

Revenue Management Games: Horizontal and Vertical Competition

On-line Companion

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4.2 Comparing the Competitors and a Monopolist

Proposition S1. *The game characterized by payoffs (4.3) is submodular so best response functions are decreasing and a Nash equilibrium exists. Furthermore, there is a unique Nash equilibrium in this game and it is characterized by optimality conditions (4.4).*

Proof: To show supermodularity it is sufficient to demonstrate non-negativity of the cross-partial derivatives which can be done by straightforward differentiation

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial B_i \partial B_j} &= -p_H \frac{\partial}{\partial B_j} \Pr(D_{Hi}^T > C - B_i, D_{Li} > B_i) \\ &= -p_H f_{D_{Hi}^T | D_{Hj} > C - B_j, D_{Lj} > B_j, D_{Li} > B_i} (C - B_i) \Pr(D_{Hj} > C - B_j, D_{Lj} > B_j, D_{Li} > B_i) < 0 \end{aligned}$$

It immediately follows that a Nash equilibrium exists and best responses are decreasing. We will now prove uniqueness by contradiction. Optimality conditions (4.4) must hold at any equilibrium. Suppose there are two distinct equilibria (B_i^c, B_j^c) and $(B_i^c + \Delta_i, B_j^c - \Delta_j)$ where $\Delta_i > 0$ and $\Delta_j > 0$ (without loss of generality, the case with $\Delta_i < 0$ and $\Delta_j < 0$ is analogous). For player i it must be true that

$$\Pr\left(D_{Hi}^T(B_j^c) > C - B_i^c \mid D_{Li} > B_i^c\right) = \Pr\left(D_{Hi}^T(B_j^c - \Delta_j) > C - B_i^c - \Delta_i \mid D_{Li} > B_i^c + \Delta_i\right)$$

Notice that the last probability term is increasing in Δ_i and decreasing in Δ_j . Evidently, for equality to hold it must be true that $\Delta_j > \Delta_i$. However, analysis of optimality conditions for player j reveals that we also need $\Delta_j < \Delta_i$ which is a contradiction. ■

Supplement to Section 4.3: A more detailed description of the numerical analysis. To determine whether the previous section's results apply to the full-fledged game, we calculate numerically both the competitive equilibrium and the optimal monopoly solution in a game with both low-fare and high-fare overflow. We perform these numerical experiments over a range of parameter values that is sufficiently wide so that we replicate most possible real-world scenarios. Our goal is to see whether the booking limit set by the monopoly, $B_1^a + B_2^a$, is consistently greater than or equal to the total booking limit under competition, $B_1^c + B_2^c$.

To find the appropriate range of parameters we examined a variety of sources, including published papers, unpublished PhD theses, and databases collected by the airline industry and the U.S. Department of Transportation (DOT). All of these sources will be described in more detail, below.

Our most significant primary source is the DOT's quarterly 'Passenger Origin and Destination Survey'. This database, known as either 'Data Base 1a' or 'the O&D Survey,' contains a 10% sample of all airline tickets sold for flights originating during a 3-month period. Each record of each ticket includes the cities visited by the passenger (the itinerary), the price of the ticket, a code indicating whether the ticket was for first, business, or coach-class, and finally a *fare code* indicating whether the ticket was restricted or unrestricted. In general, unrestricted, or full-fare, tickets are not subject to limitations such as advance purchase requirements, minimum/maximum stays, or refund penalties. Restricted, or discount, tickets have one or more of these limitations (for more on the O&D Survey and how the airlines classify tickets into fare categories, see DOT, 2004a and 2004b). Our analysis used O&D Survey data from the 4th quarter of 1999 for the 7 largest hub-and-spoke airlines: American, America West, Continental, Delta, Northwest, United and USAirways. For this analysis, we included in our data set only tickets in coach-class for direct flights (no connections) within the U.S. Our data set included both one-way and round-trip tickets.

We used the O&D Survey to generate rough estimates of two parameters: the ratio of the high fare to the low fare (p_H / p_L) and the proportion of demand due to low-fare passengers. To estimate these parameters, we used the database's fare code as a proxy for our high and low-fare categories. Given that our model allows for only two types of fare classes, we believe that dividing fares into restricted and unrestricted categories is a reasonable approximation.

Unfortunately, the O&D Survey data has two significant limitations. First, the data set includes only tickets sold, while our parameters refer to characteristics of exogenous demand. Second, neither the date nor the time of each flight is included in the Survey. Therefore, our smallest unit of analysis is a *market*, which is defined by a city pair (e.g., Rochester to Chicago) and the type of itinerary (one-way or round-trip). A final screen of the O&D Survey data eliminated those markets with fewer than 30 tickets sold in either the full-fare or discount categories. Our final data set included 471,000 tickets sold for 1500 different markets.

Despite the large sample size, the two limitations mentioned above imply that the data may not provide perfectly accurate estimates of the parameters for particular flights. For example, a round-trip ticket for a Saturday afternoon flight between Rochester and Chicago may have a different p_H / p_L ratio than a ticket for a Monday morning flight, but this information is lost when the data are aggregated into market categories. However, our goal is to identify parameter ranges that capture most

real-world scenarios. To our knowledge, there are no published sources of this information, and we believe that our analysis accomplishes our goal.

The following paragraphs describe additional data analysis and the parameter values:

- Capacity (C_1 and C_2): The Federal Aviation Administration collects information on the number and type of aircraft operated by the major airlines (FAA, 2004). From the data submitted by the 7 largest airlines, we calculate that an average airplane has 180 coach seats, varying from approximately 50 for a regional jet to over 400 for a Boeing 747. Given these data, we will run two sets of experiments: a symmetric case with $C_1 = C_2 = 200$, and an asymmetric case with $C_1 = 200$ and $C_2 = 100$.
- Ratio of high fare to low fare (p_H / p_L): In our data set derived from the O&D Survey, the median ratio among all 1500 markets is $p_H / p_L = 2.6$. For over 90% of all markets, the ratio fell within a range from 1.3 to 4. Therefore, we define three scenarios, $p_H / p_L = [1.3, 2.6, 4]$, for both the symmetric and asymmetric cases.
- Proportion of demand due to low-fare passengers: Let μ_{Li} (μ_{Hi}) be the average low-fare (high-fare) demand for airline i , $i=1,2$. The proportion of low-fare demand is $\mu_{Li} / (\mu_{Li} + \mu_{Hi})$. In the data from the O&D Survey, the median value for this proportion among all 1500 markets is 0.74, with 90% of all markets falling between 0.5 and 0.9. Therefore, we set $\mu_{Li} / (\mu_{Li} + \mu_{Hi}) = [0.5, 0.74, 0.9]$ for both the symmetric and asymmetric cases. Below we will also discuss experiments in which $\mu_{Li} / (\mu_{Li} + \mu_{Hi}) < 0.5$.
- Total demand and demand faced by each airline: According to the airline industry trade group the Air Transport Association (ATA), the industry load factor (the utilization of airplane seats) has hovered near 70% for the last decade (ATA, 2004). However, there is substantial flight-to-flight variation around this range, with a large majority of flights seeing load factors between 0.5 and 1. In addition, the application of revenue management techniques sometimes truncates demand for certain fare classes, so actual demand is often higher than the observed load factor. Here we assume that total exogenous demand (μ_T) is equal to total airline capacity. For the symmetric case, $\mu_T \equiv \mu_{L1} + \mu_{L2} + \mu_{H1} + \mu_{H2} = 400$, and for the asymmetric case, $\mu_T = 300$. In our experiments, this led to realized load factors between 70% and 98%, depending upon the values of the other parameters.

We also must allocate demand between the airlines. It is most convenient to describe this allocation in terms of the ‘load’ placed on each airline, where the load for airline i is $(\mu_{Li} + \mu_{Hi})/C_i$. For the symmetric case, we vary airline 1’s load through three parameters, $[0.5, 0.75, 1]$, which implies that airline 2’s complementary load is $[1.5, 1.25, 1]$. For the asymmetric case, airline 1’s load = $[0.5, 1, 1.25]$ and airline 2’s load = $[2, 1, 0.5]$ (recall that in the asymmetric case, airline 1 has 200 seats and airline 2 has 100).

- Variability: To limit the number of parameters, we assume that all four customer demand distributions have the same coefficient of variation, CV . Belobaba (1987) states that the airlines traditionally assume that $CV=0.33$. Jacobs, Ratliff and Smith (2000) describe 0.2 to 0.6 as a reasonable range for the CV . Therefore, we use $CV = [0.2, 0.33, 0.6]$.
- Correlation: There is little published information about correlation among airline demand classes, and, as far as we know, no public database that would allow us to estimate these quantities. Belobaba (1987) examines proprietary industry data and finds “a preponderance of zero or insignificant correlation coefficients between fare class demand levels. A few significant positive and negative results were obtained.” (Belobaba, 1987, pg. 164). Our own discussions with representatives of the airline industry and with consultants in yield management confirm that there is no strong pattern, but that positive correlation is probably more prevalent than negative correlation. Therefore, we assume a baseline correlation of 0 and vary the correlation among the following values: $\rho = [-0.3, 0.0, 0.3, 0.6]$. To limit the number of parameters, we assume that the correlations among all demands are equal.
- Probability density: Belobaba (1987) cites industry studies conducted by Boeing that assume that total demand for a flight is distributed according to the Normal distribution. Belobaba also conducted an analysis of data from TWA to test the Normality assumption. After controlling for seasonality, the day of the week, and other systematic variation, he found that demand within each fare class fit the Normal distribution well. For each of our scenarios, we assume that demand is distributed according to a multivariate Normal distribution and is truncated at zero; any negative demand is added to a mass point at zero.

When combined, these parameters define $2 * 3 * 3 * 3 * 3 * 4 = 648$ scenarios. Solutions were found by a simple gradient algorithm and the gradients themselves were evaluated by Monte Carlo integration (a simple search procedure was also used if the objective function was not quasi-concave).

Before we examine aggregate statistics from the 648 scenarios, we focus on a single baseline scenario that represents an average case, according to our data analysis. We choose $C_1 = C_2 = 200$, $p_H / p_L = 2.6$, $\mu_{Li} / (\mu_{Li} + \mu_{Hi}) = 0.74$, $\mu_T = 400$, the demand is split equally between the airlines so that the load on each = 1, $CV = 0.33$, and $\rho = 0$. For this scenario, there is a unique equilibrium with $B_1^c = B_2^c = 140$. Therefore, the total booking limit is 280 and the airlines reserve a total of 120 seats for high-fare customers. A monopolist, on the other hand, has an optimal total booking limit of $B_1^a + B_2^a = 290$ seats, with 110 seats set aside for high-fare customers. If we define the "service level" as the probability that a customer is able to purchase a seat on either aircraft, the difference in booking limits produces significantly different service levels for each customer class. Under competition, 41% of low-fare customers found a seat on either flight, while under a monopoly the low-fare service level rises to 46%. On the other hand, high-fare passengers benefit from competition. Their service level is 83% under competition, 75% under the monopoly.

While this particular example produced a unique equilibrium, we saw that the full-overflow game may have multiple equilibria or may not have any equilibria at all. Such an outcome would complicate the comparison between competitive and monopoly booking limits. However, by examining the airline response functions for each of the 648 scenarios, we saw that in *every* case an equilibrium exists and is unique. All response functions were continuous, and all produced a stable equilibrium, as in Figure 3. As mentioned in Section 4, an extremely low (negative) correlation between high and low-fare demands can generate the outcome shown in Figure 2, in which no pure-strategy equilibrium exists. We have also found instances of multiple equilibria when the ratio $\mu_{Li} / (\mu_{Li} + \mu_{Hi})$ is low (e.g., 0.1) and correlation is negative or zero. We will discuss these cases at the end of this section.

First we compare the total booking limits in the competitive and monopoly environments for the original 648 scenarios. In every scenario, the booking limit for the monopoly is equal to, or greater than, the sum of the booking limits for the airlines in competition. The mean difference $(B_1^a + B_2^a) - (B_1^c + B_2^c)$ across all scenarios is 9.3 seats, and the difference varies from 0 seats to 111 seats. In general, the largest differences occur when correlation is low ($\rho = -0.3$) and when both capacity and demand are balanced among airlines and classes, as when $C_1 = C_2$, $\mu_{Li} / (\mu_{Li} + \mu_{Hi}) = 0.5$, and the loads on the two airlines are equal. Table S1 displays the difference $(B_1^a + B_2^a) - (B_1^c + B_2^c)$ for

each value of ρ . Each column of Table S1 represents an average over 162 scenarios. As the correlation increases, the difference between the monopoly and competitive cases decreases.

The differences in booking limits have a significant effect on the service levels offered to each customer class. Over all cases, the service level offered to low-fare customers rose an average of 5.4% under the monopoly while the service level offered to high-fare customers declines an average of 5.3%. In addition, the range of results was extremely large. The difference in low-fare service levels was as high as 62%, while the difference in high-fare service levels reached 30%.

	$\rho = -0.3$	$\rho = 0.0$	$\rho = 0.3$	$\rho = 0.6$
Avg. monopoly total booking limit $B_1^a + B_2^a$	269	247	238	232
Avg. competitive total booking limit $B_1^c + B_2^c$	246	239	234	230
Average $(B_1^a + B_2^a) - (B_1^c + B_2^c)$	23	8	4	2
Low-fare service level (monopoly-competitive)	13.7%	4.8%	2.1%	0.9%
High-fare service level (monopoly-competitive)	-11.8%	-5.2%	-2.8%	-1.4%

Table S1. Demand correlation and the effects of competition.

In general, the difference in total profits between the monopoly and competitive cases was small. Averaged over all 648 scenarios, profits to the monopoly are just 0.2% higher than the total profits under competition, with a range from 0% to 4%. The largest differences in profit were seen when correlation is low, p_H/p_L is high, and capacities and expected demands are equally balanced among airlines and classes. These small differences in profit are not unexpected since in most cases the objective function is relatively 'flat' near the optimum.

It is more difficult to make these comparisons when the proportion of demand due to low-fare passengers is small ($\mu_{Li}/(\mu_{Li} + \mu_{Hi}) < 0.5$) because scenarios with multiple competitive equilibria begin to appear. For example, with $\mu_{Li}/(\mu_{Li} + \mu_{Hi})=0.1$, we identified one scenario with three equilibria: $(B_1^c = 6, B_2^c = 36)$, $(B_1^c = 36, B_2^c = 6)$, and $(B_1^c = 22, B_2^c = 22)$. However, under this scenario the monopoly solution is $B_1^a + B_2^a = 93$. As was true for the original 648 scenarios, at each competitive equilibrium the total booking limit is smaller than or equal to the booking limit chosen by a monopoly. This was true for *all* examined scenarios with $\mu_{Li}/(\mu_{Li} + \mu_{Hi}) < 0.5$.

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