Procurement in Supply Chains when the End-Product Exhibits the “Weakest Link” Property[1]

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Abstract: We consider a supply chain with one manufacturer who assembles an end-product using multiple outsourced parts. The end-product exhibits the “weakest-link” property, such that if any of its component parts fails, the end-product fails. The supplier of each component part can improve the (uncertain) quality of her parts by exerting costly effort that is unobservable to the manufacturer and is non-contractible. We analyze two possible contractual agreements between the manufacturer and suppliers: Acceptable Quality Level (AQL) and Group Warranty. Under AQL, the manufacturer inspects all incoming parts, but establishes different quality thresholds and pays the suppliers different amounts for achieving the different thresholds. Under Group Warranty there is no testing of the individual parts; instead all suppliers are responsible for any failed end-product. We compare the efficiency of these two contractual arrangements.

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1. Introduction

End-product manufacturers often outsource the production of many of the parts or even whole sub-assemblies that go into the manufacture of their end-products.[3] As a result, the quality of their end-products is dependent on the quality of the parts supplied by others. The need to ensure the quality of these sub-contracted parts is especially important when one defective part provided by a supplier has the potential to cause the failure of the entire end-product.[4] In a world of no supplier incentive problems (one in which suppliers will costlessly implement any decision rule specified by the manufacturer) as the number of parts increases, holding all else fixed, the probability that the end-product fails increases. But, what additional effects does the number of parts have in the presence of supplier incentive problems? Moreover, what effect does the number of parts that go into the end-product have on the performance of different types of contractual relationships between the manufacturer and suppliers?[5] These questions are the focus of our paper.

We consider a stylized game-theoretic model in which a single manufacturer (hereinafter referred to as *he*) produces a good which is made up of n different parts or components. Part i (i=1,2,…,n) is produced and supplied only by supplier i (each hereinafter referred to as *she*). Each supplier agrees to supply a batch of units of her part to the manufacturer. Each end-product is composed of one part from each supplier. Some of the parts supplied may be defective. Each supplier determines her own yield rate of non-defective parts based on her own cost-of-yield realization.

At the time of contracting, no one knows the cost-of-yield realizations, but everyone agrees on the feasible set of realizations and their probability distributions. Thus, contracting takes place under symmetric but incomplete information. After the contracts have been agreed upon, each supplier privately observes her own cost-of-yield realization. The yield rate chosen by each supplier is not observed by any other party. We assume that each supplier chooses her yield rate based on her private

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[3] Pratt & Whitney, GE and Rolls Royce manufacture most airplane engines for Boeing and Airbus. Maxtor, Western Digital and Seagate manufacture most computer hard drives for Dell, Compaq and IBM. For numerous other examples in which end-product manufacturers use suppliers to provide not just parts but large sub-assemblies, see Templin and Cole (1994).

[4] NASA estimated that in the Challenger, there were approximately 4,686 critical parts, such that if any one of them failed, the entire mission would fail (Pate-Cornell and Dillon, 2001). A comparable, but less extreme, example is a product like a VCR where if one part fails it is cheaper to replace the entire unit than to replace the defective part.

[5] We do not, however, address the more basic question of when firms would choose to insource vs. outsource the production of parts. See Monteverde and Teece (1982) for an analysis of this issue in the US automobile industry.

information so as to maximize her own expected profit rather than the manufacturer's profit. This is the suppliers' incentive problem faced by the manufacture. A key feature of the model is the "weakest-link" property of the end-product: failure of any single component part causes the failure of the end-product that is assembled by the manufacturer. The manufacturer wishes to design a contractual agreement with the suppliers that will maximize his profit taking into consideration the suppliers' incentive problem. We analyze and compare two such contracts described below.

Many companies inspect incoming parts to ensure their conformity (see, e.g., Bossert 1994, page 65). We first consider a situation in which the manufacturer, costlessly and perfectly inspects *all* incoming parts, specifies threshold(s) of acceptable yield rates of parts and pays different amounts to suppliers who achieve different acceptable yield rates. This is often referred to as "Acceptable Quality Level" contracting (hereinafter referred to as AQL, see Windham (1995)). We characterize the optimal AQL contract and analyze the distortion in the suppliers' yields arising from the underlying incentive problem. We also show that, depending on problem parameters, two different AQL contract forms can emerge. In the first, referred to as a *Separating* solution, each supplier's contract specifies multiple acceptable yield rates, one for each possible supplier cost realization. In the second, referred to as a *Pooling* solution, each supplier's contract specifies only one acceptable yield rate, regardless of the supplier's cost realization.

The AQL contract depends on the manufacturer's ability to test the incoming parts. Another possibility is not to test incoming parts, but to contract on the failure of the end-product. These are typically referred to as warranty contracts. If the failure of the end-product can be traced to the failure of a specific part, the same AQL contract can still be written.[6] However, this contract is not feasible if it is too costly (or impossible) to assign blame to a particular part based on an end-product failure.[7] For this reason we examine a "Group Warranty" contract in which *all* suppliers bear the cost of an end-product failure, regardless of whose part caused the failure.[8] Again, the resulting distortions in the suppliers' yield rates arising from the underlying incentive problems are analyzed.

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[6] GM writes these types of parts-supply contracts. To facilitate this process, GM gives its 30,000 suppliers access to a dealer warranty and repair database (Smith 1997). Lear, a major car seat manufacturer, writes these types of parts-supply contracts with its tool suppliers (Pryweller 1999).

[7] A recent example of such a case is the Ford/Firestone battle over the cause of the rollover accidents involving the Ford Explorer (see Pinedo et al. 2003).

[8] Contracts that allow the manufacturer to reward the suppliers based on *both* the outcome from testing parts and external failure are outside the scope of this paper and hence are not analyzed.

The remainder of the paper is organized as follows. In Section 2 we discuss the related literature. In Section 3 we lay out the model, discuss the assumptions and derive the benchmark Team solution when there are no supplier incentive problems. The benchmark model serves as a reference point for evaluating the AQL contract in section 4 and the Group Warranty contract in Section 5. We then compare the AQL and the Group Warranty contracts in terms of expected profits to the manufacturer, suppliers and the supply chain in Section 6. Section 7 concludes the paper by summarizing the findings and suggesting directions for further research.

2. Literature Survey

The product design and development literature discusses product architecture issues including the number, design and interaction of product components from an engineering/design perspective (Ulrich 1995, Ulrich and Eppinger 2000). However, such design decisions may also affect the incentives of the of the component parts suppliers. The incentive implications of the number of product components are the focus of this paper.

Likewise, much of the traditional quality literature studies the design of optimal testing policies without considering incentive implications. Our paper is closely related to the recent contracting on quality literature in supply chain management, e.g., Reyniers and Tapiero (1995), Baiman et al. (2000, 2001), Lim (2001), Iyer et al. (2002), and Balachandran and Radhakrishnan (2003). These papers analyze a situation in which a manufacturer contracts with a single supplier for a single part, while we consider the case of contracting for multiple parts provided by multiple suppliers. We demonstrate how the number of parts affects the extent of the supplier incentive problems when the product exhibits the weakest link property. Other papers study contracting with multiple suppliers in supply chains but, unlike the present paper, do not focus on quality issues (e.g., see Cachon 2002).

Our work is also related to the economics and accounting literature dealing with the agency problem (e.g., Alchian and Demsetz 1972, Holmstrom 1979 and Baiman and Rajan 2002). Our work differs from this literature in that we focus on situations with multiple agents and on a more specific context (the weakest link relationship). A subset of the contracting literature does allow for multiple agents, but addresses issues that are different from ours. Itoh (1991, 1994) allows for multiple agents but addresses the issue of when there are benefits to allowing the agents to help each other. The mechanism design literature (Demski and Sappington 1984 and Ma 1988) examines the effect of contracts on the implicit coordination among agents. We do not address either of these issues. The paper in this literature closest to ours is Harris et al. (1982) which analyzes resource allocation within a multi-divisional firm. Like our paper, Harris et al. (1982) assumes that the divisions (suppliers in our paper) possess private information about their productivity, their actions are unobservable, and the firm's output is the minimum of the outputs of its divisions (our weakest link assumption). However, Harris et al. (1982) assumes a linear production technology, that divisions communicate their cost observations to the manufacturer who then sets production schedules (in our model such communications are excluded), focuses on one specific contract, and does not address the effect of the number of divisions (parts in our context) on the efficiency of the firm.

In the experimental game theory literature a similar concern arises in coordination games where actions of the "weakest-link" player determine payoffs to all other players (see Cachon and Camerer 1996). Finally, our work is somewhat related to recent papers on interdependent security in which the incentive to invest in protective measures is reduced if other parties have not taken similar steps (see Kunreuther and Heal 2003, Heal and Kunreuther 2003).

3. **Assumptions, notation and solution without contracting problems**

a. **Assumptions and notation**

Our model consists of one risk-neutral manufacturer who assembles an end-product consisting of n parts, $i = 1, \ldots, n$. Each part i is supplied only by the risk neutral supplier i.[9] Each of the n suppliers provides the same fixed number of parts, Z, to the manufacturer. This fixed lot size delivered by the suppliers may result from a fixed-batch-size production technology and the ability to produce only one batch per period.[10] An example of a fixed-batch-size production technology is that employed

[9] We restrict our attention to the case in which there is a separate supplier for each part. If different parts were supplied by the same supplier then she would internalize part of the externality, partially mitigating, but not eliminating, the incentive problem which we study.
[10] This fixed-batch size technology is referred to as lot-for-lot production, see, e.g., Hopp and Spearman, 2000. Without this assumption each supplier would then have to decide on both her yield rate and the number of parts to deliver to the manufacturer.

in computer chip-making where chips are made on a standard sized wafer. Each end-product assembled by the manufacturer requires one part from each supplier and each supplier provides a different part.[11] The manufacturer's only job is to assemble the end-product from the parts delivered by the suppliers, and the manufacturer does so without affecting the probability of the end-product failing. A finished end-product will fail if any one of its parts is defective. Supplier *i* can influence the probability of her parts failing by choosing the percentage of parts which she delivers which are not defective (i.e., her yield rate) and thus will not fail if assembled into an end-product. Supplier *i*'s chosen yield rate when her cost-of-yield is c_{ij} is represented by $0 \le x_{ij} \le 1$, where the index *j* represents supplier *i*'s cost-of-yield realization. The cost to supplier *i* with a cost realization c_{ij} of delivering *Z* units with a yield rate of x_{ij} is $Zc_{ij}x_{ij}^2$.[12] We further assume that all suppliers are symmetric and that there are only two cost-of-yield realizations, c_1 and c_2, where $c_1 < c_2 < \infty$, occur with probabilities p and $1\text{-}p$, respectively, and $0<p<1$. Each supplier's cost realization is independent of the other suppliers' cost realizations. We will denote the manufacturer's expected profit as Π and each supplier's expected profit as π.

The order of events and additional assumptions (further discussed in Section 7) are:

Step 1. The manufacturer announces contract terms to the suppliers. The contract is not re-negotiable ex-post. At this point, the suppliers have not yet observed their own cost realizations. The manufacturer and the suppliers share common beliefs as to the feasible set of cost realizations $\{c_1, c_2\}$, and the probability distribution over the cost realizations {p, 1-p}. Included within the terms of the contract is the specification of the manufacturer's testing policy (incoming parts or end-product). Further, we assume that each supplier can only observe the outcome of her own test, and hence can only be compensated upon the outcome of that test.

Step 2. Each supplier privately observes her own cost-of-yield realization. A supplier cannot communicate her cost realization to the manufacturer or to other suppliers. This implies that the contract negotiated in Step 1 cannot specify that supplier *i's* compensation or desired yield rate be a function of any communications regarding supplier *i* or *j's* cost realization. At this point in the process, each supplier has the option of costlessly abrogating the contract

[11] Baiman and Netessine 2004b investigates the situation in which the same part may be sourced from multiple suppliers.
[12] Notice that the chosen yield rate is non-stochastic but whether any particular part produced is defective or non-defective is uncertain. Supplier *i*'s chosen yield rate and cost-of-yield incorporate any parts testing which she may do on her own prior to shipping the parts to the manufacturer.

negotiated in Step 1 and not delivering the promised parts. A supplier will only implement the contract negotiated in Step 1 if her expected profit from doing so (conditional on her observed cost realization) exceeds her outside opportunity cost (e.g., supplying parts to another manufacturer). That outside opportunity cost is assumed to be zero. This assumption is referred to as the supplier's Limited Liability Constraint. We assume that the manufacturer designs the contract so that each supplier, regardless of her cost realization, has the incentive to implement the contract negotiated in Step 1.

Step 3. Each supplier *i*, after having observed her cost realization c_{ij}, simultaneously chooses her x_{ij}. These choices are not observable by the other suppliers or by the manufacturer.

Step 4. The manufacturer receives the parts from all suppliers. If the manufacturer tests the parts before assembling the end-product he does so here; otherwise he assembles the end-products, tests them, and sells the non-defective ones, collecting revenue *R* for each. Suppliers are compensated based on the testing outcomes according to the terms of the contract.

In Step 1, the manufacturer commits to one of two different testing strategies (test *all* incoming parts or just test the end-product) and to one of two different contractual arrangements (AQL or Group Warranty). In Section 4 we assume that the manufacturer commits to costlessly and perfectly testing each part received from each supplier and assembles finished products from the non-defective parts. To illustrate further, if there were two suppliers, each delivering *Z* parts, and one chose a yield rate of 40% while the other chose a yield rate of 60%, the manufacturer would only have enough non-defective parts to assemble Z x minimum (.4,.6) end-products. More generally, the manufacturer's revenue function with parts testing is $RZ\min\left(x_{1\bullet}, x_{2\bullet}, ..., x_{n\bullet}\right)$, where $x_{i\bullet}$ is supplier *i*'s realized yield rate. Notice that we are assuming that, because of time, spoilage, fashion concerns, or other constraints, the manufacturer can neither ask a supplier to replace his parts which were found to be defective, nor inventory any unused non-defective parts for use at some later date.[13]

In Section 5 we assume that the manufacturer foregoes testing the incoming parts and assembles end-products from *all* of the parts delivered. Therefore, if as before there were two suppliers, each delivering *Z* parts, and one chose a yield rate of 40% while the other chose a yield rate of 60%, the manufacturer would assemble *Z* end-products, but the expected number of non-defective

[13] The qualitative results would continue to hold as long as there is a cost to the manufacturer of receiving defective parts, for example, there is a positive cost to replace a defective part or there is a cost to inventory excess parts.

end-products would be Z(.4)(.6). More generally, the manufacturer's revenue function is $RZ\left(x_{1\bullet}\right)\left(x_{2\bullet}\right)(.)(.)\left(x_{n\bullet}\right)$. We are thus assuming that the end-product is tested after assembly and before it is sold, and only the non-defective end-products are sold for the price *R* per unit.[14] Notice that because of the weakest link assumption, the different testing strategies lead to very different production technologies for the manufacturer. Given the assumed structure of the problem, we can, without loss of generality, assume that *Z*=1 and refer to x_{ij} as both the yield rate and the number of non-defective parts.

b) Team solution (no supplier incentive problems)

We begin our analysis by studying the solution to the situation in which there are no incentive problems but there is still information asymmetry in the sense that suppliers' cost realizations are private. We call this the Team solution (similar to Marschak and Radner 1972) and use it as a benchmark to interpret the results from the two contractual arrangements of interest. Therefore, in succeeding sections the optimal contracts given the presence of incentive problems can be compared to the Team solution to determine the effect that the underlying incentive problems alone have on the optimal contract.

In the Team solution the manufacturer provides each supplier with directions of the form: "if c_1 is observed, choose yield x_1^T , if c_2 is observed, choose yield x_2^T". Each supplier will do as directed as long as she is no worse off by implementing the contract than by supplying some other manufacturer and earning her outside opportunity cost of zero. Thus, the payment to supplier with cost realization c_j must be $T_j = c_j\left(x_j^T\right)^2$. Notice that the Team solution will be worse than First-Best because the former is subject to asymmetric information. Under First-Best, the manufacturer would base each supplier's yield on *all n* cost realizations. However, in our problem each supplier knows only its own cost realization and cannot communicate that information to the manufacturer. This asymmetry of information causes a loss of efficiency in the Team solution, relative to First-Best, because of the inability of the manufacturer to coordinate suppliers based on full information about their cost

[14] Alternatively, we could assume that the manufacturer charges *R* per unit sold and returns the *R* to the customer if the end-product proves to be defective. The qualitative results would continue to hold if there was a reputational cost to the manufacturer of selling defective end-products.

realizations. However, because each supplier will implement whatever yield strategy the manufacturer chooses, there is no loss of efficiency because of incentive problems.

Recall that the manufacturer is able to assemble x_1 end-products when all suppliers have the low-cost realization (the probability of this happening is p^n) and x_2 end-products otherwise. The manufacturer's objective function (as of Step 1) is therefore:

$$\Pi = Rp^n x_1 + R\left(1-p^n\right)x_2 - n\left(pc_1x_1^2 + \left(1-p\right)c_2x_2^2\right).$$

Proposition 1: *The optimal Team solution is either a* (All proofs are in Appendix A)*:*

i) Separating solution when $\left(p^{n-1}-p^n\right)/\left(1-p^n\right) \geq c_1/c_2$*:*

$$x_1^T = \frac{Rp^{n-1}}{2nc_1},\ x_2^T = \frac{R\left(1-p^n\right)}{2nc_2\left(1-p\right)}.$$

ii) Pooling solution when $\left(p^{n-1}-p^n\right)/\left(1-p^n\right) < c_1/c_2$*:*

$$x_1^T = x_2^T = x^T = \frac{R}{2n\left(pc_1 + \left(1-p\right)c_2\right)}.$$

In the Separating solution, the manufacturer specifies two yield rates, one for each cost realization. In the Pooling solution the manufacturer specifies only one yield rate, regardless of the suppliers' cost realizations. The advantage of the Pooling solution is that all suppliers provide the same yield rate and the same number of non-defective parts. As a result, the manufacturer does not end up paying for non-defective parts that cannot be used in an end-product. The disadvantage of the Pooling solution is that the manufacturer cannot benefit from the possibility that all of the suppliers are low-cost by having each produce a higher yield, resulting in more end-products that can be sold at a profit. Notice that the manufacturer chooses either the Pooling or the Separating solution based on the problem parameters.

The Team solution is a trade-off between the two effects discussed above. As the probability of the low-cost realization increases (i.e., p increases), it becomes more likely that the manufacturer is facing all low-cost suppliers. It is, then, beneficial to fully exploit each of them and therefore, the Separating solution dominates. To see this effect, notice that the derivative of the Separating/Pooling threshold with respect to p is non-negative:

$$\frac{\partial}{\partial p}\frac{p^{n-1}-p^{n}}{1-p^{n}}=\frac{\left((n-1)p^{n-2}-np^{n-1}\right)\left(1-p^{n}\right)+np^{n-1}\left(p^{n-1}-p^{n}\right)}{\left(1-p^{n}\right)^{2}}=p^{n-2}\,\frac{n\left(1-p\right)-1+p^{n}}{\left(1-p^{n}\right)^{2}}\geq 0\,.$$

Thus, holding all else fixed, the Pooling Solution will be chosen for smaller p's and the Separating Solution will be chosen for large p's.

Further, for any given p, as the number of parts increases, the probability of there being at least one high-cost supplier who will provide parts with a lower yield rate increases. This makes it more likely that the "extra parts" supplied by the low-cost suppliers will not be utilized in an end-product, making the Pooling solution more attractive. Again, to see this effect, notice that the derivative of the Separating/Pooling threshold with respect to n is negative:

$$\frac{\partial}{\partial n}\frac{p^{n-1}-p^{n}}{1-p^{n}}=\frac{\left(p^{n-1}-p^{n}\right)\left(1-p^{n}\right)\ln p+\left(p^{n-1}-p^{n}\right)p^{n}\ln p}{\left(1-p^{n}\right)^{2}}=\left(p^{n-1}-p^{n}\right)/\left(1-p^{n}\right)^{2}\ln p<0.$$

Thus, holding all other parameters fixed, the Team solution will be Separating for small values of n and Pooling for larger values of n. Likewise, as the ratio of costs, c_1/c_2, increases (i.e., the difference in costs becomes smaller), there is less reason to treat suppliers of different types differently, and the Pooling solution becomes more advantageous.

Finally, note also that optimal yields are decreasing in n due to the weakest link problem. This is because as n increases the manufacturer has to pay for more parts, the cost of the product increases, and therefore, it is less valuable to assemble a non-defective end-product.[15]

In the next two sections, we study the yields induced by two different types of contracts. Unless otherwise noted, we will assume that the exogenous parameter values $\left(c_1,c_2,p,n\right)$ are such that the optimal yields given those contracts are strictly less than 1. For instance, in terms of the Team solution this would imply that $Rp^{n-1}/\left(2nc_1\right)\leq 1$ and $R\leq\left(2n\left(pc_1+\left(1-p\right)c_2\right)\right)$. This is consistent with the philosophy of Juran who, unlike Deming, believes that less than perfect quality is optimal (Juran 1992).

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[15] We could have normalized for this effect of adding suppliers by assuming that the revenue function was nR. However, the incremental effect arising from the underlying incentive problem would remain the same.

4. AQL contract

In the remainder of this paper, we retain the assumption of asymmetric information but drop the assumption that each supplier will choose her yield as instructed but the manufacturer. Instead, each supplier will choose whichever yield rate maximizes her expected utility given her cost realization and the contract offered to her by the manufacturer. Thus, in choosing between the suppliers' contracts, the manufacturer must now consider their incentive effects on the suppliers' behavior.

In this section, we examine the AQL contract in which the manufacturer specifies alternative acceptable yield rates, each with a different payment. We assume that each supplier can only observe the outcome of the testing of her own parts. As a result, each supplier can only be compensated based on the results of her own tested parts, not on the results of other suppliers' tested parts. Further, after testing, only good parts will be used to assemble the end-products, resulting in $\min\left(x_{1\bullet}, x_{2\bullet}, \ldots., x_{n\bullet}\right)$ end-products assembled, where $x_{i\bullet}$ represents the supplier *i's* yield given her cost realization.

As a result of costless and perfect parts testing, the manufacturer can observe each supplier's yield rate. With an AQL contract the manufacturer specifies the yields for which he will compensate the suppliers; all other yields are not acceptable and not rewarded. Thus, without loss of generality (see Kreps 1990), the manufacturer can restrict himself to contracts which: 1) specify two acceptable yield rates, one for suppliers with a low-cost realization and one for suppliers with a high-cost realization; 2) induce each supplier to choose the yield rate consistent with her cost realization and anticipated by the manufacturer; and 3) assure each supplier at least her outside opportunity cost regardless of her cost realization. Denote by *P* the set of all permutations of *n* cost indices with each taking values 1 or 2. Further, denote by *P(k,n)* the subset of these permutations such that there are *k* low-cost realizations of *n* suppliers. Given this definition, the manufacturer's problem is

$$\max_{x_{ij},T_{ij}} \sum_{k=0}^{n} \left\{ Rp^k \left(1-p\right)^{n-k} \sum_{P(k,n)\in P} \min\left(x_{1\bullet}, x_{2\bullet}, \ldots, x_{n\bullet}\right) \right\} - p\sum_{i=1}^{n} T_{i1} - \left(1-p\right)\sum_{i=1}^{n} T_{i2}$$

subject to

$$T_{i1} - c_1 x_{i1}^2 \geq 0 \qquad \forall i \qquad \text{(LL1)}$$

$$T_{i2} - c_2 x_{i2}^2 \geq 0 \qquad \forall i \qquad \text{(LL2)}$$

$$T_{i1} - c_1 x_{i1}^2 \geq T_{i2} - c_1 x_{i2}^2 \qquad \forall i \qquad \text{(IC1)}$$

$T_{i2} - c_2 x_{i2}^2 \geq T_{i1} - c_2 x_{i1}^2 \quad \forall i$ (IC2)

The objective function takes the expectation over all possible permutations of low-cost and high-cost suppliers. Constraints (LL1) and (LL2) are supplier *i*'s Limited Liability constraints and assure that, regardless of supplier *i*'s cost realization, she will earn at least her outside opportunity cost and hence will implement the contract negotiated in Step 1. Constraints (IC1) and (IC2) are the Incentive Compatibility constraints which assure that the supplier behavior anticipated by the manufacturer is in each supplier's best interest. That is, if supplier *i* is a low-cost supplier she will prefer to choose x_{i1} rather than x_{i2} (IC1) and vice versa if she is a high-cost supplier (IC2). The objective function above is different from that for the Team solution because we allow for the possibility that the manufacturer will offer different contracts to otherwise identical suppliers.

Proposition 2: *The optimal AQL contract is either a:*

i) Separating solution when $p^{n-1} \geq c_1 / c_2$*:*

$x_{i1} = x_{j1} = x_1 > x_{i2} = x_{j2} = x_2$ and $T_1 = T_{i1} = T_{j1} > T_{i2} = T_{j2} = T_2 \ \forall i, j$

$x_1 = p^{n-1} R / 2nc_1, \ x_2 = \left(1 - p^n\right) R / 2n\left(c_2 - pc_1\right)$

$$T_1 = c_1 \left(\frac{p^{n-1} R}{2nc_1} \right)^2 + \left(c_2 - c_1\right) \left(\frac{\left(1 - p^n\right) R}{2n\left(c_2 - pc_1\right)} \right)^2, \ T_2 = c_2 \left(\frac{\left(1 - p^n\right) R}{2n\left(c_2 - pc_1\right)} \right)^2$$

The manufacturer's expected profit is $\dfrac{p^{2n-1} R^2}{4nc_1} + \dfrac{\left(1 - p^n\right)^2 R^2}{4n\left(c_2 - pc_1\right)}$

The supplier's expected profit is $p\left(c_2 - c_1\right) \left(\dfrac{\left(1 - p^n\right) R}{2n\left(c_2 - pc_1\right)} \right)^2$

ii) Pooling solution when $p^{n-1} < c_1 / c_2$*:*

$x_{ij} = x$ and $T_{ij} = T \ \forall i, j$

$x = R / 2nc_2$

$T = R^2 / 4nc_2$

The manufacturer's expected profit is $R^2 / 4nc_2$

The supplier's expected profit is $R^2 p\left(c_2 - c_1\right) / 4n^2 c_2^2$.

The same logic for the Separating vs. Pooling Team solutions applies in the case of AQL contract. As with the Team solution, holding all other parameters fixed, the AQL solution will be Separating for small values of *n* and Pooling for larger values of *n*. Under both types of AQL solutions, the low-cost supplier earns more than her minimum expected profit while the high-cost supplier earns just her outside opportunity. The amount that the low-cost supplier earns in excess of her outside opportunity wage is referred to as her informational rent. It is the amount that is required to get the low-cost supplier to honestly act on or reveal her cost realization. This is a standard result for such screening problems and follows from the fact that it is more expensive for a high-cost supplier to act as if she were a low-cost supplier than vice versa (see Kreps 1990).

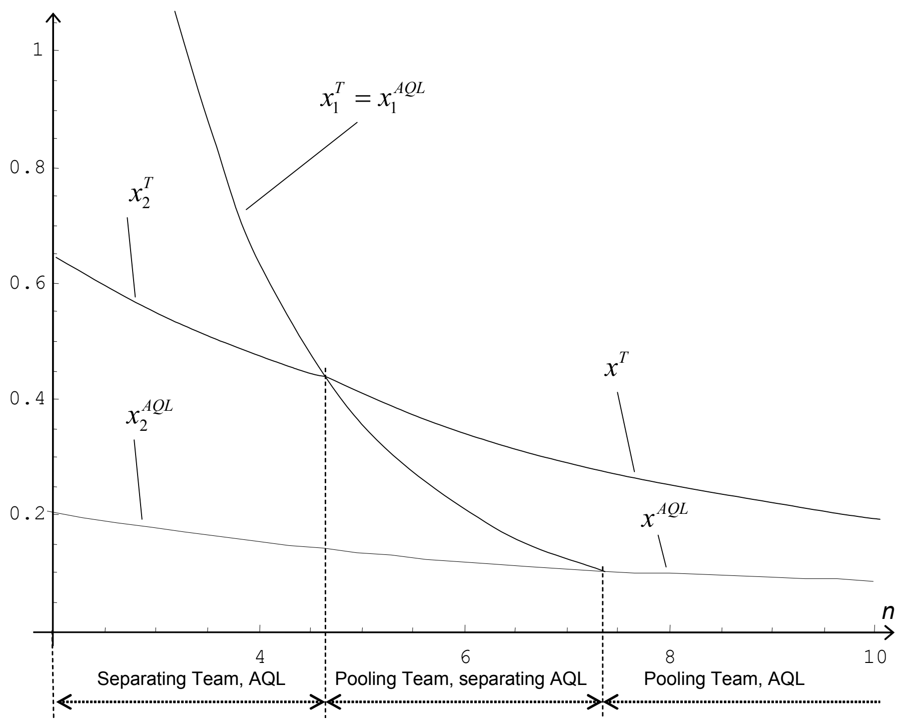

Figure 1. Optimal yield rates with the AQL contract.

We are interested in studying how the optimal AQL yield rates vary in the exogenous parameters and how that variation is affected by the existence of incentive problems. To isolate the variation in the optimal AQL yields that arises strictly from incentive problems, we restate them in terms of the Team solution yields. Recall that the Team problem has asymmetric information but no incentive problems (each supplier will implement the yield decision rule desired by the manufacturer as long as she cannot improve her expected profit by supplying the parts to another manufacturer). Therefore, any difference in the optimal yields between the Team problem and the AQL solution is due solely to incentive problems. The difficulty in making this comparison, however, is that both the AQL and Team solutions can result in Separating and Pooling solutions which may occur over different sets of parameters. We therefore begin by comparing the Separating/Pooling thresholds as functions of *n*, i.e., p^{n-1} *vs* $\left(p^{n-1}-p^{n}\right)/\left(1-p^{n}\right)$. It is straightforward to see that $p^{n-1}>\left(p^{n-1}-p^{n}\right)/\left(1-p^{n}\right)$, and thus, holding all other parameters fixed, the threshold *n* for the Pooling AQL solution is always higher than for the Pooling Team solution. As a result, separation of suppliers is maintained over a larger range of *n* for the AQL contract. Figure 1 illustrates the comparison between Team solution and AQL contract as a function of *n*. As is evident from the picture, three cases have to be considered:

<u>Case 1.</u> $p^{n-1}\geq\left(p^{n-1}-p^{n}\right)/\left(1-p^{n}\right)\geq c_1/c_2$. Both the AQL and Team solutions are Separating. We can rewrite the AQL yields as:

$$x_1=x_1^T,\ x_2=x_2^T\frac{c_2\left(1-p\right)}{\left(c_2-pc_1\right)}.$$

See Figure 1. In this case, production by low-cost suppliers is not distorted away from the low-cost Team solution and therefore there is no distortion in the low-cost supplier's production because of incentive problems. The high-cost suppliers' yield, on the other hand, is less than the Team solution because of incentive problems.[16] Interestingly, the relative distortion in x_2 is not affected by the number of parts, *n*. Hence, within the range of parameters in Case 1, changing the number of parts that make up the final end-product does not affect the incentive costs associated with the AQL contract. However, *p* does affect the distortion in x_2, relative to the Team solution. As *p* increases, the multiple for x_2^T decreases thus aggravating the production distortion for high-cost suppliers. The intuition is that an increase in *p* implies a lower probability of a high-cost supplier. Therefore, the value of the high-cost supplier's production is less, so reducing x_2 costs the manufacturer less. However, by reducing x_2, the manufacturer can now reduce the *low-cost supplier's informational rent* $\left(\left(c_2-c_1\right)\left(x_2\right)^2\right)$

<u>Case 2.</u> $p^{n-1}\geq c_1/c_2\geq\left(p^{n-1}-p^{n}\right)/\left(1-p^{n}\right)$. The AQL solution is Separating while the Team solution is Pooling. We can rewrite the AQL yields as:

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[16] In most screening problems the low-cost supplier's action choice is First-Best, while the high-cost supplier's action choice is distorted from First-Best (see Kreps 1990). The First-Best solution is not the relevant comparison for our problem, because even without incentive problems, we still have asymmetric information. Our Case 1 result is therefore similar to the standard result in screening problems where the comparison is now to the Team solution

$$x_1 = x^T \frac{\left(pc_1 + (1-p)c_2\right) p^{n-1}}{c_1}, \; x_2 = x^T \frac{\left(pc_1 + (1-p)c_2\right)\left(1-p^n\right)}{c_2 - pc_1}.$$

See Figure 1. In this case *both* the low-cost and high-cost suppliers' yield rates are distorted *below* their respective Team yield rates (see also Figure 1).[17] Now the number of parts that make up the end-product does have an incentive effect in the sense that the more parts that make up the end-product, the greater (less) the distortion induced in the low-cost (high-cost) supplier's yield, relative to the Team solution. Intuitively, as *n* increases the chance of facing low-cost realizations from all suppliers diminishes. Thus, for the Separating AQL solution it becomes optimal to reduce the difference in the low and high-cost yields and, thereby, reduce the low-cost supplier's informational rent while simultaneously increasing the production of the more-likely-to-occur outcome, x_2.

<u>Case 3.</u> $c_1 / c_2 \geq p^{n-1} \geq \left(p^{n-1} - p^n\right) / \left(1 - p^n\right)$. Both AQL and Team solutions are pooling so that

$$x = x^T \left(pc_1 + (1-p)c_2\right) / c_2.$$

Note that the AQL optimal yield is distorted below the Team yield. Like Case 1 and unlike Case 2, the impact of the incentive problem in Case 3 does not depend on the number of parts, because the effect of *n* in both the AQL and Team yields is the same (see Figure 1 and note that the difference between Team and AQL yields decreases in *n* over this interval but the ratio does not change). As *p* increases, the AQL yield decreases relative to the Team solution thus amplifying the effect of the incentive problem. The reason is that an increase in *p* results in the manufacturer paying informational rent to more suppliers. Hence, to decrease the total informational rent paid, the manufacturer further reduces the AQL yield.

To summarize, with the AQL contract the inefficiency introduced by incentive problems results in "over-separation" of supplier yields (see Figure 1) in two senses. First, the optimal yields for the low and high-cost suppliers are always farther apart under the AQL contract than under the Team solution. Second, the Separating AQL solution is implemented for a larger set of problem parameters than for the Team solution. Interestingly, the number of parts that make up the end-product affects the

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[17] Note that in Case 2 the actions of all suppliers, in particular the low-cost suppliers, are distorted. This is different than the standard result for screening contract in which the low-cost supplier's yield is always undistorted. The reason for this difference is that in the typical screening problem there are no externalities among the suppliers. Therefore, each supplier's First-Best action depends solely on her own private information, whereas in our model, there are externalities.

distortion in yields, and hence the agency costs of the supply chain, only for those problem parameters such that the AQL solution is Separating but the Team solution is Pooling. It is in this region that there is "over-separation" of supplier yields in the second sense discussed above. When the manufacturer finds himself in this region, his design choice of the number of parts which make up the end-product has substantial supplier incentive effects.

A contract widely-used in practice and modeled in the literature is a Q-Pricing contract (quality-based incentive pricing).[18] In this contract, the manufacturer accepts any yield rate but pays each supplier *T* for the proportion of her parts that are good. We do not study this contract in this paper because it can be shown that, for our problem, the AQL contract is the optimal contract and thus the manufacturer strictly prefers the optimal AQL contract to the optimal Q-Pricing contract.[19] For a detailed discussion of Q-Pricing contracts for our problem, see Baiman et al. (2004a).

5. Group Warranty

Suppose now that the manufacturer assembles the end-product without first testing the individual parts.[20] Under the Group Warranty contract, if an assembled product fails, the suppliers are collectively and equally responsible for the failure.[21] That is, each supplier is paid *T* for each assembled end-product that does not fail (the assembled product only functions if all parts are non-defective), and zero if the assembled end-product does not function.[22] We continue to assume that the suppliers cannot either help each other or coordinate their efforts. We also assume that suppliers can costlessly verify when the end-product actually fails.

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[18] See Windham 1995 for a discussion of this contract in practice and Reyniers and Tapiero (1995) for an example of its use in the modeling literature.
[19] This result is based on the Revelation Principle. See Myerson (1979) and Harris and Townsend (1981).
[20] The manufacturer might decide not to test incoming parts if assembly is cheap or testing parts is costly or it is only possible to determine if parts are good by testing the end-product (e.g., testing is destructive as is true with airbags) or it is impossible (or too costly) to discover which supplier's part was faulty if an assembled product fails.
[21] The Group Warranty contract is similar in spirit to the joint and several liability regime in the law literature and to the use of joint liability in financial lending, see Ghatak and Guinnane (1999).
[22] An alternative is to share Group Warranty costs between the manufacturer and suppliers. However, because the manufacturer does not affect product quality in our model, sharing Group Warranty costs does not introduce any additional interesting tradeoffs – it merely transfers money from the manufacturer to suppliers. It can be easily shown that if the manufacturer is in charge of the contract design (as is the case in our model), he would allocate full Group Warranty costs to suppliers. Hence, for the reminder of the paper we assume that Group Warranty costs are fully born by suppliers.

Unlike the situation with the AQL contract, under Group Warranty each supplier's payoff is not independent of decisions by other suppliers. Consider, for example, supplier 1. Supplier 1 knows only the probability distribution of the *other* suppliers' cost realizations, not the actual realizations. She also knows that when her parts are assembled with the other suppliers' parts, the yield rate of the assembled end-product is the product of the yield rates of all suppliers' parts. Hence, supplier 1 is interested in the expected yield of all suppliers other than herself. Denote this expectation by X. Dropping the supplier subscript, supplier 1 solves the following problem for each of her cost realizations, c_j

$$\max_{x_j} x_j XT - c_j x_j^2 .$$

From the first-order conditions, the optimal yield for a supplier with cost realization c_j is

$$x_j = \min\left(XT / 2c_j, 1\right). \tag{1}$$

Assuming a symmetric solution, *X* can be written as $X = \left(px_1 + (1-p)x_2\right)^{n-1}$ because the suppliers' cost realizations are independent. One can observe that *X* is an increasing function of the *other* suppliers' chosen yield rates. Hence, (1) indicates that *the higher (lower) the other n-1 suppliers' expected yield rates the higher (lower) will be supplier 1's chosen yield rates and profits*. The same is true for all suppliers so that the suppliers' chosen yield rates are strategic complements. Thus, with a Group Warranty arrangement, for any given *T* each supplier is better off if the other suppliers increase their yield rates. Notice that this effect does not arise in the cases in which there is testing before assembly, because each supplier's reward is then independent of the other suppliers' yields. The game that arises among suppliers with a Group Warranty contract is similar to "coordination games" (see Cachon and Camerer 1996) in which the payoff to each player is determined by the one player whose strategy is the "weakest link". The difference here, however, is that in our problem there is a manufacturer who can change the outcome of the game by choosing *T*. The manufacturer's profit is

$$\Pi = (R - nT)\left(px_1 + (1-p)x_2\right)^n .$$

Proposition 3: *The solution under Group Warranty is:*

i) *for $n \ge 3$, one of the following three solutions (the one resulting in the highest manufacturer's profit) is an equilibrium:*

a. $x_1 = x_2 = 0$ *with the manufacturer's profit of* $\Pi = 0$.

b. $x_1 = 1, x_2 = c_1 / c_2$ *with the manufacturer's profit of*

$$\Pi = R\left(p + (1-p)c_1 / c_2\right)^n - 2nc_1\left(p + (1-p)c_1 / c_2\right).$$

c. $x_1 = x_2 = 1$ *with the manufacturer's profit of* $\Pi = R - 2nc_2$.

ii) *for n=2, in addition to the above three options, the following equilibrium must be considered*[23]

d. $$x_1 = 1,\ x_2 = \frac{R\left(1-p\right) - 2c_2}{4c_2 - R\left(1-p\right)} \frac{p}{(1-p)} \qquad (2)$$

as long as $c_1 / c_2 < x_2 < 1$ *with the manufacturer's profit*

$$\Pi = \frac{p^2 4c_2^2}{(1-p)\left(4c_2 - R\left(1-p\right)\right)}.$$

Because the suppliers' yield choices are strategic complements, we might expect that in equilibrium all suppliers will gravitate toward the boundary solution in which all of them will either select the maximum or minimum possible quality. If *R* is relatively small, the manufacturer cannot make positive profits under Group Warranty and hence will refuse to contract with suppliers inducing equilibrium yields of zero (case a above). As *R* increases, at some point the manufacturer begins making positive profits and it becomes optimal for him to offer positive compensation $T>0$ to the suppliers. Because the suppliers' decisions are complements, it is relatively easy to induce them to select high yields. As a result, a small increase in *T* leads to a large increase in both high-cost and low-cost suppliers' yields followed by an increase in the manufacturer's profit. Note from (1) that if the solutions for both yields are interior, then $x_1 / x_2 = c_2 / c_1$ so that as the manufacturer increases *T*, yields increase while preserving this proportion until $x_1 = 1, x_2 = c_1 / c_2$ (case b above). At this point, a further increase in *T* will only increase yields of the high-cost suppliers (x_2) so that the return to increasing *T* is reduced. This is why for some problem parameters the manufacturer will choose equilibrium b for $n \geq 3$. However, it is also possible that, for some problem parameters, a further increase in *T* is warranted. If that is the case, it will be optimal to raise *T* until $x_1 = x_2 = 1$ (case c above for $n \geq 3$).

[23] The number of equilibria is actually larger for *n=2* (see the proof) but we use Pareto-dominance to eliminate most of them.

If, however, *n=2,* it is possible to have a solution with $c_1 / c_2 < x_2 < 1$. To see this in more detail, let $x_1 = 1$ and recall that $T = 2c_2 x_2 / X$ so that manufacturer's objective function becomes

$$\Pi = \left(R - \frac{2nc_2 x_2}{X} \right) \left(p + (1-p) x_2 \right)^n = R \left(p + (1-p) x_2 \right)^n - 2nc_2 x_2 \left(p + (1-p) x_2 \right).$$

Notice that the cost term is quadratic in x_2 while the revenue term has power *n.* If *n=2,* there are problem parameters such that the objective function is concave quadratic and a unique interior maximum exists.

Because up to 4 different equilibria can arise, we introduce assumptions which eliminate all but the most interesting equilibrium. The solution with $x_1 = x_2 = 0$ is uninteresting so we will assume that problem parameters are such that the manufacturer makes positive profits under Group Warranty. Furthermore, the solution with $x_1 = x_2 = 1$ is also uninteresting. We will eliminate it for the time being as well (see Proposition 7 where this solution is discussed) by assuming that $R < 2nc_2$ which implies that the manufacturer's profit is negative when $x_1 = x_2 = 1$. This is a rather mild parametric assumption; note that the optimal yield under the Pooling AQL solution is $R/(2nc_2)$ so we are simply assuming that this yield is less than 1. Finally, for *n=2*, there is only a very narrow range of problem parameters such that an interior solution characterized by *(2)* actually arises.[24] Hence, we will consider only the equilibrium in which $x_1 = 1,\ x_2 = c_1 / c_2$.

We next analyze the distortions in the yields under Group Warranty arising from the underlying supplier incentive problems. Because $x_1 = 1$ under Group Warranty, it is clear that the low-cost suppliers' yields are distorted above the Team solution and that the distortion increases in *n.* On the other hand, under Group Warranty the high-cost suppliers may have a yield that is below or above the corresponding Team solution yield. As *n* increases, the Group Warranty yield for high-cost suppliers further increases relative to the Team solution yield so that, depending on problem parameters, three situations may arise with respect to x_2: an increase in *n* may increase the distortion, may decrease the distortion, or it may initially decrease but then increase the distortion. Both the high and low-cost

[24] The following four conditions must hold simultaneously: $2c_2 < R\left(1-p\right) < 4c_2$, $\frac{c_1}{c_2} < \frac{R\left(1-p\right) - 2c_2}{4c_2 - R\left(1-p\right)} \frac{p}{(1-p)} < 1$.

distortions arise because the Team yields are decreasing in the number of suppliers while the Group Warranty yields are constant.

In equilibrium, the payment to suppliers is:

$$T = \frac{2c_1}{x_1^{n-2}\left(p+(1-p)c_1/c_2\right)^{n-1}} = \frac{2c_1}{\left(p+(1-p)c_1/c_2\right)^{n-1}},$$

each supplier's profit is

$$\pi = p\left(x_1XT - c_1x_1^2\right) + (1-p)\left(x_2XT - c_2x_2^2\right) = \left(XT - c_1\right)\left(p + \frac{c_1}{c_2}(1-p)\right) = \frac{c_1}{c_2}\left(c_1 + p\left(c_2 - c_1\right)\right),$$

and the manufacturer's profit is

$$\Pi = R\left(\frac{c_1 + p\left(c_2 - c_1\right)}{c_2}\right)^n - \frac{2nc_1}{c_2}\left(c_1 + p\left(c_2 - c_1\right)\right).$$

Note that each supplier's expected profit does not depend on the number of suppliers. However, the manufacturer's expected profit is decreasing in the number of suppliers. This is the result of the optimal quality being independent of the number of suppliers.

6. Comparison among contracts

In this section we compare the two contractual arrangements studied previously. Tables 1 and 2 in Appendix B summarize the optimal solutions for the AQL and Group Warranty contracts. We proceed by comparing relative benefits of these contracts for the manufacturer, suppliers, and the supply chain as a whole.

Proposition 4: The following comparisons hold:

1) *The manufacturer's preference ordering is: AQL≥ Group Warranty.*
2) *For suppliers, Group Warranty≥AQL.*
3) *For the supply chain, Pooling AQL≥Group Warranty.*

The manufacturer is worse off under the Group Warranty contract than under the AQL contract. The Group Warranty contract affords the manufacturer so little control regardless of the parameters ($x_1 = 1, x_2 = c_1 / c_2$) that it is less preferred by him than the AQL contract. At first glance, we should be able to use the Informativeness Condition (Holmstrom 1979) to argue that the manufacturer prefers the AQL contract to the Group Warranty contract. Clearly the former allows the manufacturer to base the suppliers' contracts on strictly more informative variables. However, the argument doesn't hold here because going from parts testing to end-product testing with an end-product that is characterized by the "weakest link" actually changes not just the set of contractible variables but also the production function from the minimum over all yields to the product of all the yields.

Part 1) of Proposition 4 is appealing from a practical point of view. While the AQL contract may not be easy to state and enforce, for a large set of parameters (i.e., $p^{n-1} < c_1 / c_2$) the AQL contract will yield a Pooling solution in which the manufacturer need only specify one acceptable yield rate and one payment. Furthermore, the Pooling solution under the AQL contract coincides with what is widely observed in industrial practice (see Windham 1995).

Intuition suggests that suppliers should not find the Group Warranty contract attractive because it makes them solely responsible for external product failures – even failures caused by other suppliers. However, as we see from part 2) of Proposition 4, the suppliers prefer a Group Warranty to the AQL contract because the former induces complementarities among suppliers that result in higher yield rates. The suppliers prefer Group Warranty to AQL because under the former the suppliers are able to earn rents for both cost realizations while under the latter they are able to earn rents only for the high-cost realization.

Parts 1 and 2 of Proposition 4 indicate that the manufacturer's preferences over contracts and suppliers' preferences over contracts are in conflict. The Supply Chain profits are merely the sum of the manufacturer's and suppliers' profits. Therefore part 3 of Proposition 4 indicates that the advantage of the Pooling AQL to the manufacturer more than offsets its disadvantage to suppliers. As a result, the manufacturer might be willing to pay a lump-sum to the suppliers to agree to an AQL contract. Notice also that the result of Proposition 4 includes only a comparison of Group Warranty and Pooling AQL. Figure 2 below indicates that there exist cases in which the supply chain is actually better off with a Group Warranty contract (solid line) contract than with an AQL contract that result in a Separating solution (dashed lines). Figure 2 indicates that Group Warranty is most preferred by the supply chain only when there are few parts and the probability of a low-cost realization is high. This combination of problem parameters leads to small distortion of Group Warranty yields from the Team solution and hence better contract performance. For a majority of problem parameters we find that the Separating AQL contract does better, especially when a large number of parts is involved.

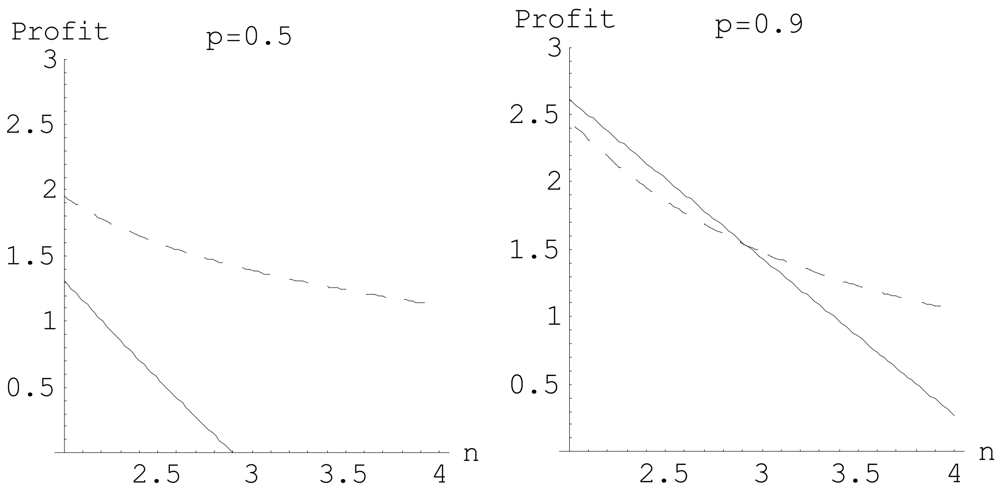

Figure 2. Supply chain profits under low and high ***p*** ***(c_1=1, c_2=2, R=5).***

To this point, we have assumed that less than perfect quality is optimal. If it is optimal to induce $x_1 = x_2 = 1$, what would be the most efficient contractual relationship for doing so? Proposition 5 below addresses that question.

Proposition 5: *Assuming that the manufacturer wants to induce perfect quality regardless of the cost-of-yield realizations, then:*

a) *The supply chain profits are the same under both contracts*

b) *The manufacturer strictly prefers AQL to the Group Warranty.*

The supply chain profits are the same under both contracts because both are inducing perfect quality. The cost of doing so is irrelevant to supply chain profits. The cost of doing so just affects the manufacturer's and suppliers' shares of the supply chain profits. The manufacturer strictly prefers AQL to Group Warranty because in order to induce perfect quality, the payment under AQL is $T = c_2$ while under Group Warranty it is $T = 2c_2$.

7. Discussion

In this paper we have analyzed the issue of structuring contracts for multiple outsourced parts in the presence of externalities among suppliers in the form of the "weakest-link". We modeled two contracts, compared their advantages and disadvantages to supply chain members and analyzed the underlying incentive problems under both by comparing their yield rates to those of the Team solution that does not have incentive problems. With a minor exception, under both contracts the yield rates are distorted from the Team solution but the nature of the distortions differ between the contracts.

In the AQL contract the optimal yield rates decrease as the number of parts increase. This result arises because of the weakest link property. Therefore, among those products having the weakest link property, we would expect to see a negative relationship between the number of parts/subassemblies and the cost of failure, if those contracts were in place. Of course, while we have treated the weakest link property as a dichotomous variable, in practice it is a continuous variable. Hence, we would expect the strength of this negative relationship to vary in the "weakness" of the link.

Our results strongly suggest that the AQL contract works well for both the manufacturer and suppliers when the number of parts is large. This conjecture is supported by our finding that over a large range of problem parameters the incentive problems are unaffected when parts are added under AQL contract. This is because in these regions adding suppliers has the same effect on the Team solution as on the AQL solution. Further, we found that when the number of parts is small the AQL contract specifies several different acceptable levels of quality, while when the number of parts is large, there is only one acceptable level of quality. One way of interpreting this result is that the band of acceptable quality should narrow as the number of parts increases. It would be interesting to see if this relationship holds for different products that are subject to the weakest link property.

One of our interesting results was that the Group Warranty contract may be preferred by the supply chain, but only when there are few parts. We interpret this result as being consistent with the observed relationship between Japanese car manufacturers and their first – tier suppliers. In that relationship, the manufacturer does very little testing of incoming parts, works closely with the few first-tier suppliers providing large sub-assemblies, and implicitly promises to share the benefits of collaboration. This relationship is similar to our Group Warranty in that there is little testing of incoming parts and an aligning of the suppliers' interests on the outcome of the end-product rather than their specific parts.

Our results rely on several assumptions. First, we assumed that the testing of incoming parts is costless. The reason for doing so is that the relative cost of parts testing vs. product testing is unclear. Further, adding different costs of testing parts versus end-products would not change our basic analysis in Sections 1-5 but might affect the comparison of contracts in Section 6. Second, we assumed that the results of the testing and the choice of yield rates were non-stochastic. Given that the manufacturer and suppliers were all risk-neutral, adding a random component to either testing or the choice of yield rates should not qualitatively change the results.

A third major assumption was symmetry of parts. Without this assumption, the complexity of the model greatly increases making it difficult to analyze the effect of adding parts. Another simplification was to assume that each part is only provided by one supplier. However, multi-sourcing can have benefits that our present model does not capture. This issue is analyzed in Baiman and Netessine (2004b). Finally, our analysis was based on a production function which we characterized as having the weakest link property, so that the manufacturer's end-product failed if any one part failed. The qualitative results would continue to hold if we examined a setting in which the end-product would fail if any fixed number of components failed.

There are several possible extensions to this line of research. Analyzing the case in which suppliers differ in their ex ante cost characteristics would be an interesting extension. This would lead to different contracts being offered to the different suppliers. However, this would greatly complicate the analysis of the AQL contracts. Second, it would be interesting to see when and if the manufacturer would choose to implement a combination of internal and external testing, rather than each in isolation, as we have assumed.

It would also be useful to undertake empirical studies in industries which manufacture complex products, such as airlines and automobiles, where a poor quality part can cause the product to malfunction. What type of testing and inspection systems have been established to deal with this weakest link problem? Are firms in these industries utilizing variations of the AQL and Group Warranty systems or some other procedures? Are there special long-term relationships between suppliers and manufacturers that minimize the chances of such failures occurring? Answers to these questions will enable us to understand more fully the challenges firms face when dealing with supply procurement where the end-products exhibits a weakest link property.

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Appendix A: Proofs of Propositions

Proof of Proposition 1 (Team solution).

Two cases need to be considered.

Case 1. $x_1 > x_2$. After differentiating the objective function, we obtain optimality conditions:

$$\frac{\partial \Pi}{\partial x_1} = Rp^n - 2npc_1x_1 = 0,$$
$$\frac{\partial \Pi}{\partial x_2} = R\left(1-p^n\right) - 2n(1-p)c_2x_2 = 0,$$

from which optimal yields follow. It remains to ensure that $x_1 > x_2$ which leads to the condition for the separating solution. Objective function at optimality is

$$\Pi = Rp^n x_1 + R\left(1-p^n\right)x_2 - n\left(pc_1x_1^2 + \left(1-p\right)c_2x_2^2\right) = R^2\frac{p^{2n-1}\left(1-p\right)c_2 + \left(1-p^n\right)^2 c_1}{4nc_1c_2\left(1-p\right)}.$$

Case 2. $x_1 = x_2 = x$. In this case straightforward differentiation leads to the optimal yield. The objective function at optimality is

$$\begin{aligned}\Pi &= Rx - nx^2\left(pc_1 + \left(1-p\right)c_2\right)\\ &= \frac{R^2}{2n\left(pc_1 + \left(1-p\right)c_2\right)} - \frac{R^2 n}{4n^2\left(pc_1 + \left(1-p\right)c_2\right)^2}\left(pc_1 + \left(1-p\right)c_2\right)\\ &= \frac{R^2}{2n\left(pc_1 + \left(1-p\right)c_2\right)} - \frac{R^2}{4n\left(pc_1 + \left(1-p\right)c_2\right)}\\ &= \frac{R^2}{4n\left(pc_1 + \left(1-p\right)c_2\right)}.\end{aligned}$$

Because as long as $\left(p^{n-1} - p^n\right)/\left(1-p^n\right) \geq c_1/c_2$ both solutions are feasible, we need to verify that the separating solution indeed dominates over this range of parameters. The difference in expected profits (the Separating solution – the Pooling solution) =

$$= R^2\frac{p^{2n-1}\left(1-p\right)c_2 + \left(1-p^n\right)^2 c_1}{4nc_1c_2\left(1-p\right)} - \frac{R^2}{4n\left(pc_1 + \left(1-p\right)c_2\right)}$$

$$= R^2 p\frac{\left(p^{n-1}\left(1-p\right)\ c_2 - \left(1-p^n\right)c_1\right)^2}{4nc_1c_2\left(1-p\right)\left(pc_1 + \left(1-p\right)c_2\right)} \geq 0.$$

Hence, the manufacturer will choose the separating solution everywhere it is feasible and the pooling solution otherwise.

Proof of Proposition 2 (AQL contract).

Proposition 2 is proved in a series of steps.

Step 1. Constraints (LL1) cannot bind

Proof: Note that $T_{i1} - c_1 x_{i1}^2 \geq T_{i2} - c_1 x_{i2}^2 > T_{i2} - c_2 x_{i2}^2 \geq 0$ The first inequality follows from (IC1), the second from $c_1 < c_2$ and the third from (LL2).

Step 2. Constraints (LL2) must be binding.

Proof: Assume otherwise and suppose that we start with an optimal solution so that all of the constraints were satisfied and (LL1) is not binding. Reduce both T_{i1} and T_{i2} by ε. None of the constraints will be violated by the change and the manufacturer increases his objective function.

Step 3. $x_{i1} \geq x_{i2}\ \forall i$.

Proof: From (IC1) and (IC2) $c_1(x_{i2}^2 - x_{i1}^2) \geq T_{i2} - T_{i1} \geq c_2(x_{i2}^2 - x_{i1}^2)$. Given, $c_2 > c_1 \Rightarrow x_{i1} \geq x_{i2}$.

Step 4. The optimal solution must be of the form $x_{i1} = x_{j1} = x_1\ \forall i,j$.

Proof: Assume otherwise, $\exists i, j$ such that $x_{i1} > x_{j1}$. Let $\lambda_{ij}\ i = 1,..,n\ j = 1,..,4$ be the Lagrange multiplier for the *ith* supplier's *jth* constraint. The First-Order conditions with respect to x_{i1} are $-2\lambda_{i3} c_1 x_{i1} + 2\lambda_{i4} c_2 x_{i1} = 0 \Rightarrow \lambda_{i3} c_1 = \lambda_{i4} c_2 \Rightarrow \lambda_{i3} = \lambda_{i4} = 0$, where the latter implication follows from $c_2 > c_1$. Notice that the FOC with respect to x_{i1} will not involve terms from the objective function because the objective function is of the form min(•) and $x_{i1} > x_{j1} \geq x_{j2}$. But if constraints (LL1), (IC1) and (IC2) for supplier *i* are all not binding, then the manufacturer can reduce T_{i1} by $\varepsilon > 0$, and make the manufacturer better off without violating any of the constraints. Hence we have a contradiction.

Step 5. The optimal solution must be of the form $x_{i2} = x_{j2} = x_2\ \forall i,j$.

Proof:

We begin by showing that if x_{j2} and x_{k2} are the two smallest yields in the high-cost state then $x_{j2} = x_{k2}$. The reminder of the proof then follows by induction. Assume otherwise, $x_{j2} > x_{k2}$. The manufacturer's problem becomes:

$$\max_{x_{ij}, T_{ij}} \left(\left[(1-p) x_{k2} + p(1-p) x_{j2} + ... \right] R - pT_{j1} - pT_{k1} - (1-p) T_{j2} - (1-p) T_{k2} - ... \right)$$

subject to

$T_{i1} - c_1 x_{i1}^2 \geq 0 \qquad \forall i \qquad$ (LL1)

$T_{i2} - c_2 x_{i2}^2 \geq 0 \qquad \forall i \qquad$ (LL2)

$T_{i1} - c_1 x_{i1}^2 \geq T_{i2} - c_1 x_{i2}^2 \quad \forall i \qquad$ (IC1)

$T_{i2} - c_2 x_{i2}^2 \geq T_{i1} - c_2 x_{i1}^2 \quad \forall i \qquad$ (IC2)

Note that the objective function explicitly includes all the terms involving suppliers *j* and *k*. Let $\lambda_{ij}\ i = 1,..,n\ j = 1,..,4$ be the Lagrange multiplier for the *i*th supplier's *j*th constraint. The First-Order conditions with respect to T_{k1} are $-p + \lambda_{k3} - \lambda_{k4} = 0$. Assume that $x_{k1} > x_{k2}$. If both (IC1) and (IC2) were binding then $c_1(x_{i2}^2 - x_{i1}^2) = T_{i2} - T_{i1} = c_2(x_{i2}^2 - x_{i1}^2) \Rightarrow x_{i2} = x_{i1}$, which is a contradiction. Hence $\lambda_{k4} = 0$ and $\lambda_{k3} > 0$. Alternatively, assume that $x_{k1} = x_{k2}$. In this case the two IC constraints are identical and we can drop one, say (IC2), and hence, again, $\lambda_{k4} = 0$ and $\lambda_{k3} > 0$. Using the fact that constraints (LL2) and (IC1) are binding for suppliers *j* and *k* we can now use the constraints to solve for the optimal payments to suppliers *j* and *k*, substitute them into the objective function and find the following optimal yields: $x_{k2} = (1-p)/2(c_2 - pc_1), x_{j2} = p(1-p)/2(c_2 - pc_1)$. But this implies that $x_{k2} > x_{j2}$ which contradicts our original hypothesis. The remainder of the proof follows by Induction on the next highest yield in the high-cost state.

At this point we are left with two different possible optimal solutions:

a). Pooling = $x_{ij} = x\ \forall i, j$

and

b). Separating $x_{11} = x_{21} = x_{31} = ... = x_{n1} = x_1 > x_{12} = x_{22} = x_{32} = ... = x_{n2} = x_2$

<u>Step 6.</u> Pooling $x_{ij} = x\ \forall i, j$

With Pooling and the symmetry among the agents, $T_{ij} = T$ and the (IC) constraints can be dropped. We can use the binding (LL2) constraint for each supplier to solve for T in terms of *x*. We then substitute that into the objective function which results in an unconstrained maximization in *x*. The resulting solution is: $x = R/2nc_2$ and $T = c_2\left(R/2nc_2\right)^2$ from the binding (LL2) constraint.

The manufacturer's expected profit is $\dfrac{R^2}{4nc_2}$

Each supplier's expected compensation is $c_2\left(\dfrac{R^2}{4n^2c_2^2}\right)=\dfrac{R^2}{4n^2c_2}$

Each agent's expected profit is $\dfrac{R^2}{4n^2c_2}-\left(pc_1+(1-p)c_2\right)\dfrac{R^2}{4n^2c_2^2}=\dfrac{R^2}{4n^2c_2^2}p\left(c_2-c_1\right)$

Finally the total supply chain profit is $\dfrac{R^2}{4nc_2}+\dfrac{R^2}{4nc_2^2}p\left(c_2-c_1\right)=\dfrac{R^2}{4nc_2^2}\left(c_2+p\left(c_2-c_1\right)\right)$.

<u>Step 7.</u> Separating $x_{11}=x_{21}=x_{31}=.....=x_{n1}=x_1>x_{12}=x_{22}=x_{32}=.....=x_{n2}=x_2$

With Separation and the symmetry among the agents, $T_{ij}=T_{kj}=T_j\ j=1,2$ and $\forall i,k$. Recall that the optimal solution is such that for each supplier the (LL2) and (IC1) constraints are binding. This allows us to solve for T_1 and T_2 in terms of x_1 and x_2. Substituting these into the objective function results in an unconstrained optimization in x_1 and x_2. The optimal yield rates are: $x_1=p^{n-1}R/2nc_1$ and $x_2=\left(1-p^n\right)R/2n\left(c_2-pc_1\right)$. Notice that consistency requires that $x_1>x_2\Rightarrow p^{n-1}c_2>c_1$. Hence the Separating solution can only hold when $p^{n-1}c_2>c_1$.

Using the binding (LL2) and (IC1) constraints to solve for the compensations results in:

$$T_1=c_1\left(\frac{p^{n-1}R}{2nc_1}\right)^2+\left(c_2-c_1\right)\left(\frac{\left(1-p^n\right)R}{2n\left(c_2-pc_1\right)}\right)^2 \text{ and } T_2=c_2\left(\frac{\left(1-p^n\right)R}{2n\left(c_2-pc_1\right)}\right)^2.$$

The manufacturer's expected profit is $\dfrac{p^{2n-1}R^2}{4nc_1}+\dfrac{\left(1-p^n\right)^2R^2}{4n\left(c_2-pc_1\right)}$

Each supplier's expected profit is

$$\left(1-p\right)c_2\left(\frac{\left(1-p^n\right)R}{2n\left(c_2-pc_1\right)}\right)^2+p\left[c_1\left(\left(\frac{p^{n-1}R}{2nc_1}\right)^2-\left(\frac{\left(1-p^n\right)R}{2n\left(c_2-pc_1\right)}\right)^2\right)+c_2\left(\frac{\left(1-p^n\right)R}{2n\left(c_2-pc_1\right)}\right)^2\right]$$

$$-pc_1\left(\frac{p^{n-1}R}{2nc_1}\right)^2-(1-p)c_2\left(\frac{\left(1-p^n\right)R}{2n\left(c_2-pc_1\right)}\right)^2$$

$$= p(c_2 - c_1)\left(\frac{(1-p^n)R}{2n(c_2 - pc_1)}\right)^2$$

Finally, the supply chain profit is $\frac{p^{2n-1}R^2}{4nc_1} + \frac{(1-p^n)^2 R^2}{4n(c_2 - pc_1)} + p(c_2 - c_1)\left(\frac{(1-p^n)^2 R^2}{4n(c_2 - pc_1)^2}\right)$.

Step 8. Comparison of Pooling and Separating Equilibria

Only the Pooling solution can hold when $p^{n-1} < c_1 / c_2$, while both are feasible when $p^{n-1} \geq c_1 / c_2$. To determine which the manufacturer will choose (recall that it is the manufacturer who is offering the contract) we compare his expected profit under both. The manufacturer's expected profit under (the Separating solution – the Pooling solution) =

$$\left(\frac{R^2(1-p^n)^2}{4n(c_2 - pc_1)}\right) + \left[\frac{R^2 p^{2n-1}}{4nc_1}\right] - \left(\frac{R^2}{4nc_2}\right) = \left(\frac{R^2 p}{4nc_1c_2(c_2 - pc_1)}\right)\left[p^{n-1}c_2 - c_1\right]^2 > 0$$

Hence, the manufacturer will choose the Separating solution when $p^{n-1} \geq c_1 / c_2$ and the Pooling solution otherwise.

Proof of Proposition 3 (Group Warranty).

Case *i), n* ≥ 3. From (1), when $x_1 < 1$, then $x_2 = x_1 c_2 / c_1$. In addition, when $x_1 = 1$, it is possible that $x_2 = x_1 c_2 / c_1$. Therefore, we start by assuming $x_2 = x_1 c_2 / c_1$ and hence we can write

$$x_1 = \frac{XT}{2c_1} = \frac{(px_1 + (1-p)x_2)^{n-1} T}{2c_1} = \frac{x_1^{n-1}(p + (1-p)c_1/c_2)^{n-1} T}{2c_1} = \left(\frac{2c_1}{(p+(1-p)c_1/c_2)^{n-1} T}\right)^{\frac{1}{n-2}}.$$

$$\Rightarrow T = \frac{2c_1}{x_1^{n-2}(p + (1-p)c_1/c_2)^{n-1}}. \tag{3}$$

The expression for T can be substituted into the manufacturer's objective function so that the manufacturer effectively sets x_1 rather than *T*. The manufacturer's expected profit is

$$\Pi^* = (R - nT)(px_1 + (1-p)x_2)^n = (R - nT)(p + (1-p)c_1/c_2)^n x_1^n$$
$$= \left(Rx_1^n - \frac{2c_1 n x_1^2}{(p + (1-p)c_1/c_2)^{n-1}}\right)(p + (1-p)c_1/c_2)^n.$$

The first and second derivatives are:

$$
\begin{aligned}
\frac{\partial \Pi}{\partial x_1} &= \left(Rnx_1^{n-1} - \frac{4c_1 n x_1}{\left(p+(1-p)c_1/c_2\right)^{n-1}} \right)\left(p+(1-p)c_1/c_2\right)^n, \\
\frac{\partial^2 \Pi}{\partial x_1^2} &= \left(Rn\left(n-1\right)x_1^{n-2} - \frac{4c_1 n}{\left(p+(1-p)c_1/c_2\right)^{n-1}} \right)\left(p+(1-p)c_1/c_2\right)^n.
\end{aligned} \tag{4}
$$

From (4) we find that $x_1^{n-2} = 4c_1 / \left(R\left(p+(1-p)c_1/c_2\right)^{n-1} \right)$ which we further substitute into the second derivative:

$$
\begin{aligned}
\left.\frac{\partial^2 \Pi}{\partial x_1^2}\right|_{\frac{\partial \Pi}{\partial x_1}=0} &= \left(Rn\left(n-1\right)\frac{4c_1}{R\left(p+(1-p)c_1/c_2\right)^{n-1}} - \frac{4c_1 n}{\left(p+(1-p)c_1/c_2\right)^{n-1}} \right)\left(p+(1-p)c_1/c_2\right)^n \\
&= \left(\left(n-1\right)-1\right)4nc_1\left(p+(1-p)c_1/c_2\right) \geq 0.
\end{aligned}
$$

The second derivative is strictly positive (FOC characterizes a global minimum). Hence, the global maximum will be on the boundary and either ($x_1 = 1$, $x_2 = c_1/c_2$) or ($x_1 = x_2 = 0$). We have therefore established parts a and b.

To establish part c, assume that $x_1 = 1$ but drop the previous assumption that $x_2 = x_1 c_2 / c_1$, and allow $c_1/c_2 \leq x_2 \leq 1$. Assuming an interior solution for x_2, from (1), $x_2 = \left(p+(1-p)x_2\right)^{n-1} T / \left(2c_2\right)$. The manufacturer's profit is

$$
\begin{aligned}
\Pi &= (R-nT)\left(p+(1-p)x_2\right)^n \\
&= R\left(p+(1-p)x_2\right)^n - 2nc_2 x_2\left(p+(1-p)x_2\right).
\end{aligned}
$$

The first and second derivatives are

$$
\begin{aligned}
\frac{\partial \Pi}{\partial x_2} &= Rn\left(1-p\right)\left(p+(1-p)x_2\right)^{n-1} - 2nc_2 p - 4nc_2(1-p)x_2, \\
\frac{\partial^2 \Pi}{\partial x_2^2} &= Rn\left(n-1\right)\left(1-p\right)^2\left(p+(1-p)x_2\right)^{n-2} - 4nc_2(1-p) \\
&= n\left(1-p\right)\left(R\left(n-1\right)\left(1-p\right)\left(p+(1-p)x_2\right)^{n-2} - 4c_2 \right).
\end{aligned} \tag{5}
$$

From (5) we find that $R\left(1-p\right)\left(p+(1-p)x_2\right)^{n-2} = \dfrac{2c_2 p + 4c_2(1-p)x_2}{\left(p+(1-p)x_2\right)}$ and substitute this into the second derivative:

$$\left.\frac{\partial^2\Pi}{\partial x_2^2}\right|_{\frac{\partial\Pi}{\partial x_2}=0} = n(1-p)\left((n-1)\frac{2c_2 p + 4c_2(1-p)x_2}{p+(1-p)x_2} - 4c_2\right)$$
$$= 2c_2 n(1-p)\left(\frac{p(n-3)+2(1-p)x_2(n-2)}{p+(1-p)x_2}\right). \qquad (6)$$

The second derivative is strictly positive so that the global maximum is on one of the boundaries, $x_2 = c_1/c_2$ or $x_2 = 1$. This establishes case c.

Case *ii), n=2*. To establish parts a and b we repeat the proof with the assumption that $x_1 = x_2 c_1/c_2$ but notice from (3) that $T = 2c_1/\left(p+(1-p)c_1/c_2\right)$. This implies that both first-order conditions simplify to $x_1/x_2 = c_2/c_1$ so that any yields satisfying this equality will be an equilibrium. However, it is straightforward to verify that the largest of these equilibria is Pareto-optimal so that $x_1 = 1$ and $x_2 = c_1/c_2$ which establishes b. Notice that in this case *T* does not depend upon the yields so effectively the manufacturer either pays suppliers a fixed *T* according to (3) and induces the above equilibrium or pays nothing and induces $x_1 = x_2 = 0$ which establishes a.

To establish c and d, we repeat the proof based on the assumption that $x_1 = 1$ and $c_1/c_2 \le x_2 \le 1$. Observe from (6) that the second derivative is strictly negative so that there is a unique global maximum for x_2 found from (5) which is $x_2 = \frac{R(1-p)-2c_2}{4c_2 - R(1-p)}\frac{p}{(1-p)}$, assuming that $c_1/c_2 \le x_2 = \frac{R(1-p)-2c_2}{4c_2 - R(1-p)}\frac{p}{(1-p)} \le 1$ which establishes d. Otherwise, the solution will be on either boundary, $x_2 = c_1/c_2 \; or \; x_2 = 1$ thereby establishing b and c.

Proof of Proposition 4 (profit comparisons).

Part 1). Manufacturer's profit comparison. We will prove this part by showing that, for any problem parameters, there is a feasible AQL contract that, while incurring the same cost as Group Warranty, generates higher revenues. Hence, the optimal AQL contract will do even better under the same problem parameters.

Recall that the manufacturer's profit under the Group Warranty contract is

$$R\left(\frac{c_1+p(c_2-c_1)}{c_2}\right)^n-\frac{2nc_1}{c_2}\left(c_1+p(c_2-c_1)\right)$$

where the first term represents revenues and the second term represents payments to suppliers. Re-write this second term as follows

$$n\left(2c_1p+\frac{2c_1^2}{c_2}(1-p)\right)$$

We see that every supplier gets paid $2c_1$ with probability p and $2c_1^2/c_2$ with probability *(1-p)*. One way to *interpret* this result is that each low-cost supplier gets $T_1=2c_1$ for providing $x_1=1$ while each high-cost supplier gets $T_2=2c_1^2/c_2$ for providing $x_2=c_1/c_2$.

We will next replicate this Group Warranty contract with a Separating AQL contract. That is, we will structure an AQL contract so that $T_1=2c_1$ and $T_2=2c_1^2/c_2$. If we do so, total payments to suppliers will be the same under both contracts. Hence, it will remain to compare revenues. If we can show that this feasible AQL contract results in higher revenues than the Group Warranty contract, then the manufacturer will strictly prefer the optimal AQL contract to the optimal Group Warranty contract.

Recall that IR constraint is binding for the high-cost supplier so that $T_2=c_2x_2^2$. Letting $y=c_1/c_2$, it follows that $T_2=c_2x_2^2=2c_1^2/c_2 \Leftrightarrow x_2=\min\left(1,y\sqrt{2}\right)$. If $x_2=1$, then the proof is complete since $x_1\geq x_2=1$ and the manufacturer will get the maximum possible revenue R. We will therefore assume that $x_2=y\sqrt{2}<1$. For the low-cost supplier we know that $T_1=c_1x_1^2+(c_2-c_1)x_2^2$. But this implies that

$$T_1=c_1x_1^2+(c_2-c_1)x_2^2=2c_1 \Leftrightarrow x_1=\min\left(\sqrt{2\left(y^2-y+1\right)},1\right)=1.$$

Hence, the low-cost supplier under such an AQL contract will provide $x_1=1$.

We next calculate revenues for the AQL contract

$$R\left(p^nx_1+\left(1-p^n\right)x_2\right)=R\left(p^n+y\sqrt{2}\left(1-p^n\right)\right).$$

We need to show that this revenue is higher than the revenue for the Group Warranty contract that we started with, i.e., that the following difference is non-negative

$$AQL-GW=R\left(p^n+y\sqrt{2}\left(1-p^n\right)-\left(y+p(1-y)\right)^n\right). \qquad (7)$$

We first show that this difference is decreasing in *p:*

$$\frac{\partial}{\partial p}(AQL - GW) = Rn\left(p^{n-1}\left(1 - y\sqrt{2}\right) - \left(y + p(1-y)\right)^{n-1}(1-y)\right)$$

or that

$$p^{n-1}\left(1 - y\sqrt{2}\right) \le \left(y + p(1-y)\right)^{n-1}(1-y).$$

To establish this, it is sufficient to observe that $p^{n-1} \le \left(y + p(1-y)\right)^{n-1}$ and $\left(1 - y\sqrt{2}\right) \le (1-y)$.

Hence, it is sufficient to show that the difference (7) is non-negative at *p=1*.

$$AQL - GW\big|_{p=1} = R(1-1) = 0.$$

To summarize, we have demonstrated that, for any problem parameters, a feasible AQL contract is preferred by the manufacturer over the optimal Group Warranty contract since it generates higher revenues while resulting in the same costs. Hence, the manufacturer will prefer the optimal AQL contract to the optimal Group Warranty contract.

Part 2). Suppliers' profit comparison. We consider pooling and separating equilibria separately. But first recall the assumption that yields do not exceed 1 for any problem parameters. To ensure that this is the case it suffices to ensure that under the separating AQL $x_1 \le 1$ which is equivalent to assuming that $R/(2c_1) \le 1$.

1) For the pooling AQL we need to show

$$\frac{nc_1}{c_2}\left(c_1 + p(c_2 - c_1)\right) \ge \frac{R^2}{4nc_2^2} p(c_2 - c_1). \tag{8}$$

Inequality (8) can be re-written in terms of the optimal yields for the AQL contract (see Proposition 2) as follows $n\left(c_1 + p(c_2 - c_1)\right)c_1/c_2 \ge np(c_2 - c_1)x_2^2$. We know that $x_2 = R/(2nc_2) \le c_1/(nc_2)$ due to the above assumption. It remains to show that $n\left(c_1 + p(c_2 - c_1)\right)c_1/c_2 \ge np(c_2 - c_1)(c_1/nc_2)^2$ or similarly $n^2\left(c_1 + p(c_2 - c_1)\right) \ge p(c_2 - c_1)(c_1/c_2)$. But this holds for any problem parameters.

2) Group Warranty≥Separating AQL. To this point we have demonstrated that Group Warranty≥pooling AQL without using the fact that $p^{n-1} < c_1/c_2$. Separating AQL and Pooling AQL are defined over different ranges of parameters. However, as we show next, the suppliers' profit under Pooling AQL is more than suppliers' profit under Separating AQL even over the range of parameters

for which the Separating AQL holds. Thus, the proof will be completed if we show that for $p^{n-1} > c_1 / c_2$, we have

$$\frac{R^2}{4nc_2^2} p(c_2 - c_1) > \frac{p(c_2 - c_1)}{n} \left(\frac{(1-p^n)R}{2(c_2 - pc_1)} \right)^2 .$$

To see this, notice that profits for both Pooling and Separating AQL contracts can be written $np(c_2 - c_1)x_2^2$ in terms of the their respective optimal yield rates. Therefore, it suffices to show that x_2 is higher under Pooling AQL than under Separating AQL:

$$\frac{R}{2nc_2} \geq \frac{R(1-p^n)}{2n(c_2 - pc_1)} \Leftrightarrow \frac{c_1}{c_2} \leq p^{n-1}.$$

To summarize, we have already shown that, for the suppliers, Group Warranty≥Pooling AQL for any problem parameters and we have also shown that pooling AQL≥Separating AQL when $p^{n-1} > c_1 / c_2$. Hence, for the suppliers, Group Warranty≥Separating AQL.

Part 3). Supply chain profit comparison. The difference in total expected profits between AQL pooling and Group Warranty is:

$$\frac{R^2}{4nc_2^2}\left(c_2 + p(c_2 - c_1)\right) - R\left(\frac{c_1 + p(c_2 - c_1)}{c_2}\right)^n + \frac{nc_1}{c_2}\left(c_1 + p(c_2 - c_1)\right)$$

which is clearly positive for large enough n. This difference is quadratic in *R*. Therefore to prove that the above is always strictly positive all we need to do is prove that there are no real roots in R or equivalently that the determinant is less than zero – this would imply that the above parabola is always above the x – axis.

$$\left(\frac{c_1 + p(c_2 - c_1)}{c_2}\right)^{2n} < \frac{c_1}{c_2^3}\left(c_2 + p(c_2 - c_1)\right)\left(c_1 + p(c_2 - c_1)\right),$$

$$\left(y + p(1-y)\right)^{2n-1} < y\left(1 + p(1-y)\right), \; where \; y = c_1 / c_2. \tag{9}$$

Recall that under pooling AQL, $p^{n-1} < y$. Observe that the RHS of (9) is linearly increasing in *p* and the LHS is increasing and convex in *p*. Moreover, for the smallest *p*, *p=0* we have $y^{2n-1} < y$ because *y<1*. Hence, if we can show that the inequality also holds for the largest possible $p^{n-1} = y$ the proof is complete because the convex function (LHS) will always be below the linear function (RHS). Notice that the LHS is decreasing in *n* (since the expression in the brackets is less than 1). So all we need to do is prove this result for the smallest *n, n=2*. To summarize, we only need to prove the inequality for *n=2* and the maximum *p* allowed by the constraint $p^{n-1} = p < y$. We substitute the maximum possible *p* into the initial expression (9)

$$\left(y + y(1-y)\right)^3 < y\left(1 + y(1-y)\right),$$
$$\left(y + y(1-y)\right)^3 < \left(y + y(1-y)\right)^2 < y\left(1 + y(1-y)\right),$$

since the term inside the cube is less than 1. Further,

$$\left(2y - y^2\right)^2 < y + y^2 - y^3,$$
$$y^3 - 3y^2 + 3y - 1 = \left(y-1\right)^3 < 0,$$

which always holds for y<1.

Proof of Proposition 5 (perfect quality).

To establish the results we solve each contract separately.

Part i) Group Warranty

Given that the suppliers are induced to produce perfect quality regardless of their types, $X_{-i} = 1\ \forall i$.

supplier expected profit = $2c_2 - \left(pc_1 + (1-p)c_2\right)$, manufacturer expected profit = $R - 2nc_2$

supply chain expected profit = $R - 2nc_2 + 2nc_2 - n\left(pc_1 + (1-p)c_2\right) = R - n\left(pc_1 + (1-p)c_2\right)$

Part ii) AQL

With AQL, the manufacturer pays the supplier only if parts are perfect and zero otherwise. So the Incentive Compatibility constraints require only that the supplier prefers to supply perfect quality to zero quality, i.e., $T - c_1 \geq 0$ and $T - c_2 \geq 0$. Hence $T = c_2$. This results in the following:

supplier expected profit = $p\left(c_2 - c_1\right)$, manufacturer expected profit = $R - nc_2$,

supply chain expected profit = $R - nc_2 + nc_2 - n\left(pc_1 + (1-p)c_2\right) = R - n\left(pc_1 + (1-p)c_2\right)$.

All the results immediately follow by comparing the above profit expressions.

Appendix B

Table 1. Solution in terms of problem parameters.

	Separating AQL $p^{n-1} \geq c_1 / c_2$	Pooling AQL $p^{n-1} < c_1 / c_2$	Group Warranty
Quality x_1	$\frac{p^{n-1}R}{2nc_1}$	$\frac{R}{2nc_2}$	1
Quality x_2	$\frac{\left(1-p^n\right)R}{2n\left(c_2-pc_1\right)}$	$\frac{R}{2nc_2}$	$\frac{c_1}{c_2}$
Manufacturer's profit	$\frac{p^{2n-1}R^2}{4nc_1}+\frac{\left(1-p^n\right)^2 R^2}{4n\left(c_2-pc_1\right)}$	$\frac{R^2}{4nc_2}$	$R\left(\frac{c_1+p\left(c_2-c_1\right)}{c_2}\right)^n - \frac{2nc_1}{c_2}\left(c_1+p\left(c_2-c_1\right)\right)$
Suppliers' total profit	$\frac{p\left(c_2-c_1\right)}{n}\left(\frac{\left(1-p^n\right)R}{2\left(c_2-pc_1\right)}\right)^2$	$\frac{R^2}{4nc_2^2}p\left(c_2-c_1\right)$	$\frac{nc_1}{c_2}\left(c_1+p\left(c_2-c_1\right)\right)$
Supply chain profit	$\frac{p^{2n-1}R^2}{4nc_1}+\frac{\left(1-p^n\right)^2 R^2}{4n\left(c_2-pc_1\right)} + \frac{p\left(c_2-c_1\right)}{n}\left(\frac{\left(1-p^n\right)R}{2\left(c_2-pc_1\right)}\right)^2$	$\frac{R^2}{4nc_2^2}\left(c_2+p\left(c_2-c_1\right)\right)$	$R\left(\frac{c_1+p\left(c_2-c_1\right)}{c_2}\right)^n - \frac{nc_1}{c_2}\left(c_1+p\left(c_2-c_1\right)\right)$