Supply Chain Design and the Cost of Greenhouse Gas Emissions

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Abstract

The density, size and location of stores in a retailer’s network influences both the retailer’s cost to operate its stores as well as its customer’s costs - with stores few and far between, consumers must travel a long distance to shop, whereas shopping trips are shorter with a dense network of stores. Although the layout of the retail supply chain has received little attention in the academic literature, it is of interest to retailers who have emission reduction targets and urban planners concerned with sprawl - are small local shops preferred over large, “big-box”, retailers? In a model of the downstream supply chain, this paper considers the retailer’s transportation costs (a Traveling Salesmen problem), the consumers’ transportation costs (a $k$-median problem), and the retailer’s space costs (an inventory problem). Explicit operating costs are included (such as rent for retail space, vehicle depreciation and fuel) as well as a cost for environmental externalities associated with carbon emissions. Several supply chain designs are compared: designs that minimize operating costs, emissions costs or total cost (that includes both operating costs and emissions). If only transportation costs are considered (e.g., retail space costs are ignored or are small), then these

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designs perform essentially equally well no matter the objective - the design that minimizes operating costs nevertheless nearly minimizes (within 1%) total emissions. Greater variation in performance is observed when retail space costs are considered - a focus on minimizing exclusively operating costs may substantially increase emissions (up to 75% in one scenario) relative to the minimum level of emissions. To reduce emissions, a price on carbon could be charged, but it is found that substantial reductions only occur if the carbon price is extremely high (e.g., greater than $1,000 per metric tonne), suggesting that this approach will not be effective. Alternatively, emissions could be reduced through regulation limiting the size of stores, thereby yielding dense networks with many small stores located close to consumers. Among other options, the most attractive appears to be improving consumer fuel efficiency (doubling fuel efficiency of cars reduces long run emissions by about one third), increasing the load consumers carry (doubling their load per shopping trip reduces long run emissions by about 30%) and either reductions in the retailer electricity consumption or electricity emissions (cutting either one by half yields a total long run reduction of about 13%). An improvement in truck fuel efficiency has little impact on total emissions.

Keywords: Sustainability, k-median, traveling salesman problem, carbon tax, cap-and-trade
1 Introduction

The combustion of fossil fuels is believed to contribute to climate change by adding carbon dioxide and other greenhouse gases to the atmosphere (IPCC 2007). Electric power generation and transportation are two major sources of these emissions - electric power accounts for 39%, and transportation accounts for 31% of CO₂ emissions in the U.S. in 2009 (EPA 2011). Among transportation induced emissions, about 65% is due to the consumption of gasoline by personal vehicle use (EPA 2011).

Factors that influence the emissions generated by a supply chain network include the size of its facilities, the fuel efficiency of the vehicles used, the weight of the loads they carry, and the distance they travel. This paper focuses on the downstream supply chain - the portion that includes inbound replenishments to retail stores, the stores themselves and the “final mile” segment between the retail stores and consumers’ homes. As the number of stores increases, thereby creating a dense network, stores shrink in size and consumers find themselves closer to some store, so they need not travel far to make their purchases. However, with more stores, the retailer must travel farther to replenish its stores and its total floor space grows as inventory productivity falls.

The density of retail stores is of interest to urban planners. One concern, among others, is that large retail stores contribute to a car culture that encourages consumers to drive further than what they would drive if smaller store formats were available closer to where they live (Owen 2009; Duany and Speck, 2010; Glaeser 2011; Shoup 2011). As an increase in vehicle miles traveled by consumers leads to additional emissions, environmentalists have been critical of “big-box” stores. The issue of retail store density is also relevant for a retailer’s strategic planning. For one, a retailer’s store density influences its desirability to consumers - all else equal, a consumer favors the convenience of a nearby store over a more distant store. For example, Pancras, Sriram and Kumar (2012) estimate in the context of a fast food retailer that consumers behave as if each mile of travel costs $0.60. Second, the density decision is also important to retailers with emission reduction targets. For example, Walmart has pledged that it will remove 20 million metric tons of CO₂ from its supply chain by 2015. These reductions can occur from the supply chain it directly controls (e.g., reductions in its fuel consumption), which are often called “scope 1” emissions. Alternatively, reductions can come from further down the supply chain (e.g., reductions in its consumer’s fuel consumption), which are called “scope” 3 emissions. While it is probably too costly to make major modifications to store density to achieve short term emission reduction targets, the size and location of stores can be substantially modified over a long run horizon of five or more years. For example, from 2006 to 2010, Walmart decreased the number of discount stores in the U.S. by a
third (1209 to 803), increased the number of its larger supercenters by 60% (1713 to 2747) while also expanding its small store format, Neighborhood Markets, by nearly 60% (100 to 158) (Source: 2010 10k filing.)

In this paper a model is developed in which a retailer chooses the size, location and number of stores to serve a region of customers. The retailer incurs costs for its physical space as well as costs that are proportional to the distance it travels to replenish its stores. Consumers also incur transportation costs that are proportional to the distance they must travel to shop at the nearest store to their home. While the retailer uses a truck for its replenishments and consumers use passenger vehicles (i.e., cars), both of their transportation costs are divided into three components: a variable operating cost (e.g., wear and tear on brakes and tires), a fuel consumption cost and an emissions cost (due to the consumption of fuel). Similarly, the retailer’s space cost has three components: a variable operating cost (e.g., rent), an energy consumption cost (e.g., electricity and natural gas) and an emissions cost. Given a set of \( n \) stores, an inventory model is developed to determine the amount of physical space the retailer needs. The retailer’s transportation cost can be modeled by the well studied Traveling Salesman Problem (TSP). The consumer’s transportation problem is equivalent to the continuous version of the well studied \( k \)-median problem. Thus, the supply chain design problem combines an inventory model with the TSP and the \( k \)-median problem.

An optimal supply chain design is presented for three different objectives. The first ignores the cost of emissions and instead minimizes just the explicit operating costs incurred by the retailer and consumers, such as wear-and-tear on vehicles, fuel usage, rent and electricity consumption. This roughly represents that status quo in the United States in which there are no required taxes on carbon emissions. Alternatively, a supply chain design could be chosen to minimize total emissions. This objective avoids the debate as to the true cost of the externalities caused by emissions (e.g., Cline, 1992; Shelling 1992; Stern 2007; Arrow 2007; Weitzman 2007) by merely acting as if that cost is quite large. Comparing the supply chain that minimizes operating costs with the one that minimizes emissions allows for a measure of the consequence of choosing one objective over the other. For example, one can ask by how much emissions increase if operating costs are minimized, or by how much operating costs increase if emissions are minimized.

Between the two extremes of just minimizing operating costs or just minimizing emissions, a supply chain design is developed that minimizes total costs given an explicit price for carbon emissions. Naturally, if the price of carbon is assumed to be low, this objective leads to a supply chain that resembles the one that minimizes operating costs, whereas if the price of carbon is assumed to be high, it recommends a supply chain that is similar to the one that minimizes
emissions. This model addresses the question of how high that price needs to be for the system to approximately minimize emissions.

There are several reasons to believe that failing to account for the cost of emissions leads to a poor supply chain design. The emission of carbon is an example of a negative production externality (Varian 1984) - the action of one reduces the utility of others. In this setting all agents emit carbon through their actions, and therefore they all contribute to this negative externality. It has been well established that self-interested agents tend to do "too much" of their action when there is a negative externality, i.e., they drive too much and use too much electricity and natural gas (Varian 1984). Of course, the importance of the distortion in actions from the socially optimal ones depends on the intensity of the externality.

In the context of a supply chain, it is reasonable to conjecture that the negative effects of the carbon externality are substantial. For one, as already mentioned, it is possible that the true cost of emitting carbon is high due to its potential to have numerous negative consequences on our environment, such as climate change, ocean acidification, sea-level rise, and others. Next, it is already understood in the context of transportation that a retail truck is substantially more efficient at hauling goods than a passenger vehicle. A commonly used metric is the "cost per tonne-kilometer" - the cost to transport one tonne of goods the distance of one kilometer. In terms of this metric, a truck can be several orders of magnitude better than a car. Hence, in a product’s journey from source to consumer home, it has been argued that costs and emissions can be reduced considerably by replacing one kilometer of car transport with a kilometer of truck transport (McKinnon and Woodburn, 1994). In short, supply chain emissions are probably minimized if the retailer builds many small stores close to consumers. Finally, a retailer faces a tradeoff between store size and inventory productivity. Large stores that serve large market areas require less space per unit of product than small stores. Consequently, the energy intensity of small stores is higher. If retailers and consumers are not explicitly charged for the cost of carbon, it is plausible that a retailer will choose store locations and sizes that will lead to an undesirable level of emissions.

To foreshadow the paper’s results, in terms of transportation costs, it is discovered that the supply chain design problem is remarkably robust. In particular, given reasonable parameter estimates, the supply chain that minimizes explicit operating costs also happens to nearly minimize emissions. Similarly, the supply chain that minimizes emissions nearly minimizes operating costs - it essentially does not matter which of two objectives are chosen. As a result, there is little value for explicitly charging for carbon or for any other approach designed to modify the supply chain design. There are several reasons for this robustness. To begin, stores and consumers live in a two dimensional world. In a linear world, as discussed, moving a store closer to a consumer
might swap one car \( km \) for a truck \( km \), thereby producing a substantial efficiency gain. But in a two dimensional world this one-for-one swapping is not possible - as the number of stores increases, each additional \( km \) traveled by truck replaces a smaller and smaller distance traveled by car. Next, trucks have an advantage, primarily due to their large load, both in terms of operating costs as well as emissions. Hence, whether the retailer considers the tradeoff between a dense and a sparse network of stores from the perspective of operating costs or from emissions, the tradeoffs in transportation are sufficiently similar that a good decision is obtained with either objective, again, given reasonable parameter estimates.

The tensions in the supply chain design become more substantial when retail space costs are included. Now there are additional opportunities for cost asymmetries. While both trucks and cars use combustion engines, operating costs and emissions for retail space are based on very different technologies. In fact, when retail space costs are considered, there are cases in which ignoring emissions leads to substantially higher emissions, and ignoring operating costs results in a similarly bad penalty. The model indicates that the asymmetries are most consequential when the retailer operates in a high rent market and utilizes electricity from a low emissions source. In that case, the supply chain that minimizes operating costs builds a sparse network to exploit the inventory productivity advantage of large stores. In contrast, the supply chain that minimizes emissions builds a dense network of small stores to exploit the retailer’s “clean” electricity relative to the consumers’ “dirty” vehicles. In one scenario, minimizing operating costs increases emissions by 75% while minimizing emissions yields the same penalty on operating costs.

In situations in which there is a substantial difference in the structure of the supply chain that minimizes operating costs relative to the one that minimizes emissions, it is natural to consider the value of explicitly charging a price for carbon emissions. If carbon is explicitly charged, then it becomes part of operating costs, so the objective of minimizing operating costs begins to consider the quantity of emissions. Furthermore, using the supply chain design model, it is possible to estimate how large a price needs to be imposed on carbon so that the supply chain approaches one that minimizes emissions. Unfortunately, that price is considerably higher than most estimates of the cost of carbon, well over \$1,000 per metric tonne (Tol, 2008). Put another way, even when retail space costs are considered, hefty prices need to be imposed on carbon to induce changes that are substantial enough to lead to a significantly lower emissions. In terms of the supply chain design, a carbon price is probably not an effective mechanism for change.

Taxing carbon is not the only approach to influence change. As mentioned earlier, the supply chain that minimizes emissions generally has a dense network of small stores located close to consumers. Hence, land use regulations could be considered to limit the construction of large format
retail stores. Technological improvement provides another opportunity. The model indicates that doubling consumer fuel efficiency (which is technologically feasible), would reduce short term emissions by over one third and long run emissions (after the supply chain network can be adjusted) by about 30%. In contrast, fuel efficiency improvements for trucks have little impact on overall emissions in this context. But substantial emissions reductions are also possible by increasing the load consumers carry per store visit, or by reducing retailer electricity consumption (e.g., more efficient lighting) or emissions (e.g., through renewable energy sources).\footnote{Clean energy could be produced locally by the retailer, as in roof solar panels, or more distant renewable power, as in hydro-electric production. Clean electricity could also be produced by carbon capture and storage technology - see İşlegen and Reichelstein (2011).}

2 The Supply Chain Design Problem

The supply chain design problem in this paper focuses on the retailer’s downstream supply chain. The retailer’s task is to decide (i) the number, $n$, and location of stores to have in a single (polygon) region (ii) the size of the stores, (iii) how to route replenishment deliveries to these stores, (iv) the quantity and timing of deliveries to the stores and (v) the shape of the region. Without loss of generality, the area of the region is normalized to one unit.

Consumers live uniformly throughout the region and they travel a straight line (i.e., the Euclidean norm, $L_2$) with their cars to shop at the nearest store from their home. The mean and standard deviation of demand per unit of time per unit of area are $\lambda$ and $\sigma$ respectively. Hence, $\lambda$ is total demand rate in the region. Consumers incur a cost $c_c$ per unit of distance travelled per unit of product purchased. (The subscript “c” refers to the fact that consumers travel with “cars”.) Let $d_c$ be the average round-trip distance a consumer travels to a store. Thus, the average travel cost per consumer per unit purchased is $c_c d_c$.

The retailer directly incurs two types of costs, one due to the transportation needed to replenish its stores and the other due to the operation of retail floor space. In terms of transportation, the retailer has a single warehouse where it receives goods from an outside supplier. The warehouse is collocated with one of the $n$ stores. The retailer has a single truck that is used to transport goods from the warehouse to the stores - all deliveries must start at the warehouse and end at the warehouse. Furthermore, the truck travels in straight lines between stores, delivers to all $n$ stores on each route, and completes its route instantly (i.e., fast enough such that transit time is not a major issue). The retailer incurs a cost of $c_t$ per unit of product delivered per unit of distance the truck travels. (As with $c_c$, the “t” in $c_t$ refers to the type of vehicle used.) $c_t$ is a commonly used
measure of transportation efficiency - it is analogous to “$ per tonne-km”. Let \( d_t \) be the length of the truck’s route, so the retailer’s transportation cost per unit sold is \( c_t d_t \).

Several components are included in the transportation costs of vehicle type \( j, j \in \{c, t\} \):

\[
c_j = \frac{v_j + f_j (p_j + e_j p_e)}{q_j},
\]

where \( v_j \) is the non-fuel variable cost to transport the vehicle per unit of distance (e.g., $ km^{-1} ); \( f_j \) is the amount of fuel used to transport the vehicle per unit of distance (e.g., l km\(^{-1} \)); \( p_j \) is the cost of fuel per unit of fuel (e.g., $ l^{-1} ); \( e_t \) is the amount of emission released by the consumption of one unit of fuel (e.g., kg CO\(_2\) l\(^{-1} \)); \( p_e \) is the cost of emissions per unit released (e.g., $ CO\(_2\) kg\(^{-1} \)); and \( q \) is the maximum load carried by the vehicle (e.g., kg). The variable cost, \( v_j \), includes depreciation on the vehicle, maintenance (such as tire replacement) and other costs that can be linked to the distance the vehicle is driven. Fuel usage, \( f_j \), is measured at the average load of the vehicle, which yields an accurate measure of true fuel usage when fuel usage is linear in the vehicle's load (which is approximately true), the vehicle always carries the same amount per trip, the vehicle makes deliveries at a constant rate (e.g., the stores on the route are equally distant from each other), and the vehicle travels at the same speed during the trip (Kellner and Igl 2012).

The cost of emissions, \( p_e \), is assumed to be independent of the source of the emissions, which is accurate given the focus on carbon emissions: a kilogram of CO\(_2\) has the same impact if emitted by a car or a truck. We refer to \( p_e \) as the “cost of carbon”.\(^2\) This cost is interpreted as the true cost of emissions including all externalities. Depending on which objective is used to design the supply chain, it may or may not be included in the analysis.

It is useful to divide \( c_j \) into two categories. The operating costs include variable costs and fuel consumption, i.e., \( (v_j + f_j p_j) / q_j \). Emissions refers to the quantity of carbon emissions, \( f_j e_j / q_j \), and emissions costs include the price of carbon, \( f_j e_j p_e / q_j \).

Retail space is proportional to the amount of inventory the retailer holds in each store - doubling the amount of inventory doubles the needed footprint area of a store. Thus, space costs are related to inventory quantities. Let \( c_s \) be the retailer’s space costs per unit per unit of time the unit is held in the store’s inventory and let \( t_s \) be the average time a unit spends in inventory at a store.

\(^2\) The focus is on carbon emissions that impact climate change rather than other types of emissions (e.g., particulates, lead) that may impact health. See Currie and Walker (2011) for a discussion of other pollutants due to vehicle traffic. Other gases (such as methane) also contributed to climate change. Hence, emissions are often measured in terms of “CO\(_2\) equivalents” (the amount of CO\(_2\) that has the same global warming potential as one unit of the reference gas). The results presented would not change if this broader measure were used.
(The subscript “s” refers to retail “space” throughout.) Hence, the retailer’s cost of space per unit sold is $c_s t_s$. Neither the cost of inbound deliveries to the retailer nor warehouse space costs are considered in this analysis.

The space cost of inventory is also divided into several components:

$$c_s = v_s + f_s (p_s + e_s p_e) / q_s,$$

where $v_s$ is the variable cost per unit of retail space per unit of time, $f_s$ is the amount of energy needed to maintain one unit of space for one unit of time, $p_s$ is the per unit price of energy, $e_s$ is the carbon emissions from each unit of energy and $q_s$ in the number of units of product stored per unit of retail space. The variable cost, $v_s$, primarily includes the cost of rent (which is generally quoted as a cost per unit of area per unit of time, such as $\$ m^{-2} yr^{-1}$), but it could also include the opportunity cost of capital or inventory obsolescence costs. The retailer uses energy primarily from two sources: electricity for lighting, cooling and operating office equipment; and natural gas for heating. Thus, $f_s, p_s$ and $c_s$ are taken to be averages weighted by the relative usage of electricity and natural gas. This does not restrict other measures of energy which could be appropriate given the context. As before, operating costs include the variable and fuel consumption costs, and emissions are proportional to fuel consumption.

Let $C$ be the average cost per unit sold, which includes the average cost to transport a unit from the retailer’s warehouse to a consumer’s home as well as the cost of the space needed for the retail stores:

$$C = c_c d_c + c_t d_t + c_s t_s.$$

If $p_e$ is the correct cost of carbon and included in $c_c$, $c_t$, and $c_s$, then minimizing $C$ minimizes society’s total cost. Minimizing $C$ is also an appropriate objective for the retailer when the retailer must compensate consumers for their travel costs. Pancras, Sriram and Kumar (2012) provide evidence that consumers do account for travel costs in their shopping decisions. The retailer provides this compensation to consumers by modifying its price - with many stores the retailer charges a higher price because consumers are offered the convenience of a store near their home, but with few stores, the retailer charges a lower price to motivate consumers to travel the longer distance. In this formulation the price reduction is exactly one-for-one with any additional transportation costs the consumers’ incur. This leaves aggregate demand constant.

If carbon is not charged, i.e., $p_e = 0$, the objective to “minimize $C$” is equivalent to “minimize operating costs”. Alternatively a “minimize emissions” objective could be adopted, which is equivalent to “minimize $C$” under the assumption of a very high price for carbon (e.g., as $p_e \to \infty$).
Parts of the supply chain design problem are familiar. Given a set of \( n \) stores, the routing sub-problem is the well-known Traveling Salesman Problem (TSP) - find a route through \( n \) locations, starting and ending at the same location, and visiting each location exactly once so as to minimize the total transportation cost. There exists an extensive literature on heuristics and solutions to the TSP (see Bramel and Simchi-Levi 1997; Lawler, Lenstra, Rinnooy Kan and Shmoys 1985). More generally, there is a considerable literature on vehicle routing, such as when a fleet of vehicles (as opposed to a single vehicle) must be used to make deliveries to a set of known points in a region so as to minimize travel distances (e.g., Dantzig and Ramser 1959; Daganzo 1984; Haimovich and Rinnooy Kan 1985). This literature is further extended by work that includes inventory management along with vehicle routing (e.g., Federgruen and Zipkin 1984; Burns, Hall, Blumenfeld, Daganzo 1985; Gallego and Simchi-Levi 1990). The key differences between this supply chain design problem and the TSP and its extensions are (i) the retailer can choose the location of the stores and (ii) the retailer accounts for the consumers’ transportation costs (i.e., the “final mile” of the supply chain is not ignored).

Focusing on just the consumers’ transportation costs, the problem is analogous to the well known \( k \)-median problem (which is also referred to as the \( p \)-median problem or multi-source Weber problem or location-allocation problem). In the \( k \)-median problem there exists a set of demand locations. The objective is to choose \( k \) locations - call them stores - to minimize the total transportation cost from the demand locations to their nearest store. The \( k \)-median problem is generally studied in its discrete form (i.e., a finite number of possible demand and store locations) but there has also been some work on the continuous \( k \)-median problem, which is the retailer’s supply chain design problem when the retailer’s transportation and space costs are ignored (see Papadimitriou, 1981). Work on the \( k \)-median problem has focused on good solution procedures, rather than on the structure of the solution. See Daskin (1995) for an overview of the \( k \)-median problem. Brimberg, Hansen, Mladenović, Taillard (2000) study numerous solution algorithms for the discrete \( k \)-median problem and Fekete, Mitchell and Beurer (2005) do the same for the continuous version of the problem.

This combination of the TSP and the \( k \)-median problem has not been previously studied (with or without considering inventory/space). It extends the traditional boundary of supply chain analysis, which typically incorporates just the firm, to include the final leg of transportation performed by consumers.

There is some work on the interaction between operational decisions and emissions. Hoen, Tan, Fransoo and van Houtum (2010) analyze a single location inventory model in which the firm can select from a set of transportation modes that vary in their per unit delivery cost, level of emission
and replenishment lead time. Benjaafar, Li and Daskin (2010) analyze a single location model in which inventory management decisions (the timing and quantity of orders) influence supply chain holding costs, backorder costs and emissions which include the following: (i) a fixed amount per unit held on average in inventory; (ii) a fixed amount per unit sold and (iii) a fixed amount per delivery. Unlike those two papers, this model has multiple locations. Unlike Hoen, Tan, Fransoo and van Houtum (2010), the retailer in this model has a single mode of transportation - the focus is on distances travelled rather than lead time. Unlike Benjaafar, Li and Daskin (2010), this model does not include a fixed amount of emissions per unit sold (this would not influence the decisions considered), and the amount of emissions per delivery is not fixed (it depends on the number of stores and their locations). Similar to Hoen, Tan, Fransoo and van Houtum (2010), this model finds that charging an explicit price for carbon emissions, unless unreasonably high, is unlikely to influence decisions enough to reduce emissions substantially. Gillerlain, Fry and Magazine (2011) also study a model in which a retailer and consumers incur transportation costs and emission costs based on distances. They too evaluate the cost minimizing number of stores, but they impose a different spacial geometry - in their model, stores and consumers are located on the boundary of a circle. They find that imposing a constraint on a retailer that limits its emissions may actually lead to an increase in total emissions (because the constraint on the retailer leads to higher consumer emissions). These papers consider the effectiveness of different methods for providing incentives to reduce emissions (e.g., taxes or constraints) but they do not measure the impact for failing to provide incentives.

Caro, Corbett, Tan and Zuidwijk (2011) consider a supply chain in which firms have the opportunity to make investments to reduce emissions. They do not explicitly consider store locations, replenishment routes or inventory. Instead, they study how different methods for allocating carbon emissions to various processes influences the amount of investment in emission reductions. Keskin and Plambeck (2011) also study carbon allocation rules and find that a poorly chosen rule may lead self interested firms to decisions that raise overall emissions. Ata, Lee and Tongarlak (2012) study the upstream structure of the fresh produce supply chain, holding the downstream structure (the number and location of stores) fixed.

There is a large literature on facility location problems (see Daskin 1995 and Snyder and Shen 2011) that generally focuses on the fixed cost of opening facilities and the transportation cost of serving a set of customers from the opened facilities. Shen and Qi (2007) also include inventory costs into the facility location problem. However, their model includes only one transportation segment of the supply chain (and the delivery to customers is a TSP rather than a k-median problem) and they do not study emissions.
3 Problem analysis

The supply chain design problem involves a number of related decisions, primarily store locations, replenishment routes and store sizing. This analysis first considers transportation costs and then evaluates retail space costs.

Given a set of \( n \) store locations, the region of customers can be partitioned into \( n \) sub-regions that represent the stores’ “service areas”, i.e., all customers in a service area shop at the store in their service area because that store is the closest to them among the \( n \) available choices. This partitioning is also called a Voronoi diagram. Figure 1 displays one possible partitioning: In addition to store locations, the retailer must choose a TSP route to minimize its transportation costs. Figure 2 presents two possible routes in the Figure 1 partitioning.

The optimal locations for stores in the supply chain problem is unknown and may involve a complex geometry. Therefore, this model includes store configurations in which the resulting Voronoi diagram is a tessellation of a single regular polygon. Carlsson (2012) considers additional tilings and finds that total transportation costs can be reduced with an Archimedean spiral relative to the three tilings considered here. If the retailer’s transportation cost is sufficiently low (so it is desirable to have many facilities) then he shows that the Archimedean spiral can reduce transportation costs relative to the triangle tiling by approximately 7%.
Figure 3 displays a tiling with equilateral triangles. There are two other feasible tilings that consist of a single regular polygon, one with squares and the other with hexagons. Given an area, the number of stores, and one of the three regular polygon tiling patterns, it is straightforward to find an optimal TSP route to replenish the stores. To finish the evaluation of transportation costs, it remains to find the average distance each unit travels to the stores, $d_t$, and the average round trip distance to each consumer’s home, $d_c$.

Consider one of the service regions. Within that region create a tessellation of right triangles by connecting the store to each vertex and to each face via a line perpendicular to its edge, to form sub-regions. Figure 4 demonstrates this for the triangle tessellation.

In a regular polygon with $s$ sides, an “$s$-gon”, there are $2s$ sub-regions. Consider one of the (identical) sub-regions. Let $\theta$ be the degree measure (in radians) of the acute angle formed at the vertex where the store is located and let $b$ be the shortest distance to the edge of the sub-region (i.e., $b$ is the height of one of the right triangles in Figure 4). For an $s$-gon, $\theta = \pi/s$. The average round-trip distance of customers within a sub-region to the store is:

$$d_c(b, \theta) = 2\int_0^b\int_0^{\tan\theta}\frac{\sqrt{x^2 + y^2}dxdy}{\sqrt{1/2}b^2\tan\theta}$$

where the denominator is the area of the sub-region. Using a change of variables ($t = y/x$), we
obtain
\[ d_c(b, \theta) = \frac{2}{3} \left( \sqrt{1 + (\tan \theta)^2} + \frac{\ln \left( \tan \theta + \sqrt{1 + (\tan \theta)^2} \right)}{\tan \theta} \right) b. \]

The area of the service region is \( 1/n \) and the area of the sub-region is then \( 1/(2sn) \). (Recall that the total area has been normalized to 1 unit.) Thus, \( (1/2)b^2 \tan \theta = 1/(2sn) \), and \( d_c(b, \theta) \) can be written as
\[ d_c = \phi_c n^{-1/2} \] (1)
where \( \phi_c \) is a constant that depends on the tessellation (i.e., the number of sides, \( s \)):
\[ \phi_c = \frac{2}{3} \sqrt{\frac{1}{s \tan \theta}} \left( \sqrt{1 + (\tan \theta)^2} + \frac{\ln \left( \tan \theta + \sqrt{1 + (\tan \theta)^2} \right)}{\tan \theta} \right) \]

The retailer’s truck must travel into and out of each region at least once (assuming there are at least two stores, \( n \geq 2 \)). The truck’s minimum distance within a sub-region is \( 2b \), so the length of the TSP tour is no shorter than \( 2bn \). Hence, a lower bound estimate for the retailer’s transport distance is
\[ d_t = \phi_t n^{1/2} \] (2)
where the constant \( \phi_t \) also only depends on the chosen tessellation:
\[ \phi_t = \frac{2}{\sqrt{s \tan \theta}}. \]

The estimate (2) is exact for triangles with \( n \in \{2, 6, 10, \ldots\} \), for squares with \( n \in \{2, 4, 6, \ldots\} \) and for all \( n \) with hexagons.3

Using \( d_c = \phi_c n^{-1/2} \) and \( d_t = \phi_t n^{1/2} \) to solve for \( d_c \) in terms of \( d_t \) reveals that \( d_c'(d_t) = -\phi_c \phi_t / d_t^2 \) and \( d_c''(d_t) = 2\phi_c \phi_t / d_t^3 \); increasing the distance the truck travels in the supply chain reduces the distance cars drive, but at a decreasing rate. Curiously, the products of the average distances remains a constant no matter the design chosen, i.e., \( d_c d_t = \phi_c \phi_t \).

Now turn to the cost of retail space. Given that retail space is directly proportional to the amount of inventory held, an inventory model is needed. In this context the optimal policy to choose quantities and dispatches is unknown and likely complex. Heuristic policies in similar situations are developed by Cachon (2001) and Gürbüz, Moinzadeh and Zhou (2007), but their results do not provide closed form estimates of inventory levels. Hence, the approach taken here is to develop a tractable approximation of this inventory system.

3 With triangles an upper bound on the TSP distance is \( 2b(n + 1) \), which is exact for \( n \in \{4, 8, 12, \ldots\} \). For \( n \in \{3, 5, 7, 9, \ldots\} \) the TSP distance is \( 2b(n + \sqrt{3} - 1) \). With squares, the TSP distance is \( 2b(n + \sqrt{2} - 1) \) for odd \( n \).
Assume the retailer’s truck is dispatched with \( q_t \) units every \( q_t/\lambda \) units of time, which is called a "period". Each store places an order every period and receives a replenishment with a zero lead time. Each store operates with a base stock policy: order enough inventory to ensure that the store has \( S \) units immediately after a delivery. For store \( j \), let \( \lambda_{j,\tau} \) be its demand in period \( \tau \), and let \( I_{\tau} \) be its inventory at the end of period \( \tau \). It follows that

\[
I_{j,\tau+1} = \left[ I_{j,\tau} + \left( (S - I_{j,\tau})^+ + \omega_{j,\tau+1} \right)^+ - \lambda_{j,\tau+1} \right]^+
\]

where \( \omega_{j,\tau} \) is an adjustment to store \( j \)'s order to ensure a full truck load delivery:

\[
\sum_{j=1}^{n} \omega_{j,\tau+1} = q_t - \sum_{j=1}^{n} (S - I_{j,\tau})^+.
\]

For example, if the sum of the orders from the stores is less than a truckload, additional inventory is sent to fill the truck, and if orders exceed the truck’s capacity, then some stores receive less than their order. This inventory policy is difficult to analyze primarily because the adjustment \( \omega_{j,\tau} \) may cause a delivery in one period to be more or less than demand in the previous period. However, if the store operates such that stockouts are rare and stores rarely start a period with more than \( S \) units of inventory, then

\[
E[I_j] \approx S + E[\omega_j] - E[\lambda_j]
\]

\[
\approx S - E[\lambda_j]
\]

because on-average the average adjustment is zero, \( E[\omega_j] = 0 \). If demand is taken to be normally distributed, then \( S = E[\lambda_j] + z\sigma_j \), where \( \sigma_j \) is the standard deviation of a store’s demand in one period and \( z \) is a constant chosen by the firm to influence the service level (i.e., the probability of being in-stock). Thus,

\[
E[I_j] = z\sigma_j
\]

is an estimate of a store’s inventory. Each store services an area of size \( 1/n \), so

\[
\sigma_j = \sqrt{\frac{1}{n}} \sqrt{\frac{q_t}{\lambda} \sigma} = \sqrt{\frac{q_t}{n\lambda} \sigma}
\]

The average time a unit spends in the retail store, \( t_s \), is \( nE[I_j]/\lambda \), so the average space cost per unit sold is

\[
c_s t_s = c_s \frac{nE[I_j]}{\lambda} = \phi_s c_s n^{1/2}
\]

where

\[
\phi_s = z \sqrt{\frac{q_t}{\lambda} \left( \frac{\sigma}{\lambda} \right)}.
\]

There are three terms in the \( \phi_s \) constant: inventory increases as the retailer chooses a higher service level (\( z \)), or if deliveries become less frequent (\( \sqrt{q_t/\lambda} \) increases) or if demand becomes more variable (the coefficient of variation of aggregate demand, \( \sigma/\lambda \), increases).
From (3), the inventory cost per unit grows proportional to $n^{1/2}$: as more stores are added units stay in the store longer, thereby incurring greater space costs. This reflects the notion that there are economies of scale in managing inventory: stores with larger service areas have higher inventory productivity.

Combining the retail space costs, (3), with the transportation distances, (1) and (2), the supply chain cost function is

$$C(n) = \phi_c c_c n^{-1/2} + (\phi_t c_t + \phi_s c_s) n^{1/2}. \quad (4)$$

Considering only the transportation costs, the cost function $C(n)$ is consistent with several studies of probabilistic versions of the $k$-median and TSP problems. For example, Fisher and Hochbaum (1980) consider the $k$-median problem of selecting $n$ store locations from a set of randomly chosen sites to minimize the total distance consumers must travel to the closest of the $n$ stores. They find that the value of the optimal cost grows proportional to $\sqrt{1/n}$, as in (4). For the TSP, Beardwood, Halton and Hammersley (1959) show that the shortest distance through $n$ randomly selected points in an unit area is asymptotically proportional to $\sqrt{n}$, again, as in (4).

Table 1 summarizes the transportation constants in the cost function.

<table>
<thead>
<tr>
<th>tessellation</th>
<th>$\phi_c$</th>
<th>$\phi_t$</th>
<th>$\phi_c/\phi_t$</th>
<th>$\phi_c c_c / (\phi_t c_t + \phi_s c_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>0.807293</td>
<td>0.877383</td>
<td>0.708</td>
<td>0.920</td>
</tr>
<tr>
<td>square</td>
<td>0.765196</td>
<td>1.000000</td>
<td>0.765</td>
<td>0.765</td>
</tr>
<tr>
<td>hexagon</td>
<td>0.754393</td>
<td>1.074570</td>
<td>0.811</td>
<td>0.702</td>
</tr>
</tbody>
</table>

For a given number of stores ($n$) and a fixed region size, consumers prefer hexagons because the hexagon pattern has the lowest $\phi_c$. (Others have observed that the “honeycomb” pattern is effective for the $k$–median problem - Papadimitriou, 1981). The firm, on the other hand, travels the farthest with the hexagon tessellation. In all cases, consumers on average travel a shorter distance than the retailer ($\phi_c < \phi_t$) despite the fact that some consumers must travel farther (e.g., as much as twice as far with the triangle tessellation). Consequently, if it were equally costly for consumers and the retailer to transport goods, then one would expect the optimal tessellation to have few stores, forcing consumers to drive long distances.

If we allow $n$ to be non-integer, minimization of $C(n)$ is straightforward, given that it is quasi-convex in $n$. Let $n^*$ be the cost minimizing number of stores:

$$n^* = \frac{\phi_c c_c}{\phi_c c_c / \phi_t c_t + \phi_s c_s}. \quad (5)$$

From Table 1, $\phi_c/\phi_t < 1$, so $n^* > 2$ only if $c_c >> c_t$, which is likely because the load carried by a truck is substantially larger than the load carried by a car, $q_t >> q_c$. Section 4 provides specific estimates to confirm that $c_c >> c_t$. 

14
The minimum cost is

\[ C(n^*) = 2(\phi_c c_c)^{1/2}(\phi_t c_t + \phi_s c_s)^{1/2}. \]

If the cost of retail space is low (or ignored), then the triangle tessellation is best no matter the relative transportation costs, because, according to Table 1, that minimizes \( \phi_c \phi_t \).\(^4\) In fact, the transportation cost with the triangle tessellation is about 7% lower than with hexagons \( (1 - \sqrt{0.708/0.811}) \). It is also possible to show that \( \phi_c \phi_t \) is decreasing for all \( s \geq 3 \), suggesting that the triangle tessellation may perform well even with tessellations that include more than one regular polygon. However, if the retailer’s transportation costs are low (or ignored) relative to space costs, then the hexagon tessellation is best, as it minimizes \( \phi_c \).

The distances \( d_c \) and \( d_t \) are measured by the Euclidian norm, \( L_2 \). Another common norm is \( L_1 \), which is sometime referred to as the "Manhattan" norm or the "city-block" norm in which the distance between points \( \{x_1, y_1\} \) and \( \{x_2, y_2\} \) is taken to be \( |x_2 - x_1| + |y_2 - y_1| \), i.e., travel occurs along a square grid. It is possible to show that for consumers, even with the \( L_1 \) norm, their round-trip distance to the nearest store is proportional to \( n^{-1/2} \). With the \( L_1 \) norm, the retailer’s travel distance is easiest to estimate with the square tessellation, and in that case the distance continues to be proportional to \( n^{1/2} \). Hence, these results do not appear to be sensitive to how distance is measured.

The solution \( n^* \) minimizes total supply chain costs, but the retailer and consumers will choose this solution only if they are fully charged for the cost of carbon, \( p_c \). There are a number of reasons why this may not incur, including the fact that a precise measure of \( p_c \) is difficult to obtain. Hence, it is worthwhile to consider two alternative approaches for selecting a supply chain design. The first minimizes total emissions while ignoring explicit operating costs. In that case a design is chosen as if the cost of emissions is extremely high relative to operating costs (i.e., operating costs are dwarfed by emissions costs, so they can be effectively ignored). The second minimizes explicit operating costs while ignoring emission costs. That case is relevant if one believes emissions externalities are small or there is no explicit means in place that charges for carbon emissions. This can be taken to be the current status quo.

Let \( C^e(n) \) be total emissions per unit,

\[ C^e(n) = \phi_c c_c^e n^{1/2} + (\phi_t c_t^e + \phi_s c_s^e) n^{-1/2}, \]

\(^4\) When exact values of the TSP are utilized (and retail space costs are ignored), then triangles are best (among the three) for \( n = 2 \) and \( n \geq 5 \), hexagons are best for \( n = 3 \) and squares are best for \( n = 4 \).
and \(C^o(n)\) be total operating costs per unit,

\[
C^o(n) = \phi_c c^o_c n^{1/2} + (\phi_t c^o_t + \phi_s c^o_s) n^{-1/2},
\]

where, for \(j \in \{c, t, s\}\),

\[
e^c_j = \frac{f^c_j e^c_j}{q_j},
\]

and

\[
o^o_j = \frac{v_j + f^o_j c^o_j}{q_j}.
\]

Note that \(C^e(n)\) is not actually a cost, as it is total emissions, but for notational consistency it is represented with a “\(C\)” nevertheless.

As with \(C(n)\), both \(C^e(n)\) and \(C^o(n)\) are quasi-convex and a cost minimizing \(n\) can be found:

\[
n^e = \arg \min_n C^e(n) = \frac{\phi_e c^e_e}{\phi_t c^e_t + \phi_s c^e_s}
\]

and

\[
n^o = \arg \min_n C^o(n) = \frac{\phi_o c^o_c}{\phi_t c^o_t + \phi_s c^o_s}
\]

As one would expect, as carbon becomes expensive, the optimal design approaches the emissions minimizing design,

\[
\lim_{p_e \to \infty} n^* = n^e,
\]

and as carbon becomes cheap, the optimal design approaches the operating cost minimizing design,

\[
\lim_{p_e \to 0} n^* = n^o.
\]

Further, the optimal design falls somewhere between the two extreme designs: \(\min\{n^o, n^e\} \leq n^* \leq \max\{n^o, n^e\}\). Whether the emissions minimizing design is dense with stores \((n^o < n^e)\) or sparse with stores \((n^e < n^o)\) depends on the parameter values of the technologies. In particular, if \(\gamma > 1\), then the emissions minimizing design is dense with stores, where\(^5\)

\[
\gamma = \frac{\beta_c}{\delta_t + \delta_s \beta_t + \delta_t + \delta_s \beta_s},
\]

where, for \(j \in \{c, t, s\}\),

\[
\beta_j = \frac{f^c_j e^c_j}{v_j + f^o_j p_j} = \frac{e^c_j}{\delta_j},
\]

and

\[
\delta_j = \frac{\phi_j (v_j + f^o_j p_j)}{q_j}.
\]

\(^5\) Differentiating \(n^*\) with respect to \(p_e\) yields:

\[
\frac{dn^*}{dp_e} = \frac{\delta_t \beta_t + \delta_s \beta_s}{(\delta_t + \beta_t p_e + \delta_s + \beta_s p_e)^2} \gamma (\gamma - 1).
\]

Hence, \(n^*\) is increasing if \(\gamma > 1\).
The parameters $\beta_j$ and $\delta_j$ are introduced for notational convenience but $\beta_j$ are referred to as an *emissions to operating cost ratio* - it is the ratio of emissions to operating cost for one vehicle or unit of space for one unit of time, independent of the amount of product carried or stored (i.e., independent of $q_j$). For example, it is the emissions to move a car one km relative to the operating cost to move the car one km. To summarize, minimizing emissions surely requires a denser network with more stores when $\beta_c > \max\{\beta_t, \beta_s\}$, but minimizing emissions does not always involve a denser network - when $\beta_c < \min\{\beta_t, \beta_s\}$, the emissions minimizing supply chain may have few stores because the consumers’ cars have low emissions relative to their operating costs.

Now consider the supply chain’s emissions, operating costs and total costs with these different designs. Define the *operating cost penalty for minimizing emissions* as $C^o(n^e)/C^o(n^o) - 1$ - this is the percentage increase in operating costs that a supply chain incurs when the emissions minimizing design is chosen. This provides a measure of the explicit cost to adopt a "minimize emissions" objective. Some algebra yields,

$$\frac{C^o(n^e)}{C^o(n^o)} - 1 = \frac{1}{2} \left( \frac{n^e}{n^o} \right)^{1/2} + \frac{1}{2} \left( \frac{n^o}{n^e} \right)^{-1/2} - 1.$$

If $\gamma = 1$, minimizing emissions also minimizes operating costs. This could occur in the unlikely case in which $\beta_c = \beta_t = \beta_s$. Otherwise, there generally is a penalty. The magnitude of this penalty can be bounded:

$$\frac{C^o(n^e)}{C^o(n^o)} \leq \frac{1}{2} \max \left\{ \left( \frac{\beta_c}{\beta_t} \right)^{1/2} + \left( \frac{\beta_c}{\beta_t} \right)^{-1/2}, \left( \frac{\beta_c}{\beta_s} \right)^{1/2} + \left( \frac{\beta_c}{\beta_s} \right)^{-1/2} \right\} - 1. \quad (6)$$

Therefore, if $\beta_c/\beta_t$ and $\beta_c/\beta_s$ are sufficiently close to 1, the emissions penalty is small. However, the emissions penalty can be large if there is a large discrepancy between either $\beta_c$ and $\beta_t$ or between $\beta_c$ and $\beta_s$. Nevertheless, the emissions penalty is finite, even if emissions are infinitely expensive. Furthermore, this bound is useful as it involves a somewhat limited number of parameters.

Define the *emissions penalty for minimizing operating costs* as $C^e(n^o)/C^e(n^e)$. This penalty turns out to be identical to the previous one considered, and shares the same penalty bound, (6):

$$\frac{C^e(n^o)}{C^e(n^e)} = \frac{1}{2} \left( \frac{n^e}{n^o} \right)^{1/2} + \frac{1}{2} \left( \frac{n^o}{n^e} \right)^{-1/2} - 1.$$

Hence, if there is a small emissions penalty, then there is a small operating cost penalty and a large emissions penalty is always matched with a large operating cost penalty.

The next two penalties reflect the total cost penalty for adopting one of the extreme approaches of either minimizing just emissions or just operating costs:

$$\frac{C^*(n^o)}{C^*(n^*)} = \frac{1}{2} \left( \frac{n^o}{n^*} \right)^{1/2} + \frac{1}{2} \left( \frac{n^o}{n^*} \right)^{-1/2} - 1 \quad \text{and} \quad \frac{C^*(n^e)}{C^*(n^*)} = \frac{1}{2} \left( \frac{n^e}{n^*} \right)^{1/2} + \frac{1}{2} \left( \frac{n^e}{n^*} \right)^{-1/2} - 1.$$
Refer to these as the total cost penalties. If the cost of carbon is extremely high or extremely low, then these total cost penalties approach the emissions and operating cost penalties:

\[
\lim_{p_e \to -\infty} \frac{C^*(n^o)}{C^*(n^e)} = \lim_{p_e \to 0} \frac{C^*(n^o)}{C^*(n^e)} = \frac{C^o(n^o)}{C^o(n^e)}.
\]

4 Parameter Estimates

This section provides estimates for the parameters in the supply chain design problem. The first objective is to determine if the emissions minimizing design has a denser network of smaller stores relative to the operating cost minimizing design (i.e., if \(n^e > n^o\)). The second objective is to determine the magnitude of the various penalties. Baseline estimates are given as well as several scenarios that indicate plausible ranges. The section end with a discussion of options for reducing emissions.

Begin with the transportation parameters. The average mileage of passenger vehicles in the U.S. in 2009 was 21.1 miles gal\(^{-1}\) or 8.97 km l\(^{-1}\) (EPA 2009). Fuel consumption was then \(f_c = 1/8.97 = 0.111 \text{ l km}^{-1}\). A typical retailer truck travels 6 miles per gallon of diesel, which yields a fuel consumption of \(f_t = 0.392 \text{ l km}^{-1}\).

From EPA (2005), 8.8 kg of CO\(_2\) are emitted per gallon of gasoline, or \(e_c = 2.325 \text{ kg CO}_2 \text{ l}^{-1}\). From the same report 10.1 kg of CO\(_2\) are emitted per gallon of diesel, or \(e_t = 2.669 \text{ kg CO}_2 \text{ l}^{-1}\). On May 21, 2012 the national average price per gallon of gasoline and diesel, respectively, were \(p_c = 0.98 \text{ $ l}^{-1}\) and \(p_t = 1.05 \text{ $ l}^{-1}\) (http://www.eia.gov/petroleum/gasdiesel/). For the variable operating cost of a car, Barnes and Langworthy (2003, Table 4.2) provide an estimate of $0.0644 km\(^{-1}\) in 2003 dollars. Adjusting for inflation of 2.5\% per year, yields a 2012 estimate of \(v_c = $0.0804 \text{ km}^{-1}\). For trucks, the comparable estimates in Barnes and Langworthy (2003) are $0.1375 km\(^{-1}\) in 2003 dollars and the 2012 estimate of $0.172 km\(^{-1}\), which does not include

6 The Bureau of Transportation Statistics (2011) Table 4-14 reports that truck fuel consumption in the United States was 5.9 in 2004 but dropped to 5.4 by 2008. Bonney (2009) reports that Walmart’s fuel efficiency in 2005 was 5.9 miles per gallon, which increased to 7.1 miles per gallon by 2008.

7 The American Automobile Association (2011) suggests \(v_c = $0.0621 \text{ km}^{-1}\): the average cost per kilometer to drive a sedan includes $0.0278 for maintenance, $0.006 for tires, and $0.0283 for distance related depreciation (in $ km\(^{-1}\)) (Three observations are provided for the annual depreciation cost \{10000,$3471\}, \{15000,$3728\} and \{20000,$3924\}, where the first term is the miles driven during the year and the second is the total depreciation cost. A linear regression through these observations yields a slope of $0.0453 per mile, or $0.0283 km\(^{-1}\)) The estimate
driver wages. If driver wages are included, at $50,000 per year and 100,000 miles, their estimate is $v_t = $0.484 $km^{-1}$.

McKinnon and Woodburn (1994) report that the average consumer in a survey carried $q_c = 18$ kg of goods with each shopping trip. A retail truck can carry up to 45,000 lbs in the U.S., which is about $q_t = 20,000$ kg. Even if consumers carried 40 kgs per shopping visit, the retailer’s truck carries a load that is 500 times greater. In terms of variable operating costs, the car incurs $v_c + f_c p_c = $0.189 $km^{-1}$, or $0.30$ per mile, and the truck incurs $v_t + f_t p_t = $0.896 $km^{-1}$, or $1.43$ per mile. Given that a truck has a variable operating costs per km that is less than 5 times that of a car, but carries at least 500 times more product, it follows that $c_c >> c_t$ : if the price of carbon is ignored ($p_c = 0$) and $q_c = 18$ and $q_t = 20,000$, then $c_c = $0.0105 $km^{-1}$ and $c_t = $0.000045 $km^{-1}$, yielding a ratio of $c_c/c_t = 235$.

### Table 2: Transportation parameters*

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$f_c$</th>
<th>$e_c$</th>
<th>$v_c$</th>
<th>$p_c$</th>
<th>$f_t$</th>
<th>$e_t$</th>
<th>$v_t$</th>
<th>$p_t$</th>
<th>$\beta_c$</th>
<th>$\beta_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.111</td>
<td>2.325</td>
<td>0.0804</td>
<td>0.98</td>
<td>0.392</td>
<td>2.669</td>
<td>0.484</td>
<td>1.05</td>
<td>1.364</td>
<td>1.168</td>
</tr>
<tr>
<td>High consumer fuel efficiency</td>
<td>0.056</td>
<td>2.325</td>
<td>0.0804</td>
<td>0.98</td>
<td>0.392</td>
<td>2.669</td>
<td>0.484</td>
<td>1.05</td>
<td>0.957</td>
<td>1.169</td>
</tr>
<tr>
<td>High retailer fuel efficiency</td>
<td>0.111</td>
<td>2.325</td>
<td>0.0804</td>
<td>0.98</td>
<td>0.196</td>
<td>2.699</td>
<td>0.484</td>
<td>1.05</td>
<td>1.364</td>
<td>0.758</td>
</tr>
<tr>
<td>High consumer and retailer fuel efficiency</td>
<td>0.056</td>
<td>2.325</td>
<td>0.0804</td>
<td>0.98</td>
<td>0.196</td>
<td>2.669</td>
<td>0.484</td>
<td>1.05</td>
<td>0.957</td>
<td>0.758</td>
</tr>
<tr>
<td>High fuel prices</td>
<td>0.111</td>
<td>2.325</td>
<td>0.0804</td>
<td>1.96</td>
<td>0.392</td>
<td>2.669</td>
<td>0.484</td>
<td>2.10</td>
<td>1.001</td>
<td>0.982</td>
</tr>
</tbody>
</table>

* Fuel efficiency scenarios double fuel efficiency and the high fuel scenario doubles the price of gasoline and diesel

To consider alternative scenarios, it is not necessary to vary the emissions per liter for gasoline and diesel, $e_c$ and $e_t$, because these are estimated with little error. It is possible that future fuel efficiency could change. For example, current U.S. standards require that automobile manufacturers achieve 54.5 miles per gallon among the cars in their fleet by 2025 (Vlasic 2011), which would represent a 158% improvement over the 2009 level. Three additional scenarios can be constructed in which consumer fuel efficiency doubles, retail fuel efficiency doubles, or both double. Increasing fuel efficiency is analogous to increasing variable operating costs (because $\beta_j = e_j/(v_j/f_j + p_j)$, cutting fuel usage in half is the same as doubling variable operating costs). The final parameters $v_c = $0.0837 $km^{-1}$ is comparable. The AAA focuses on the first five years of vehicle life, so it is expected to underestimate the variable cost of operating the existing fleet, which has an average age of approximately 10.8 years (Meier 2012).
are the cost of gasoline, $p_c$, and diesel, $p_t$. Substantial asymmetries in these two prices have not occurred historically, but they do vary considerably. Suppose both could increase by a factor of 2: $p_c = 1.96$ and $p_t = 2.10$. Table 2 provides the transportation parameters for the various scenarios described and the resulting emissions to operating cost ratios.

Now consider the retail space parameters: $v_s$, $f_s$, $p_s$, and $e_s$. The cost of retail space, $v_s$, varies considerably by the quality and type of the location - according to loopnet.com, among the top 20 metropolitan areas in the United States, in March 2012, the average retail lease space was $19.8$ per square foot per year, with a low of $12.26$ (Detroit) and a high of $36.46$ (San Francisco). Retail space in a successful shopping mall, can be considerably higher, as in $50$ per square foot per year. For the baseline scenario, take $19.8$ per square foot per year, which is $v_s = 1.84 \text{ m}^{-2}\text{ yr}^{-1}$.

Table 3 provides electricity and natural gas usage by type of retailer in the United States. The average February 2012 price of electricity to commercial customers in the U.S. was $0.101 \text{ kW} h^{-1}$ (Table 5.6.A. Average Retail Price of Electricity to Ultimate Customers by End-Use Sector, by State, February 2012 and 2011). The price of natural gas for commercial firms has ranged from $5$ to $15$ per 1000 cubic feet over the period of 1984-2012, with a February 2012 average of $7.97$ (http://www.eia.gov/dnav/ng/hist/n3020us3m.htm). For the baseline scenario, combine these prices with the average mercantile consumption levels reported in Table 3 to yield $f_s p_s = 1.78 \times 0.101 + 2.15 \times 0.00797 = 0.197 \text{ m}^{-2}\text{ yr}^{-1}$.

<table>
<thead>
<tr>
<th>Table 3: Electricity and natural gas usage by retail type</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Merchantile</strong></td>
</tr>
<tr>
<td><strong>Non-mall</strong></td>
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<tr>
<td><strong>Mall</strong></td>
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<tr>
<td><strong>All</strong></td>
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<td><strong>Food Sales</strong></td>
</tr>
<tr>
<td>Electricity ($\text{kWh m}^{-2}\text{ yr}^{-1}$)</td>
</tr>
<tr>
<td>1.33</td>
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<tr>
<td>2.07</td>
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<td>1.78</td>
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<td>4.52</td>
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<tr>
<td>Natural gas ($\text{ft}^3 \text{ m}^{-2}\text{ yr}^{-1}$)</td>
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<tr>
<td>1.92</td>
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<tr>
<td>2.28</td>
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<tr>
<td>2.15</td>
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<tr>
<td>2.83</td>
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<tr>
<td>Source: Energy Information Administration, 2003 Commercial Buildings Energy Consumption Survey: Table CA1</td>
</tr>
</tbody>
</table>

From eGRID2012 Version (1.0) (year 2009 data), average U.S. emissions is 0.55 kg CO$_2$ kW$^{-1}$. The lowest emissions region has 0.23 kg CO$_2$ kW$^{-1}$ (Upstate NY) and the highest has 0.83 kg CO$_2$ kW$^{-1}$ (Rockies). Natural gas emits 0.05 kgs CO$_2$ ft$^{-3}$ (EPA emissions calculator 2012). For the baseline scenario, combine the U.S. average electricity emissions and the natural gas emissions with the average mercantile consumption levels reported in Table 3 to yield $f_s e_s = 1.78 \times 0.55 + 2.15 \times 0.05 = 1.087 \text{ kg CO}_2 \text{ m}^{-2}\text{ yr}^{-1}$.

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8 Personal communication with Randy Whitaker, Executive Vice President Store Operations, Victoria’s Secret Stores.
Table 4 provides baseline and alternative scenarios for the parameters relevant to $\beta_s$. In all cases, natural gas emissions are taken to be $0.05 \text{ kgs CO}_2 \text{ ft}^{-3}$, the price of electricity is $0.101 \text{ kWh}^{-1}$ and the price of natural gas is $7.97 \text{ per 1000 cubic feet}$.

Table 4: Baseline and alternative scenarios for parameters that yield $\beta_s$

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>$v_s$</th>
<th>Electricity usage</th>
<th>Gas usage</th>
<th>Electricity emissions</th>
<th>$\beta_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1.84</td>
<td>1.78</td>
<td>2.15</td>
<td>0.55</td>
<td>0.534</td>
</tr>
<tr>
<td>Low electricity emissions</td>
<td>1.84</td>
<td>1.78</td>
<td>2.15</td>
<td>0.23</td>
<td>0.254</td>
</tr>
<tr>
<td>High electricity emissions</td>
<td>1.84</td>
<td>1.78</td>
<td>2.15</td>
<td>0.83</td>
<td>0.778</td>
</tr>
<tr>
<td>High rent</td>
<td>3.68</td>
<td>1.78</td>
<td>2.15</td>
<td>0.55</td>
<td>0.280</td>
</tr>
<tr>
<td>Low electricity emissions and</td>
<td>3.68</td>
<td>1.78</td>
<td>2.15</td>
<td>0.23</td>
<td>0.133</td>
</tr>
<tr>
<td>high rent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High fuel usage</td>
<td>1.84</td>
<td>4.52</td>
<td>2.83</td>
<td>0.55</td>
<td>1.130</td>
</tr>
<tr>
<td>High fuel usage and high rent</td>
<td>1.84</td>
<td>4.52</td>
<td>2.83</td>
<td>0.55</td>
<td>0.632</td>
</tr>
</tbody>
</table>

The “low electricity emissions” scenarios corresponds to the emissions of the lowest emissions region in the U.S. and the “high electricity emissions” scenario corresponds to the highest emissions region. The “high rent” scenarios double the cost to rent per square meter per year relative to the baseline (to approximately $40 per square foot per year). The “high fuel usage” scenarios correspond to the usage of Food Sales retailers (i.e., groceries), which are higher in large part due to the need for product refrigeration.

The remaining parameters to estimate are part of $\delta_t$ and $\delta_s$. Assume a triangle tessellation, so $\phi_t = 0.877$, and a retailer’s truck load is $q_t = 20,000 \text{ kg}$. Including baseline parameters, $v_t = 0.484$, $f_t = 0.392$, and $p_t = 1.05$, yields $\delta_t = 3.92 \times 10^{-5}$. To evaluate $\phi_s$, use $z = 3.00$ (which corresponds to a 99.9% in-stock probability), $\sqrt{q_t/\lambda} = \sqrt{1/52} = 0.139$ (which implies one delivery per week), and $\sigma/\lambda = 1$ (which implies a coefficient of variation of demand equal to 1): $\phi_s = 3 \times 0.139 \times 1 = 0.416$. $q_s$ is a challenging parameter to estimate as it is the kg of product per $m^2$. Using 2011 data from Walmart, $q_s = 141 \text{ kg} \text{ m}^{-2}$. Hence, using baseline estimates for $v_s$ and $f_s p_s$, yields $\delta_s = 0.006$, which is 153 times larger than $\delta_t$. It seems reasonable to conclude,

---

In 2011, Walmart’s annual inventory turns were 8.75. U.S. sales revenue was $309B. Hence, U.S. inventory (in sales $s$s) was $309B/8.75 = $35.3B. U.S. square footage of retail space was 698M, so Walmart held $35.3B / 698M = $50.59 of inventory per square foot. From their website, they report that they made 4M deliveries to their stores, which implies $309B / 4M = $77,250 per delivery. Given 20,000 kg per delivery implies $77,250 / 20,000 = $3.86 \text{ kg}^{-1}$. Walmart then has $50.59 sqft^{-1} /$3.86 kg$^{-1} = 13.1 \text{ kg sqft}^{-1}$. With 10.76 sqft $m^{-2}$, there is $10.76 \times 13.1 = 141 \text{ kg} \text{ m}^{-2}$.
even with some substantial estimation error in $q_s$ and the other parameters, that $\delta_s \gg \delta_t$.

The first issue to address is whether the emissions minimizing supply chain is more dense with smaller stores than the operating cost minimizing supply chain, i.e., is $n^e > n^o$? Taking $\delta_s \gg \delta_t$, this occurs if $\beta_c > \beta_s$. Comparing Tables 2 and 4, that holds for the baseline scenarios, and occurs for most combinations of scenarios with the exception of high consumer fuel efficiency ($\beta_c = 0.957$) and high retailer space fuel usage ($\beta_s = 1.13$). Thus, the emissions minimizing supply chain generally has more stores, located closer to consumers, than the supply chain that minimizes operating costs. This is particularly true if the retailer has lower electricity emissions and high rent ($\beta_s = 0.133$). In those cases, a retailer that minimize operating costs builds large stores that are far from customers (to economize on its high rent costs) whereas a retailer that minimizes emissions builds many small stores close to customers (to exploit it low emissions). Table 5 indicates that the emissions minimizing supply chain may have many more stores, upwards of seven times more, than the operating cost minimizing supply chain.

Table 5: Ratio of the number of stores that minimize emissions to number that minimize operating costs, $n^e/n^o$

<table>
<thead>
<tr>
<th>Retail space scenario</th>
<th>Transportation scenario</th>
<th>Baseline</th>
<th>High consumer fuel efficiency</th>
<th>High fuel prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td></td>
<td>2.5</td>
<td>1.8</td>
<td>1.9</td>
</tr>
<tr>
<td>Low electricity emissions and high rent</td>
<td></td>
<td>7.4</td>
<td>3.7</td>
<td>7.4</td>
</tr>
<tr>
<td>High fuel usage</td>
<td></td>
<td>1.2</td>
<td>0.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Now turn to the evaluation of the penalty bound, (6). Considering just the transportation parameters, $\beta_c$ and $\beta_t$, the bound for the baseline scenario is remarkably small, about 0.3%:

$$\frac{1}{2} \left( \left( \frac{1.364}{1.168} \right)^{1/2} + \left( \frac{1.364}{1.168} \right)^{-1/2} \right) - 1 = 0.003.$$  

Recall, this applies to the operating cost penalty for minimizing emissions, the emissions penalty for minimizing operating costs and the total cost penalty. This penalty with respect to transportation costs is small because cars and trucks have similar emissions to operating cost ratios: $\beta_c = 1.364$ for cars and $\beta_t = 1.168$ for trucks. There is a larger discrepancy between these ratios in the other transportation scenarios described in Table 2, but even in the scenario with the largest discrepancy (high retailer fuel efficiency), the bound is still only 4.3%. Thus, from the perspective of transportation costs, the supply chain design is remarkably robust given reasonable parameters. If emissions are minimized while operating costs are ignored, operating costs will nevertheless not
increase by much (e.g., by 0.3% in the baseline scenario). Similarly, if operating costs are minimized while emissions costs are ignored, total emissions increase by the same relatively small amount.

It is important to note that this robustness result does not occur for any all parameters. According to the penalty bound, (6), there is a substantial penalty whenever there is an asymmetry in the emissions to operating cost ratios. It happens to be that those ratios are similar for cars and trucks. Furthermore, the substantial difference in loads carried, which one might presume would great an asymmetry that leads to a large penalty, has no role in the penalty bound - the substantial difference in the loads carried is accounted for no matter which objective is selected, and thus they do not factor into the penalty bound.

Considering the retailer’s space cost, Table 6 evaluates the penalty bound for each \( \beta_s \) scenario and two value of \( \beta_c \): the baseline, \( \beta_c = 1.36 \) and consumers double their fuel efficiency, \( \beta_c = 0.957 \).

| Scenarios                                      | \( \beta_s \) | \( \beta_c = 0.957 \) | \( \beta_c = 1.36 \) |
|------------------------------------------------|----------------|------------------------|
| Baseline                                       | 0.534          | 4.3%                   | 11.2%                 |
| Low electricity emissions                       | 0.254          | 22.9%                  | 37.5%                 |
| High electricity emissions                      | 0.778          | 0.5%                   | 4.0%                  |
| High rent                                      | 0.280          | 19.5%                  | 33.0%                 |
| Low electricity emissions and high rent         | 0.133          | 52.6%                  | 75.6%                 |
| High fuel usage                                 | 1.130          | 0.4%                   | 0.4%                  |
| High fuel usage and high rent                   | 0.632          | 2.2%                   | 7.5%                  |

We observe a somewhat substantial penalty bound, 11.2%, for the baseline scenario. This is due to the asymmetry in the emissions to operating cost ratios for cars, \( \beta_c = 1.36 \), and for retailer space, \( \beta_s = 0.534 \). The penalties in the other scenarios vary considerably, depending on how the \( \beta_s \) and \( \beta_c \) parameters change relative to each other. The highest penalty bound, 75.6%, occurs when the retailer uses electricity from a low emissions source (nearly comparable to renewable energy) and operates in an environment with high retail space costs. This combination also created the largest penalty when only the transportation parameters are considered, \( \beta_c \) and \( \beta_{rt} \), but the penalty is clearly even larger when retail space is included in the analysis. Hence, in these situations outcomes are considerably different depending on the selected objective. If operating costs are minimizing while ignoring emissions, then emissions are 75.6% higher than their minimal level. Similarly, if emissions are minimizing while ignoring operating costs, then operating costs are 75.6% higher than their lowest level. In this scenario there is a clear tension between environmental and financial preferences - as long as there is no explicit charge for carbon (so that carbon costs are not part of
minimizing operating costs), one cannot have both low operating costs and low emissions.

Given the potential tension between financial and environmental preferences, a natural solution is to include an explicit charge for carbon emissions into the financial objective function - with an explicit price for carbon, i.e., \( p_e > 0 \), the negative externalities associated with carbon emissions can be properly accounted for in the supply chain design that minimizes operating costs (which now include a portion of the cost of carbon emissions). Estimates of \( p_e \) vary considerably, but generally in the range between $20 and $1000 per metric tonne (see Tol 2008). How large does \( p_e \) have to be to induce substantial reductions in emissions in the context of the supply chain design problem? Ideally, a small fee for carbon would induce a sufficiently dramatic shift in the supply chain that a large portion of the potential emissions reduction can be achieved. Unfortunately, this does not appear to be the case. To explain, define the emissions gap reduction as the percentage of the potential emissions reduction that is achieved when carbon is explicitly priced, i.e., when \( p_e > 0 \). In particular, the emissions gap reduction is

\[
\frac{C^e(n^o) - C^e(n^*(p_e))}{C^e(n^o) - C^e(n^e)};
\]

the denominator is the reduction in emissions that occurs with a switch from minimizing operating costs to minimizing emissions, and the numerator is the amount of carbon reduced as the price of carbon is increased from zero, \( C^e(n^o) \), to \( p_e \). Figure 5 plots the emissions gap reduction curve for two scenarios: in the first the baseline parameters are used for both transportation and space, whereas in the second the baseline parameters for transportation are paired with the "low usage and high rent" scenario for retail space. In the first case the maximum reduction in carbon emissions is 11.2% whereas in the second it is 75.6%. The figure demonstrates that the gap is reduced at a decreasing rate in the price of emissions - that is the good news, as the initial price increase for carbon has the largest effect. Unfortunately, a substantially high carbon price is needed to close the emissions gap by a significant amount. For example, if the price of carbon is $1000 per metric tonne, which is in the upper limit of most estimates, then the emissions gap reductions are 16% and 11% in the two scenarios. (These reductions are 1.8% and 8.3% of total emissions.) To achieve a 90% emissions gap reduction requires a carbon price of approximately $3,000 and $5,000 per metric tonne. To put these numbers in perspective, $1000 per metric tonne, corresponds to a fee of $8.8 per gallon of gasoline and $0.55 per kWh for electricity (at the U.S. average emissions rate), which would increase the cost of gasoline by 337% (to $12.50 per gallon) and increase the average cost of electricity by 645% (to $0.651 per kWh). The conclusion from this observation is that while charging for carbon can induce a supply chain design that reduces emissions, very high (and quite possibly unreasonably high) prices are needed to achieve substantial reductions in
Figure 5: Reduction in the emissions gap (the difference in emissions when minimizing operating costs relative to minimizing emissions) as the price of carbon is increased.

Even though the model suggests that pricing carbon will not be an effective strategy for reducing emissions (because very high prices are needed to have a significant impact), there are alternative strategies to reduce emissions. Land use regulations that restrict the size of retail stores will force retailers to build networks with more stores, closer to consumers. As indicated earlier, the supply chain that minimizes emissions generally has considerably more, smaller stores, than the supply chain that minimizes operating costs. Alternatively, technological improvements in fuel usage or emissions can lead to substantial reductions in carbon emissions. Table 7 provides total emission reductions that occur due to various improvements to the system assuming the baseline parameters as the starting point. The “short term” column assumes the supply chain design does not change before and after the improvement, on the assumption that supply chain design changes take time to implement. The “long term” column evaluates the emissions reduction after the improvement changes the supply chain design as well (i.e., the number of stores adjusts). In all cases the objective is to minimize supply chain operating costs and there is no price for carbon, $p_c = 0$. 
The table reveals that changing consumer behavior (large quantities per shopping trip) or improving consumer fuel efficiency are the most effective strategies for reducing emissions. The short term impact of these changes are the most substantial, but even the long term changes are significant. The long term benefit of these consumer improvements is dampened relative to the short term because they induce a supply chain with fewer, larger stores - taken to the extreme, if consumers drove near zero-emissions vehicles, then the supply chain that minimizes emissions has one very large store located very far from customers, which mitigates some of the benefit of the low emissions vehicle.

Reducing retailer electricity or emissions provides the next largest set of reductions in emissions. Now the long term impact is even greater (because the improvement encourages additional stores in the case of electricity usage), but only marginally so. While improving truck fuel efficiency does reduce emissions, it does so only marginally. This is because trucks are already considerably more efficient than cars. Improvements in truck fuel efficiency may be more beneficial in long-haul deliveries to the retailer, which are outside the scope of this model.

The robustness result for the supply chain design problem is reminiscent of results developed for the well-known EOQ problem that studies the trade-off between fixed ordering costs and inventory holding costs. In the EOQ problem, costs are the sum of two terms, one linearly increasing in the decision variable, call it $x$, and the other decreasing in the inverse of $x$ (i.e., it is $1/x$). In the supply chain design problem with fixed quantities, costs are again the sum of two terms, but now one is linearly increasing in $\sqrt{x}$ and the other decreasing in the inverse of $\sqrt{x}$. It has been established that the EOQ problem has a flat objective function (see Dobson 1988, Porteus 2002), but it follows that the objective function in the supply chain design problem is even flatter. Nevertheless, even with a relatively flat function, a poorly chosen decision can lead to a substantial increase in costs. In particular, in the EOQ problem, if one parameter is grossly misestimated, then the cost penalty can essentially be unlimited. In the supply chain design problem this does not occur - the penalty bound is finite, and can be small, even if the true cost of carbon is enormous yet ignored in the decision. The reason is that erring in the cost of carbon causes errors in multiple parameters. For

<table>
<thead>
<tr>
<th>Improvement</th>
<th>Short term</th>
<th>Long term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double consumer load: $q_c \rightarrow 2q_c$</td>
<td>35.9%</td>
<td>29.3%</td>
</tr>
<tr>
<td>Double consumer fuel efficiency: $f_c \rightarrow \frac{1}{2}f_c$</td>
<td>40.3%</td>
<td>33.6%</td>
</tr>
<tr>
<td>Double truck fuel efficiency: $f_t \rightarrow \frac{1}{2}f_t$</td>
<td>0.23%</td>
<td>0.20%</td>
</tr>
<tr>
<td>Half retailer electricity usage: $f_s \rightarrow \frac{1}{2}f_s$</td>
<td>12.6%</td>
<td>13.8%</td>
</tr>
<tr>
<td>Half retailer electricity emissions: $e_s \rightarrow \frac{1}{2}e_s$</td>
<td>12.6%</td>
<td>12.6%</td>
</tr>
</tbody>
</table>

Table 7: Total supply chain emissions reductions under various improvements

<table>
<thead>
<tr>
<th>Total emissions reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short term</td>
</tr>
<tr>
<td>-------------</td>
</tr>
</tbody>
</table>


example, if the cost of emissions from gasoline are ignored, so is the cost of emissions from diesel. Given that the emissions to operating cost ratios for cars and trucks are similar, these errors can net out, thereby leading to nearly optimal decisions for the system. However, even though the objective function is relatively flat, when the emissions to operating cost ratios differ substantially (such as when retailers use renewable electricity and operates in high rent areas) then a focus exclusively on emissions or operating costs (or if carbon is mispriced) can lead to a substantial cost penalty for the system (about 75% in one scenario from Table 6).

To emphasize a point again, the supply chain design problem incurs a high penalty only when there is an asymmetry in the emissions to operating cost ratios, $\beta_j$. This suggests that other forms of asymmetry could lead to substantial penalties. For example, suppose the cost of carbon is fully charged to consumers and the retailer (probably through higher energy prices). If everyone fully acknowledges these costs in their decisions, they will behave as if $c_c$ and $c_t$ are the costs to haul one kg of product one km for the consumer’s car and the retailer’s truck respectively. Furthermore, the retailer will recognize that $c_s$ is the cost to store one kg of product for one unit of time. The retailer has a strong incentive to fully acknowledge these costs and to make appropriate decisions. However, possibly due to cognitive limitations, consumers may not fully consider their transportation costs in their shopping decisions. Specifically, say consumers behave as if their costs are $\alpha_c c_c$ to transport one kg one km, where $\alpha_c \in (0,1]$, yet the retailer fully accounts for its costs, and knows that consumers do not fully account for their costs. (See Attari, Dekay, Davidson and Bruine de Bruin 2010 for further discussion of consumer perception of energy expenditure and potential savings.) The optimal store configuration with $\alpha_c < 1$ has

$$\hat{n}(\alpha_c) = \frac{\alpha_c c_c}{\phi_t c_t + \phi_s c_s} = \alpha_c n^*.$$  
Thus, if consumers partially ignore the costs of driving to a store, then the retailer builds a sparser network than optimal (i.e., stores that are too big), which requires consumers to drive farther than optimal. The cost penalty is then

$$\frac{C(\hat{n}(\alpha_c)) - C(n^*)}{C(n^*)} = \frac{\alpha_c^{-1/2} + \alpha_c^{1/2}}{2} - 1.$$  
If consumers account for only 50% of their costs ($\alpha_c = 0.5$), the penalty is 6%, but it increases to 34% if they account for only 20% of their actual costs ($\alpha_c = 0.2$) and 134% if they account of only 5% of their costs ($\alpha_c = 0.05$).

5 Conclusion

The supply chain design problem involves choosing the size and location of retail stores in a region
of consumers so as to minimize the sum of the retailer’s inbound replenishment costs, the retailer’s space costs and the consumers’ travel costs. These costs include variable operating costs, such as wear and tear on vehicles, fuel, and rent and electricity for retail space. These costs could also reflect the externalities associated with carbon emissions, presumably in the form of higher fuel and electricity prices. However, it is possible (and maybe even likely) that the full cost of carbon emissions is not included in current energy prices.

If only transportation costs are considered (i.e., space costs are low or ignored), it is found that the supply chain design is remarkably robust given current parameters. Consequently emissions increase by very little over their minimal level even if the supply chain is designed to optimize costs. Similarly, operating costs are nearly optimal even if the supply chain minimizes emissions. The key to this result is the discrepancy between the emissions to operating cost ratios for cars (the consumers’ vehicle) and for trucks (the retailer’s vehicle). Given current technology, even though a truck can haul several orders of magnitude more product than a car, its ratio of carbon emissions per km to variable operating costs per km is about the same as for a car, surely of the same order of magnitude (1.168 kg CO$_2$ $\$^{-1}$ v.s. 1.364 kg CO$_2$ $\$^{-1}$). This conclusion remains even with several alternative cost scenarios (e.g., consumer fuel efficiency doubles, fuel prices increase, etc.)

If retail space costs are included, the results are different. In this case, the baseline penalty is about 11% - the supply chain that minimizes operating costs increases emissions by about 11%. This occurs because the retailer’s emissions per m$^2$ to variable operating costs per m$^2$ is substantially lower than the comparable ratio for a consumer’s car (0.534 kg CO$_2$ $\$^{-1}$ v.s. 1.364 kg CO$_2$ $\$^{-1}$). The gap in this critical metric increases further if the retailer uses relatively renewable electricity and operates in a high rent region (0.133 kg CO$_2$ $\$^{-1}$), leading to a penalty of about 75%. In that case, minimizing operating costs yields emissions that are 75% higher than the minimum. Furthermore, there is over seven times more stores in the supply chain that minimizes emissions relative to the supply chain that minimizes operating costs.

The large emissions penalty for minimizing exclusively operating costs, can be alleviated by charging a sufficiently high price for carbon emissions. Unfortunately, the estimated prices needed to substantially reduce this emissions gap are very high, well above $1,000 per metric tonne. This suggests that a direct pricing mechanism may not be effective - a politically acceptable price for carbon, or even current carbon prices, are not sufficient to have a major impact on the supply chain design. Better alternatives include increasing consumer fuel efficiency, and reducing retailer electricity usage and emissions.
References


