatrix} 2(\beta^2 - \gamma^2) \\ (\beta^2 - \gamma^2) + \beta\gamma - \gamma^3/\beta \end{bmatrix}. \\
\pi &= \frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2} \begin{bmatrix} (2\beta - \gamma^2/\beta) - \gamma \\ \beta - \gamma \end{bmatrix}^T \begin{bmatrix} 2(\beta^2 - \gamma^2) \\ (\beta^2 - \gamma^2) + \beta\gamma - \gamma^3/\beta \end{bmatrix} \\
&= \frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2} \begin{bmatrix} 4\beta^3 - 2\gamma^2\beta - 2\beta^2\gamma - 4\beta\gamma^2 + 2\gamma^4/\beta + 2\gamma^3 \\ +\beta^3 - \gamma^2\beta + \beta^2\gamma - \gamma^3 - \beta^2\gamma + \gamma^3 - \beta\gamma^2 + \gamma^4/\beta \end{bmatrix} \\
&= \frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2} [5\beta^3 - 8\gamma^2\beta - 2\beta^2\gamma + 3\gamma^4/\beta + 2\gamma^3].
\end{aligned}$$

Manufacturer i increases F_i as long as π is greater than what the retailer's profit would be if it carried only product j , i.e., $\pi_i(d_i, 0|w_i)$. If only j were carried, then

$$\begin{aligned}
\hat{\pi}_j &= \frac{(\theta_j - w_j)^2}{4\beta} = \frac{(\theta_i - (\theta_j + c_j)/2 + \gamma(\theta_i - c_i)/2\beta)^2}{4\beta} \\
&= \frac{(\theta_i(1 + \gamma/2\beta) - c_i\gamma/2\beta - (\theta_j + c_j)/2)^2}{4\beta}, \\
\text{if symmetric} &: \hat{\pi}_j = \frac{(\theta - c)^2(1 + \gamma/\beta)^2}{16\beta}.
\end{aligned}$$

Hence, the manufacturers' profits are as follows.

$$\begin{aligned}
\Pi_1^{TP-W} &= F_1^* = \pi - \hat{\pi}_2 \\
&= \frac{(\theta - c)^2}{16} \left[\frac{[5\beta^3 - 8\gamma^2\beta - 2\beta^2\gamma + 3\gamma^4/\beta + 2\gamma^3]}{(\beta^2 - \gamma^2)^2} - \frac{(1 + \gamma/\beta)^2}{\beta} \right] \\
&= \frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2\beta} \left[[5\beta^4 - 8\gamma^2\beta^2 - 2\beta^3\gamma + 3\gamma^4 + 2\gamma^3\beta] - (\beta^2 - \gamma^2)^2(1 + \gamma/\beta)^2 \right] \\
&= \frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2\beta} \left[[5\beta^4 - 8\gamma^2\beta^2 - 2\beta^3\gamma + 3\gamma^4 + 2\gamma^3\beta] - (\beta^4 - 2\beta^2\gamma^2 + \gamma^4)(1 + 2\gamma/\beta + \gamma^2/\beta^2) \right] \\
&= \frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2\beta} \left[\begin{array}{l} [5\beta^4 - 8\gamma^2\beta^2 - 2\beta^3\gamma + 3\gamma^4 + 2\gamma^3\beta] - (\beta^4 - 2\beta^2\gamma^2 + \gamma^4) \\ -2\gamma/\beta(\beta^4 - 2\beta^2\gamma^2 + \gamma^4) - \gamma^2/\beta^2(\beta^4 - 2\beta^2\gamma^2 + \gamma^4) \end{array} \right] \\
&= \frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2\beta} \left[\begin{array}{l} [5\beta^4 - 8\gamma^2\beta^2 - 2\beta^3\gamma + 3\gamma^4 + 2\gamma^3\beta] - (\beta^4 - 2\beta^2\gamma^2 + \gamma^4) \\ - (2\gamma\beta^3 - 4\beta\gamma^3 + 2\gamma^5/\beta) - (\gamma^2\beta^2 - 2\gamma^4 + \gamma^6/\beta^2) \end{array} \right] \\
&= \frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2\beta} \left[\begin{array}{l} 4\beta^4 - 7\gamma^2\beta^2 - 4\beta^3\gamma + 4\gamma^4 + 6\gamma^3\beta \\ -2\gamma^5/\beta - \gamma^6/\beta^2 \end{array} \right], \\
\Pi_2^{TP-W} &= (w_2^* - c_2) d_2 = \frac{(\beta - \gamma)(\theta - c)^2}{8\beta(\beta + \gamma)}.
\end{aligned}$$

For the symmetric case, the following implies that it is always optimal for a manufacturer to switch to two-part tariff, even if the competitor is committed to wholesale-price contracts.

$$\begin{aligned}
\Pi_1^{TP-W} &> \Pi_1^{W-W} \tag{17} \\
\frac{(\theta - c)^2}{16(\beta^2 - \gamma^2)^2\beta} \left[\begin{array}{l} 4\beta^4 - 7\gamma^2\beta^2 - 4\beta^3\gamma + 4\gamma^4 + 6\gamma^3\beta \\ -2\gamma^5/\beta - \gamma^6/\beta^2 \end{array} \right] &> \frac{\beta(\beta - \gamma)(\theta - c)^2}{2(\beta + \gamma)(2\beta - \gamma)^2} \\
(2\beta - \gamma)^2 \left[\begin{array}{l} 4\beta^4 - 7\gamma^2\beta^2 - 4\beta^3\gamma + 4\gamma^4 + 6\gamma^3\beta \\ -2\gamma^5/\beta - \gamma^6/\beta^2 \end{array} \right] &> 8(\beta^2 - \gamma^2)\beta^2(\beta - \gamma)^2 \\
(4\beta^2 - 4\beta\gamma + \gamma^2) \left[\begin{array}{l} 4\beta^4 - 7\gamma^2\beta^2 - 4\beta^3\gamma + 4\gamma^4 + 6\gamma^3\beta \\ -2\gamma^5/\beta - \gamma^6/\beta^2 \end{array} \right] &> 8(\beta^4 - \beta^2\gamma^2)(\beta^2 - 2\beta\gamma + \gamma^2) \\
\left[\begin{array}{l} 16\beta^6 - 28\beta^4\gamma^2 - 16\beta^5\gamma + 16\beta^2\gamma^4 + 24\beta^3\gamma^3 \\ -8\beta\gamma^5 - 4\gamma^6 \\ -16\beta^5\gamma + 28\beta^3\gamma^3 + 16\beta^4\gamma^2 - 16\beta\gamma^5 - 24\beta^2\gamma^4 \\ +8\gamma^6 + 4\gamma^7/\beta \\ +4\beta^4\gamma^2 - 7\gamma^4\beta^2 - 4\beta^3\gamma^3 + 4\gamma^6 + 6\gamma^5\beta \\ -2\gamma^7/\beta - \gamma^8/\beta^2 \end{array} \right] &> 8 \left[\begin{array}{l} (\beta^6 - 2\beta^5\gamma + \beta^4\gamma^2) \\ + (-\beta^4\gamma^2 + 2\beta^3\gamma^3 - \beta^2\gamma^4) \end{array} \right] \\
\left[\begin{array}{l} 16\beta^6 + \beta^5\gamma(-16 - 16) + \beta^4\gamma^2(-28 + 4 + 16) \\ + (24 + 28 - 4)\beta^3\gamma^3 + (16 - 24 - 7)\beta^2\gamma^4 \\ + (-8 - 16 + 6)\beta\gamma^5 + (-4 + 8 + 4)\gamma^6 \\ + (4 - 2)\gamma^7/\beta - \gamma^8/\beta^2 \end{array} \right] &> 8[\beta^6 - 2\beta^5\gamma + 2\beta^3\gamma^3 - \beta^2\gamma^4] \\
\left[\begin{array}{l} 16\beta^6 - 32\beta^5\gamma - 8\beta^4\gamma^2 + 48\beta^3\gamma^3 - 15\beta^2\gamma^4 \\ -18\beta\gamma^5 + 8\gamma^6 + 2\gamma^7/\beta - \gamma^8/\beta^2 \end{array} \right] &> 8\beta^6 - 16\beta^5\gamma + 16\beta^3\gamma^3 - 8\beta^2\gamma^4 \\
(\beta - \gamma)^3(\beta + \gamma)(8\beta^4 - 8\beta^2\gamma^2 + \gamma^4) &> 0.
\end{aligned}$$