Supplement to
Can We All Get Along?
Incentive Contracts to Bridge the Marketing and Operations Divide

S.1 Introduction

In this supplement, we provide the detailed analyses for the extensions of the basic model to the case when agents are risk averse (in Section S.2) and to the case when the firm is selling two products (in Section S.3). For the purpose of making this a coherent document, we allow considerable redundancy between the main document and this supplement.

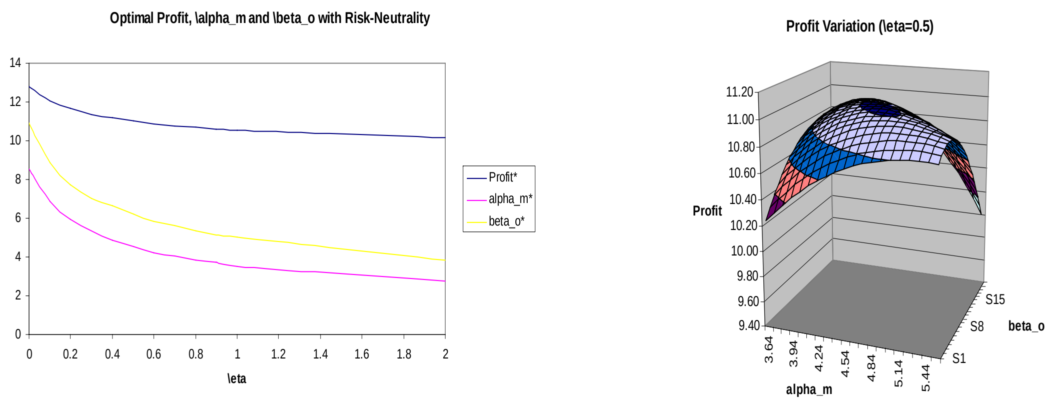
S.2 Risk-Averse Agents

The literature on salesforce compensation has typically assumed agents to be risk averse and the principal to be risk neutral. In our basic model, we assume the agents to be risk neutral. In this section, we show that assuming agents to be risk averse does not change the insights from the basic model – risk aversion affects compensation contracts exactly (and only) in the manner proposed in the literature and is an independent driver of the characteristics of compensation contracts.

A risk-averse agent does not like uncertainty in the outcome and her actions will therefore be sub-optimal from the point-of-view of a risk-neutral firm. When the firm employs this agent, it has to give her some ownership in the outcome (the commission) to motivate effort from her and has to pay her a risk premium. (Due to the presence of the risk premium, the firm cannot achieve the first-best profit.) The expected profit of the firm, after adjusting for compensation paid, has an inverted-U shape as commission rate increases.[17] Furthermore, the more risk-averse the agent is, the greater is the divergence between the optimal actions of the firm and the agent, and the lesser is the optimal commission rate and the optimal expected profit of the firm. In this section, we replicate the above results for the coordinating contracts for one case; the results are qualitatively the same for all other cases.

We assume that the firm is risk-neutral and both the agents are risk-averse with expected mean-variance utility $E[\mathcal{U}(X)] = E[X] - \frac{1}{2}\eta Var[X]$ where η is the risk-aversion parameter. A higher η implies a more risk-averse agent. We consider the simplest interdependent contracts, i.e., the sales manager has the contract $S_m = w_m + \alpha_m Y$ and the operations manager has the contract $S_o = w_o + \beta_o Y$. Given their contracts, the sales manager and the operations manager

[17] If the commission rate is small, the agent does not exert sufficient effort but the risk premium is small. As the commission rate increases, the agent exerts more effort leading to a better expected outcome, but the firm also has to pay her a higher risk premium. Hence, the optimal point occurs at an intermediate value of commission.



(a) Profit variation with η

(b) Profit variation with α_m and β_o (fixed η)

Figure S.1: Figure (a) shows that optimal profit and optimal commission rates decrease with η (coefficient of risk aversion). Figure (b) shows variation of profit with α_m and β_o for fixed $\eta = 0.5$. All graphs are for the parameter values $r = 10, c_u = c_b = 3, k_o = 2, \sigma = 1$.

simultaneously solve for A^* and Q^*:

$$
\begin{aligned}
A^* &= \arg\max_A E\left[\mathcal{U}\left(w_m + \alpha_m Y - \frac{1}{2}A^2\right)\Big|A, Q^*\right] \\
&= \arg\max_A w_m + \alpha_m E\left[Y\Big|A, Q^*\right] - \frac{1}{2}A^2 - \frac{1}{2}\eta\alpha_m^2 Var\left[Y\Big|A, Q^*\right] \\
Q^* &= \arg\max_Q E\left[\mathcal{U}\left(w_o + \beta_o Y - \frac{1}{2}k_o Q^2\right)\Big|A^*, Q\right] \\
&= \arg\max_Q w_o + \beta_o E\left[Y\Big|A^*, Q\right] - \frac{1}{2}k_o Q^2 - \frac{1}{2}\eta\beta_o^2 Var\left[Y\Big|A^*, Q\right]
\end{aligned}
$$

Anticipating these values of A^* and Q^* in terms of the contract parameters, the firm solves for the optimal contracts S_m and S_o to maximize its own profits:

$$
\max_{\{S_m, S_o\}} E\left[\Pi_f|Q^*, A^*\right] = rE\left[Y|Q^*, A^*\right] - E\left[C|Q^*, A^*\right] - E\left[S_m|Q^*, A^*\right] - E\left[S_o|Q^*, A^*\right]
$$

$$
\begin{aligned}
\text{such that: } &\max_A E\left[\mathcal{U}\left(w_m + \alpha_m Y - \frac{1}{2}A^2\right)\Big|A, Q^*\right] \geq 0 \\
&\max_Q E\left[\mathcal{U}\left(w_o + \beta_o Y - \frac{1}{2}k_o Q^2\right)\Big|A^*, Q\right] \geq 0
\end{aligned}
$$

The analytical solution to the above problem is intractable, so we resort to a numerical simulation. The result for one case ($r = 10, c_u = c_b = 3, k_o = 2, \sigma = 1$) is shown in Figure S.1. Figure S1(a) shows the trends with increasing η. (The profit with $\eta = 0$ is the first-best profit.) It is

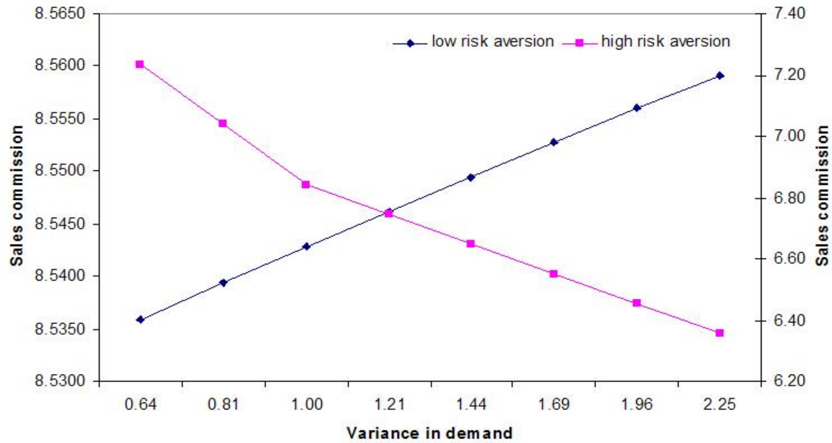


Figure S.2: The figure shows how the sales commission for the sales manager varies with variance in demand for different risk aversion tendencies of the sales manager. The above plots are for the parameter values $r = 10, c_u = c_b = 3, k_o = 2$. The low-risk-aversion graph (left vertical axis) is for $\eta = 0.001$ and the high-risk-aversion graph (right vertical axis) is for $\eta = 0.1$.

clear from the plot that, as predicted by classical theory, as the agents become more and more risk averse: (1) the optimal profit that the firm can achieve decreases, and (2) the optimal reward rates for sales decrease for both agents. Figure S1(b) shows, for $\eta = 0.5$, the variation in firm profit as α_m and β_o increase. Along both these axes, the profit has an inverted-U shape, as predicted by classical theory, and firm profits are maximized at intermediate values of α_m and β_o (i.e. these values are less than the values when agents are risk neutral, but larger than zero).

In our model with risk-neutral agents, coordination between sales and operations leads to the result that as the uncertainty in the outcome increases the optimal commission rate increases. In reality, one would expect both forces (risk-aversion and coordination issues) to work simultaneously, and the observed direction of change will be determined by the one that dominates. Figure S.2 shows an example of exactly this effect. When the agents are only slightly risk-averse ($\eta = 0.001$) the commission rate increases with increasing variance in demand because the coordination effect is stronger, while when the agents are more risk-averse ($\eta = 0.1$) the commission rate decreases with increasing variance in demand because the risk-aversion effect is stronger. Lastly, note that with risk-averse agents, the firm cannot achieve the first-best outcome under delegation. The issue then is not whether coordination is achieved, but which contract gets closer to the first-best outcome. We find that, consistent with the results of the basic model, interdependent contracts and information-based contracts get closer to the first-best outcome as compared to traditional cost-based contracts.

S.3 Model for Two Products

When a firm carries more than one product, the distortion in incentives that arises by contracting on total cost with the operations manager discussed in the one-product case is present, and can be corrected in the same way. However, we see another distortion, the source of which is that one manager now has to allocate selling or supplying effort among two or more products. This allocation will be suboptimal from the point of view of the firm if incentives are not properly aligned. In this supplement, we study this distortion and show how it leads a firm to allocate a larger part of its workforce to either its sales arm or its operations arm.

We extend our notation used in the one-product case by adding 1 and 2 as subscripts to the symbols to distinguish between the two products. The only change is in the demand system, since the demand for the two products can now be related. We assume that effort exerted to promote one product has an adverse effect on the demand for the other product. We incorporate this by using the following demand system:

$$D_i(A_i, A_j) = A_i - \theta A_j + \mu_i + \varepsilon_i, j = 3 - i, \tag{S.1}$$
$$\text{where } 0 \leq \theta < 1.$$

Our analysis for the two-product case proceeds as follows. First, we show that interdependent contracts and information-based contracts can always coordinate the marketing and operations arms of the firm and achieve the first-best outcome. The coordinating contracts, however, are quite complex, and can be difficult to implement. Compensation schemes observed in reality are much simpler. For practical considerations, we build simplifying assumptions into the incentive contracts, leading to distortions in the managers' incentives due to multitasking, and show that the firm's attempts to mitigate these distortions leads to interesting implications for the choice of workforce allocation.

S.3.1 Coordinating contracts

Consider the case when every product has its own sales manager and its own operations manager. We showed in the one-product case that if the firm contracts with the operations manager on total inventory cost, it cannot always achieve the first-best outcome. Clearly, this distortion will be seen in the two-product case as well. We also showed for the one-product case that if the firm contracts with the operations manager on sales alone or backorders alone, then it can always achieve coordination. In the two-product case, if the demand for the products is unrelated, it is easy to see that this result will also hold. We now briefly show that even if the demand is interdependent (as in (S.1)) the firm can achieve coordination by using either interdependent contracts or information-based contracts. Further, contracting with the operations manager only on sales (the simplest interdependent contract) or only on backorders (the simplest information-based contract) can also achieve the first-best outcome for the firm.

Given contracts $S_{m1}, S_{m2}, S_{o1}, S_{o2}$ the managers solve the following problems simultaneously to obtain $A_1^*, A_2^*, Q_1^*, Q_2^*$:

$$A_i^* = \arg\max_{A_i} E\left[\Pi_{mi}|Q_i^*, A_i, A_j^*\right] = E\left[S_{mi}|Q_i^*, A_i, A_j^*\right] - V_{mi}(A_i) \ , i = 1, 2$$
$$Q_i^* = \arg\max_{Q_i} E\left[\Pi_{oi}|Q_i, A_i^*, A_j^*\right] = E\left[S_{oi}|Q_i, A_i^*, A_j^*\right] - E\left[V_{oi}(\chi_i\zeta_i Q_i)|Q_i\right] \ , i = 1, 2. \quad \text{(S.2)}$$

Given these values in terms of the contract parameters, the firm solves for optimal values of the contract parameters to maximize its own profits:

$$\max \sum_{i=1,2} r_i E\left[Y_i|Q_i^*, A_1^*, A_2^*\right] - E\left[C_i|Q_i^*, A_1^*, A_2^*\right] - E\left[S_{mi}|Q_i^*, A_1^*, A_2^*\right] - E\left[S_{oi}|Q_i^*, A_1^*, A_2^*\right]$$
$$\text{such that: } \max_{A_i} E\left[S_{mi}|Q_i^*, A_i, A_j^*\right] - V_{mi}(A_i) \ , i = 1, 2$$
$$\max_{Q_i} E\left[S_{oi}|Q_i, A_i^*, A_j^*\right] - E\left[V_{oi}(\chi_i\zeta_i Q_i)|Q_i\right] \ , i = 1, 2. \quad \text{(S.3)}$$

The first-best problem for the firm (when it dictates the actions of the managers) is:

$$\max_{\{A_1, A_2, Q_1, Q_2\}} \sum_{i=1,2} r_i EY_i(Q_i, A_1, A_2) - EC_i(Q_i, A_1, A_2) - V_{mi}(A_i) - EV_{oi}(\chi_i\zeta_i Q_i)$$

which can be written as

$$\max_{\{A_1, A_2, Q_1, Q_2\}} \sum_{i=1,2} \Big[(r_i + c_{ui} + c_{bi})E\min\{Q_i, D_i(A_1, A_2)\}$$
$$- (c_{oi} + c_{ui})Q_i - c_{bi}ED_i(A_1, A_2) - V_{mi}(A_i) - EV_{oi}(\chi_i\zeta_i Q_i)\Big]. \quad \text{(S.4)}$$

Note that the above is independent of the contract parameters. We assume that the optimal solution to the first-best problem is unique, and denote it by $\{A_1^{FB}, A_2^{FB}, Q_1^{FB}, Q_2^{FB}\}$[18].

Now, when the firm does not dictate the actions of the managers, suppose the sales managers are offered the contracts $S_{mi} = w_{mi} + \alpha_i Y_i$ and the operations managers are offered the contracts $S_{oi} = w_{oi} - \alpha_{oi} C_i + \beta_{oi} Y_i$ (i.e., interdependent contracts). We obtain the following proposition.

Proposition S.1 *Suppose the firm sells two heterogeneous products with demand specified in (S.1) and interdependent contracts specified above. Then, the firm can always determine contracts to achieve the first-best solution. The simplest interdependent contract that achieves the first-best outcome compensates the operations managers only on the sales for their respective products.*

Proof Refer to Appendix S.I. ■

[18]The first-best solution is straightforward to compute using the first-order conditions, but is cumbersome. Except for the effect of the parameter θ, this solution is otherwise similar to the solution of the one-product case. The Hessian determines the conditions under which the first-best solution is unique. See Appendix S.III for the first-best solution in a simplified case.

As in the one-product case, the firm can also achieve coordination using information-based contracts, with the simplest contract compensating the operations managers only on backorders.

We have assumed above that, for each product, the firm employs separate managers for sales and stocking. Suppose now that the managers have to multitask, i.e., one sales manager manages both products and/or one operations manager manages both products, and the firm uses interdependent contracts or information-based contracts for the operations manager. The firm can still structure the contracts to achieve the first-best outcome, but the coordinating contracts in the most general case are complex, in one of two ways:

- the contracts have a separate variable component for each product, or
- the performance metrics contracted upon are a specific weighted combination of the metrics for each product. (When sales are contracted upon, these weights may not simply be the retail prices, and when backorders are contracted upon, these weights may not simply be the backorder costs for the products.) In other words, contracting on simple aggregate measures like total dollar sales or total backorder costs cannot achieve the first-best outcome.

We provide a mathematical sketch for the above argument in Appendix S.II.

Comparative statics: The comparative statics for the different optimal contract parameters remain the same as in the one-product case with respect to all parameters. However, we have one new parameter θ denoting the degree to which sales effort for one product hurts the demand for the other product. To focus on the effect of θ, we assume that the two products are identical in every respect. Then, as θ increases, the first-best levels of sales effort as well as stocking quantity decrease. The optimal reward for sales for the sales manager decreases with θ when a separate manager sells each product but increases with θ when one manager sells both products (otherwise, as θ increases, he will exert suboptimal effort). For the operations manager, the reward for sales (under the simplest interdependent contract) and the penalty for backorders (under the simplest information-based contract) decrease with increasing θ.[19] See Appendix S.III for details.

S.3.2 Simple contracts and implications for organization design

We saw in the previous section that interdependent contracts and information-based contracts can help the firm incentivize the managers to achieve the first-best outcome. The above theoretical result, as we argued, requires one of three conditions to hold:

1. Every product has its own sales manager and operations manager, so that one manager does only one job (e.g., selling her product or stocking his product).
2. If one manager manages both products (i.e., the sales manager sells both products, or the operations manager supplies both products), her/his compensation plan needs to have a separate variable component for each product.

[19] All of these results depend upon the assumption that $(r - \mu)(r + c_u + c_b) + \sigma(r + c_u - c_b) > 0$. Since μ (mean demand when sales effort is zero) is small and $r > c_b$, this assumption holds.

3. If one manager manages both products and the compensation plan for each manager includes only one variable component, the performance metric contracted upon is a specific weighted function of the sales or cost metrics for the two products, where the weights, in general, differ from retail prices for sales metrics and costs for cost metrics.

In the real world, we usually observe that even when one person manages more than one product, the compensation plan has a single variable component, and the measures contracted upon to determine this variable component are simple measures like total dollar value of sales achieved. This is contrary to conditions two and three above. We restrict ourselves to contracts on sales only for both managers (information-based contracts can achieve the same outcomes, so the choice does not matter). This already removes the distortion due to contracting on total costs discussed earlier. We make the following assumptions:

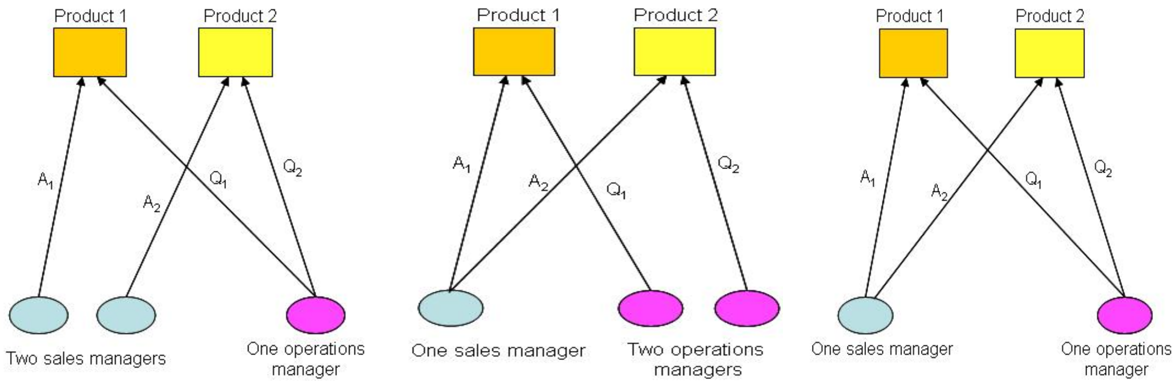
1. The firm wants to employ as few managers as possible, but not go "too far" from the first-best solution.

2. The firm wants to keep the contracts simple. This means that (1) for every manager there should only be one variable component in the salary, and (2) the performance metric that determines this variable component should be a simple aggregate metric like total dollar sales of the two products.

Under these simple contracts, the firm cannot achieve the first-best outcome in the most general case. One agent makes more than one decision, and from the perspective of the firm, there is an optimal level for each decision. However, under the simple contract offered, the incentives of the managers are misaligned – the effort allocation between the two products that best pays her/him does not best pay the firm. This distortion, tied to the phenomenon of multitasking, is the problem identified in Baker (2000).

What should the firm do to alleviate this problem? Should it always go with one manager for every job? Clearly, we do not see that in the real world and often observe that one employee manages more than one product. What we do see, however, is that firms usually have either a "sales image," i.e., they focus on the sales side and have more employees working as sales managers than employees managing operations, or they have a "cost image," i.e., they focus on the costs side and have more employees managing operations. As an example, compare chain-store retailers like Neiman Marcus and Wal-mart. A visit to Wal-mart makes it quite evident that in-store employees are more focused towards managing inventory and less towards providing sales-enhancing customer service. Neiman Marcus, on the other hand, appears to employ several sales managers at its retail outlets who work closely with customers to try to sell more to them. To explain these observations, we propose that the firm can opt for one of the following three forms of workforce allocation:

1. *Sales-focused organization:* Under this organization scheme, the firm opts for more flexibility on the sales side and has two sales managers (one for each product) and one operations manager. The sales managers decide the sales efforts for their respective products and are



(a) Sales-focused organization (b) Operations-focused organization (c) Minimal organization

Figure S.3: Schematic representation of the sales-focused, operations-focused and minimal organizations.

rewarded on the dollar sales achieved for their respective products. The operations manager jointly decides the quantity supplied for each product and is rewarded on the total dollar sales achieved for the two products. Figure 3(a) shows a schematic representation of this design. Formally, the firm offers each sales manager the contract $S_{mi} = w_{mi} + \alpha_{mi} r_i Y_i$ and the operations manager the contract $S_o = w_o + \beta_o(r_1 Y_1 + r_2 Y_2)$. In the backward induction solution of the associated game, the three managers, given their contracts, simultaneously determine the four quantities $A_1^*, A_2^*, Q_1^*, Q_2^*$ by maximizing their utilities. Given these quantities, the firm determines the parameters $w_{m1}, w_{m2}, \alpha_{m1}, \alpha_{m2}, w_o$ and β_o.

2. *Operations-focused organization:* Under this organization scheme, the firm opts for more flexibility on the operations side and has two operations managers (one for each product) and one sales manager. The operations managers decide the stocking levels for their respective products and are rewarded on the dollar sales achieved for their respective products. The sales manager jointly decides the sales effort for each product and is rewarded on the total dollar sales achieved for the two products. Figure 3(b) shows a schematic representation of this design. Formally, the firm offers the sales manager the contract $S_m = w_m + \alpha_m(r_1 Y_1 + r_1 Y_2)$ and the operations managers the contracts $S_{oi} = w_{oi} + \beta_{oi} r_i Y_i$. The associated game proceeds in the same manner as above.

3. *Minimal organization:* Under this organization scheme, the firm has one sales manager and one operations manager. The sales manager jointly decides the sales effort for each product and is rewarded on the total dollar sales achieved for the two products. The operations manager jointly decides the quantity supplied for each product and is rewarded on the total dollar sales achieved for the two products. Figure 3(c) shows a schematic representation of this design. Formally, the firm offers the sales manager the contract $S_m = w_m + \alpha_m(r_1 Y_1 + r_2 Y_2)$ and the operations manager the contract $S_o = w_o + \beta_o(r_1 Y_1 + r_2 Y_2)$. The associated game proceeds in the same manner as above.

Under a sales-focused allocation, the sales levels are close to first-best, but both stocking quantities are distorted away from first-best – one product is supplied higher than the optimal level and the other is supplied lower than the optimal level. Under an operations-focused allocation, the sales are distorted in a similar way, but the stocking levels are close to optimal. Based on the product and demand characteristics for the two products, distortions in either the sales arm or the operations arm will be more harmful for the firm, so it will prefer one of these two allocation schemes. The firm's choice is to finely control one side but to settle for a distortion on the other to limit the size of the salesforce. The point of interest here is how close the firm can get to the fully coordinated outcome using one of these schemes. Following the intuition above, we provide below some broad results regarding when it will choose which design and why. The analytical solutions to the games outlined above are intractable so we resort to a numerical maximization procedure to determine the optimal contract parameters and associated actions for the managers.

Numerical analysis: For the numerical analysis, note that the major components of the firm's net profits are the revenues from sales and the salaries of the managers to compensate them for their efforts. We assume marginal costs to be zero so that the retail prices effectively act as margins. The inventory cost due to the mismatch between demand and quantity supplied is much smaller. This is due to two reasons: (1) this component is determined by the expected *difference* between quantity supplied and demand (both for unsold inventory and backorders) and these differences are small compared to expected sales, and (2) per-unit costs are lower than per-unit retail prices.

We vary the parameters in the following manner to generate wide asymmetry between the two products: (1) $\theta \in \{0, 0.1, 0.2, 0.3\}$, (2) $r_1 = 8, r_2 \in \{5, 6, 7, 8, 9, 10, 11\}$, (3) $\sigma_1 = 2, \sigma_2 \in \{1, 1.5, 2, 2.5, 3\}$, and (4) $\omega_1^2 = 1, \omega_2^2 \in \{0.5, 0.75, 1, 1.25, 1.5\}$. The values for the other parameters are $c_{u1} = c_{u2} = 4, c_{b1} = c_{b2} = 4$ and $\mu_1 = \mu_2 = 0.1$. The parameter values are chosen to parallel the numerical analysis presented in Netessine and Rudi (2004) who have a similar model.

The most important result from our numerical analysis is that the choice of workforce allocation can have a major influence on the profits of the firm. Hence, it is important for a firm to correctly decide its "organization focus."

Result 1: Under certain scenarios, the firm has a clear preference for one kind of workforce allocation over the other.

There are several scenarios that we identify where, between a sales-focused and an operations-focused allocation, the better choice can lead to a profit that is very close to the first-best profit (99% of first-best), but the wrong allocation choice can lead to greatly lower profits (over 50% lower than first-best). This result arises because, as mentioned earlier, a sales-focused allocation distorts the decisions on the operations side while an operations-focused allocation distorts the decisions on the sales side, and one kind of distortion can lead to a more severe impact than the other. For instance, when $\theta = 0.2, r_1 = 8, r_2 = 6, \sigma_1 = 2, \sigma_2 = 3, \omega_1^2 = \omega_2^2 = 1$, a sales-focused allocation can achieve a profit 99% of the first-best outcome, while an operations-focused allocation achieves a profit only 40% of the first-best outcome.

Result 2: For the parameter values considered above, the firm can get very close to the first-best outcome when it has the choice of sales-focused or operations-focused allocation. However, when the firm is restricted to a minimal organization, the resulting profit is much farther away from the first-best profit.

Always choosing a minimal organization can take the firm very far from the first-best profit (mean deviation 22.0%). Always choosing a sales-focused or always choosing an operations-focused allocation can lead to a substantial difference from the first-best outcome (overall mean deviation 4.5%). However, choosing the better of sales-focused or operations-focused allocation for each scenario gives a mean deviation of only 0.8%. This suggests that most firms will opt for either a sales-focused or operations-focused allocation and supports the observation pointed out earlier that almost all firms have either a "sales image" or a "cost image." Broadly speaking, when retail prices are low the firm prefers an operations-focused allocation, because the firm needs to "get it right" on the costs side. If the allocation is operations-focused, the firm settles for suboptimal sales but can set the stocking quantities such that overall costs are lower. If the allocation is sales-focused, the firm takes a hit on costs. Since retail prices are low, a little is lost in revenue, while the benefit from cost reduction is more significant.

The above results provide some insight into how the underlying objective of aligning marketing and operations, while keeping the size of the salesforce small and compensation contracts simple, can influence the organization of a firm. The purpose of this section was to show the existence of these effects rooted in distortions in incentive structures due to multitasking; characterizing the exact conditions governing the preference of one allocation over another requires a more detailed study which we propose as a direction for future research.

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Appendix to Supplement

S.I Proof of Proposition S.1

Let $A_1^{FB}, A_2^{FB}, Q_1^{FB}, Q_2^{FB}$ denote the first-best solution.

Given the contracts $S_{mi} = w_{mi} + \alpha_i Y_i$ and $S_{oi} = w_{oi} - \alpha_{oi} C_i + \beta_{oi} Y_i$, the first derivatives for the managers' net utility functions are

$$\frac{dE\Pi_{mi}}{dA_i} = \alpha_{mi} \Pr\{D_i > Q_i\} - A_i, \qquad i = \{1, 2\}$$

$$\text{and } \frac{dE\Pi_{oi}}{dQ_i} = \big(\alpha_{oi}(c_{ui} + c_{bi}) + \beta_{oi}\big) \Pr\{D_i > Q_i\} - \alpha_{oi}(c_{oi} + c_{ui}) - \omega_i^2 Q_i, \qquad i = \{1, 2\}$$

The second derivatives are

$$\frac{d^2 E\Pi_{mi}}{dA_i^2} = -\left(\alpha_{mi} \frac{1}{2\sigma_i} + A_i\right), \qquad i = \{1, 2\}$$

$$\text{and } \frac{d^2 E\Pi_{oi}}{dQ_i^2} = -\left(\big(\alpha_{oi}(c_{ui} + c_{bi}) + \beta_{oi}\big) \frac{1}{2\sigma_i} + \omega_i^2\right), \qquad i = \{1, 2\}.$$

Solving the first-order conditions for the optimal quantities and sales efforts and equating them to the first-best levels, we obtain the following specification for the contract parameters:

$$\alpha_{mi} = \frac{2A_i^{FB}\sigma_i}{Q_i^{FB} - (A_i^{FB} - \theta A_j^{FB} + \mu_i - \sigma_i)}$$

$$\alpha_{oi} + \frac{(A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i) - Q_i^{FB}}{(c_{bi} + c_{ui})\big((A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i) - Q_i^{FB}\big) - 2\sigma_1(c_{oi} + c_{ui})} \beta_{oi}$$

$$= \frac{2\omega_i^2 \sigma_i Q_i^{FB}}{(c_{bi} + c_{ui})\big((A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i) - Q_i^{FB}\big) - 2\sigma_1(c_{oi} + c_{ui})}, i = \{1, 2\}, j = 2 - i.$$

The simplest interdependent contract for the operations manager is given by

$$\alpha_{oi,s} = 0$$

$$\beta_{oi,s} = \frac{2\omega_i^2 \sigma_i Q_i^{FB}}{(A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i) - Q_i^{FB}}$$

which can always coordinate, since $A_i^{FB} - \theta A_j^{FB} + \mu_i - \sigma_i \le Q_i^{FB} \le A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i$. ■

S.II Coordinating Contracts When Managers Multitask

We consider the case of interdependent contracts. The argument for information-based contracts proceeds along similar lines. When a single sales manager sells both products and a single operations manager supplies both products, the firm offers them the contracts $S_m = w_m + \alpha_{m1}Y_1 + \alpha_{m2}Y_2$ and $S_o = w_o - \alpha_{o1}C_1 + \beta_{o1}Y_1 - \alpha_{o2}C_2 + \beta_{o2}Y_2$ respectively. The first derivatives for the managers' utility functions are given by

$$\frac{\partial E\Pi_m}{\partial A_i} = \alpha_{mi}\Pr\{D_i > Q_i\} - A_i - \theta\alpha_{mj}\Pr\{D_j > Q_j\}, \qquad i = \{1,2\}, j = 2-i$$
$$\text{and } \frac{\partial E\Pi_o}{\partial Q_i} = \big(\alpha_{oi}(c_{ui}+c_{bi}) + \beta_{oi}\big)\Pr\{D_i > Q_i\} - \alpha_{oi}(c_{oi}+c_{ui}) - k_{oi}Q_i, \qquad i = \{1,2\}.$$

Solving the first-order conditions for the optimal quantities and sales efforts and equating them to the first-best levels, we obtain the following specification for the contract parameters:

$$\alpha_{mi} = \frac{2(A_i^{FB} + \theta A_j^{FB})\sigma_i}{(1-\theta^2)\big(Q_i^{FB} - (A_i^{FB} - \theta A_j^{FB} + \mu_i - \sigma_i)\big)}$$
$$\text{and } \alpha_{oi} + \frac{(A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i) - Q_i^{FB}}{(c_{bi}+c_{ui})\big((A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i) - Q_i^{FB}\big) - 2\sigma_1(c_{oi}+c_{ui})}\beta_{oi}$$
$$= \frac{2k_{oi}\sigma_i Q_i^{FB}}{(c_{bi}+c_{ui})\big((A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i) - Q_i^{FB}\big) - 2\sigma_1(c_{oi}+c_{ui})}, i = \{1,2\}, j = 2-i.$$

The simplest interdependent contract for the operations manager is given by

$$\alpha_{oi,s} = 0$$
$$\text{and } \beta_{oi,s} = \frac{2k_{oi}\sigma_i Q_i^{FB}}{(A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i) - Q_i^{FB}}$$

which can always coordinate, since $A_i^{FB} - \theta A_j^{FB} + \mu_i - \sigma_i \le Q_i^{FB} \le A_i^{FB} - \theta A_j^{FB} + \mu_i + \sigma_i$.

The contracts above assume a separate piece-rate reward (i.e., a separate variable component) for each product for both managers. If the performance metric of the sales manager is defined as $P_m = \sum_{i=1,2}\lambda_{mi}Y_i$, where $\lambda_{mi} = \alpha_{mi}$ above, and the performance metric of the operations manager is defined as $P_o = \sum_{i=1,2}\lambda_{oi}Y_i$, where $\lambda_{oi} = \beta_{oi,s}$ above, then the contracts $S_m = w_m + P_m$ and $S_o = w_o + P_o$ achieve coordination. However, since $\lambda_{\cdot i} \neq r_i$ in general, these metrics differ from aggregate dollar sales.

S.III Effect of θ

The solution for the case of two products is similar to the solution for the one-product case, but is cumbersome to write. The main difference is the effect of the parameter θ, which measures the extent to which the sales effort for one product hurts the demand for the other product. To focus on how θ influences the solution, we provide the expressions for the first-best sales efforts and stocking levels under the assumption that the two products are identical in all respects, but sales effort for one product has a negative impact on the demand for the other product (as in (**??**)).

Using the notation $Q^{2FB} = Q_1^{FB} = Q_2^{FB}$ and $A^{2FB} = A_1^{FB} = A_2^{FB}$, the first-best solution is given by the following expressions:

$$Q^{2FB} = \frac{(1-\theta)^2(r-c_o)(r+c_u+c_b)+(r+c_b)(\mu+\sigma)+c_u(\mu-\sigma)-2\sigma c_o}{(r+c_u+c_b)(1+\omega^2(1-\theta)^2)+2\sigma\omega^2}$$

$$\text{and } A^{2FB} = \frac{(1-\theta)\big((r-c_o)(r+c_u+c_b)-((r+c_u)(\mu-\sigma)+c_b(\mu+\sigma))\omega^2\big)}{(r+c_u+c_b)(1+\omega^2(1-\theta)^2)+2\sigma\omega^2}.$$

Using the above, we can characterize the parameters of the coordinating contracts. The optimal piece-rate rewards for sales for the sales managers when one manager manages a single product are given by (refer to Appendix S.I)

$$\alpha_{m\cdot} = \frac{2A^{2FB}\sigma}{Q^{2FB}-((1-\theta)A^{2FB}+\mu-\sigma)}.$$

The optimal piece-rate rewards for sales for the sales managers when one manager manages both products are given by (refer to Appendix S.II)

$$\alpha_{m\cdot} = \frac{2A^{2FB}\sigma}{(1-\theta)\big(Q^{2FB}-((1-\theta)A^{2FB}+\mu-\sigma)\big)}.$$

Under the simplest interdependent contract, the piece-rate rewards for sales for the operations manager(s) are (refer to Appendices S.I and S.II)

$$\beta_{o\cdot} = \frac{2\omega^2\sigma Q^{2FB}}{\big((1-\theta)A^{2FB}+\mu+\sigma\big)-Q^{2FB}}.$$

To obtain the comparative statics for the above parameters w.r.t. θ, we can simply take the derivatives w.r.t. θ.