

Robust Newsvendor Competition under Asymmetric Information

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We generalize analysis of competition among newsvendors to a setting in which competitors possess asymmetric information about future demand realizations, and this information is limited to knowledge of the support of demand distribution. In such a setting, traditional expectation-based optimization criteria are not adequate and therefore we focus on the alternative criterion used in the robust optimization literature: the absolute regret minimization. We show existence and derive closed-form expressions for the robust-optimization Nash equilibrium solution for a game with an arbitrary number of players. This solution allows us to gain insight into the nature of robust asymmetric newsvendor competition. We show that the competitive solution in the presence of information asymmetry is an intuitive extension of the robust solution for the monopolistic newsvendor problem, which allows us to distill the impact of both competition and information asymmetry. In addition, we show that, contrary to the intuition, a competing newsvendor does not necessarily benefit from having better information about its own demand distribution than its competitor has.

Key words: Robust optimization, newsvendor competition, absolute regret, asymmetric information, robust optimization equilibrium.

1. Introduction and Literature Review

Game-theoretic models in operations management primarily rely on the assumption that players possess complete and symmetric information and then utilize Nash's approach to find the equilibria of the game (Nash (1951)). However, in practice firms often know more details about their own operations (including costs, demand distribution, productivity, etc.) while their competitors may have only an approximate idea. For that reason, numerous research firms offer (often expensive) information products focused on market trends and competitor sales forecasts. These products, while mitigating the problem of information asymmetry, do not solve it completely. For example, SoundScan is a company that electronically tracks every single music record sold by 85% of retailers in the US (Singer (1999)). ACNielsen tracks similar information for the sales of VHS tapes and DVDs. At the same time, neither company currently has data from Wal-Mart, which no longer shares its sales data with outside companies (Hays (2004)).

The issue of informational structure in games was first addressed by Harsanyi, who introduced the notion of the Bayesian Nash equilibrium in his seminal work (Harsanyi (1967)). In Bayesian games, each player possesses incomplete information about other players' payoff functions but forms beliefs (in the form of the full prior probability distribution) about other players. Although the Bayesian Nash equilibrium approach is now a gold standard for analyzing games with incomplete information, the limiting part of this approach is that it requires each player to come up with an entire probability distribution of an unknown parameter for the competitor (Myerson (2004)). This limitation is especially apparent in business settings involving new products, where it is hard to

expect decision makers to come up with a detailed distribution of demand not only for competing firms, but even for their own products.

To deal with settings in which competitors have incomplete information about each others' pay-offs, we propose a different approach that does not rely on a knowledge of complete prior demand distributions. Instead, we invoke the robust optimization paradigm of Savage (1951), Vairaktarakis (2000), Perakis and Roels (2008), Aghassi and Bertsimas (2006), Yue et al. (2006)), which ensures a certain level of performance irrespective of the underlying distributions of the involved random parameters. At present, the literature on the analysis of robust policies has been almost exclusively limited to the monopolistic setting, with two exceptions being Perakis and Sood (2006) and Perakis and Roels (2007). However, none of the papers on robust optimization considers information asymmetry among competitors. On the other hand, a stream of papers starting with Li (1985) models information asymmetry but focuses on the incentives to share information and does not utilize the robust optimization approach. In the present work, we assume that competitors minimize the absolute regret, as defined in Savage (1951), while knowing only the support (minimum and maximum values) of demand distributions. Furthermore, we enrich the robust optimization approach by incorporating the asymmetry of the information available to different players: while the firm knows the support of its own distribution, it does not know the precise support of competitors' demand distributions but forms a belief about the intervals in which the upper and lower bounds of the distributions are located.

We apply this approach to the now classical newsvendor inventory competition model with stock-out-based demand substitution (as classified in Bitran et al. (2006)). Stock-out-based demand substitution is ubiquitous, for example, in retail settings, and its importance is well-documented. A recent survey of retailers has found that, of the customers who do not find what they want on the shelf, 40% either defer the purchase or go to a competitor's store to find the item, while the rest simply do not make the purchase (see Andraski and Haedicke (2003)). Extensive literature on demand substitution has been developed which relies on knowledge of the joint probability of the primary demand distribution as well as the substitution behavior of consumers. As a basis of our analysis, we use the demand substitution model pioneered by McGillivray and Silver (1978) and first studied in the competitive framework by Parlar (1988). Wang and Parlar (1994), Netessine and Rudi (2003), Kraiselburd et al. (2004), Netessine and Zhang (2005) and Netessine and Shumsky (2005) are examples of more recent papers which focus on retail competition in the presence of demand substitution. In the present work, we focus on the competitive, single-period version of this problem, which is often referred to as "competitive newsvendors."

Applying the absolute regret minimization criterion to this problem produces closed-form solutions which can be interpreted as intuitive modifications of the noncompetitive newsvendor solutions incorporating both information asymmetry and competition in a transparent way. In particular we demonstrate that the well-known result on inventory overstocking by competing newsvendors is strengthened by the presence of information asymmetry. In fact, newsvendors collectively might stock even more than the upper bound on the total demand distribution, and this effect becomes stronger as information asymmetry increases. Further, we find that having better information about its own demand distribution than competitors do does not necessarily benefit the newsvendor. This is because more information asymmetry leads to higher stocking quantities for the competitors and, as a result, lower overflow demand and lower profit for the newsvendor. The key contribution of this paper is introducing the robust optimization approach (regret minimization) to formulate and solve a game-theoretic model with asymmetric information. We analytically demonstrate the impact of information asymmetry and competition on newsvendors' decisions. Advantages of our approach include the minimal number of problem parameters (we require only knowledge of the support of the demand distributions) and intuitive, closed-form solutions which lead to novel insights. The rest of the paper is organized as follows. In section 2, we outline our modeling assumptions. In

section 3, we present an analysis of a newsvendor game in which competitors minimize the absolute regret. We conclude with a summary of our findings in section 4. All proofs can be found in the on-line appendices.

2. The Model

Consider a market served by N newsvendors, each selling a different product. We denote the product selling price and procurement costs for newsvendor i as p_i and c_i , respectively, $p_i > c_i > 0$, $i = 1, \dots, N$. We assume that customers arrive with a product preference in mind so that each newsvendor faces random primary demand denoted by D_i . Here all D_i are assumed to be stochastically independent. Moreover, if product i is out of stock, a proportion $0 \leq o_{ji} \leq 1$ of customers which cannot be satisfied by newsvendor i “spills over” (substitutes) to newsvendor j , where we naturally require that $\sum_{j \neq i} o_{ji} \leq 1$. In our analysis we consider a setting in which selling prices, procurement costs and spill-over coefficients for all products are common knowledge for all competing firms. For example, prices might be pre-announced in advance and spill-over coefficients might be estimated using historical consumer choice behavior based on sales data for past products. While these assumptions are clearly a simplification, we make them to keep the number of variables at a manageable level and to stay consistent with existing literature.

With respect to demand distribution, we assume that newsvendors have limited and asymmetric knowledge about the possible primary demand values for their own product and for the product offered by their competitor. This would be the case when the products are new and/or innovative and therefore historical demand data are not readily available. In particular, as far as the demand for firm i is concerned, each firm $j \neq i$ believes that the lowest (highest) possible value for firm’s i demand belongs to an interval $\hat{\mathcal{A}}_i \left(\hat{\mathcal{B}}_i \right)$, where $\hat{\mathcal{A}}_i$ and $\hat{\mathcal{B}}_i$ are such that $\left(a \in \hat{\mathcal{A}}_i, b \in \hat{\mathcal{B}}_i \right) \Rightarrow a \leq b$. In other words, the knowledge that each firm has about its competitor’s demand is reduced to knowing the intervals to which the lower and upper demand support values belong. For example, $\hat{\mathcal{A}}_2 = [10, 20]$ and $\hat{\mathcal{B}}_2 = [50, 80]$ describe the situation in which firm 1 believes that the lower boundary of the demand support interval for firm 2 can be anywhere between 10 and 20, while the upper boundary of that interval can be anywhere between 50 and 80. Similarly, firm 2 believes that the lowest (highest) possible value for firm 1’s demand belongs to an interval $\hat{\mathcal{A}}_1 \left(\hat{\mathcal{B}}_1 \right)$, where $\hat{\mathcal{A}}_1$ and $\hat{\mathcal{B}}_1$ are such that $\left(a \in \hat{\mathcal{A}}_1, b \in \hat{\mathcal{B}}_1 \right) \Rightarrow a \leq b$. We assume that the intervals $\hat{\mathcal{A}}_i, \hat{\mathcal{B}}_i$ are common knowledge for all firms - in other words, each firm i knows about the beliefs of any firm $j \neq i$ regarding firm’s i demand support. At the same time, we also assume that firm i has a *private* knowledge about the actual support for its demand, $[A_i, B_i]$, where $A_i \in \hat{\mathcal{A}}_i$ and $B_i \in \hat{\mathcal{B}}_i$, $i = 1, \dots, N$. Such private knowledge that each firm has about its own demand support creates an asymmetric information setting in which competitors have limited information about future demand (expressed in terms of demand support), yet the information they possess about their own demand is more precise than the information about competitor’s demand. Note that if $\hat{\mathcal{A}}_i$ and $\hat{\mathcal{B}}_i$ are singletons for each $i = 1, \dots, N$ (i.e., if $A_i \equiv \hat{\mathcal{A}}_i$ and $B_i \equiv \hat{\mathcal{B}}_i$), our setting reduces to that of a robust competition under symmetric information.

Our model might reflect a business setting in which competing firms (e.g., Sony and Nintendo) introduce successive competing generations of products (e.g., Playstation and Wii). Over time, competitors might be able to estimate/learn substitution fractions between two brands which would largely depend on customer brand loyalty. On the other hand, demand for each individual product generation would remain highly uncertain, with each company possessing a better knowledge about its own demand. In other words, demand substitution process is governed by long-term product brand values, known to competitors with higher degree of certainty than the short-term market characteristics which drive uncertainty in actual product demand realizations. Nevertheless, our

results can be easily extended to the case in which the competing firms only possess knowledge about the support of the spill-over coefficients rather than their actual values.

In our analysis we combine the Bayesian framework with the robust optimization framework. The resulting equilibrium concept is closest to the “robust-optimization equilibrium” described in Aghassi and Bertsimas (2006). It is similar to the classical Bayesian Nash equilibrium approach in that a tuple of strategies is an equilibrium if each player selects the best response to the other players’ strategies using his beliefs about uncertainty in payoffs for competitors as well as his private information about his own uncertainty in payoffs. Yet, it differs from the Bayesian Nash equilibrium in that it does not involve calculation of expectations since players do not possess distributional information. For brevity, we will refer to this equilibrium approach as “equilibrium”.

3. The Analysis

We apply the absolute regret criterion to the newsvendor competition problem under asymmetric information. This robust optimization approach was studied in the context of the classical monopolistic newsvendor problem by Vairaktarakis (2000) and further developed by Perakis and Roels (2008). In our setting, we define the absolute regret for newsvendor i as

$$\begin{aligned} & \Delta_i \left(Q_i, \{Q_j(\hat{A}_j, \hat{B}_j), j \neq i\}, D_i, \{D_j, j \neq i\} \right) \\ &= \max_{\hat{Q}_i \geq 0} \left(\Pi_i \left(\hat{Q}_i, \{Q_j(\hat{A}_j, \hat{B}_j), j \neq i\}, D_i, \{D_j, j \neq i\} \right) \right. \\ & \quad \left. - \Pi_i \left(Q_i, \{Q_j(\hat{A}_j, \hat{B}_j), j \neq i\}, D_i, \{D_j, j \neq i\} \right) \right), \end{aligned} \quad (1)$$

where

$$\begin{aligned} \Pi_i(Q_i, \{Q_j(\hat{A}_j, \hat{B}_j), j \neq i\}, D_i, \{D_j, j \neq i\}) &= -c_i Q_i \\ & \quad + p_i \min(D_i + \sum_{j \neq i} o_{ij}(D_j - Q_j(\hat{A}_j, \hat{B}_j))^+, Q_i). \end{aligned} \quad (2)$$

Given this definition, the minimax absolute regret minimization problem for newsvendor i can be stated as follows:

$$\min_{Q_i \geq 0} \left(\max_{\substack{\hat{A}_j \in \hat{\mathcal{A}}_j, \\ \hat{B}_j \in \hat{\mathcal{B}}_j, \\ j \neq i}} \left(\max_{\substack{D_i \in [A_i, B_i], \\ D_j \in [A_j, B_j], \\ j \neq i}} \left(\Delta_i \left(Q_i, \{Q_j(\hat{A}_j, \hat{B}_j), j \neq i\}, D_i, \{D_j, j \neq i\} \right) \right) \right) \right). \quad (3)$$

Note that the above problem definition involves calculation of regret after demand realizations are observed. In Appendix B we demonstrate that this definition is without loss of generality since in our problem setting problems with ex-post and ex-ante regrets are equivalent. The equilibrium in the strategy space is defined as a set of strategies $Q_i^*(A_i, B_i)$, $i = 1, \dots, N$ defined on $\hat{\mathcal{A}}_i \otimes \hat{\mathcal{B}}_i$ such that

$$\begin{aligned} Q_i^*(A_i, B_i) &= \arg \min_{Q_i \geq 0} \left(\max_{\substack{\hat{A}_j \in \hat{\mathcal{A}}_j, \\ \hat{B}_j \in \hat{\mathcal{B}}_j, \\ j \neq i}} \left(\max_{\substack{D_i \in [A_i, B_i], \\ D_j \in [A_j, B_j], \\ j \neq i}} \left(\Delta_i \left(Q_i, \{Q_j^*(\hat{A}_j, \hat{B}_j), j \neq i\}, D_i, \{D_j, j \neq i\} \right) \right) \right) \right), \\ & \text{for } \forall A_i \in \hat{\mathcal{A}}_i, \text{ and } B_i \in \hat{\mathcal{B}}_i, i = 1, \dots, N. \end{aligned} \quad (4)$$

The following proposition identifies the equilibria of this problem and is the key result of this section.

PROPOSITION 1. *There exists an equilibrium for the newsvendor game under the absolute regret criterion. Further, let E_i , $i = 1, \dots, N$ be a set of constants satisfying the following system of non-linear and non-smooth equations:*

$$E_i = \sum_{j \neq i} o_{ij} \left(1 - \frac{c_j}{p_j}\right) \left(\frac{c_j}{p_j - c_j} (\bar{B}_j - \bar{A}_j) - E_j\right)^+, \quad i = 1, \dots, N, \quad (5)$$

where

$$\bar{A}_j = \min(A_j | A_j \in \hat{\mathcal{A}}_j), \quad \bar{B}_j = \max(B_j | B_j \in \hat{\mathcal{B}}_j), \quad j = 1, \dots, N. \quad (6)$$

Then, an equilibrium defined by (4) satisfies

$$Q_i^*(A_i, B_i) = \frac{c_i}{p_i} A_i + \left(1 - \frac{c_i}{p_i}\right) (B_i + E_i), \quad i = 1, \dots, N. \quad (7)$$

Note first that (5)-(7) generalizes the solution of the classical newsvendor problem under the absolute regret minimization criterion, which can be obtained by letting $o_{ij} \equiv 0$, $\forall i, j$. In this case, $Q_i^{NV} = \frac{c_i}{p_i} A_i + \frac{p_i - c_i}{p_i} B_i$, $i = 1, \dots, N$, a solution that coincides with that of Vairaktarakis (2000). This simple newsvendor solution can be thought of as the weighted average of the lower and the upper bounds of demand distribution, with weights reflecting the cost-revenue trade-off of the classical newsvendor problem. As Vairaktarakis notes, this solution can also be obtained by solving the classical newsvendor problem assuming that demand is distributed uniformly on $[A_i, B_i]$. The reason is that, in the absence of testable information, the uniform distribution maximizes entropy so that, if the newsvendor knows nothing about the shape of the distribution except its support, the uniform distribution is reflective of such information. However, Vairaktarakis' remark no longer holds for the competitive newsvendor problem. Namely, when the competitive newsvendor solution (5)-(7) is not the same as the optimal Nash solution under the expected criterion in the framework of Netessine and Rudi (2003) when demand is a uniform distribution, which is still true even when asymmetry does not exist and can be easily verified by a numerical example.

Under competition, in the formula for the equilibrium order quantities, the upper bound for demand B_i is replaced by $B_i + E_i$ to account for demand substitution. It is straightforward to see that, as a result, each competing newsvendor stocks a higher quantity than Q_i^{NV} because of the higher upper bound on the demand distribution. However, the expression for the additional stocking quantity $\frac{p_i - c_i}{p_i} E_i$ is a relatively complicated expression which depends on all cost/revenue/demand parameters of the problem. By examining the solution, it is quite apparent that the addition to the right-most point of the demand distribution E_i is not simply the sum of overflow demands from other newsvendors as one might expect given the structure of the non-competitive robust solution. Another way to look at the solution is as follows. Note that E_i which appeared in (7) and which is defined in (5) can be rewritten as $\sum_{j \neq i} o_{ij} (B_j - Q_j + c_j/p_j \Theta_j)^+$, where $\Theta_j = (\hat{B}_j - \hat{A}_j) - (B_j - A_j)$. This new formula shows that overstock increase E_i in (7) is the sum of the overflow $B_j - Q_j$ due to competition and $c_j/p_j \Theta_j$ due to information asymmetry. When information asymmetry does not exist, $\Theta = 0$, and the increase of overstock only comes from competition.

Note that our solution incorporates information asymmetry in a simple and intuitive form: the newsvendor takes into account the lowest possible point \bar{A}_j of his information about the lower bound of the support for competitors and the highest possible point \bar{B}_j of his information about the upper bound of the support for competitors. The reason is that the highest regret is observed when the newsvendor (i) over-orders and the overflow demand from his competitors is minimal or (ii) when he under-orders and the overflow demand from his competitors is maximal. Note also that, given this observation, it is possible to define the problem in a simplified way by saying that newsvendor i knows that the range of its demand lies in $[A_i, B_i]$ but other newsvendors know that

its demand lies in $[\bar{A}_i, \bar{B}_i] \supseteq [A_i, B_i]$. Whenever \hat{A}_i and \hat{B}_i are singletons such that $A_i \equiv \hat{A}_i$ and $B_i \equiv \hat{B}_i$, our setting reduces to that of a robust competition under symmetric information and \bar{B}_j, \bar{A}_j should be simply replaced by B_j, A_j throughout.

Let us ignore information asymmetry for the moment and consider the impact of competition only (i.e., replace \bar{B}_j, \bar{A}_j by B_j, A_j throughout). It is interesting to observe how competition is reflected in our solution: the additional stocking quantity due to demand overflow depends only on the difference $B_j - A_j$, and not on the individual values of the lower and upper bounds of the competitor's demand distribution. The intuition is as follows: consider the stand-alone newsvendor who stocks $Q_i^{NV} = \frac{c_i}{p_i} A_i + \frac{p_i - c_i}{p_i} B_i$, and consider what we know about the excess demand of this newsvendor (which would create an overflow for the competitors). The lower bound of the overflow is clearly 0 and the upper bound is $B_i - Q_i^{NV} = B_i - \frac{c_i}{p_i} A_i - \frac{p_i - c_i}{p_i} B_i = \frac{c_i}{p_i} (B_i - A_i)$. Thus, since the stocking quantity of the newsvendor is the weighted average of the lower and upper bounds of the demand distribution, the excess demand depends only on the spread of the demand distribution but not on its upper/lower bounds.

If we now return to the problem with information asymmetry, it is clear that, as a consequence of the previous observation, the impact of information asymmetry is also captured by the differences in the upper and lower bounds of the perceived demand distribution $\bar{B}_j - \bar{A}_j$. As we observe, the solution is determined by this quantity, and not by the asymmetry in the information on the individual values of the upper or lower bounds of the demand distribution.

The key question that we would like to shed some light on is: what is the impact of information asymmetry upon the optimal stocking quantities? Or more specifically, if information asymmetry increases (i.e., Θ_i increases), how does this affect Q_i and Q_j , $j \neq i$? While expression (5) is compact, it is not straightforward to see how the order quantity behaves with respect to the problem parameters since this seemingly transparent solution is implicit: equilibrium decisions appear both on the left- and the right-hand sides of (5). Thus, we proceed to consider simplified versions of this solution to gain further insights. In two special cases we are able to obtain closed-form solutions for this game with an arbitrary number of players.

PROPOSITION 2. (a) Suppose that

$$\sum_{j \neq i} \frac{o_{ij} c_j}{p_j} (\bar{B}_j - \bar{A}_j) \leq \frac{c_i}{p_i - c_i} (B_i - A_i), i = 1, \dots, N, \quad (8)$$

and let

$$M_{ij} = \delta_{ij} + o_{ij} \left(1 - \frac{c_j}{p_j}\right), \quad n_i = \sum_{j=1}^N o_{ij} \frac{c_j}{p_j} (\bar{B}_j - \bar{A}_j), i, j = 1, \dots, N,$$

where δ_{ij} is a Kronecker delta symbol, and $o_{ii} = 0, i = 1, \dots, N$. Define \mathbf{M} as an $N \times N$ matrix whose (i, j) -th element is M_{ij} , $i, j = 1, \dots, N$, and \mathbf{n} as an N -dimensional vector whose i -th element is n_i , $i = 1, \dots, N$. Then, the unique equilibrium order quantity Q_i^* for any firm i does not exceed B_i and is given by

$$Q_i^* = \frac{c_i}{p_i} A_i + \left(1 - \frac{c_i}{p_i}\right) (B_i + (\mathbf{M}^{-1} \mathbf{n})_i), i = 1, \dots, N. \quad (9)$$

(b) Consider a symmetric game such that $A_i = A, B_i = B, \bar{A}_i = \bar{A}, \bar{B}_i = \bar{B}, c_i = c, p_i = p$, and $o_{ij} = o, \forall i \neq j, i, j = 1, \dots, N$. The unique equilibrium order quantities for such a game are given by

$$Q_i^* = \frac{c}{p} A + \left(1 - \frac{c}{p}\right) B + \frac{o(N-1) \left(1 - \frac{c}{p}\right)}{1 + o(N-1) \left(1 - \frac{c}{p}\right)} \frac{c}{p} (\bar{B} - \bar{A}), i = 1, \dots, N. \quad (10)$$

In (a), we have a closed-form solution to the game in a case when each optimal stocking quantity does not exceed the upper bound on the primary demand distribution, which is ensured by (8). This assumption is quite reasonable in settings where substitution proportions o_{ij} are relatively small, for example. In these settings, the solution is obtained by merely solving a system of linear equations. When the problem is symmetric, the equilibrium stocking quantity is clearly increasing in the substitution fraction o and in the number of newsvendors ($N - 1$) due to stronger demand overflow. More information asymmetry (higher $\Theta = (\bar{B} - \bar{A}) - (B - A)$) results in higher stocking quantities for all newsvendors. Another way to re-write the solution is $Q_i^* = B + \frac{c}{p} \left((\bar{B} - \bar{A}) \frac{o(N-1)(1-c/p)}{1+o(N-1)(1-c/p)} - (B - A) \right)$, where the second term is the value by which the stocking quantity exceeds the “true” upper bound B on demand distribution. This second term is higher the more asymmetry there is in demand (i.e., $\bar{B} - \bar{A} \gg B - A$). Note that this result underscores an interesting consequence of information asymmetry in the competitive environment we consider: it is possible that each competitor stocks more than her estimate of the upper bound of her own demand, so that all competitors together may significantly overstock as compared to the overall total demand (if there is a lot of asymmetry in demand). It is easy to see that in the symmetric problem this happens if and only if

$$\frac{\bar{B} - \bar{A}}{B - A} > 1 + 1 / (o(N - 1) (1 - c/p)). \quad (11)$$

Thus, severe overstocking occurs when (1) information asymmetry is strong (so that $\bar{B} - \bar{A} \gg B - A$), (2) substitution is more likely (o is large), (3) the number of newsvendors is large ($N \gg 1$), and/or (4) profit margins are high ($(p - c) \gg p$). Although overstocking by competitive newsvendors is a well-known result (Netessine and Rudi (2003), Lippman and McCardle (1997), Mahajan and van Ryzin (2001)), it is reassuring that this result holds under a different optimization criterion as well. Further, we show that this results can be strengthened due to information asymmetry.

To further separate out the effect of competition vs. that of information asymmetry on stocking decisions, let us re-write the equilibrium solution as follows:

$$Q_i^* = \frac{c}{\hat{p}} A + \left(1 - \frac{c}{\hat{p}} \right) B + \frac{\hat{p} - p}{\hat{p}} \frac{c}{p} \Theta, \quad i = 1, \dots, N, \quad \text{where } \hat{p} = p + o(p - c)(N - 1). \quad (12)$$

If we let $\Theta = 0$ (no information asymmetry), then the first two terms in (12) are equivalent to the solution in Vairaktarakis (2000) except for an adjustment of the unit price \hat{p} , which captures the effect of competition but not the effect of information asymmetry. The equilibrium stocking quantity is clearly increasing in \hat{p} so that the effect of competition is to increase stocking quantity as the number of newsvendors N , the substituting proportion o and the profit margin $p - c$ grow. With respect to the impact of information asymmetry (captured by the size of the third term in (12)), we see that it also increases the stocking quantity as N and o increase: a higher number of newsvendors have the potential to increase the upper bound of demand support and a higher substitution fraction amplifies this effect. What is not obvious is what happens when p and c increase. It turns out that the impact of information asymmetry decreases in p and increases in c whenever $p \geq c \left(1 + \sqrt{1 - o(N - 1) / (1 + o(N - 1))} \right)$, and vice versa. To understand this condition better, note that when $N = 1$ or $o = 0$ (no competition), it reduces to a straightforward $p \geq 2c$. As the number of competing newsvendors and/or the substitution fraction increase, the condition is more and more likely to hold. We therefore conclude that the non-linearity of the value of information in cost/revenue parameters is due to the moderation effect of competition. When competition is strong ($oN \rightarrow 1$), we see that the impact of information asymmetry is more likely to be decreasing in price and increasing in cost. However, when competition is weak, this effect is more likely to be reversed.

While the symmetric solution allows for sharper insights into the problem, it remains unclear how information asymmetry affects the solution when newsvendors are not identical. For example, will all stocking quantities increase when demand asymmetry for one newsvendor increases? For the case of $N = 2$ newsvendors, it is possible to obtain a closed-form solution for a problem with arbitrary cost/revenue parameters and show that the answer to this question is negative.

PROPOSITION 3. Let $N = 2$, and for $i, j = 1, 2, i \neq j$, define

$$\bar{o}_{12} = \left(\frac{c_1}{p_1 - c_1} \right) \frac{p_2 (\bar{B}_1 - \bar{A}_1)}{c_2 (\bar{B}_2 - \bar{A}_2)}, \quad \bar{o}_{21} = \left(\frac{c_2}{p_2 - c_2} \right) \frac{p_1 (\bar{B}_2 - \bar{A}_2)}{c_1 (\bar{B}_1 - \bar{A}_1)}. \quad (13)$$

Note that it is impossible to have both $o_{12} > \bar{o}_{12}$ and $o_{21} > \bar{o}_{21}$.

(a) If $o_{12} \leq \bar{o}_{12}$ and $o_{21} \leq \bar{o}_{21}$, the unique equilibrium is

$$\begin{aligned} Q_1^* &= \frac{c_1}{p_1} A_1 + \left(1 - \frac{c_1}{p_1} \right) B_1 \\ &\quad + o_{12} \left(1 - \frac{c_1}{p_1} \right) \left(1 - \frac{c_2}{p_2} \right) \frac{c_1 (\bar{B}_1 - \bar{A}_1)}{p_1} \left(\frac{\bar{o}_{21} - o_{21}}{1 - o_{12} o_{21} \left(1 - \frac{c_1}{p_1} \right) \left(1 - \frac{c_2}{p_2} \right)} \right), \\ Q_2^* &= \frac{c_2}{p_2} A_2 + \left(1 - \frac{c_2}{p_2} \right) B_2 \\ &\quad + o_{21} \left(1 - \frac{c_2}{p_2} \right) \left(1 - \frac{c_1}{p_1} \right) \frac{c_2 (\bar{B}_2 - \bar{A}_2)}{p_2} \left(\frac{\bar{o}_{12} - o_{12}}{1 - o_{12} o_{21} \left(1 - \frac{c_1}{p_1} \right) \left(1 - \frac{c_2}{p_2} \right)} \right). \end{aligned} \quad (14)$$

(b) If $o_{12} \leq \bar{o}_{12}$ and $o_{21} > \bar{o}_{21}$, the unique equilibrium is

$$\begin{aligned} Q_1^* &= \frac{c_1}{p_1} A_1 + \left(1 - \frac{c_1}{p_1} \right) B_1, \\ Q_2^* &= \frac{c_2}{p_2} A_2 + \left(1 - \frac{c_2}{p_2} \right) B_2 + \frac{o_{21}}{\bar{o}_{21}} \left(\frac{c_2}{p_2} \right) (\bar{B}_2 - \bar{A}_2). \end{aligned} \quad (15)$$

If $o_{21} \leq \bar{o}_{21}$ and $o_{12} > \bar{o}_{12}$, the unique equilibrium is

$$\begin{aligned} Q_1^* &= \frac{c_1}{p_1} A_1 + \left(1 - \frac{c_1}{p_1} \right) B_1 + \frac{o_{12}}{\bar{o}_{12}} \left(\frac{c_1}{p_1} \right) (\bar{B}_1 - \bar{A}_1), \\ Q_2^* &= \frac{c_2}{p_2} A_2 + \left(1 - \frac{c_2}{p_2} \right) B_2. \end{aligned} \quad (16)$$

The last proposition provides additional insights into the equilibrium solution for non-identical newsvendors. Threshold values for substitution fractions (13) play the same role as condition (8): when both substitution fractions are small enough, as in case (a), both competing newsvendors select stocking quantities which not only exceed the newsvendor solution but which can also exceed the upper bound on the primary demand value. For example, Q_1^* exceeds B_1 if and only if

$$\frac{\bar{B}_1 - \bar{A}_1}{B_1 - A_1} > \left(\frac{1}{o_{12} o_{21} \left(1 - \frac{c_1}{p_1} \right) \left(1 - \frac{c_2}{p_2} \right)} - 1 \right) / \left(\frac{\bar{o}_{21}}{o_{21}} - 1 \right), \quad (17)$$

i.e., in cases when, for fixed value of \bar{o}_{21} , the perceived (by newsvendor 2) width of newsvendor 1's demand support considerably exceeds its true width (a similar observation, of course, applies to Q_2^*

and B_2). Note that the expressions in (14) are quite intuitive. The first two terms, $(c_i/p_i)A_i + (1 - c_i/p_i)B_i$, reflect the classical newsvendor solution that the newsvendor would follow when faced with primary demand only. The third term accounts for demand overflow from retailer j to retailer i , where $i, j = 1, 2$. Clearly, the stocking quantity is higher if more customers are willing to substitute (higher o_{ij}). In case (b), when substitution fractions are asymmetric (one higher and one lower than the thresholds), one of the newsvendors may elect to stock more than the upper bound on its primary demand. For example, when $o_{21} \leq \bar{o}_{21}$ and $o_{12} > \bar{o}_{12}$, newsvendor 1 always stocks more than B_1 . In this setting, the second newsvendor does not expect any demand spillovers and elects to stock the classical newsvendor quantity Q_2^{NV} . In this case, the solution does not depend on the substitution fraction from newsvendor 1 to newsvendor 2, o_{21} . This simple and intuitive reflection of competitive interactions is not available in newsvendor competition models that assume knowledge of demand distribution (Netessine and Rudi (2003), Lippman and McCardle (1997)).

We shall now focus on the impact of information asymmetry while considering solution (14). It is straightforward to verify that, for example, Q_1^* increases in newsvendor 1's perception of the competitor's distribution spread ($\bar{B}_2 - \bar{A}_2$) and decreases in the competitor's perception of newsvendor 1's own distribution spread ($\bar{B}_1 - \bar{A}_1$). This differentiated impact of own vs. competitor's information asymmetry was not available in the previous proposition. Intuitively, companies should benefit by knowing more about their demand than their competitors know. However, it is quite easy to see that any increase in the competitor's stocking quantity Q_i results in lower profits for newsvendor j since overflow demand stochastically decreases. Thus, the less information newsvendor 2 has about demand for newsvendor 1 (i.e., the larger the $\Theta_1 = (\bar{B}_1 - \bar{A}_1) - (B_1 - A_1)$), the more newsvendor 2 stocks and the less profit newsvendor 1 appropriates. Therefore, newsvendor 1 has an explicit incentive not to hoard information about its own demand distribution. At the higher level, it is convenient to think that information asymmetry increases competition and leads to more overstocking. By sharing information, competing newsvendors reduce competition and therefore reduce overstocking (note that this effect is symmetric for the newsvendors).

At this point an open question remains: what happens in the problem with multiple asymmetric newsvendors? To be specific, what happens with the stocking quantity Q_i^* when Θ_j , $j \neq i$ increases for $N > 2$? Unfortunately, as our numerical experiments demonstrate, the answer to this question is ambiguous: Q_i^* may increase or decrease. The reason is as follows. There are two effects of increasing Θ_j on Q_i^* . The first effect is direct: higher Θ_j means higher perceived demand overflow and hence higher Q_i^* (which is consistent with the last proposition). However, there is also a second effect due to $N > 2$: higher Q_k^* means less demand overflow and hence lower Q_i^* . Depending on the problem parameters and demand substitution coefficients, one or the other effect can dominate, which can be determined using our closed-form solutions and numerical experiments.

4. Conclusion

We have established equilibrium existence/uniqueness results for robust newsvendor games and derived closed-form solutions. These analytical results allow us to better understand the consequences of robust solutions when limited information about demand is available. Our results indicate that, from a practical standpoint, the absolute regret minimization approach is both analytically tractable and leads to solutions that are along the lines of the expected profit maximization solutions even though we only use information about support of demand distribution. But the main advantage of our approach is that information asymmetry about demand distributions is incorporated in our solutions in a simple and intuitive way. For example, the existing literature on Bayesian games usually limits information asymmetry to a single parameter (e.g., cost of effort, product quality, a single parameter of the consumer utility function, etc.) because otherwise analytical tractability is hard to achieve. To the best of our knowledge, ours is the first paper to consider information asymmetry regarding the entire probability distribution. We envision that our

approach can be used to tackle many classical Bayesian game problems as well as more complex problems.

One possible future research topic is to consider other robust optimization paradigms and additional information about demand distribution. Recently, Perakis and Roels (2008) and Zhu et al. (2006) have investigated robust newsvendor optimization with additional demand information (e.g., mean and standard deviation, symmetry, etc.) and have also obtained promising results. Another interesting direction would be to extend our analysis by investigating a centralized system with newsvendors under robust performance criteria although our preliminary results indicate that this is a very hard problem that is unlikely to possess analytical tractability. Other promising avenues for research include incorporation of uncertainty regarding substitution coefficients, addition of supply chain considerations, consideration of other information asymmetry types, as well as incorporation of information-sharing considerations into the model.

5. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://or.journal.informs.org/>.

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