

VOLUME FLEXIBILITY, PRODUCT FLEXIBILITY OR BOTH: THE ROLE OF DEMAND CORRELATION AND PRODUCT SUBSTITUTION

Manu Goyal

mgoyal@rhsmith.umd.edu

Robert H. Smith School of Business, University of Maryland

Serguei Netessine

netessine@wharton.upenn.edu

The Wharton School, University of Pennsylvania

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Abstract

We analyze volume flexibility – the ability to produce above/below the installed capacity for a product – under endogenous pricing in a *two-product* setting. We discover that the value of volume flexibility is a function of demand correlation between products, an outcome which *cannot* be explained by classical risk-pooling arguments. Furthermore, while the value of product flexibility *always* decreases in demand correlation, we show that the value of volume flexibility can increase or decrease in demand correlation depending on whether the products are strategic *complements* or *substitutes*. We further find that volume flexibility better combats *aggregate* demand uncertainty for the two products, while product flexibility is better at mitigating *individual* demand uncertainty for each product. Our results thus underscore the *necessity* of analyzing volume flexibility with more than one product, and emphasize the contrast with product flexibility. Furthermore, we highlight the possible pitfalls of combining flexibilities: we show that while adding volume flexibility to product flexibility never hurts performance, adding product flexibility to volume flexibility is not always beneficial, even when such an addition is costless.

1 Introduction

The business environment today is characterized by increasing product variety and escalating levels of demand uncertainty. With capacity decisions typically frozen far in advance of the actual product launch, when demand is still uncertain, misalignments between capacity installed and demand realized are inevitable. Manufacturing *flexibility* helps mitigate the economically severe consequences of such a demand-supply mismatch by allowing firms to appropriately *reallocate or customize* capacity in accordance with the realized demand.

Firms in a variety of industries can choose from a plethora of flexibility types, such as product (or mix) flexibility, volume flexibility, new product flexibility, etc. (Suarez *et al.*, 1996; Gerwin, 1993). Since investment in flexibility requires commitments in terms of time and money, choosing the right flexibility type is critical. To aid this decision, as Vokurka and Leary-Kelly (2000) argue, it is important to study the inter-relationships, trade-offs and synergies among the different flexibility types. However, there is a dearth of research in this domain. As Parker and Wirth (1999) note, “[...] [there is] so little research into these relationships or trade-offs [among flexibility types]... information regarding the trade-offs and relationships among the flexibility types will aid decisions.”

The goal of the paper is to analytically compare two such flexibilities (product and volume) and to study the impact of combining them. *Product flexibility* (same as mix flexibility, process flexibility or product-mix flexibility in the extant literature) entails the ability to manufacture multiple products on the same capacity, and the ability to reallocate capacity between products in response to realized demand. In contrast, a firm with *volume flexibility* manufactures a *single product* on a given capacity. However, volume flexibility enables the firm to produce above/below capacity (at a cost) in response to realized demand (the ability to profitably increase production above capacity is often referred to as “upside” flexibility and the ability to profitably decrease production below capacity is referred to as “downside” flexibility).

Our choice of flexibilities – product and volume – is neither whimsical nor particularly restrictive. Much attention has been given to product flexibility in the analytical literature (*cf.* Fine and Freund, 1990; Jordan and Graves, 1995) as well as in the popular press (Mackintosh, 2003). Numerous studies have also argued for the importance of volume flexibility (*cf.* Jack and Raturi, 2002). Arguably then, of the many flexibilities that firms have at their disposal, product flexibility and volume flexibility are amongst the most potent (Hayes and Wheelright, 1984). Moreover, both flexibilities are related since they help mitigate demand uncertainty by customizing capacity (hence production volumes) once demand is known. Even though the end objective is the same (to mitigate the misalignment between

supply (capacity) and demand), the underpinnings of the two flexibility types, in the way capacity is customized in response to demand realization, are very different. As we demonstrate, these differences endow inherent advantages to the two flexibility types under various environmental conditions. It is therefore important for firms to understand the relative advantages/disadvantages of the two flexibility types, and the possible pitfalls of combining them. The analysis and the insights presented in this paper help managers achieve this goal.

We model a firm manufacturing two products and facing an uncertain demand curve for each. The firm can utilize two types of flexibility: volume and/or product. Using these two flexibilities as building blocks, the firm has a choice of four different technologies: volume-flexible technology (V), product-flexible technology (P), both volume and product-flexible technology (VP) or neither volume nor product-flexible technology (D or dedicated). Having settled on the choice of technology, the firm invests in capacity. If the firm selects P or VP technologies, it invests in a single capacity that produces both products; while if the firm selects D or V technologies, it invests in two capacities, one for each product. The firm then observes the demand curve realizations and, finally, adjusts/allocates capacity to the two products constrained both by the choice of technology and the capacity decisions. The market price is a function of the firm's output quantities. We derive insights by comparing capacity investment decisions and profits for each type of technology.

Our model setup is relevant for many industries, such as electronic manufacturing (Nakomoto, 2003), airplane manufacturing (Lunsford, 2007), etc. We illustrate our model from current automotive praxis where a choice of technology can cover the entire range from no-flexibility (D technology) to full flexibility (VP technology). Firms typically choose technology/capacity well in advance of the actual manufacturing (in fact, this decision can be made as early as 5 years from the planned manufacturing date, as detailed in Fleischmann *et al.*, 2006). Consequently, the choices of technology/capacity are based on sales forecasts for the products that will be manufactured, rather than on actual demand information (*ibid.*). Not surprisingly then, mismatches between supply and demand are common and can often have dire economic consequences. In a celebrated example, in year 2000, Chrysler faced higher than anticipated demand for the just launched PT Cruiser, but on the flip side, dampened demand for the hitherto well-selling Town & Country minivan. However, Chrysler could not align production volumes to demand (since both products were manufactured on non-flexible dedicated technology), and this imbalance between supply and demand for the two products cost the company about \$2 billion (Greenberg, 2001). Firms in the industry have therefore splurged on different kinds of flexibilities to mitigate such demand supply mismatches. These flexibilities (or technologies) may be installed on existing capacity (*retro-fitting*), or may involve setting up an entirely new capacity or plant. For

instance, not only is Ford investing heavily in making its existing plants product-flexible (i.e., installing P technology on *existing* capacity), the company is also investing in *new* product-flexible capacity (their new Rouge Plant can manufacture 9 different products) – Mackintosh (2003). Similarly, BMW located its new \$1.6 billion factory in Germany, where labor costs are extremely high, to leverage unprecedented labor flexibility which would allow the company to increase/decrease production for its multiple products depending on the realized demand (VP technology) – Edmondson (2005).

Our main findings can be summarized as follows:

(i) Since a firm with volume flexibility manufactures just the one product on a given capacity, volume flexibility has typically been analyzed for a single product in the extant literature. In this paper, we analyze a firm with volume flexibility manufacturing *two* products (with each product manufactured on a separate capacity), and discover the following dynamics absent in a single product setting:

(a) The value of volume-flexible technology depends on the demand correlation between the two products, an outcome which cannot be explained by the classical risk-pooling arguments (unlike for product-flexible technology where this dependence is a direct outcome of risk-pooling, *cf.* Eppen, 1979; Van Mieghem, 1998). Moreover, unlike product-flexible technology where the profit *always* decreases in demand correlation, the profit for volume-flexible technology can increase or decrease with demand correlation, depending on whether the two products are strategic ‘complements’ (such as TVs and DVD players) or strategic ‘substitutes’ (such as different models of TVs). Furthermore, while product flexibility is ineffective in mitigating demand uncertainty for positively correlated demands (*cf.* Fine and Freund, 1990; Jordan and Graves, 1995), we find that volume flexibility is indeed a potent tool to harness high (positive) demand correlation between products.

(b) We formally show that volume flexibility can be best used to combat *aggregate* demand uncertainty for a *set of products*, such as uncertainty in demand for an entire line of new apparels being launched, while product flexibility is best used to mitigate *individual* demand uncertainty, such as uncertainty in each style of the apparel being launched.

(ii) We unambiguously delineate the optimal choice of technology (V, P or D) for the myriad environmental variables at play.

(iii) By combining two different flexibility types – volume and product – we show that:

(a) The value of flexibility is not absolute and depends critically on the *existing* technology deployed by the firm. For instance, the value of each flexibility type (volume or product) depends crucially on whether flexibility is added to dedicated technology or to an existing base of flexibility (product or volume, correspondingly).

(b) While adding volume flexibility to product flexibility never hurts performance, adding prod-

uct flexibility to volume flexibility is not always beneficial, even when such an addition is costless.

The rest of the paper is organized as follows. Related literature is surveyed below in Section 2. Models of flexibility are presented in Section 3. Volume and product flexibilities are analyzed in Section 4. The optimal choice of technology is solved in Section 5, including an analysis where the two flexibilities are combined. Robustness of our insights is discussed in Section 6. Finally, Section 7 concludes.

2 Literature Review

We survey three streams of literature below: volume flexibility, product flexibility and the literature that studies both (see Gerwin (1993) for a more extensive survey on manufacturing flexibility).

Volume Flexibility

Much of the work on volume flexibility (or *output* flexibility) comes from economics. Stigler (1939) pioneered the study, and he measured the *degree* of volume flexibility by the steepness of the average cost curve: the steeper the average cost curve around the minimum (i.e., as it becomes costlier to deviate away from the ‘sweet-spot’ of the average cost curve *on either side*), the less flexible is the firm. Our model of volume flexibility, suitably adapted to a price-dependent demand model for two products with capacity constraints, incorporates this fundamental idea of increasing average cost on either side of the minimum. Oi (1961) showed that the profit of a firm with volume flexibility increases in the variance of the price fluctuations. Sheshinski and Dreze (1976), and Mills (1984) generalize and extend the findings of Oi (1961) to a competitive landscape.

Vives (1989) models a two-stage-two-firm game with cost increasing *quadratically* in the output for a single product. The firms select the level of flexibility by setting the steepness of the average cost curve. Similar to our model, Vives utilizes a linear demand model with a random intercept and endogenous pricing. However, he considers only a single product and does not focus on the tradeoffs between volume and product flexibilities.

In the Operations Management literature, analytical work on volume flexibility is eclectic. Tomlin (2006) considers the risk of supply chain disruption in a single-product setting: a firm can either source from an unreliable supplier, or a reliable supplier who has volume flexibility (which is defined as the amount and speed with which extra capacity becomes available). Rajagopalan and Soteriou (1994) formulate capacity acquisition, disposal and replacement problem as an integer program in which capacity comes in a few discrete sizes. Other papers analyze volume flexibility in forms such as ‘Quick Response’, where firms can make additional (typically more expensive) ordering decisions after obtaining better demand information (*cf.* Fisher *et al.* (2001), Iyer and Bergen (1997) and Donohue (2000)).

Although the fundamental idea in all the above papers is similar to our notion of volume flexibility, product prices are exogenous to the model and only upside flexibility is modeled. Cachon and Kok (2007) consider flexibility in decreasing the order by utilizing the salvage market with an endogenously determined clearing price, which can be interpreted as downside flexibility. Anupindi and Jiang (2008) establish the strategic equivalence of price and quantity competition under demand uncertainty, and demonstrate that, under competition, investment in (downside) volume flexibility depends on whether demand shock is additive or multiplicative. Lastly, models of aggregate planning focus on the most cost effective way of meeting *short-term* demand fluctuations by hiring/firing workers, accumulating inventories, etc., without accounting for end-market prices (see Nam and Logendran (1992) for a survey). On the other hand, volume flexibility in our paper mitigates long-term demand uncertainty against the backdrop of product-markets interacting through endogenous pricing.

In the empirical stream, Parker and Wirth (1999) develop measures for volume flexibility, and identify linkages among flexibility types. Jack and Raturi (2002) use case studies to isolate sources of volume flexibility. Vokurka *et al.* (2000) provide a detailed survey of this literature.

Product Flexibility

Unlike volume flexibility, product flexibility has been given much more attention in the operations literature. Seminal work on product flexibility by Fine and Freund (1990) models a firm manufacturing n products, which optimizes the capacity levels for n dedicated resources and one flexible resource that can manufacture all n products. The decision to invest in flexible capacity is based on the cost differential between the dedicated and flexible technologies as well as demand uncertainty and demand correlation. Jordan and Graves (1995) look at total flexibility vs. partial flexibility through the concept of *chaining* (a chain consist of product-plant ‘links’: more links correspond to higher flexibility). They find that adding limited flexibility in the right place can achieve nearly all the benefits of total flexibility. They also show that, unless capacity is very small or very large compared to the mean demand, there are significant benefits to adding product flexibility. Graves and Tomlin (2003) adapt the single-stage chaining strategy of Jordan and Graves (1995) to multistage supply chains. Bassamboo *et al* (2009) establish the value of flexibility under a fairly general framework of newsvendor networks. Van Mieghem (1998) finds that flexibility is beneficial even under perfect (positive) demand correlation if one product is more profitable than the other. Tomlin and Wang (2005) highlight the ‘resource aggregation’ risk of product flexibility where a capacity-disruption could halt the production of *both* products. In all these papers, product prices are exogenous to the model. Chod and Rudi (2005), and Bish and Wang (2004) analyze the value of product flexibility with endogenous pricing. Goyal and Netessine (2007), and Roller and Tombak (1993) investigate the impact of competition on the adoption of product flexibility.

Boyabatli and Toktay (2007) consider the impact of capital market imperfections on the value of product flexibility for a monopolist firm. Bish and Chen (2008) consider two alternate strategies for hedging against supply and demand risks: investing in product flexibility, or demand management through product differentiation and pricing. Bish *et al* (2010) demonstrate under very general conditions that the insights on product flexibility are robust to the typical assumptions made in extant literature (such as always producing to capacity).

Our model of product flexibility is in the spirit of Chod and Rudi (2005), and Goyal and Netessine (2007). However, we also consider the notion of ‘demand management’ through product differentiation and pricing, similar in spirit to Bish and Chen (2008).

Several flexibility types

In the empirical stream, Suarez *et al.* (1996) study three different types of flexibility – mix, volume and new-product – for printed-circuit-board assembly. Anand and Ward (2004) argue, as we do, that there should be a match between the flexibility type and the environmental variables that the firm faces.

Chod *et al* (2006) analytically consider the tradeoff between product and volume flexibility. They look at mix, volume and time flexibility in a related setting. Their notion of volume flexibility is different from ours – volume flexibility is measured by the proportion of manufacturing costs incurred after demand realization (in contrast, the use of the average cost curves in our setup to model and analyze volume flexibility is closer in spirit to the economics literature). Moreover, while Chod *et al* (2006) ignore cross-price effects across markets, an explicit consideration of cross-price effects in our setup reveals the key interplay between demand correlation and the nature of the products (strategic substitutes or complements) for volume flexibility.

Summary

To summarize, though much is known about product flexibility, there is a dearth of studies on volume flexibility especially in a multiproduct setting. Moreover, although the empirical literature has hinted at interrelationships among the different flexibility types, there are no formal models (barring Chod *et al*, 2006) that analyze *both* volume and product flexibility under one framework. We fill this gap by (i) explicitly analyzing some fundamental properties of volume flexibility in a multiproduct setting with endogenous pricing, (ii) by bringing forth the contrast between volume and product flexibility under various environmental conditions, and (iii) by studying the impact of combining flexibilities.

3 Model and Notation

A single firm manufactures two products indexed by $y = 1, 2$. We assume that the inverse demand curve for product y is linear of the form: $P_y(q_y, q_{3-y}) = A_y - q_y - \beta q_{3-y}$, where q_y is the quantity put in the market for product y . We refer to the parameter $\beta \in (-1, 1)$ as the *substitutability parameter*, where $\beta > 0$ signifies that the two products are strategic ‘substitutes’ while $\beta < 0$ signifies that the two products are strategic ‘complements’ (Singh and Vives, 1984). When $\beta = 0$, the products are ‘independent’. For instance, Sony manufactures different models of camcorders (substitutes), as well as televisions and DVD players (complements) often on the same manufacturing facility (Nakamoto, 2003). Substitutability implies that the demand for a product increases with an increase in the price of the other product, and vice versa for complementarity (there is no such cross-price effect when the products are independent). Lus and Muriel (2009) show that, compared to other linear demand models, our model is more appropriate for evaluating investments in flexible technologies because it does not lead to unnatural comparative statics results. The demand intercepts, $A_y \in \mathfrak{R}_+$, are random draws from a bivariate continuous distribution function $F(., .)$ with a density function $f(., .)$. Denote the mean of the marginal distribution by μ_y , the variance by σ_y^2 and the covariance of the joint distribution by $\sigma_{12} = \rho\sigma_1\sigma_2$, where $-1 \leq \rho \leq 1$ is the correlation coefficient. When uncertainty is resolved, the firm noiselessly observes the realization of the random intercepts (A_1, A_2) . The intercepts (and hence the means of the distribution) indicate the sizes of the two markets: the higher the intercepts, the larger the markets.

First, the firm selects one of four technologies: D (not flexible), V (volume flexible), P (product flexible), or VP (both volume and product flexible). Given the choice of technology, the firm invests in capacity. Capacity is denoted by K and is called the *installed capacity*. A product-flexible firm (P or VP) invests in a single production line of capacity K_p or K_{vp} and manufactures both products on this line. A firm without product flexibility (V or D) invests in two production lines with capacities K_{yv} or K_{yd} , $y = 1, 2$, correspondingly. Capacity is costly and the firm purchases capacity at a linear cost c_x per unit, $x \in \{V, VP, P, D\}$; linear capacity investment cost is standard in the literature. We assume that the cost of D technology is the lowest, but place no restriction on the costs of V, P or VP technologies: which technology actually costs more is a question best answered empirically. The firm’s profit at this stage is given by Π_x , where $x \in \{V, VP, P, D\}$.

After uncertainty in demand curves is resolved, the firm with technology $x \in \{V, VP, P, D\}$ manufactures and puts in the market a quantity q_{yx} for product y (consistent with extant literature, marginal production costs are normalized to zero). For the firm with dedicated technology this decision is trivial

because the firm is neither product nor volume flexible, and hence it manufactures at installed capacity for each product, $q_{yx} = K_{yx}$. The firm using a product-flexible technology decides how much capacity to allocate to each product, although total capacity cannot be altered because there is no volume flexibility (neither upside nor downside), $q_{1p} + q_{2p} = K_p$. This allocation is modeled as costless in line with the literature on product flexibility (*cf.* Fine and Freund, 1990; Roller and Tombak, 1993; Van Mieghem, 1998). The firm with volume-flexible technology manufactures two products on two distinct production lines and, in this respect, is similar to the firm with dedicated technology. However, volume flexibility allows the firm to produce above or below capacity (for example by temporarily hiring/firing workers, by shutting down/starting up machines, or by restructuring upstream/downstream supply contracts, which also leads to administrative and operational costs). In a sense, the firm with volume-flexible technology alters capacity (at a cost) after the demand curves are realized, and then produces to capacity. Denote the adjusted capacity by q_{yv} , which is also the output. The *frictional* cost of adjusting capacity under volume flexibility is modeled as a quadratic function of the deviation from the installed capacity, $c \sum_{y=1}^2 (K_{yv} - q_{yv})^2$, where c is the *friction parameter*. This functional form gives rise to convex average cost curves, which have traditionally been used to model and study volume flexibility in economics (*cf.* Mills, 1984). Lastly, the firm with both volume and product flexibility makes two decisions *ex post*: first, the total capacity at which to produce and, second, the allocation of this capacity between the two products. Hence, if the *ex post* capacity is Q_{vp} , then $q_{1vp} + q_{2vp} = Q_{vp}$ and the frictional cost is $c(K_{vp} - Q_{vp})^2$. Profits *ex post* demand curve realizations are denoted by π_x . E denotes the expectation operator with respect to the random variables A_y , $y = 1, 2$. To avoid trivialities, we impose the assumption that $c_x < \mu_y$, $y = 1, 2$ so that the marginal cost of capacity investment is lower than the expected maximum price, or else capacity investment is not profitable in expectation.

3.1 The general problem formulation

We now formulate the optimization problem. The optimal capacity investment decision is common to all technologies and can be written as

$$\Pi_x^* = \max_{K_x > 0} (E(\pi_x) - c_x K_x), \quad (1)$$

where $x \in \{V, VP, P, D\}$ and K_x is the total capacity. The *ex post* optimization problem (π_x) for all four technologies is summarized in the following table where, for analytical tractability, we invoke relatively innocuous bounds on the support of the distribution of demand intercepts so that it is optimal to produce both products, and such that prices are non-negative for P technology (these bounds have precedence in literature, *cf.* Chod and Rudi (2005), and greatly simplify all subsequent derivations without altering

insights). The technical details are in the appendix (Lemma A.1 in the proof of Proposition 2).

	Volume Flexibility	No Volume Flexibility
Prod. Flex.	$\max_{q_{yvp} > 0} \sum_{y=1}^2 \left(A_y - q_{yvp} - \beta q_{(3-y)vp} \right) q_{yvp}$ $-c (K_{vp} - Q_{vp})^2,$ $s.t. q_{1vp} + q_{2vp} = Q_{vp}.$	$\max_{q_{yp} > 0} \sum_{y=1}^2 \left(A_y - q_{yp} - \beta q_{(3-y)p} \right) q_{yp},$ $s.t. q_{1p} + q_{2p} = K_p.$
No Prod. Flex.	$\max_{q_{yv} > 0} \sum_{y=1}^2 \left(A_y - q_{yv} - \beta q_{(3-y)v} \right) q_{yv}$ $-c \sum_{y=1}^2 (K_{yv} - q_{yv})^2.$	$\sum_{y=1}^2 \left(A_y - K_{yd} - \beta K_{(3-y)d} \right) K_{yd}.$

4 Analysis of volume (and product) flexibility

While product flexibility has been well analyzed in the literature, volume flexibility has received far less attention. We therefore begin with only a brief discussion on P technology, and focus mainly on developing insights for V technology.

4.1 Analysis of Product Flexibility

The following Proposition derives the optimal capacities and profits for P and D technologies, also analyzed previously in Goyal and Netessine (2007), and Chod and Rudi (2005).

Proposition 1 (i) *The optimal (total) installed capacity and profit for a firm investing in dedicated technology (D) are:*

$$K_d^* = K_{1d}^* + K_{2d}^* = \frac{\mu_1 + \mu_2}{2(1 + \beta)} - \frac{c_d}{(1 + \beta)},$$

$$\Pi_d^* = \left(\frac{(\mu_1 + \mu_2 - 2c_d)^2}{8(1 + \beta)} + \frac{(\mu_1 - \mu_2)^2}{8(1 - \beta)} \right).$$

(ii) *The optimal installed capacity and profit for a firm investing in product-flexible technology (P) are:*

$$K_p^* = \frac{\mu_1 + \mu_2}{2(1 + \beta)} - \frac{c_p}{(1 + \beta)},$$

$$\Pi_p^* = \left(\frac{(\mu_1 + \mu_2 - 2c_p)^2}{8(1 + \beta)} + \frac{(\mu_1 - \mu_2)^2}{8(1 - \beta)} \right) + \left(\frac{(\sigma_1^2 + \sigma_2^2) - 2\rho\sigma_1\sigma_2}{8(1 - \beta)} \right).$$

We briefly explain the relevant profit components for P technology (a detailed discussion on profit and capacity for P and D technologies can be found in Goyal and Netessine, 2007). The profit for P technology is non-decreasing in the variance of the distribution of the demand intercepts (as long as $\sigma_i \geq \rho\sigma_j$, which subsumes the case of symmetric moments; see Chod and Rudi (2005) for details).

Increasing the variance permits realizations of higher demand states. P technology enables a production shift *to* the product with high demand *from* a product with low demand; consequently, the firm accrues increasing gains for the high-demand product, but at the same time, losses increase for the low-demand product. However, because of endogenous pricing, the gains outpace the losses and the firm benefits. The stochastic component decreases as correlation ρ rises – an increase in correlation allows for fewer opportunities for reallocating capacity between products.

4.2 Analysis of volume flexibility

We now fully characterize and analyze the capacity and profit for volume flexibility (V technology).

Proposition 2 *The optimal (total) installed capacity and the expected profit for a firm investing in volume-flexible technology (V) are:*

$$K_v^* = K_{1v} + K_{2v} = \frac{(\mu_1 + \mu_2)}{2(1 + \beta)} - \frac{c_v}{c} - \frac{c_v}{1 + \beta},$$

$$\Pi_v^* = \left(\frac{(\mu_1 + \mu_2 - 2c_v)^2}{8(1 + \beta)} + \frac{(\mu_1 - \mu_2)^2}{8(1 - \beta)} \right) + \left(\frac{c_v^2}{2c} \right) + \left(\frac{(1 + c)(\sigma_1^2 + \sigma_2^2) - 2(\beta\rho)\sigma_1\sigma_2}{4((1 + c)^2 - \beta^2)} \right).$$

The optimal capacity for V technology is similar to the expressions for technologies P and D , with the exception of the additional term (c_v/c) . This term reflects the extra ‘leverage’ enjoyed by V technology: by utilizing volume flexibility, a firm may want to lower the ex ante investment and then increase the capacity ex post demand realization¹. Hence, the capacity for V technology is lower than that for P technology for the same cost of capacity. Furthermore, capacity does not depend on demand variance due to symmetric (upside and downside) frictional cost of adjustment and risk-neutrality.

Similar to P technology, the profit of volume-flexible technology is non-decreasing in the variance of the demand distribution (as long as $\sigma_i \geq \rho\beta\sigma_j$, which subsumes the case of symmetric moments). A higher realized demand intercept for a product is responded to by increasing production on the product’s *own production line* (in contrast, for product flexibility, the *same* capacity is *redistributed* across products). Coupled with endogenous pricing, the firm gains more in higher demand states than it loses in lower demand states for each product. Consequently, a high variance is preferred by the firm.

¹Notice that the higher the ‘leverage’ term (c_v/c) , the smaller the installed capacity. Hence, for small enough c or large enough c_v , the optimal capacity becomes negative, implying that the firm sells capacity to the outside market since the upside adjustment is cheap. We wish to avoid these issues in the present work and from here on assume that c is sufficiently large (or that c_v is sufficiently small) so that the installed capacity is always positive. This implies that $c \geq 2c_v(1 + \beta) / (\mu_1 + \mu_2 - 2c_v)$ or $c_v \leq (\mu_1 + \mu_2)c / 2(1 + \beta + c)$ for V technology.

What is a little surprising is that the stochastic component of V technology is a function of correlation between the random demand intercepts. To appreciate this finding, recall the classical arguments on the role of correlation from the risk-pooling literature (*cf.* Eppen, 1979) or the product-flexibility literature (*cf.* Van Mieghem, 1998). Because the supply (production quantities) of the two products are linked together by a common shared capacity, a firm *needs* to account for the demand for *both* products while deciding a *feasible* supply for each. For instance, when correlation is negative, the demand for one product rises as the demand for the other product falls. Hence, the firm redistributes capacity so as to allocate more capacity to the product with a higher demand, and thereby less to the product with a lower demand. This ability of a firm to allocate the *same* capacity across multiple products (or to satisfy demands from multiple sources from the same inventory/capacity) forms the leitmotif behind the classical risk-pooling argument. It is in this spirit that the profit of the firm with P technology increases as demand correlation decreases in Proposition 1-(ii).

However, notice that with V technology, there is *no* shared capacity as each product is manufactured on its very own independent production line (or capacity), and each production line manufactures just the one product. Hence a firm with volume flexibility can decide a feasible supply for each product independent of the demand realization of the other. Consequently, the classical reasons from the risk-pooling literature fail to explain the impact of correlation on V technology.

Moreover, quite unlike P technology where the profit *always* decreases in correlation (ρ), the strength of the impact of correlation on V technology depends on the value, and more importantly, on the *sign* of β (recall that the numerator of the stochastic component for V technology is a function of $-\rho\beta$): volume flexibility is best utilized for complementary products with positive correlation ($\beta < 0, \rho > 0$), or substitutable products with negative correlation ($\beta > 0, \rho < 0$).

The following “thought experiment” best summarizes the intuition on why (and how) correlation impacts the profitability of volume flexibility.

A thought experiment: Suppose that for V technology, the quantities can adjust instantaneously and costlessly to any realization of the random demand intercepts – in effect, ignore the *mechanism* (and cost) of quantity adjustment. Then, a firm would solve the following program based on the realization of intercepts (A_1, A_2) :

$$\pi_v^* = \max_{q_{1v}, q_{2v}} ((A_1 - q_{1v} - \beta q_{2v}) q_{1v} + (A_2 - q_{2v} - \beta q_{1v}) q_{2v})$$

The optimal order quantities and profit are:

$$q_y^* = \frac{A_y - \beta A_{(3-y)}}{2(1 - \beta^2)}; \pi_v^* = \frac{A_1^2 + A_2^2 - 2A_1A_2\beta}{4(1 - \beta^2)} \quad (2)$$

Now if $\beta = 0$, clearly correlation has no impact on the firm's profit since each product's optimal order quantity is a function of only its own demand intercept. However, $\beta \neq 0$ links A_1 and A_2 together thereby introducing ρ in the picture. Now suppose that products are substitutes, i.e., $\beta > 0$. With positive correlation, as both A_1 and A_2 increase together, any increase in the intercept for product y gets 'eaten away' by the corresponding increase in the intercept for product $(3 - y)$. Hence, the two markets tend to cannibalize one another – because the products are *substitutes*, there is an implicit competition between the markets. Hence, the firm prefers one large intercept (or market) and one small – even though the smaller market gets mauled by the larger one, the larger market survives with minimal cannibalization. Consequently the firm prefers negative correlation with substitutable products. Similarly for $\beta < 0$ (complementary products) a positive correlation is preferred as it helps grow *both* markets. Mathematically, taking the expectation of the profit function results in the above mentioned interaction of ρ and β .

A specific cost structure merely tweaks the optimal quantities without changing the fundamental interaction of ρ and β in equation 2. The optimal order quantities (equation 3 in the Technical Appendix) readily verify this for quadratic adjustment costs of our model. In fact, note that the key driving force behind the result is that the optimal order quantity for a product decreases with an increase in the intercept of the other substitutable product ($\beta > 0$), and the optimal order quantity for a product increases with an increase in the intercept of the other complementary product ($\beta < 0$). As the following lemma shows, it suffices to have (weakly) convex adjustment costs independent of any specific functional form to achieve this dynamic.

Lemma 1 *Suppose the cost of adjustment for product y is of the general form $C_y(q_y, K_y)$ such that $\partial^2 C_y(q_y, K_y) / \partial q_y^2 \geq 0$ for all $y \in \{1, 2\}$. Then,*

$$\frac{\partial q_y^*}{\partial A_{3-y}} = \frac{-2\beta}{2 \left(\frac{\partial^2 C_y(q_y, K_y)}{\partial q_y^{*2}} + 2(1 - \beta^2) \right) + \frac{\partial^2 C_{3-y}(q_{3-y}, K_{3-y})}{\partial q_{3-y}^{*2}} \left(2 + \frac{\partial^2 C_y(q_y, K_y)}{\partial q_y^{*2}} \right)} \quad \forall \beta \in (-1, 1)$$

Moreover, $\partial q_y^* / \partial A_{3-y} \leq 0$ iff $\beta \geq 0$.

In summary, while the impact of correlation is hardwired into product flexibility by virtue of the products sharing the same capacity, for volume flexibility such dependence is achieved only for non-independent products ($\beta \neq 0$) through endogenous pricing. Hence, the impact of correlation on volume flexibility, unlike the case for product flexibility, is predicated to a large extent on the firm being able

to set prices in two non-independent markets. The following implications, absent in single-product settings, are thus imminent.

First, with volume flexibility, a firm can ‘link’ products on the supply side through shared markets rather than (physically) shared capacity, as in product flexibility. In fact, these production lines may be located thousands of miles apart, but as long as the end products share a common market, such linkages are possible through endogenous pricing. Second, to the extent that β (which reflects how customers view the products as being substitutes/complements) is controlled by the firm through product positioning/marketing efforts, a firm’s marketing and manufacturing strategies (specifically the management of volume-flexible capacity) interact through β . A formal analysis is beyond the scope of the paper, but is an interesting avenue for future research. Third, even if β is exogenous and outside the control of the firm, there is a need to estimate β (Cachon and Olivares (2008) demonstrate the empirical methodology) and manage capacities ‘together’ for V technology. Since, these production lines are often distinct and disparate, unlike for P technology, such recognition often proves elusive. Fourth, while the literature has unequivocally highlighted the inefficacy of product flexibility in coping with high (positive) correlation, (*cf.* Fine and Freund, 1990; Chod and Rudi, 2005), we find that volume flexibility is a potent tool to harness positive demand correlation between products (at the very least when $\beta \leq 0$, a more formal analysis is undertaken in Section 5.1). At first glance, while volume flexibility is better suited for complementary products with positive correlation, both flexibility types could be used for substitutable products with negative correlation. Hence, there is a need for a more formal analysis for the choice of flexibility, which is undertaken next in section 5.

5 The Flexibility Choice

While we analyzed and highlighted the key interaction between β and ρ for volume flexibility, and contrasted the same with product flexibility, this is still just one piece of the puzzle and the choice of technology is by no means evident. For that, we must juxtapose the three technologies, D, V and P , together with *all* the variables at play in Section 5.1. Thereafter, in Section 5.2, we look at the value of flexibility in response to the kind of uncertainty that a firm faces (the analysis in Sections 5.1 and 5.2 follows the traditional paradigm where flexibilities are not combined). Finally in Section 5.3, we analyze the implications of combining flexibility types. To reduce the number of variables, we limit the analysis to distributions with symmetric first and second moments, i.e., $\mu_1 = \mu_2 = \mu$ and $\sigma_1 = \sigma_2 = \sigma$.

5.1 The choice among technologies D, V and P

We first compare the three technologies *pairwise* by developing cost thresholds (as a function of all other problem parameters) such that one technology is preferred over the other as long as its cost is below this threshold (parts (i) – (iii) of Proposition 3 below). This line of analysis was initiated by Fine and Freund (1990), and continued by Van Mieghem (1998), Bish and Wang (2004), Chod and Rudi (2005), and Goyal and Netessine (2007). Finally, part (iv) of Proposition 3 culls out the choice of technology when the choice-set includes all three technologies.

Proposition 3 (i) *A firm selects product-flexible (P) technology over dedicated (D) technology if the cost of P technology, c_p , is lower than c_p^* where,*

$$c_p^* = \mu - \sqrt{\left((\mu - c_d)^2 - \frac{\sigma^2}{2} (1 - \rho) \left(\frac{1 + \beta}{1 - \beta} \right) \right)^+},$$

which is convex non-decreasing in σ^2 . Moreover, $\partial c_p^/\partial \rho \leq 0$ and $c_p^*/\partial \beta \geq 0$.*

(ii) *A firm selects volume-flexible (V) technology over dedicated (D) technology if the cost of V technology, c_v , is lower than c_v^* where,*

$$c_v^* = \frac{c}{1 + \beta + c} \left(\mu - \sqrt{\left(\frac{(1 + \beta + c) c_d^2 - 2(1 + \beta + c) \mu c_d + c \mu^2}{c} - \sigma^2 \frac{(1 + \beta)(1 + c - \rho \beta)}{(1 - \beta + c)c} \right)^+} \right),$$

which is convex non-decreasing in σ^2 . Moreover, $\partial c_v^/\partial \rho \leq 0$ iff $\beta \geq 0$.*

(iii) (a) *A firm selects product-flexible (P) technology over volume-flexible (V) technology if the cost of P technology, c_p , is lower than c_{p-v}^* , where*

$$c_{p-v}^* = \mu - \sqrt{\frac{\left(\frac{2(1 - \beta) \left((1 + c)^2 - \beta^2 \right) \left((1 + \beta) c_v^2 + c(\mu - c_v)^2 \right)}{-\sigma^2 (2\beta(1 + c + \rho) - \beta^2(1 + \rho) + (1 + c)(c(1 - \rho) - (1 + \rho)))} \right)^+}{2(1 - \beta)c \left((1 + c)^2 - \beta^2 \right)}}.$$

Moreover, $\partial c_{p-v}^/\partial \rho \leq 0$.*

(b) *If $c_v = c_p = c_z$, then $c_{p-v}^* = c_z^*$ where*

$$c_z^* = \sqrt{c\sigma^2 \left(\frac{1 - \rho}{2(1 - \beta)} - \frac{1 + c - \beta\rho}{(1 + c)^2 - \beta^2} \right)^+}.$$

Moreover, $\partial c_z^/\partial c \geq 0$, $\partial c_z^*/\partial \rho \leq 0$ and $\partial c_z^*/\partial \beta \geq 0$.*

(iv) Suppose a firm faces a choice between all three technologies, D, V and P such that $c_v = c_p = c_z$. Then, the firm selects product-flexible technology whenever $c_p < \min(c_p^*, c_z^*)$. It selects volume flexibility if $c_z^* \leq c_v \leq c_v^*$. Else it selects dedicated technology. However, under high enough demand correlation ($\rho \geq \rho^*(\beta, c)$), where $\rho^*(\beta, c) = \left(\frac{(1+c)^2 - \beta^2 - 2(1-\beta)(1+c)}{(1+c)^2 - \beta^2 - 2\beta(1-\beta)}\right)$, $c_z^* = 0$ and the firm never selects product-flexible technology. Moreover, $\partial \rho^*(\beta, c) / \partial c \geq 0$, $\partial \rho^*(\beta, c) / \partial \beta \geq 0$.

When flexible technologies V and P are compared to dedicated technology, D , the cost thresholds in parts (i) – (ii) are convex non-decreasing in demand variance. This is because higher demand variance permits realizations of higher demand states, leading to higher profit and higher sustainable cost of investment. Consistent with the previous discussion, while the threshold c_p^* always decreases in demand correlation, the threshold c_v^* can increase or decrease in correlation depending on whether products are complements or substitutes.

Part (iii) (a) of the Proposition compares the two technologies, V and P through the cost threshold c_{p-v}^* . As demand correlation increases, this threshold decreases implying that volume flexibility is increasingly preferred with a rise in correlation. This is because with increasing demand correlation, it becomes progressively more difficult for P technology to shift (shared) capacity across products, and hence to mitigate demand uncertainty. In contrast, V technology does better since capacity adjustments are needed on individual production lines (recall discussion in Section 4.2, and also see related discussion under Proposition 4).

In contrast, as β increases and ρ decreases², the threshold c_{p-v}^* increases, favoring product flexibility. Although we argued earlier that volume flexibility too performs well with substitutable products and negative correlation, product flexibility does a better job at this, and hence is preferred. There are two reasons for this: (i) Unlike P technology, the impact of negative ρ for V technology is moderated by β , and is maximized only when $\beta \rightarrow 1$, and (ii) any capacity change necessitated by the interaction of ρ and β is costly for V technology, whereas it is costless for P technology (c_{p-v}^* is increasing in c . See also related discussion at the end of Proposition 4).

Finally from part (iii) (b), when capacity investment costs are high ($c_v = c_p = c_z > c_z^*$), a firm with V technology can make by with a smaller capacity investment since the firm has the possibility of increasing capacity ex post, and hence V technology is preferred over P technology.

The aforementioned isolated interactions come together in part (iv) of the proposition, which delineates the optimal choice of technology given all three technologies (V, P and D). For simplicity, this analysis is conducted for the same capacity investment costs for P and V technologies – this aids visual

²Comparative statics w.r.t β and c are obtained from extensive numerical experiments when $c_v \neq c_p$, or analytically for the special case when $c_v = c_p$, as in part (iii) (b) of Proposition 3.

representation, and moreover, it is rather straightforward to extend the analysis to different capacity investment costs for V and P technologies using part (iii) (a) of the Proposition. In Figures 1-3 below, we plot the cost thresholds c_v^* and c_p^* against demand variance (σ^2) on the horizontal axis and the cost of capacity ($c_v = c_p = c_z$) on the vertical axis. The cost of dedicated technology is normalized to zero ($c_d = 0$) so that costs of capacity for all other technologies are interpreted as premiums over this ‘vanilla’ technology.

Figure 1 is drawn for the case where ρ is relatively large ($\rho \geq \rho^*(\beta, c)$). For such a high correlation, $c_v^* > c_p^*$ and $c_z^* = 0$, from part (iv). Therefore, three areas (denoted I, II and III in the figure) emerge, where D technology is chosen in Area I and V technology elsewhere. Hence P technology is never a preferred option even for substitutable products under high enough demand correlation, and volume flexibility is overwhelmingly preferred. However, rising c or β increases $\rho^*(\beta, c)$ thereby increasing the attractiveness of P technology. On the other hand, a low enough c decreases $\rho^*(\beta, c)$ to the point where P technology may not be a viable choice even for negative correlation.

Figure 2 is drawn for moderate values of the correlation coefficient, while Figure 3 is drawn for low value of the correlation coefficient. In both these cases, if the cost of flexibility is high (Area I), dedicated technology is preferred. However, the two figures differ in how thresholds c_v^* and c_p^* are intertwined (relevant technical details appear in the proof of part (iv) of Proposition 3 in the Technical Appendix).

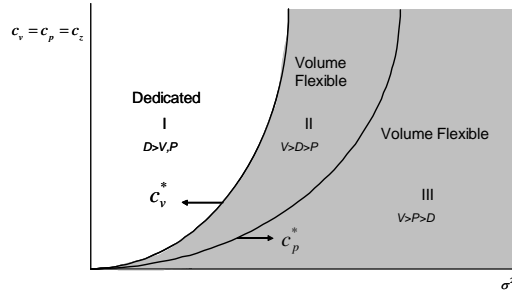


Figure 1. Technology choice for High Correlation

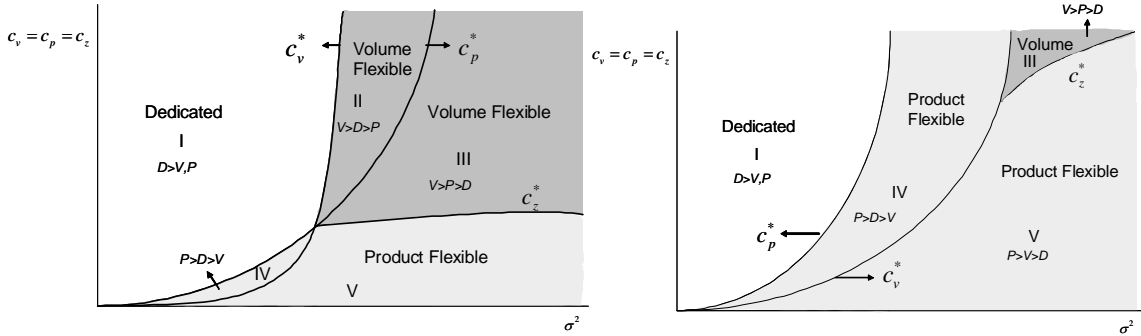


Figure 2. Technology choice for Medium Correlation.

Figure 3. Technology choice for Low Correlation

In Figure 2, even as the cost threshold curve c_v^* for V technology starts *below* that of P technology, it rapidly overtakes c_p^* . Hence, in Area II, since $c_v^* > c_p^*$, the firm invests in volume flexibility. Likewise in Area IV, $c_p^* > c_v^*$ results in the choice of product flexibility. In Areas III and V, since the investment cost is below both c_v^* and c_p^* , both volume and product flexibility may be selected. The tie, however, is broken by c_z^* from part (iii) (b) of Proposition 3. Volume flexibility is selected in Area III where the capacity investment cost is higher, while product flexibility is selected in Area V.

In contrast, in Figure 3, the cost threshold curve c_v^* always lies below c_p^* . Hence, in Area IV product flexibility is chosen. Again the tie between Areas V and III is broken by c_z^* .

In summary, while the intuition and the analysis of Section 4.2, specifically the interaction of β and ρ , serves us well for understanding the fundamental forces at play, and perhaps also to guide us towards the right choice of technology for some ‘obvious’ cases, it is ill-equipped to cull out unambiguously the choice of technology in the context of all the myriad variables at play. For instance, V technology is always preferred over P technology even for substitutable products as long as c is not too high (Figure 1). For low demand correlation one would think that product flexibility would always be the weapon of choice. However, in Figure 3 we do see a smattering of an area where volume flexibility is still preferred if the cost of capacity investment is high enough. Similarly in Figure 2 where inspite of correlation being relatively high, we see product flexibility being chosen ahead of volume flexibility for low enough cost of capacity investment. Hence, what we need is an unambiguous pointer to the technology that creates the most value for a firm operating under various environmental variables. This job is undertaken and accomplished by Proposition 3.

5.2 Mitigating aggregate and individual demand uncertainty

So far we have analyzed both volume and product flexibility at a fairly detailed level pulling various levers, such as $\sigma, \rho, \beta, c, c_v, c_p$, to ascertain the value of each flexibility type. From a more managerial perspective, when managing a portfolio of products together, a firm may face the following two extreme types of uncertainties or risks: Customers may like or dislike the entire portfolio of products on offer, in which case either all products within the portfolio have high demand or none do; or customers may like/dislike individual products within the portfolio, i.e., the total demand for the portfolio is relatively stable, but there is uncertainty in demand across individual products. For instance, small-sized firms, such as Sports Obermeyer, often face uncertainty for the entire line of products that they introduce in any given year (Hammond and Raman, 1996). On the other hand, Benetton, well known for products such as sweaters, may not face as much uncertainty at the portfolio level (sweaters typically sell well), but the uncertainty is often in terms of which colors of the sweater sell well (Heskett and Signorelli,

1984). We refer to the case where uncertainty pertains to the entire portfolio of products as *aggregate* demand uncertainty, while uncertainty pertaining to individual products within the portfolio is referred to as *individual* demand uncertainty. Our aim in this section is to study the efficacy of volume and product flexibilities in mitigating aggregate and individual demand uncertainties.

Define aggregate demand uncertainty (σ_T) for the two products as $\sigma_T^2 = 2\sigma^2(1 + \rho)$, where σ is the measure of individual demand uncertainty (for tractability, we consider symmetric second moments for the two products). Then, increasing aggregate demand uncertainty, while keeping the individual demand uncertainty fixed, simply amounts to increasing correlation. However, increasing individual demand uncertainty, while keeping the aggregate uncertainty fixed, amounts to a *measured* decrease in correlation (see technical appendix). While in reality both types of uncertainties may simultaneously change, a *ceteris paribus* analysis of change in only one type of uncertainty generates sharper insights on the differences and similarities between volume and product flexibilities. Moreover, an analysis with both types of uncertainties at play, accomplished by changing σ^2 and ρ , has been undertaken in Section 5.1, especially through Figures 1-3. Define $\Delta_{x-y} = \Pi_x^* - \Pi_y^*$ as the value of technology $x \in \{V, P\}$ compared to a benchmark technology $y \in \{D, P, V\}$. The following Proposition summarizes the impact of aggregate and individual demand uncertainties on the value of flexibility (notice that the cost of capacity investment plays no role in this analysis and may be different across technologies).

Proposition 4 (i) *If the aggregate demand uncertainty is increased while the individual demand uncertainty is held constant, then:*

$$\begin{aligned}\frac{\partial \Delta_{p-d}}{\partial (\sigma_T^2)} &= \frac{-1}{8(1-\beta)} < 0, \\ \frac{\partial \Delta_{v-d}}{\partial (\sigma_T^2)} &= \frac{-\beta}{4((1+c^2) - \beta^2)} \leq 0 \text{ iff } \beta \geq 0, \\ \frac{\partial \Delta_{p-v}}{\partial (\sigma_T^2)} &= \frac{\partial(\Delta_{p-d} - \Delta_{v-d})}{\partial (\sigma_T^2)} = \frac{(2\beta - (1+c)^2 - \beta^2)}{8(1-\beta)((1+c)^2 - \beta^2)} < 0.\end{aligned}$$

(ii) *If the individual demand uncertainty is increased while the aggregate demand uncertainty is held constant, then:*

$$\begin{aligned}\frac{\partial \Delta_{p-d}}{\partial (\sigma^2)} &= \frac{1}{2(1-\beta)} > 0, \\ \frac{\partial \Delta_{v-d}}{\partial \sigma^2} &= \frac{1}{2(1+c-\beta)} > 0, \\ \frac{\partial \Delta_{p-v}}{\partial \sigma^2} &= \frac{\partial(\Delta_{p-d} - \Delta_{v-d})}{\partial (\sigma^2)} = \frac{c}{2(1-\beta)(c+1-\beta)} > 0.\end{aligned}$$

When compared with dedicated technology, the impact of aggregate demand uncertainty on the value of product and volume flexibility is very similar to the impact of correlation, which was analyzed

in Proposition 3. Namely, as aggregate uncertainty increases, the value of product flexibility decreases (since ρ increases), and the value of volume flexibility increases only if the products are strategic substitutes (the incremental value is zero if $\beta = 0$).

In contrast, notice that as individual uncertainty increases with the aggregate uncertainty held constant (part (ii)), the value of both product and volume flexibility – compared to dedicated technology – *always* increases. Any change in individual demand uncertainty with aggregate uncertainty held constant, facilitated by a carefully measured change in correlation, decouples the two products. This necessitates corresponding changes in production quantities for *individual* products – both types of flexibilities are adept at this, although such changes with product flexibility are frictionless. Hence, not surprisingly, when $c \rightarrow 0$, the value of both volume and product flexibility in response to an increase in individual demand uncertainty is identical.

Finally, comparing volume and product flexibilities directly (Δ_{p-v}) now becomes intuitive given the previous discussion: For mitigating aggregate demand uncertainty, migrating to product flexibility from an installed base of volume flexibility yields decreasing returns (recall part (iii) (a) of Proposition 3). However, with an increase in individual demand uncertainty, since we now know that both flexibility types are on a relatively even keel, the scale is tipped in favor of product flexibility on account of the frictional cost of adjustment for volume flexibility.

In summary, although of much practical importance, studying the impact of demand uncertainty (σ) alone conflates individual and aggregate demand uncertainties. By isolating the response of volume and product flexibilities to each type of uncertainty, with the other held fixed, we have distilled out fundamental distinctions between the two flexibilities. As we proved above, volume and product flexibilities behave very differently when it comes to mitigating aggregate demand uncertainty. However, the two flexibility types are on a much more even keel as regards individual demand uncertainty – although product flexibility does better than volume flexibility due to frictionless changes.

5.3 Combining flexibilities – the VP technology

The analysis until now was based on technologies with at most a single flexibility type. In this section, we analyze the value of volume flexibility when incrementally added to product flexibility, and vice versa. We first develop the optimal profit (and capacity) for *VP* technology in the proposition below.

Proposition 5 *The optimal installed capacity and the expected profit for a firm investing in volume*

and product-flexible technology (VP) are:

$$K_{vp}^* = \frac{(\mu_1 + \mu_2)}{2(1 + \beta)} - \frac{c_{vp}}{2c} - \frac{c_{vp}}{1 + \beta},$$

$$\Pi_{vp}^* = \left(\frac{(\mu_1 + \mu_2 - 2c_{vp})^2}{8(1 + \beta)} + \frac{(\mu_1 - \mu_2)^2}{8(1 - \beta)} \right) + \left(\frac{c_{vp}^2}{4c} \right) + \left(\frac{(1 + c)(\sigma_1^2 + \sigma_2^2) - 2(\beta + c)\rho\sigma_1\sigma_2}{4(1 - \beta)(1 + \beta + 2c)} \right).$$

Not surprisingly, VP technology takes its flavor from both V and P technologies. Similar to volume flexibility, the profit (and capacity) of VP technology depends on the leverage term. However, unlike V technology, but similar to P technology, the profit for VP technology depends on correlation even when $\beta = 0$.

We now look at the impact of changes in aggregate and individual demand uncertainties on the value of VP technology compared to P or V technologies, thereby distilling the impact of *incrementally* adding flexibility.

Proposition 6 (i) *If the aggregate demand uncertainty is increased while the individual demand uncertainty is held constant, then:*

$$\frac{\partial \Delta_{vp-p}}{\partial (\sigma_T^2)} = 1 / (8(1 + \beta + 2c)) > 0,$$

$$\frac{\partial \Delta_{vp-v}}{\partial (\sigma_T^2)} = \frac{1}{4} \left(-((\beta + c) / (1 - \beta)(1 + \beta + 2c)) + \beta / ((1 + c)^2 - \beta^2) \right) \leq 0.$$

(ii) *If the individual demand uncertainty is increased while the aggregate demand uncertainty is held constant, then:*

$$\frac{\partial \Delta_{vp-p}}{\partial (\sigma^2)} = 0,$$

$$\frac{\partial \Delta_{vp-v}}{\partial (\sigma^2)} = c/2(1 - \beta)(1 + c - \beta) \geq 0.$$

Compared to P technology, the value of VP technology increases with a rise in the aggregate uncertainty, while compared to V technology it falls. Hence, the following Corollary is imminent.

Corollary 1 *VP technology always dominates P technology if they cost the same. However, VP technology does not always dominate V technology – for high enough aggregate demand uncertainty, there is a negative value of adding product flexibility to volume flexibility, even when additional flexibility comes at no cost.*

To better understand this finding, note that although both V and VP technologies have volume flexibility, the cost incurred due to capacity adjustments is different. For V technology, adjustment costs are incurred on two different production lines, while for VP technology, the cost is incurred on

one production line. This turns out to be a double-edged sword for VP technology. It is an advantage for VP technology when there is a need to increase production of one product and decrease the production of the other: V technology incurs two costs, but for VP technology these adjustments can offset each other thereby lowering capacity-adjustment costs. But whenever there is limited opportunity of offsetting capacity adjustments against one another, such as when demand correlation is positive (high aggregate demand uncertainty), VP technology incurs a higher cost of capacity adjustment. For example, suppose the capacity needs to be increased (or decreased) by 5 units for each product. Then the total frictional cost for V technology is $(5^2 + 5^2) \times c = 50c$, while that same cost for VP technology is $(5 + 5)^2 \times c = 100c$. This downside of manufacturing more than one product on the same capacity limits the benefits of adding product flexibility to volume flexibility³.

Notice that when combating aggregate demand uncertainty, *incrementally* adding volume flexibility to product flexibility is always valuable; this is in contrast to Proposition 4 where the value of adding volume flexibility to dedicated technology in response to an increase in aggregate demand uncertainty was contingent on the sign of β .

Furthermore, from part (ii), if aggregate uncertainty is fixed, then it is pointless adding volume flexibility to product flexibility to combat individual demand uncertainty, although adding product flexibility to volume flexibility is valuable. With an increase in individual demand uncertainty, as discussed in Section 5.2, volume and product flexibility behave very similarly, with a slight disadvantage to volume flexibility on account of frictional adjustment costs. Hence, incrementally adding volume flexibility is useless, but not product flexibility. Again this is in contrast to Proposition 4 where, in reference to dedicated technology, volume flexibility is always beneficial in combating individual demand uncertainty.

Taking a holistic view of Propositions 4 and 6, we can conclude that volume flexibility is better than product flexibility at mitigating aggregate demand uncertainty (product flexibility is universally bad, while volume flexibility is always better when added to product flexibility and sometimes better, especially when $\beta < 0$, when added to dedicated technology). However, product flexibility is unambiguously better than volume flexibility in mitigating individual demand uncertainty, whether it is added to volume flexibility or to product flexibility. Moreover, as Propositions 4 and 6 show, the value of a given type of flexibility, product or volume, is not absolute and critically depends on whether flexibility

³This result is indeed predicated on convex adjustment costs for volume flexibility. Such costs are seen in many industries, and moreover, a similar cost structure is used to model volume flexibility in most papers (see Section 6 for references). The thrust of the finding is that flexibilities may not reside independently in VP technology where a firm can simply “call upon” the desired flexibility; rather our results suggest that the two flexibilities intermingle giving VP technology a distinct flavor which may not always dominate one of the pure technologies.

is added to dedicated technology, or to an existing base of flexibility. In contrast, the extant literature values flexibilities only in reference to no-flexibility (or dedicated technology) as the benchmark.

6 Robustness of the model and insights

Our model is stylized and relies on a few assumptions, namely: (A1) quadratic and symmetric (upside and downside) adjustment costs for volume flexibility, (A2) a firm with product flexibility always produces to capacity. These assumptions are discussed below (relevant numerical details are in the Technical Appendix, Part B).

A1: Quadratic and symmetric (upside and downside) adjustment costs for volume flexibility: Given that the interplay between ρ and β is immune to a particular functional form of the adjustment costs (see the ‘thought experiment’ and Lemma 1 in Section 4.2), our assumption on any increase (or decrease) in production away from installed capacity as being *equally and quadratically* expensive merely serves to make the model tractable enabling sharp analytical insights (this assumption actually has a strong precedence in the economics literature, *cf.* Stigler (1939), Vives (1989), Sheshinski and Dreze (1974), Mills (1984)). Moreover, consonant with practice, we allow for lower cost of decreasing production compared to the cost of increasing production and numerically verify that the insights are intact.

A2: A firm with product flexibility always produces to capacity. This assumption is an offshoot of our model of ‘pure’ flexibilities – volume flexibility (i.e., V technology) has no infusion of product flexibility (i.e., P technology), and vice versa. The VP technology, which has features of both product and volume flexibilities, is analyzed separately. While this enables a sharp insightful comparison between the two flexibility types, it also implies that a firm with product flexibility cannot produce below installed capacity (no downside volume flexibility). This assumption has been made and justified in much of the literature –*cf.* Chod and Rudi (2005), Goyal and Netessine (2007), Anand and Girotra (2007). In fact, Bish *et al.* (2010) show under very general conditions that fundamental insights on product flexibility are invariant to this assumption. Nonetheless, we numerically allow a firm with P technology to produce below capacity and verify that the insights are untainted.

7 Conclusion

With the spotlight on product flexibility, volume flexibility has been given short-shrift in the academic literature. In this paper we show that although both volume and product flexibilities mitigate demand uncertainty by adjusting production to match supply and demand, there are fundamental points-of-departure.

A firm with volume flexibility manufactures just the one product on a production capacity. Hence all insights on volume flexibility, carved by models that analyze volume flexibility for a single product (or multiple *independent* products), ignore inter-product linkages. Such myopia obfuscates key dynamics between products, such as the interplay between demand correlation (ρ) and the product substitutability parameter (β), which directly impacts the profitability of firms. We posit the need for firms with volume flexible capacities to take a more holistic view of the entire product portfolio and to manage products ‘together’. Because many of these products may be manufactured on completely different production lines, perhaps located thousands of miles apart, such recognition often proves elusive for firms deploying volume flexibility (on the contrary, such cognizance comes easy for product flexibility since by its very nature, it involves manufacturing multiple products together on the same capacity).

The vast literature on flexibility has unequivocally established the efficacy of product flexibility in mitigating demand uncertainty as long as product demands are not positively correlated. At the same time, the literature has been rather silent on strategies/tools for mitigating uncertainty with positively correlated demands. We show that volume flexibility is indeed a potent tool for mitigating demand uncertainty when demands for products are positively correlated. Lastly, we highlighted possible pitfalls of adding product flexibility to volume flexibility.

References

- [1] Anand, G. and P.T. Ward. 2004. Fit, flexibility and performance in manufacturing: coping with dynamic environments. *Production and Operations Management*, Vol.13, 369-385.
- [2] Anand, K. S. and K. Girotra. 2007. The strategic perils of delayed differentiation, *Management Science*, Vol 53 (5), pp. 697-712.
- [3] Anupindi, R. and L. Jiang. 2008. Capacity investment under postponement strategies, market competition and demand uncertainty. *Management Science*, Vol. 54(11), pp. 1876-1890.
- [4] Bassamboo, A., Randhawa, R. S. & van Mieghem, J. A. 2009. Optimal flexibility configurations in newsvendor networks: Going beyond chaining and pairing. Working Paper, Northwestern University.
- [5] Bish, E.K. and Q. Wang. 2004. Optimal investment strategies for flexible resources considering pricing and correlated demands. *Operations Research*, Vol.52, 954-964.

- [6] Bish, E.K. and W. Chen. 2008. The firm's optimal resource portfolio under consumer choice, and supply and demand risks. Working paper, Virginia Tech.
- [7] Bish, E.K., J. Liu and D.R. Bish. 2010. A Note on "Resource Flexibility with Responsive Pricing". *Naval Research Logistics*, Vol 57(2), pp. 119-126.
- [8] Boyabatli, O. and B. Toktay. 2007. Stochastic capacity investment and technology choice in imperfect capital markets. Working Paper.
- [9] Cachon, G.P. and G. Kok. 2007. Implementation of the Newsvendor Model with Clearance Pricing: How to (and How Not to) Estimate a Salvage Value. *Manufacturing and Service Operations Management*, Vol 9(3), pp.276-290.
- [10] Cachon, G.P. and M. Olivares. 2008. Drivers of finished goods inventory performance in the US auto industry. Working Paper, University of Pennsylvania.
- [11] Chod, J. and N. Rudi. 2005. Resource flexibility with responsive pricing. *Operations Research*. 53(3).
- [12] Chod, J., N. Rudi and J.A. Van Mieghem. 2006. Mix, time and volume flexibility: valuation and corporate diversification. Working Paper.
- [13] Donohue, K. 2000. Efficient supply contracts for fashion goods with forecast updating and two production modes. *Management Science*, Vol.46, 1397-1411.
- [14] Edmondson, G. 2005. BMW keeps the home fires burning. *Businessweek*, March 30.
- [15] Eppen, G.D. 1979. Effects of centralization on expected costs in a multi-location Newsboy problem. *Management Science*, Vol 25(5), 498-501.
- [16] Fine, C. and R. Freund. 1990. Optimal investment in product flexible manufacturing capacity. *Management Science*, Vol.36, 449-466.
- [17] Fisher, M., K. Rajaram and A. Raman. 2001. Optimizing inventory replenishment of retail fashion products. *Manufacturing & Service Operations Management*, Vol.3, 230-241.
- [18] Fleischmann, B. S. Ferber and P. Henrich. 2006. Strategic planning of BMW's global production network. *Interfaces*. Vol 36(3), pp. 194-208.
- [19] Gerwin, D. 1993. Manufacturing flexibility: a strategic perspective. *Management Science*, Vol.39, 395-410.

- [20] Goyal M. and S. Netessine. 2007. Strategic technology choice and capacity investment under demand uncertainty. *Management Science*. Vol 53(2), 192-207.
- [21] Graves, S. & Tomlin, B. (2003), 'Process flexibility in supply chains', *Management Science*, Vol. 49, 907-919.
- [22] Greenberg, K. 2001. Much ado about Chrysler. *Brandweek*, April 9, 32-40.
- [23] Hammond, J. and A. Raman. 1996. Sports Obermeyer, Ltd. Harvard Business School Case # 9-695-022.
- [24] Hayes, R. and Wheelright, S. 1984. Restoring our competitive edge. Wiley, New York.
- [25] Heskett, J.L. and S. Signorelli. 1984. Benetton (A), *Harvard Teaching Case* 9-685-014.
- [26] Iyer, A.V. and M.E. Bergen. 1997. Quick response in manufacturer-retailer channels. *Management Science*, Vol.43, 559-570.
- [27] Jack, E.P. and A. Raturi. 2002. Sources of volume flexibility and their impact on performance. *Journal of Operations Management*, Vol.20, 519-548.
- [28] Jordan, W.C. and S. Graves. 1995. Principles on the benefits of manufacturing process flexibility. *Management Science*, 41(4), 577-594.
- [29] Lunsford, J.L. 2007. Burned by last boom, Boeing curbs its pace. *The Wall Street Journal*, March 26.
- [30] Lus, B. and A. Muriel. 2009. Measuring the impact of increased product substitution on pricing and capacity decisions under Linear Demand Models. *Production and Operations Management*, Vol 18(1), pp. 95-113.
- [31] Mackintosh, J. 2003. Ford learns to bend with the wind. *Financial Times*, February 14.
- [32] Mills, D.E. 1984. Demand fluctuations and endogenous firm flexibility. *Journal of Industrial Economics*. Vol 33(1), 55-71.
- [33] Nakamoto, M. 2003. A faster route from order to camcorder. *Financial Times*, February 12.
- [34] Nam, S. and R. Logendran. 1992. Aggregate production planning – a survey of models and methodologies. *European Journal of Operational Research*, Vol 61(3), pp. 255-272.

- [35] Oi, W. 1961. The desirability of price stability under perfect competition. *Econometrica*, Vol.29, 58-64.
- [36] Parker, R.P. and A. Wirth. 1999. Manufacturing flexibility: measures and relationships. *European Journal of Operational Research*, Vol.118, 429-449.
- [37] Rajagopalan, S. and A.C. Soteriou. 1994. Capacity acquisition and disposal with discrete facility sizes. *Management Science*, Vol.40, 903-917.
- [38] Roller, L.H. and M.M. Tombak. 1993. Competition and investment in flexible technologies. *Management Science*, Vol.39, 107-114.
- [39] Sheshinski, E. and J.H. Dreze. 1976. Demand fluctuations, capacity utilization, and costs. *American Economic Review*. Vol 66(5), 731-742.
- [40] Singh, N. and X. Vives. 1984. Price and quantity competition in a differentiated duopoly. *Rand Journal of Economics*. Vol 15 (4), 546-554.
- [41] Stigler, G. 1939. Production and distribution in the short run. *The Journal of Political Economy*. Vol.47, 305-327.
- [42] Suarez, F.F., Cusumano, M.A. and Fine, C.H. 1996. An empirical study of manufacturing flexibility in printed circuit board assembly. *Operations Research*, Vol.44, 223-240.
- [43] Tomlin, B., and Y. Wang. 2005. On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing and Service Operations Management*. Vol.7(1), 37-57.
- [44] Tomlin, B. 2006. On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science*, Vol 52(5), pp. 639-657.
- [45] Van Mieghem, J. 1998. Investment strategies for flexible resources. *Management Science*, Vol.44, 1071-1078.
- [46] Vives, X. 1989. Technological competition, uncertainty and oligopoly. *Journal of Economic Theory*, Vol.48, 386-415.
- [47] Vokurka, R.J. and S.W. O'Leary-Kelly. 2000. A review of empirical research on manufacturing flexibility. *Journal of Operations Management*, Vol.18, 485-501.

Technical Appendix Part (A): Proofs

Proof of Proposition 2.

Proof. For V technology, the firm maximizes the following objective function ex post demand realization:

$$\pi_v = (A_1 - q_{1v} - \beta q_{2v}) q_{1v} - c(K_{1v} - q_{1v})^2 + (A_2 - q_{2v} - \beta q_{1v}) q_{2v} - c(K_{2v} - q_{2v})^2.$$

The objective function can be verified to be globally concave and the first-order conditions are

$$A_y - 2q_{yv} - 2\beta q_{(3-y)v} + 2c(K_{yv} - q_{yv}) = 0, y = 1, 2.$$

Solving these two equalities simultaneously results in the following solution:

$$q_{yv}^* = \frac{A_y(1+c) - A_{3-y}\beta + K_{yv}(2c)(1+c) - 2\beta cK_{(3-y)v}}{2(1+c(2+c) - \beta^2)}. \quad (3)$$

We ignore the case when $A_{3-y} \gg A_y$, $y = 1, 2$ which would result in $q_{yv}^* < 0$ for $\beta > 0$ (more formal analysis is undertaken in the Lemma A.1 below). Recall from equation (1) in the main paper that the expected profit is

$$\Pi_v = E(\pi_v) - c_v(K_{1v} + K_{2v}).$$

After substituting q_{iv}^* into the expression for π_v , taking expectation by noting that $E(A_i^2) = \mu_i^2 + \sigma_i^2$ and $E(A_i A_j) = \mu_i \mu_j + \sigma_{ij}$, we obtain

$$\Pi_v = \frac{((1+c)((\sigma_1^2 + \mu_1^2) + (\sigma_2^2 + \mu_2^2)) - 2\beta\sigma_{12} + 4\mu_1 c(K_{1v}(1+c) - \beta K_{2v}) - 4c((1+c-\beta^2)(K_{1v}^2 + K_{2v}^2) + 2\beta cK_{1v}K_{2v}) - 2\beta\mu_1\mu_2)}{4((1+c)^2 - \beta^2)} - c_v(K_{1v} + K_{2v}).$$

Differentiating w.r.t. capacities and simplifying, we obtain the optimal capacities. The total capacity and the optimal expected profit follow after algebraic simplifications.

Lemma A.1: (i) Under symmetry, if the following conditions hold on the support of the distribution of the demand intercepts, then the quantities are always positive for both products for the respective technologies iff:

(a) For V technology: $\frac{1}{1+c} \leq \frac{A_1}{A_2} \leq 1+c$.

(b) For P technology: $|A_1 - A_2| \leq 2\frac{(1-\beta)}{(1+\beta)}(\mu - c_p)$.

(ii) If $|A_1 - A_2| < c_p$, then prices are non-negative for P technology.

Proof: (i) (a) For V technology, $q_{yv}^* \geq 0$ if

$$\begin{aligned} & \frac{A_y(1+c) - A_{3-y}\beta + K_{yv}(2c)(1+c) - 2\beta cK_{(3-y)v}}{2(1+c(2+c) - \beta^2)} \geq 0 \\ \Rightarrow & A_y(1+c) - A_{3-y}\beta + K_{yv}(2c)(1+c) - 2\beta cK_{(3-y)v} \geq 0. \end{aligned}$$

Under symmetric capacities, $K_{yv}(2c)(1+c) - 2\beta cK_{(3-y)v} \geq 0 \forall c \in \mathfrak{R}^+$ and $\forall \beta \in (-1, 1)$. Hence, $q_{yv}^* \geq 0 \Rightarrow A_y(1+c) - A_{3-y}\beta$. Clearly, if this condition is satisfied for $\beta = 1$, it is satisfied for all β . Hence, we check for $\beta = 1$.

$$\begin{aligned} & A_y(1+c) - A_{3-y} \geq 0 \\ \Rightarrow & \frac{A_y}{A_{3-y}} \geq \frac{1}{1+c}. \end{aligned}$$

and the result follows.

To prove the other direction, if $A_y \geq (1+c)A_{3-y}$, then the optimal order quantities are positive as long as $K_{yv}(1+c) \geq \beta K_{(3-y)v}$, which subsumes the case of symmetric capacities.

(i) (b) Follows from Goyal and Netessine (2007), Lemma EC.1 in the Technical Appendix.

(ii) Follows from Goyal and Netessine (2007), Lemma EC.1 in the Technical Appendix.

The intersection of the technical conditions of part (i) is shown in the figure below (non-negativity of prices is not particularly restrictive as negative prices (selling below cost) is seen in many industries). As can be seen, the intersection is non-empty and covers a fairly expansive set of realizations for the demand intercepts.

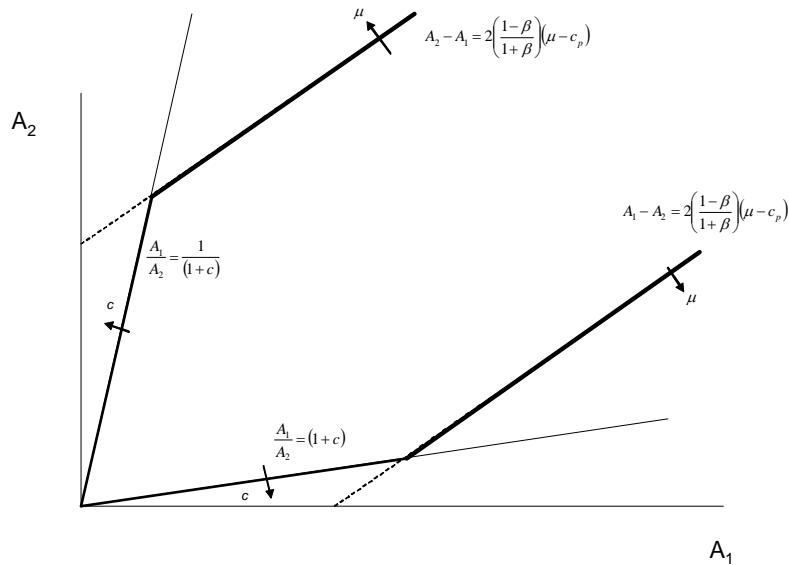


Fig A.1. Intersection of the technical conditions for V and P technologies

■

Proof of Lemma 1

Proof. The ex post profit function for the firm with V technology is

$$\pi_v = \sum_{y=1}^2 \left(\max_{q_y} (A_y - q_y - \beta q_{3-y}) q_y - C_y(q_y, K_y) \right),$$

where $C_y(q_y, K_y)$ is a general cost of adjustment for product y as a function of the production quantity and capacity of product y . For simplicity, denote $\partial C_y(q_y, K_y) / \partial q_y \equiv C'_y$ and $\partial^2 C_y(q_y, K_y) / \partial (q_y)^2 \equiv C''_y$. Then, the FOC for product y is

$$A_y - 2q_y^* - 2\beta q_{3-y}^* - C'_y = 0, y = \{1, 2\} \quad (4)$$

To obtain the derivative of q_y w.r.t the *realized* value of the intercept A_{3-y} , we need:

$$\frac{\partial q_y^*}{\partial A_{3-y}} = \frac{\partial q_y^*}{\partial q_{(3-y)}^*} \frac{\partial q_{(3-y)}^*}{\partial A_{3-y}}. \quad (5)$$

From the Implicit Function Theorem (IFT), as applied to equation (4), we obtain

$$\frac{\partial q_y^*}{\partial q_{(3-y)}^*} = \frac{-2\beta}{(C''_1 + 2)}. \quad (6)$$

Moreover, to obtain $\partial q_{(3-y)}^* / \partial A_{3-y}$, apply IFT to product $(3-y)$ treating q_{3-y}^* as a function of A_{3-y} . This results in:

$$\frac{\partial q_{(3-y)}^*}{\partial A_{3-y}} = \frac{1}{\left[2 \left(1 + \beta \frac{\partial q_y^*}{\partial q_{(3-y)}^*} \right) + C''_{3-y} \right]}. \quad (7)$$

From equations (5), (6) and (7) we obtain the desired result. ■

Proof of Proposition 3.

Proof. Proof of parts (i)–(iii): The expression for each cost threshold can be obtained by comparing pair-wise profits. In each case, equating the two profit expressions results in an equality of the form $g(\cdot) = 0$. In each case two roots can be calculated but the upper root is discarded because it results in negative capacity. The remaining root is stated in the proposition.

For the comparative statics, it is straightforward to show that $\partial c_p^* / \partial \rho \leq 0$ and $\partial c_p^* / \partial \beta \geq 0$ in part (i).

For comparative statics in part (ii), define the expression under the square root as

$$g(\rho, \beta, c) = \left(\frac{(1 + \beta + c) c_d^2 - 2(1 + \beta + c) \mu c_d + c \mu^2}{c} - \sigma^2 \frac{(1 + \beta)(1 + c - \rho \beta)}{(1 - \beta + c) c} \right).$$

Then $\partial g(\rho, \beta, c) / \partial \rho = -\sigma^2 (1 + \beta) (-\beta) / (1 - \beta + c) c$. The denominator is always positive and the numerator is positive whenever $\beta \geq 0$. Therefore when $\beta \geq 0$, $\partial g(\rho) / \partial \rho \geq 0$. This implies that whenever $\partial g(\rho) / \partial \rho \geq 0$, $\partial f(g(\rho)) / \partial \rho \geq 0$, where $f(\cdot)$ is the square root function (monotone). Hence, $c(\mu - f(g(\rho))) / (1 + \beta + c)$ is decreasing in ρ whenever β is positive, and vice versa. *Q.E.D.*

For comparative statics in part (iii) (a) define

$$g(\rho, \beta, c) = \left((1 + \beta) \left(c_v^2 / c \right) + (\mu - c_v)^2 \right) - \sigma^2 \frac{(2\beta(1+c+\rho) + (1+c)(c(1-\rho) - (1+\rho)) - \beta^2(1+\rho))}{2(1-\beta)c((1+c)^2 - \beta^2)}.$$

Then $\partial g(\rho, \beta, c) / \partial \rho = \sigma^2 \left((1+c)^2 - 2\beta - \beta^2 \right) \geq 0$. Hence, $\partial c_{p-v}^* / \partial \rho \leq 0$.

For part (iii) (b), define the bracketed expression under the square root as

$$h(\rho, \beta, c) \equiv \left(\frac{1-\rho}{2(1-\beta)} - \frac{1+c-\beta\rho}{(1+c)^2 - \beta^2} \right).$$

Then,

$$\frac{\partial h(\rho, \beta, c)}{\partial \rho} = \left(\frac{-1}{2(1-\beta)} + \frac{\beta}{(1+c)^2 - \beta^2} \right).$$

This is negative iff

$$\beta^2 + 2\beta(1-\beta) \leq (1+c)^2.$$

The LHS of the inequality is concave in β with the maximum at $\beta = 1$, and the inequality holds true at $\beta = 1$. Hence, $\partial c_z^* / \partial \rho \leq 0$.

To show that $\partial c_z^* / \partial \beta \geq 0$, it is sufficient to show that $\partial h(\rho, \beta, c) / \partial \beta \geq 0$, where

$$\frac{\partial h(\rho, \beta, c)}{\partial \beta} = \frac{1-\rho}{2(1-\beta)^2} - \frac{2\beta(1+c)}{\left((1+c)^2 - \beta^2 \right)^2} + \frac{\rho \left((1+c)^2 + \beta^2 \right)}{\left((1+c)^2 - \beta^2 \right)^2}.$$

We prove that $\partial h(\rho, \beta, c) / \partial \beta$ is monotone in ρ (in the weak sense), either increasing or decreasing. This is accomplished in Lemma A.2 below. Hence, when $\partial h(\rho, \beta, c) / \partial \beta$ is monotone in ρ , then it suffices to check $h(\rho, \beta, c) / \partial \beta \geq 0$ for $\rho \in \{-1, 1\}$. This is done below.

We first show the result for $\rho = 1$.

$$\left. \frac{\partial h(\beta, c)}{\partial \beta} \right|_{\rho=1} = \frac{\left((1+c)^2 + \beta^2 \right) - 2\beta(1+c)}{\left((1+c)^2 - \beta^2 \right)^2}.$$

Clearly, for $\beta \leq 0$, $\partial h(\beta, c) / \partial \beta|_{\rho=1} \geq 0$. For $\beta \in [0, 1)$, the numerator is convex decreasing in β . Hence, the infimum for the numerator as $\beta \rightarrow 1$ is:

$$\lim_{\beta \rightarrow 1} \left(\left((1+c)^2 + \beta^2 \right) - 2\beta(1+c) \right) = c^2 \geq 0.$$

Next we verify the statement for $\rho = -1$. Then,

$$\left. \frac{\partial h(\beta, c)}{\partial \beta} \right|_{\rho=-1} = \frac{1}{(1-\beta)^2} - \frac{1}{(1-\beta+c)^2} \geq 0.$$

Hence, $\partial c_z^* / \partial \beta \geq 0 \forall \beta \in (-1, 1)$. *Q.E.D.*

Lastly, to show that $\partial c_z^*/\partial c \geq 0$, it suffices to show that $\partial(ch(\rho, \beta, c))/\partial c \geq 0$. This translates to showing $h(\rho, \beta, c) + c(\partial h(\rho, \beta, c)/\partial c) \geq 0$. Since $h(\rho, \beta, c) \geq 0$ (otherwise c_z^* is not defined) and $c \geq 0$, it is sufficient to show that $\partial h(\rho, \beta, c)/\partial c \geq 0$,

$$\partial h(\rho, \beta, c)/\partial c = \frac{\beta^2 + (1+c)^2 - 2\beta(1+c)\rho}{\left((1+c)^2 - \beta^2\right)^2}.$$

If $\beta \geq 0$, then $\partial h(\rho, \beta, c)/\partial c$ is decreasing in ρ . Since $\partial h(\rho, \beta, c)/\partial c|_{\rho=1} \geq 0$, $\partial h(\rho, \beta, c)/\partial c \geq 0 \forall \beta \geq 0$. If $\beta < 0$, then $\partial h(\rho, \beta, c)/\partial c$ is increasing in ρ . Again, since $\partial h(\rho, \beta, c)/\partial c|_{\rho=-1} \geq 0$, $\partial h(\rho, \beta, c)/\partial c \geq 0 \forall \beta < 0$. Hence, $\partial c_z^*/\partial c \geq 0 \forall \beta \in (-1, 1)$ and $\forall \rho \in [-1, 1]$. *Q.E.D.*

Lemma A.2. $\partial h(\rho, \beta, c)/\partial \beta$ is monotone in ρ in the weak sense.

Proof: Taking the cross-partial derivative w.r.t ρ , we obtain

$$H(\beta, c) \equiv \frac{\partial^2 h(\rho, \beta, c)}{\partial \beta \partial \rho} = \frac{-\left((1+c)^2 - \beta^2\right)^2 + 2(1-\beta)^2\left((1+c)^2 + \beta^2\right)}{2(1-\beta)^2\left((1+c)^2 - \beta^2\right)^2}.$$

The function $H(\beta, c)$ can be written as:

$$H(\beta, c) = \frac{1}{2} \left(\frac{1}{(1+c-\beta)^2} - \frac{1}{(1-\beta)^2} + \frac{1}{(1+c+\beta)^2} \right).$$

Define

$$G \equiv \frac{1}{(1+c-\beta)^2} + \frac{1}{(1+c+\beta)^2},$$

$$J \equiv -\frac{1}{(1-\beta)^2}.$$

Since G is symmetric about $\beta = 0$, it either increases or decreases around 0. We show that G increases in $|\beta|$. This is done by showing that $\partial G/\partial \beta > 0$ if $\beta > 0$ and $\partial G/\partial \beta < 0$ if $\beta < 0$.

Now clearly J is decreasing in β . Hence, for $\beta < 0$, $H = G + J$ is decreasing in β . We now show that for $\beta > 0$, $\partial H/\partial \beta < 0$.

$$\frac{\partial H}{\partial \beta} = 2 \left(\frac{1}{(1+c-\beta)^3} - \frac{1}{(1+c+\beta)^3} - \frac{1}{(1-\beta)^3} \right).$$

Also,

$$\frac{\partial^2 H}{\partial \beta \partial c} = 6 \left(\frac{-1}{(1+c-\beta)^4} + \frac{1}{(1+c+\beta)^4} \right) < 0 \text{ for } \beta > 0.$$

Hence, $(\partial H/\partial \beta)_{\max}|_{\beta>0} = -1/(1+\beta)^3 < 0$ when $c = 0$. Hence, $(\partial H/\partial \beta) < 0$ for $\beta > 0$.

Hence, the function $H(\beta, c)$ is monotone decreasing in β . The function is positive for $\beta < \beta^*(c)$ and negative for $\beta \geq \beta^*(c)$. Hence, $\partial h(\rho, \beta, c)/\partial \beta$ is monotone increasing in ρ for $\beta < \beta^*(c)$ and monotone

decreasing in ρ for $\beta \geq \beta^*(c)$.

Proof of part (iv)

First note that the cost thresholds c_p^* and c_v^* are convex in demand variance σ^2 . Due to the convexity of the cost thresholds, for high enough variance, the thresholds may increase to a point where $c_p^* = \mu$ or $c_v^* = (\mu c / (1 + \beta + c))$, the highest value that these cost thresholds can take (footnote 1 in the main paper). In this case the benefit of flexibility is so high that it is attractive at any feasible cost. We denote these threshold variances by $\tilde{\sigma}_v^2$ and $\tilde{\sigma}_p^2$ respectively for V and P technologies.

Now to prove part (iv), we parse correlation into ‘high’ (Figure 1 in the main paper), ‘medium’ (Figure 2 in the main paper) and ‘low’ (Figure 3 in the main paper). The following lemma is useful:

Lemma A.3: (i) High Correlation:

(a) $\partial c_v^* / \partial \sigma^2 \big|_{\sigma=0} \geq \partial c_p^* / \partial \sigma^2 \big|_{\sigma=0}$ iff $\rho \geq \rho^*(\beta, c)$ where

$$\rho^*(\beta, c) = \left(\frac{(1+c)^2 - \beta^2 - 2(1-\beta)(1+c)}{(1+c)^2 - \beta^2 - 2\beta(1-\beta)} \right),$$

and $\partial \rho^*(\beta, c) / \partial c \geq 0, \partial \rho^*(\beta, c) / \partial \beta \geq 0$.

(b) Moreover, $\rho \geq \rho^*(\beta, c) \Rightarrow c_z^*(\sigma, \rho, \beta, c) = 0$.

(c) $\partial c_v^* / \partial \sigma^2 \big|_{\sigma=0} \geq \partial c_p^* / \partial \sigma^2 \big|_{\sigma=0} \Rightarrow \tilde{\sigma}_v^2 \leq \tilde{\sigma}_p^2$.

(ii) *Medium Correlation*

$\hat{\rho}(\beta, c) \leq \rho \leq \rho^*(\beta, c) \Rightarrow \tilde{\sigma}_v^2 \leq \tilde{\sigma}_p^2$ where

$$\hat{\rho}(\beta, c) = \left(\frac{c(1-\beta+c) - 2(1-\beta)(1+c)}{c(1-\beta+c) - 2\beta(1-\beta)} \right).$$

(iii) *Low Correlation* $\rho \leq \hat{\rho}(\beta, c) \Rightarrow \tilde{\sigma}_v^2 \geq \tilde{\sigma}_p^2$.

Proof of Lemma A.3: Part (i): High Correlation.

To show part (a), note that

$$\begin{aligned} \frac{\partial c_p^*(\mu, \beta, \sigma, \rho, c_d)}{\partial \sigma^2} \bigg|_{\sigma=0} &= \frac{1}{2\mu} \left(\frac{1-\rho}{4\mu} \right) \left(\frac{1+\beta}{1-\beta} \right), \\ \frac{\partial c_v^*(\mu, \beta, \sigma, c, c_d)}{\partial \sigma^2} \bigg|_{\sigma=0} &= \frac{1}{2\mu} \left(\frac{(1+\beta)(1+c-\rho\beta)}{(1+c)^2 - \beta^2} \right). \end{aligned}$$

Comparing the two, we obtain $(\partial c_v^* / \partial \sigma^2) \big|_{\sigma=0} \geq (\partial c_p^* / \partial \sigma^2) \big|_{\sigma=0}$ whenever

$$\rho \geq \rho^*(\beta, c) = \frac{(1+c)^2 - \beta^2 - 2(1-\beta)(1+c)}{(1+c)^2 - \beta^2 - 2\beta(1-\beta)}.$$

Furthermore,

$\partial\rho^*(\beta, c)/\partial\beta = (2c(3 - \beta(2 + \beta)) + 4c - 2\beta c + c^2) / \left((1+c)^2 - \beta^2 - 2\beta(1-\beta) \right)^2 \geq 0$, since the denominator is always positive and the numerator is positive whenever $\beta \leq 0$. For $\beta > 0$, the minimum value of the numerator is at $\beta = 1$, which is positive. Finally, $\partial\rho^*/\partial c = \left(2(1-\beta) \left((1+c)^2 - \beta^2 - 2\beta c \right) \right) / \left((1+c)^2 - \beta^2 - 2\beta(1-\beta) \right)^2 \geq 0$.

To show part (b), notice that for $c_z^*(\sigma, \rho, \beta, c) \geq 0$, the argument inside the square-root must be positive. That is,

$$\left(\frac{1-\rho}{2(1-\beta)} - \frac{1+c-\beta\rho}{(1+c)^2 - \beta^2} \right) \geq 0.$$

Simplifying we obtain $c_z^*(\sigma, \rho, \beta, c) \geq 0 \Rightarrow \rho < \rho^*(\beta, c)$.

To show part (c), notice that $\tilde{\sigma}_v^2 = \mu^2 c(1-\beta+c) / ((1+c-\rho\beta)(1+\beta))$ and $\tilde{\sigma}_p^2 = (2\mu^2(1-\beta)) / ((1-\rho)(1+\beta))$. Hence, $\tilde{\sigma}_v^2 \leq \tilde{\sigma}_p^2$ if

$$c(1-\beta+c) - 2(1-\beta)(1+c) \leq \rho(c(1-\beta+c) - 2\beta(1-\beta)).$$

Define

$$\hat{\rho}(\beta, c) \equiv \frac{c(1-\beta+c) - 2(1-\beta)(1+c)}{c(1-\beta+c) - 2\beta(1-\beta)}.$$

Hence, if $(c(1-\beta+c) - 2\beta(1-\beta)) > 0$, we obtain $\tilde{\sigma}_v^2 \leq \tilde{\sigma}_p^2$ if $\rho \geq \hat{\rho}$ and if $(c(1-\beta+c) - 2\beta(1-\beta)) < 0$, we obtain $\tilde{\sigma}_v^2 \leq \tilde{\sigma}_p^2$ if $\rho \leq \hat{\rho}$. However, if $(c(1-\beta+c) - 2\beta(1-\beta)) < 0$ then $\hat{\rho} \geq 1$ hence $\tilde{\sigma}_v^2 \leq \tilde{\sigma}_p^2$ is always true in this case. To show that $\hat{\rho} \geq 1$, notice that $c(1-\beta+c) - 2\beta(1-\beta) < 0 \Rightarrow c(1-\beta+c) < 2\beta(1-\beta) < 2(1-\beta)(1+c)$.

Hence, $\tilde{\sigma}_v^2 \leq \tilde{\sigma}_p^2$ when

$$\rho \geq \frac{c(1-\beta+c) - 2(1-\beta)(1+c)}{c(1-\beta+c) - 2\beta(1-\beta)} = \hat{\rho}(\beta, c).$$

To compare the two thresholds, rewrite $\rho^*(\beta, c)$ and $\hat{\rho}(\beta, c)$ as:

$$\begin{aligned} \rho^*(\beta, c) &= \left(1 - \frac{2(1-\beta)(1+c-\beta)}{(1+c+\beta)(1+c-\beta) - 2\beta(1-\beta)} \right), \\ \hat{\rho}(\beta, c) &= \left(1 - \frac{2(1-\beta)(1+c-\beta)}{c(1-\beta+c) - 2\beta(1-\beta)} \right). \end{aligned}$$

Since $(1+c+\beta)(1+c-\beta) - 2\beta(1-\beta) \geq c(1-\beta+c) - 2\beta(1-\beta)$, $\rho^*(\beta, c) \geq \hat{\rho}(\beta, c)$.

Hence, $(\partial c_v^*/\partial \sigma^2)|_{\sigma=0} \geq (\partial c_p^*/\partial \sigma^2)|_{\sigma=0} \Rightarrow \rho \geq \rho^*(\beta, c) \Rightarrow \rho \geq \hat{\rho}(\beta, c) \Rightarrow \tilde{\sigma}_v^2 \leq \tilde{\sigma}_p^2$.

Part (ii) Medium Correlation and Part (iii) Low Correlation.

Follows from part (c) above.

Q.E.D.

Part (iv) of Proposition follows from part (i) of Lemma A.3. Moreover, Figures 1, 2 and 3 in the main paper are based on parts (i), (ii) and (iii) of Lemma A.3 above.

■

Proof of Proposition 4

Proof. For this proposition, only the differences in the stochastic terms play a role. Hence,

$$\begin{aligned}\Delta_{p-d} &= \frac{\sigma^2(1-\rho)}{4(1-\beta)}, \\ \Delta_{v-d} &= \left(\frac{\sigma^2((1+c)-\beta\rho)}{2((1+c)^2-\beta^2)} \right), \text{ and} \\ \Delta_{p-v} &= \frac{\sigma^2}{4} \left(\frac{1-\rho}{(1-\beta)} - \frac{2(c+1-\rho\beta)}{((c+1)^2-\beta^2)} \right).\end{aligned}$$

Proof of part (i). The total variance is $\sigma_T^2 = 2\sigma^2 + 2\rho\sigma^2$. Hence,

$\partial\Delta_{x-y}/\partial(\sigma_T^2) = (\partial\Delta_{vp-p}/\partial\rho)(\partial\rho/\partial((\sigma_T^2)))$. The result follows from straightforward algebraic manipulation. To prove that $\partial\Delta_{p-v}/\partial\sigma_T^2 < 0$, note that the denominator is positive and the numerator can be rewritten as $-(1-\beta)^2 - c^2 - 2c \leq 0$.

Proof of part (ii). We keep the total variance fixed when we change σ by changing ρ accordingly. Hence, treating ρ as $\rho(\sigma^2)$, we have

$$\begin{aligned}\frac{\partial\sigma_T^2}{\partial(\sigma^2)} &= 2 + 2\rho(\sigma^2) + 2\sigma^2\frac{d\rho(\sigma^2)}{d(\sigma^2)} = 0 \\ \Rightarrow \frac{d\rho(\sigma^2)}{d\sigma^2} &= -\left(\frac{(1+\rho(\sigma^2))}{\sigma^2}\right).\end{aligned}\tag{8}$$

Now

$$\begin{aligned}\frac{\partial\Delta_{p-d}}{\partial(\sigma^2)} &= \frac{(1-\rho)}{4(1-\beta)} + \frac{(1+\rho)}{4(1-\beta)} = \frac{1}{2(1-\beta)}, \\ \frac{\partial\Delta_{v-d}}{\partial\sigma^2} &= \frac{((1+c)-\beta\rho)}{2((1+c)^2-\beta^2)} + \frac{(\beta(1+\rho))}{2((1+c)^2-\beta^2)} = \frac{(1+c+\beta)}{2((1+c)^2-\beta^2)}, \\ \frac{\partial\Delta_{p-v}}{\partial\sigma^2} &= \frac{1}{4} \left(\frac{1-\rho}{(1-\beta)} - \frac{2(c+1-\rho\beta)}{((c+1)^2-\beta^2)} \right) + \frac{\sigma^2}{4} \left(\frac{\left(\frac{(1+\rho)}{\sigma^2}\right)}{1-\beta} - \frac{2\beta\left(\frac{(1+\rho)}{\sigma^2}\right)}{((c+1)^2-\beta^2)} \right) = \frac{c}{2(1-\beta)(c+1-\beta)}.\end{aligned}$$

■

Proof of Proposition 5

Proof. For *VP* technology the firm maximizes the following objective function ex post demand realization:

$$\pi_{vp} = (A_1 - q_{1vp} - \beta q_{2vp})q_{1vp} + (A_2 - q_{2vp} - \beta q_{1vp})q_{2vp} - c(K_{vp} - (q_{1vp} + q_{2vp}))^2.$$

The objective function can be verified to be globally concave and the first-order conditions are

$$A_y - 2q_{yvp} - 2\beta q_{(3-y)vp} + 2c \left(K_{vp} - (q_{yvp} + q_{(3-y)vp}) \right) = 0, \quad y = 1, 2.$$

After solving these two equations simultaneously we obtain

$$q_{yvp}^* = \frac{A_y(1+c) - A_{3-y}(\beta+c) + 2cK_{vp}(1-\beta)}{2(1-\beta^2 + 2c(1-\beta))}. \quad (9)$$

We ignore the case when $A_{3-y} \gg A_y$, $y = 1, 2$ which would result in $q_{yvp}^* < 0$ and instead assume that the realizations of (A_1, A_2) are such that the firm manufactures both products (the exact technical condition is detailed in Lemma A.4 below). Recall that the expected profit from equation (1) in the main paper is

$$\Pi_{vp} = \max_{K_{vp}} E(\pi_{vp}) - c_{vp}K_{vp}.$$

We can substitute q_{yvp}^* into π_{vp} and take expectation by noting that $E(A_i^2) = \mu_i^2 + \sigma_i^2$ and $E(A_i A_j) = \mu_i \mu_j + \sigma_{ij}$ to obtain

$$\begin{aligned} \Pi_{vp} = & \frac{1}{4(1-\beta)(1+\beta+2c)} \left((1+c) (\mu_1^2 + \mu_2^2 + \sigma_1^2 + \sigma_2^2) - 2(c+\beta)(\mu_1\mu_2 + \sigma_{12}) \right. \\ & \left. + 4(1-\beta)(\mu_1 + \mu_2)cK_{vp} - 4cK_{vp}^2 \right) - c_{vp}K_{vp}, \end{aligned}$$

from which the optimal capacity K_{vp}^* and then Π_{vp}^* can be obtained.

Lemma A.4: Under symmetry, the quantities are always positive for both products iff the following conditions hold on the support of the distribution of the demand intercepts:

$$|A_1 - A_2| \leq \frac{2c}{1+c} \frac{(1-\beta)}{(1+\beta)} \left(\mu - \frac{c_{vp}}{2c} - c_{vp} \right).$$

Proof: From equation (9), for $q_{yvp}^* \geq 0$ it suffices to show that

$$A_y(1+c) - A_{3-y}(\beta+c) \geq -2cK_{vp}(1-\beta).$$

The LHS of the inequality is decreasing in β . Hence, it suffices to prove the following inequality (obtained by putting $\beta = 1$ in the LHS of the above inequality):

$$A_{3-y} - A_y \leq \frac{2cK_{vp}(1-\beta)}{(1+c)}.$$

Using arguments similar to Goyal and Netessine (Lemma EC.1), it can be shown that K_{vp}^* (from Proposition 2) is less than the optimal unconstrained capacity without invoking the bound. Hence, it suffices to check the above inequality at K_{vp}^* . We obtain

$$A_{3-y} - A_y \leq \frac{2c(1-\beta)}{(1+c)(1+\beta)} \left(\mu - \frac{c_{vp}(1+\beta)}{2c} - c_{vp} \right) \quad (10)$$

and the result follows.

The reverse direction is straightforward to prove as well. Suppose inequality (10) holds. Then, it must be true that the inequality would hold for any capacity $K_{vp} \geq K_{vp}^*$. Reverse the sequence of arguments and the result follows. ■

Proof of Proposition 6.

Proof. For this proposition, only the difference in the stochastic term plays a role. Hence, Δ_{vp-p} can be written as:

$$\Delta_{vp-p} = \frac{\sigma^2}{4} \left(\frac{2(c+1)}{(1-\beta)(1+\beta+2c)} - \frac{1}{(1-\beta)} \right) + \frac{\rho\sigma^2}{4} \left(\frac{1}{(1-\beta)} - \frac{2(\beta+c)}{(1-\beta)(1+\beta+2c)} \right),$$

while Δ_{vp-v} can be written as

$$\Delta_{vp-v} = \frac{\sigma^2}{2} \left(\frac{(1+c) - (\beta+c)\rho}{(1-\beta)(1+\beta+2c)} - \frac{1+c-\beta\rho}{((1+c)^2 - \beta^2)} \right).$$

(i) The total variance is $\sigma_T^2 = 2\sigma^2 + 2\rho\sigma^2$. Hence, $\partial\Delta_{vp-p}/\partial(\sigma_T^2) = (\partial\Delta_{vp-p}/\partial\rho) (\partial\rho/\partial((\sigma_T^2)))$.

The result follows from straightforward algebraic manipulation. Similarly

$$\begin{aligned} \partial\Delta_{vp-v}/\partial(\sigma_T^2) &= \frac{1}{4} \left(\frac{-(\beta+c)}{(1-\beta)(1+\beta+2c)} + \frac{\beta}{((1+c)^2 - \beta^2)} \right) \\ &= \frac{1}{4} \left(\frac{-c(\beta^2 + \beta c + (1+c)^2)}{(1-\beta)(1+\beta+2c)((1+c)^2 - \beta^2)} \right) \leq 0. \end{aligned}$$

(ii) We keep the total variance fixed when we change σ by changing ρ accordingly. Using equation (8) we obtain

$$\begin{aligned} \frac{\partial\Delta_{vp-p}}{\partial\sigma} &= \left(\frac{1}{4} \left(\frac{2(c+1)}{(1-\beta)(1+\beta+2c)} - \frac{1}{(1-\beta)} \right) \right. \\ &\quad \left. + \frac{1}{4} \left(\frac{1}{(1-\beta)} - \frac{2(\beta+c)}{(1-\beta)(1+\beta+2c)} \right) \left(\rho(\sigma^2) + \sigma^2 \left(- \left(\frac{(1+\rho(\sigma^2))}{\sigma^2} \right) \right) \right) \right) \\ &= \frac{1}{4} \left(\frac{2(c+1)}{(1-\beta)(1+\beta+2c)} - \frac{2}{(1-\beta)} + \frac{2(\beta+c)}{(1-\beta)(1+\beta+2c)} \right) \\ &= \frac{1}{2} \left(\frac{(c+1) + \beta + c}{(1-\beta)(1+\beta+2c)} - \frac{1}{(1-\beta)} \right) = 0. \end{aligned}$$

Similarly for Δ_{vp-v} we obtain

$$\begin{aligned}
\frac{\partial \Delta_{vp-v}}{\partial \sigma^2} &= \frac{1}{2} \left(\frac{(1+c) - (\beta+c)\rho(\sigma)}{(1-\beta)(1+\beta+2c)} - \frac{1+c-\beta\rho(\sigma)}{((1+c)^2-\beta^2)} \right) \\
&\quad + \frac{\sigma^2}{2} \left(\frac{(\beta+c) \left(\frac{(1+\rho(\sigma^2))}{\sigma^2} \right)}{(1-\beta)(1+\beta+2c)} - \frac{\beta \left(\frac{(1+\rho(\sigma^2))}{\sigma^2} \right)}{((1+c)^2-\beta^2)} \right) \\
&= \frac{1}{2} \left(\frac{(1+\beta+2c)}{(1-\beta)(1+\beta+2c)} - \frac{1+c+\beta}{((1+c)^2-\beta^2)} \right) \\
&= \frac{1}{2} \left(\frac{c}{(1-\beta)(1+c-\beta)} \right) \geq 0.
\end{aligned}$$

■

Technical Appendix Part (B): Numerical Study

At the heart of the difference between volume and product flexibility is the way demand correlation (ρ) and product substitutability parameter (β) interact for volume flexibility. To be specific, we find that demand correlation has no impact on volume flexibility whenever $\beta = 0$. For $\beta \neq 0$, the impact on profit for volume flexibility is most favorable whenever ρ and β are of the opposite signs: positive correlation resonates best with negative β (complementary products), while negative correlation resonates best with positive β (substitutable products). In contrast, with product flexibility, correlation has an impact even when $\beta = 0$, with negative correlation *always* preferred over positive correlation.

In this section, we (numerically) show that the above fundamentals remain intact when we relax the two key assumptions of our model: (i) We allow the firm with P technology to produce below capacity (holdback); (ii) We allow for lower cost of decreasing production compared to the cost of increasing production for the firm with V technology⁴.

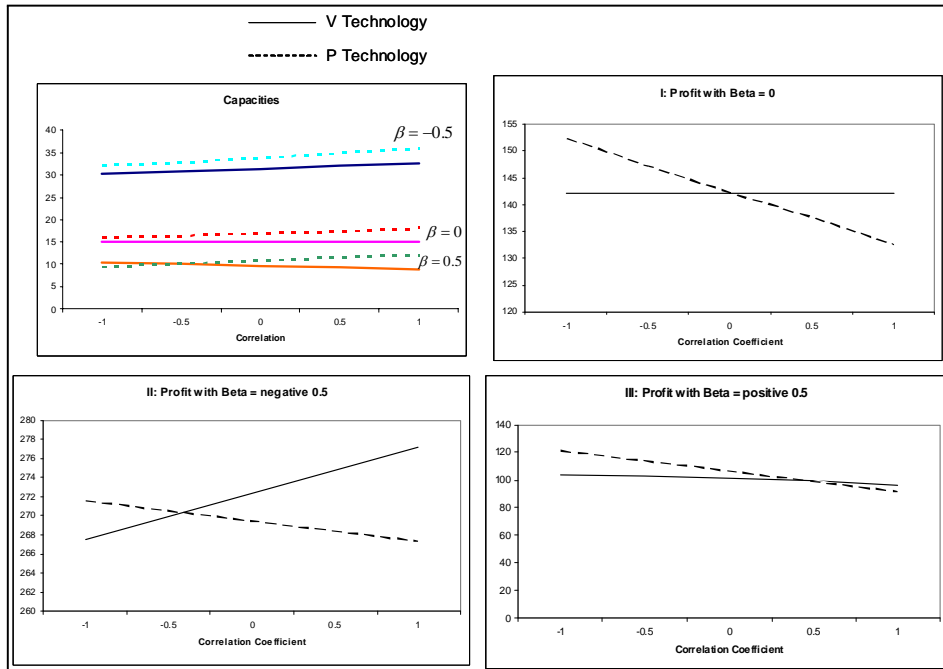


Figure A.5: Capacities and Profits for V and P Technologies as a function of β and ρ .

For the following study, the random demand intercepts are drawn from a Bivariate Normal Distribution with a mean of 50 and standard deviation of 7. The cost of capacity is assumed the same for P

⁴While we conduct a number of numerical experiments to verify the robustness of our results, we only include a small subset of key experiments below.

and V technology, i.e., $c_v = c_p = 4$. The frictional cost of increasing production (c_H) for V technology is 2, while there is no cost for decreasing production ($c_L = 0$). For P technology, there is no cost to decreasing production (costless holdback) with no possibility of increasing production. We conduct the study for $\beta \in [-0.5, 0.5]$ while $\rho \in [-1, 1]$.

We uncover the interplay between ρ and β in two ways: (i) by looking at the optimal capacity and (ii) by looking at optimal expected profits. We begin with capacity first.

Figure A.5 plots optimal capacities (top-left) as a function of ρ for various values of β . Capacities now depend on ρ since holdback is allowed for P technology, and volume flexibility has unequal costs of upside and downside capacity adjustments. The following trends are evident from the figure. First, for $\beta = 0$, correlation has *no impact* on the capacity for V technology, while the capacity for P technology increases (the latter is also known from Chod and Rudi, 2005). This reinforces the analytical finding that with $\beta = 0$, correlation has no impact on V technology as there is no conduit for redistributing capacity ex post. Second, as β increases, capacities for both technologies decrease. However, in (almost) all cases, the capacity for P technology is higher than that of V technology because V technology can increase capacity ex post demand realization (Proposition 2)⁵. Third, while the capacity for P technology *always* increases with demand correlation, the capacity for V technology increases with demand correlation only when β is negative while decreasing with correlation when β is positive.

A similar, and equally compelling, story emerges when we compare profits. This is done in Figures A.5, I-III. The profit for V technology is *flat and invariant* to correlation when $\beta = 0$ (Figure A.5-I), even as the profit for P technology always decreases with correlation. However, the profit of V technology increases with correlation for complementary products ($\beta = -0.5$, Figure A.5-II) and decreases with correlation (i.e., increases as correlation becomes negative) for substitutable products ($\beta = 0.5$, Figure A.5-III)⁶. Moreover, the profit for V technology clearly dominates the profits for P technology for high positive correlation, while the profit for P technology is higher for negative correlation. This reiterates the discussion on Figures 1-3 in the main paper.

Similar trends are observed for other parameter values, which reinforces the fundamental analytical insight on the interplay between ρ and β on the profit of V technology, which is very distinct from the impact of these parameters on the profit of P technology.

As noted earlier, since this fundamental tradeoff is at the core of the analytical insights, the numerical

⁵The exception being $\beta = 0.5$ and $\rho \approx -1$. In this range of parameter values, product-flexibility is very potent and requires comparatively low capacity investment.

⁶The deterministic component of the profit decreases as $\beta \rightarrow 1$. Hence, the effect is less noticeable for $\beta > 0$ and $\rho \rightarrow -1$ since the increase in the stochastic component is offset by the decrease in the deterministic component. The outcome is starker when only the stochastic component is plotted, but we show the more conservative results here.

study serves to provide the necessary robustness to the analytical insights even when some fundamental assumptions are relaxed.