Electronic Companion - Technical Appendix

Selling to Conspicuous Consumers: Pricing, Production and Sourcing Decisions

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Appendix A: Proofs for Sections 2 and 3

Proof of Proposition 1: The RE equilibrium conditions reduce to

$$p = \bar{F}_D(Q_s) \cdot (k + v - s) + s \tag{A1}$$

The firm will obtain the critical fractile quantity choice as:

$$\frac{\partial \Pi_N}{\partial Q} = (p - s) \cdot P(D > Q_s^*) - (c - s) = 0 \Rightarrow \bar{F}_D(Q_s^*) = \frac{c - s}{p - s} \tag{A2}$$

Solving equations (A1) and (A2) provides the equilibrium quantity:

$$\bar{F}_D(Q_s^*) = \frac{c - s}{p - s} = \frac{c - s}{\bar{F}_D(Q_s^*) \cdot (k + v - s) + s - s} = \frac{c - s}{\bar{F}_D(Q_s^*) \cdot (k + v - s)}$$

$$\Rightarrow (\bar{F}_D(Q_s^*))^2 = \tfrac{c-s}{k+v-s} \Rightarrow \bar{F}_D(Q_s^*) = \sqrt{\tfrac{c-s}{k+v-s}}$$

Putting the last term back in (A1) provides the equilibrium price:

$$p_s^* = \bar{F}_D(Q_s^*)(k + v - s) + s = \sqrt{\frac{c - s}{k + v - s}}(k + v - s) + s = \sqrt{(c - s)(k + v - s)} + s$$

□

Before we proceed with more results, we prove Lemma A1.

Lemma A1. *i) $Q_s^* < Q_0$ and $p_s^* < p_0$ when $k < \frac{(v-s)(v-c)}{c-s}$*

ii) $Q_s^* > Q_0$ and $p_s^* > p_0$ when $k > \frac{(v-s)(v-c)}{c-s}$

Proof of Lemma A1: We define $\triangle Q(k) = Q_s^*(k) - Q_0 = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) - \bar{F}_D^{-1}\left(\frac{c-s}{v-s}\right)$. Showing that $\triangle Q(k)$ is negative at $k = 0$ and $\triangle Q(k)$ strictly increases with k, is sufficient to say that there exists a unique k^* such that $\triangle Q(k^*) = 0$:

- $\mathbf{\triangle Q(0) < 0}$: Note that $\sqrt{\frac{c-s}{v-s}} > \frac{c-s}{v-s}$ since $v > c > s$. Then, $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) < \bar{F}_D^{-1}\left(\frac{c-s}{v-s}\right)$ which confirms that $\triangle Q(0) = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) - \bar{F}_D^{-1}\left(\frac{c-s}{v-s}\right) < 0$.
- $\mathbf{\frac{\partial \triangle Q(k)}{\partial k} > 0}$: $\frac{\partial \triangle Q(k)}{\partial k} = \frac{\partial Q_s^*(k)}{\partial k} = -\frac{1}{2}\frac{\sqrt{c-s}}{(k+v-s)^{3/2}}\frac{1}{-f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}))} = \frac{1}{2}\frac{\sqrt{c-s}}{(k+v-s)^{3/2}}\frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}))}$
  Note that $\sqrt{c-s} > 0$ and $k + v - s > 0$ since $v > c > s$ and $k > 0$. Then, $\frac{\partial \triangle Q(k)}{\partial k} > 0$.

Thus, there exists an unique k^* such that $\triangle Q(k^*) = 0$:

$$Q_s^*(k^*) - Q_0 = 0 \Rightarrow \bar{F}_D^{-1}\left(\sqrt{\tfrac{c-s}{k^*+v-s}}\right) = \bar{F}_D^{-1}\left(\tfrac{c-s}{v-s}\right)$$
$$\Rightarrow k^* = \tfrac{(v-s)\cdot(v-c)}{c-s}$$

$\triangle Q(0) < 0$ and $\frac{\partial \triangle Q(k)}{\partial k} > 0$ imply that $\triangle Q(k)$ changes sign only at k^* as k increases. It is easy to show that the same threshold, k^*, holds for the relation between p_s^* and p_0. This leads to the following result:

i. $Q_s^* < Q_0$ and $p_s^* < p_0$ when $k < \frac{(v-s)\cdot(v-c)}{c-s}$

ii. $Q_s^* > Q_0$ and $p_s^* > p_0$ when $k > \frac{(v-s)\cdot(v-c)}{c-s}$

□

Proof of Proposition 2(1): The firm sets the price at the reservation price of snobs, and the commoners are excluded from consideration (βD instead of β). The RE equilibrium conditions reduce to

$$p = \bar{F}_{\beta D}(Q_s^*)(k+v-s) + s = P(\beta D > Q_s^*)(k+v-s) + s \tag{A3}$$

The firm's critical fractile quantity choice is:

$$\frac{\partial \Pi_N}{\partial Q} = (p-s)P(\beta D > Q) - (c-s) = 0 \Rightarrow P(\beta D > Q_s^*) = \frac{c-s}{p-s} \tag{A4}$$

Solving equations (A3) and (A4) provides the equilibrium quantity:

$$\bar{F}_{\beta D}(Q_s^*) = \frac{c-s}{p-s} = \frac{c-s}{\bar{F}_{\beta D}(Q_s^*)(k+v-s)+s-s} = \frac{c-s}{\bar{F}_{\beta D}(Q_s^*)\cdot(k+v-s)}$$

$$\Rightarrow (\bar{F}_{\beta D}(Q_s^*))^2 = \tfrac{c-s}{k+v-s} \Rightarrow \bar{F}_{\beta D}(Q_s^*) = \sqrt{\tfrac{c-s}{k+v-s}}.$$

Putting the last term back in (A3) provides the equilibrium price:

$$p_s^* = \bar{F}_{\beta D}(Q_s^*)\cdot(k+v-s) + s = \sqrt{\frac{c-s}{k+v-s}}(k+v-s) + s = \sqrt{(c-s)(k+v-s)} + s$$

□

Proof of Proposition 2(2): The RE equilibrium conditions reduce to

$$p = \bar{F}_D(Q_c^*)(v-s) + s = P(D > Q_c^*)(v-s) + s \tag{A5}$$

The firm's critical fractile quantity choice is,

$$\frac{\partial \Pi_N}{\partial Q} = (p-s)\cdot P(D > Q_c^*) - (c-s) = 0 \Rightarrow P(D > Q_c^*) = \frac{c-s}{p-s} \tag{A6}$$

Solving equations A5 and A6 provides the equilibrium quantity:

$$\bar{F}_D(Q_c^*) = \frac{c-s}{p-s} = \frac{c-s}{\bar{F}_D(Q_c^*)(v-s)+s-s} = \frac{c-s}{\bar{F}_D(Q_c^*)(v-s)}$$

$$\Rightarrow \bar{F}_D(Q_c^*) = \sqrt{\tfrac{c-s}{v-s}}$$

Putting the last term back in (A5) provides the equilibrium price:

$$p_c^* = \bar{F}_D(Q_c^*) \cdot (v-s) + s = \sqrt{\frac{c-s}{v-s}} \cdot (v-s) + s = \sqrt{(c-s)(v-s)} + s$$

□

The ensuing Lemma A2 sheds more light on when the firm chooses *Limited Production* and sells only to snobs, and when it tries to adopt the *Regular Production* strategy to cover the whole market (subject to demand uncertainty).

Lemma A2. *There exists a unique threshold of snobs, β^*, where $\Pi_{N,s}^* \le \Pi_{N,c}^*$ when $\beta \le \beta^*$, and $\Pi_{N,s}^* > \Pi_{N,c}^*$ when $\beta > \beta^*$.*[A1]

Lemma 2 shows the firm may adopt different policies based on the concentration of conspicuous consumption in the market. The decision depends on the threshold fraction of snobs in the market (β^*).

Proof of Lemma A2: We show that the difference between profits obtained from "Limited" and "Regular" production changes sign only at a unique threshold of snobs, β^*, as β increases. Recall the implicit formulation of Q_s^* from Proposition 2(1). This leads to the following explicit solution:

$$Q_s^* = \beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$$

Then, the optimal profit for the firm is:

$$\begin{aligned}\Pi_{N,s}^* &= \sqrt{(c-s)\cdot(k+v-s)}E[\min\{\beta D, Q_s^*\}] - (c-s)Q_s^* \\ &= \sqrt{(c-s)\cdot(k+v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} \beta \cdot u \cdot f_D(u) \cdot du\end{aligned}$$

Recall the implicit formulation of Q_c^* from Proposition 2(2). This leads to the following explicit solution:

$$Q_c^* = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$$

Then the optimal profit for the firm is:

$$\begin{aligned}\Pi_{N,c}^* &= \sqrt{(c-s)\cdot(v-s)} \cdot E[\min\{D, Q_c^*\}] - (c-s)\cdot Q_c^* \\ &= \sqrt{(c-s)\cdot(v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du\end{aligned}$$

We define $\Pi_{N,s}^*(\beta)$ to represent the firm's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. We assume that $Q_s^*(\beta) > 0$ except for $\beta = 0$ and $Q_c^* > 0$ without loss of generality. Also, we define $\triangle\Pi(\beta) = \Pi_{N,s}^*(\beta) - \Pi_{N,c}^*$. Note that β is in the domain [0,1]. Then, showing that $\triangle\Pi(\beta)$ is negative at $\beta = 0$, $\triangle\Pi(\beta)$ is positive at $\beta = 1$ and

[A1]The unique threshold level is $\beta^* = \sqrt{\frac{v-s}{k+v-s}} \cdot \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u)\, du / \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u f_D(u)\, du$

$\triangle\Pi(\beta)$ strictly increases in β is sufficient to say that there exists β^* such that $\triangle\Pi(\beta^*) = 0$:

$$\begin{aligned}
\triangle\boldsymbol{\Pi}(\mathbf{0}) < \mathbf{0} : \triangle\Pi(0) &= \Pi^*_{N,s}(0) - \Pi^*_{N,c} = -\Pi^*_{N,c} \\
&= -\sqrt{(c-s)\cdot(v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du.
\end{aligned}$$

The term $\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q^*_c > 0$. Then, the last term of the equality above must be non-positive.

$$\begin{aligned}
\triangle\boldsymbol{\Pi}(\mathbf{1}) > \mathbf{0} : \ \triangle\Pi(1) &= \Pi^*_{N,s}(1) - \Pi^*_{N,c} \\
&= \sqrt{(c-s)\cdot(k+v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} u \cdot f_D(u) \cdot du \\
&\quad -\sqrt{(c-s)\cdot(v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du
\end{aligned}$$

$\sqrt{(c-s)(v-s)} \leq \sqrt{(c-s)\cdot(k+v-s)}$ since $k \geq 0$. Also, $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) \leq \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$, since $\sqrt{\frac{c-s}{v-s}} \geq \sqrt{\frac{c-s}{k+v-s}}$. Then, the last equality above must be positive.

$$\begin{aligned}
\frac{\partial\triangle\boldsymbol{\Pi}(\beta)}{\partial\beta} > \mathbf{0} : \ \frac{\partial\triangle\Pi(\beta)}{\partial\beta} &= \frac{\partial\Pi^*_{N,s}(\beta)}{\partial\beta} \\
&= \sqrt{(c-s)\cdot(k+v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} u \cdot f_D(u) \cdot du.
\end{aligned}$$

The first term of the last equality is positive since $v > c > s$ and $k \geq 0$. The support of D is non-negative and $Q^*_s(\beta)/\beta > 0$. Then, the last equality above must be positive.
Then, there exists a unique root β^* such that $\triangle\Pi(\beta^*) = 0$:

$$\begin{aligned}
&\Rightarrow \Pi^*_{N,s}(\beta^*) - \Pi^*_{N,c} = 0 \\
&\Rightarrow \sqrt{c-s}\Big(\sqrt{k+v-s} \cdot \beta^* \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} u \cdot f_D(u) \cdot du \\
&\qquad\qquad - \sqrt{v-s} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du\Big) = 0 \\
&\Rightarrow \beta^* = \sqrt{\frac{v-s}{k+v-s}} \cdot \frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u \cdot f_D(u)\, du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u \cdot f_D(u)\, du}
\end{aligned}$$

$\triangle\Pi(0) < 0$, $\triangle\Pi(1) > 0$, and $\frac{\partial\triangle\Pi(\beta)}{\partial\beta} > 0$ imply that $\triangle\Pi(\beta)$ changes sign only at β^* as β increases. This leads to the following result:

- $\Pi^*_{N,s} < \Pi^*_{N,c}$ when $\beta < \beta^*$
- $\Pi^*_{N,s} > \Pi^*_{N,c}$ when $\beta > \beta^*$

□

Proof of Proposition 3: Follows from the results of Lemma A2. If $\beta < \beta^*$ then it is more profitable to apply the "Regular Production" strategy since $\Pi^*_{N,s} < \Pi^*_{N,c}$ and if $\beta > \beta^*$ then it is more profitable to apply the "Limited Production" strategy since $\Pi^*_{N,s} > \Pi^*_{N,c}$.

□

Proof of Proposition 4: We show that the difference between Q^*_s and Q^*_c changes sign only at a particular threshold level, β_Q, as β increases. We define $Q^*_s(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. We assume that $Q^*_s(\beta) > 0$ except for $\beta = 0$ and $Q^*_c > 0$ without loss of generality. Also, we define $\triangle Q(\beta) = Q^*_s(\beta) - Q^*_c = \beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. Then, showing that $\triangle Q(0) < 0$, $\triangle Q(1) > 0$, and $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$ is sufficient to say there exists unique β_Q such that $\triangle Q(\beta_Q) = 0$:

- $\triangle \mathbf{Q}(\mathbf{0}) < \mathbf{0}:$ $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q^*_c > 0$. Then, $\triangle Q(0) < 0$.

- $\triangle \mathbf{Q}(\mathbf{1}) > \mathbf{0}:$ $\triangle Q(1) = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. We show that $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) > \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ within the proof of Lemma A2. Then, $\triangle Q(1) > 0$.

- $\frac{\partial \triangle \mathbf{Q}(\beta)}{\partial \beta} > \mathbf{0}:$ $\frac{\partial \triangle Q(\beta)}{\partial \beta} = \frac{\partial Q^*_s}{\partial \beta} = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) > 0$
  The inequality follows directly from the assumption that the support of D is non-negative and $Q^*_s(\beta)/\beta > 0$. Then, $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$.

Then, there exists a unique root β_Q such that $\triangle Q(\beta_Q) = 0$:

$$\Rightarrow Q^*_s(\beta_Q) - Q^*_c = 0 \quad \Rightarrow \quad \beta_Q \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) = 0$$
$$\Rightarrow \quad \beta_Q = \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)}$$

$\triangle Q(0) < 0$, $\triangle Q(1) > 0$, and $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$ imply that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. This leads to the following result:

- $Q^*_s < Q^*_c$ when $\beta < \beta_Q$
- $Q^*_s > Q^*_c$ when $\beta > \beta_Q$

□

Before we prove more results, we prove preparatory Proposition A3.

Proposition A3. *β_Q is larger than or equal to β^* when $\frac{x}{y} \geq \frac{\bar{F}_D(y)}{\bar{F}_D(x)}$.*

Proof of Proposition A3: We show that β_Q is larger than or equal to β^* for given parameters (s,c,v and k) under a particular condition. We define $\varrho = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ and $\omega = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$ for simplification of the system. Rewriting β^* and β_Q with the new notation leads to the following equations:

$$\beta_Q = \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} = \frac{\varrho}{\omega}$$

$$\beta^* = \sqrt{\frac{v-s}{k+v-s}} \cdot \frac{\displaystyle\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u \cdot f_D(u)\,du}{\displaystyle\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u \cdot f_D(u)\,du} = \frac{\sqrt{\frac{c-s}{k+v-s}}}{\sqrt{\frac{c-s}{v-s}}} \cdot \frac{\displaystyle\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u \cdot f_D(u)\,du}{\displaystyle\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u \cdot f_D(u)\,du}$$

$$= \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\displaystyle\int_0^{\varrho} u \cdot f_D(u)\,du}{\displaystyle\int_0^{\omega} u \cdot f_D(u)\,du}$$

The last line can be further reduced to the following equation by applying integration by parts on the numerator and the denominator of the second term:

$$\beta^* = \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\int_0^{\varrho} \bar{F}_D(u)\,du - \varrho\bar{F}_D(\varrho)}{\int_0^{\omega} \bar{F}_D(u)\,du - \omega\bar{F}_D(\omega)}$$

Showing that $\beta_Q - \beta^* \geq 0$ is sufficient for the validity of the claim:

$$\beta_Q - \beta^* = \frac{\varrho}{\omega} - \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\int_0^{\varrho} \bar{F}_D(u)\,du - \varrho\bar{F}_D(\varrho)}{\int_0^{\omega} \bar{F}_D(u)\,du - \omega\bar{F}_D(\omega)}$$

Recall that, in the Proof of Lemma A2, we showed both the first and the second term of β^* are less than or equal to 1 and non-negative. Thus, eliminating the second term will provide us a lower bound for $\beta_Q - \beta^*$:

$$\beta_Q - \beta^* \geq \frac{\varrho}{\omega} - \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)}$$

Therefore, we have shown that β_Q is larger than or equal to β^* if $\frac{\varrho}{\omega} \geq \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)}$ for given parameters (s,c,v and k). □

Proof of Lemma 1: We show that for higher k, Q_s^* increases more steeply in β. Recall $Q_s^* = \beta\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$ from the proof of Lemma A2. In Proposition 4, we show that $\frac{\partial Q_s^*}{\partial \beta} = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$. What we need to show now is that the partial derivative of $\frac{\partial Q_s^*}{\partial \beta}$ with respect to k is positive:

$$\frac{\partial^2 Q_s^*}{\partial\beta\partial k} = -\frac{1}{2}\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}\frac{1}{-f_D(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right))}$$

$$= \frac{1}{2}\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}\frac{1}{f_D(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right))}$$

The term, $\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}$, is positive by the assumptions $v > c > s$ and $k > 0$. Then, $\frac{\partial^2 Q_s^*}{\partial\beta\partial k} > 0$. Therefore, we have shown that for higher k, Q_s^* increases more steeply in β.

□

Proof of Lemma 2: Follows directly from showing that the first derivatives of both threshold levels (β^* and β_Q) with respect to k are negative:

- $\frac{\partial \beta^*}{\partial k} = -\frac{1}{2}\sqrt{\frac{v-s}{k+v-s}} \frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u) du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u f_D(u) du} \left(\frac{1}{k+v-s} + \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u f_D(u) du} \frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}} \right) < 0$
  The terms, $\sqrt{\frac{v-s}{k+v-s}}$ and $\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}$, are positive by the assumptions $v > c > s$ and $k > 0$. In Proposition 4, we show that both $\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})$ and $\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})$ are positive so both terms within the parentheses are positive as well. Then, $\frac{\partial \beta^*}{\partial k} < 0$.

- $\frac{\partial \beta_Q}{\partial k} = -\frac{1}{2} \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)^2} \frac{1}{f_D(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right))} \frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}} < 0$
  Follows directly from the proof of the previous point.

Therefore, we have shown that the threshold levels, β^* and β_Q, decrease with increase in sensitivity to stock-out, k. □

Proof of Proposition 5: We derive the conditions under which the the region of scarcity (i.e. $\beta_Q - \beta^*$) expands as snobs become more sensitive to stockouts. We define $\triangle\beta'(k) = \frac{\partial \beta^*}{\partial k} - \frac{\partial \beta_Q}{\partial k}$. We will use the same notation, ϱ and ω, which we used in the proof of Proposition A3. Showing that $\triangle\beta(k)$ is negative for all k in $[0, \infty)$ is sufficient for proving the proposition.

$$\begin{aligned}
\triangle\beta'(k) &= -\frac{1}{2}\sqrt{\frac{v-s}{k+v-s}} \frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u) du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u f_D(u) du} \left(\frac{1}{k+v-s} + \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u f_D(u) du} \frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}} \right) \\
&+ \frac{1}{2} \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)^2} \frac{1}{f_D(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right))} \frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}} \\
&= -\frac{1}{2}\frac{1}{k+v-s}\left(\frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\int_0^{\varrho} u \cdot f_D(u)\, du}{\int_0^{\omega} u \cdot f_D(u)\, du} \left(1 + \frac{\omega}{\int_0^{\omega} u f_D(u) du}\bar{F}_D(\omega)\right) - \frac{\varrho}{\omega}\frac{1}{\omega}\frac{\bar{F}_D(\omega)}{f_D(\omega)} \right) \\
&= -\frac{1}{2}\frac{1}{k+v-s}\left(\beta^* \left(\frac{\int_0^{\omega} \bar{F}_D(u)\, du}{\int_0^{\omega} \bar{F}_D(u)\, du - \omega \bar{F}_D(\omega)} \right) - \beta_Q \frac{1}{\omega}\frac{\bar{F}_D(\omega)}{f_D(\omega)} \right)
\end{aligned}$$

The last equality is less than or equal to zero if and only if the equation in parentheses is non-negative. We know from the proofs of lemma A2 and proposition 4 that all terms within the parentheses is non-negative. Thus, we require a sufficient condition that will make the equation in parentheses non-negative:

$$\begin{aligned}
&\beta^* \left(\frac{\int_0^{\omega} \bar{F}_D(u)\, du}{\int_0^{\omega} \bar{F}_D(u)\, du - \omega \bar{F}_D(\omega)} \right) - \beta_Q \frac{1}{\omega}\frac{\bar{F}_D(\omega)}{f_D(\omega)} \geq 0 \Rightarrow \frac{\omega f_D(\omega)}{\bar{F}_D(\omega)} \geq \frac{\beta_Q}{\beta^*} \frac{\int_0^{\omega} \bar{F}_D(u)\, du - \omega \bar{F}_D(\omega)}{\int_0^{\omega} \bar{F}_D(u)\, du} \\
&\Rightarrow \omega h(\omega) \geq \frac{\beta_Q}{\beta^*} \frac{\int_0^{\omega} \bar{F}_D(u)\, du - \omega \bar{F}_D(\omega)}{\int_0^{\omega} \bar{F}_D(u)\, du} \qquad (h(\omega) = f(\omega)/\bar{F}_D(\omega); \textit{the hazard rate of } D) \\
&\qquad = \frac{\beta_Q}{\beta^*} M(Q_s^*/\beta) \qquad (\textit{since } \omega = Q_s^*/\beta)
\end{aligned}$$

We define the hazard rate of D above as $h(.)$ (see Bryson and Siddiqui 1969 for details). The second term on the right-hand side of the inequality is less than or equal to 1 since $\int_0^{\omega} \bar{F}_D(u)\, du - \omega \bar{F}_D(\omega) \leq$

$\int_0^\omega \bar{F}_D(u)\,du$ and $\omega \geq 0$. The right hand side of the inequality can be more than or less than or equal to 1 depending on the relation between the first and the second term. Implicit sufficient conditions can be similarly obtained in this case. □

Appendix B: Proof for Section 4

Appendix B1: Myopic Snobs

Before we prove our main results on Myopic customers, we prove the following Proposition B1.1 and Lemma B1.1.

Proposition B1.1 (Limited Production). *In the RE equilibrium under limited production, all snobs will try to buy in the current period, and the firm's price and quantity choices are characterized by* $\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}$ *and* $p^*_{s,my} = \frac{v+s+\sqrt{(v-s)^2+4k(c-s)}}{2}$.

Proof of Proposition B1.1: The RE equilibrium conditions reduce to

$$p = v + k\bar{F}_{\beta D}(Q^*_{s,my}) \tag{B1.1}$$

The critical fractile quantity is,

$$\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{c-s}{p-s} \tag{B1.2}$$

Solving equations (B1.1) and (B1.2) provides the following quadratic equation:

$$\Rightarrow k(\bar{F}_{\beta D}(Q^*_{s,my}))^2 + (v-s)\bar{F}_{\beta D}(Q^*_{s,my}) - (c-s) = 0$$

Recall that $s < c < v$. Then, one of the solutions obtained from this quadratic equation, $\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{-(v-s)-\sqrt{(v-s)^2+4k(c-s)}}{2k}$, is infeasible since the survival function must always be non-negative. This leaves one solution which provides the equilibrium quantity:

$$\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{-(v-s)+\sqrt{(v-s)^2+4k(c-s)}}{2k} \tag{B1.3}$$

Putting (B1.3) back in (B1.1) provides the equilibrium price:

$$p^*_{s,my} = \frac{v+s+\sqrt{(v-s)^2+4k(c-s)}}{2}$$

□

Before we proceed with more results, we prove Lemma B1.1. Let $\Pi^*_{N,mySn}$ denote the firm's optimal profit obtained under the Limited Production strategy, and $\Pi^*_{N,c}$ denote the firm's optimal profit obtained under the Regular Production strategy.

Lemma B1.1. *There exists a unique threshold of snobs,* β^*_{mySn}*, where* $\Pi^*_{N,mySn} < \Pi^*_{N,c}$ *when* $\beta < \beta_{mySn}$ *and* $\Pi^*_{N,mySn} > \Pi^*_{N,c}$ *when* $\beta > \beta_{mySn}$*. The unique threshold level is*

$$\beta^*_{mySn} = \frac{\sqrt{(c-s)(v-s)}\displaystyle\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u f_D(u)\,du}{\frac{(v-s)+\sqrt{(v-s)^2+4k(c-s)}}{2}\displaystyle\int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} u f_D(u)\,du}$$

Proof of Lemma B1.1: Recall the proof of Lemma A2. We use the same technique to prove the results of Lemma B1.1. In this setting, the optimal profit of the firm serving to the whole market does not change and the optimal profit of the firm serving to snobs only becomes:

$$\Pi^*_{N,mySn} = \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \cdot \int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} \beta \cdot u \cdot f_D(u) \cdot du$$

We define $\Pi^*_{N,mySn}(\beta)$ to represent the firm's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. Also, we define $\triangle\Pi(\beta) = \Pi^*_{N,mySn}(\beta) - \Pi^*_{N,c}$. We show that the same conditions hold as in the proof of Lemma A2 under the same assumptions:

- $\mathbf{\triangle\Pi(0) < 0}$: $\triangle\Pi(0) = -\sqrt{(c-s)\cdot(v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du$ $\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q_c^* > 0$. Then, the last term of the equality above must be non-positive.

- $\mathbf{\triangle\Pi(1) > 0}$:

$$\begin{aligned}\triangle\Pi(1) &= \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \cdot \int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} \beta u f_D(u) du \\ &\quad -\sqrt{(c-s)\cdot(v-s)} \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du\end{aligned}$$

Claim: $\frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \geq \sqrt{(c-s)(v-s)}$ when $s < c < v$.

$$\begin{aligned}\left(\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2}\right)^2 &= \frac{(v-s)^2}{4}+k(c-s)+(v-s)\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{(v-s)^2}{4} \\ &> k(c-s)+(v-s)^2\end{aligned}$$

Taking square root of both sides, we have obtained the following inequality,

$$\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2} > \sqrt{k(c-s)+(v-s)^2} > \sqrt{(k+v-s)(c-s)} > \sqrt{(v-s)(c-s)} \tag{B1.4}$$

We have shown that the claim holds when $s < c < v$.

Claim: $\sqrt{\frac{c-s}{v-s}} \geq \frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}$ when $s < c < v$. We use (B1.4) to show that the claim holds.

$$\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2} > \sqrt{(k+v-s)(c-s)}$$

$$\frac{c-s}{\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2}} < \sqrt{\frac{c-s}{k+v-s}} \tag{B1.5}$$

Thus, $\sqrt{(c-s)(v-s)} \leq \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2}$ by (B1.4). Also, $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) \leq \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)$ by (B1.5). Then, $\triangle\Pi(1)$ must be positive.

- $\frac{\partial\triangle\mathbf{\Pi}(\beta)}{\partial\beta} > \mathbf{0}$: $\frac{\partial\triangle\Pi(\beta)}{\partial\beta} = \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \cdot \int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} u \cdot f_D(u) \cdot du$. The first term of the last equality is positive since $v > c > s$ and $k \geq 0$. Also, $Q^*_{s,my}/\beta > 0$. Then, the last equality above must be positive.

Then, there exists a unique root β^*_{mySn} such that $\triangle\Pi(\beta^*_{mySn}) = 0$:

$$\Rightarrow \beta^*_{mySn} = \frac{\sqrt{(c-s)(v-s)} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u \cdot f_D(u)\, du}{\frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} u \cdot f_D(u) \cdot du}$$

Hence, we have shown that $\triangle\Pi(\beta)$ changes sign only at β^*_{mySn} as β increases. □

Proposition B1.2. *There exists a unique threshold of snobs,* β^*_{mySn} [B1.1]. *If* $\beta < \beta^*_{mySn}$*, in the RE equilibrium, the firm's price and quantity choices are characterized by* $\bar{F}_D(Q^*_c) = \sqrt{\frac{c-s}{v-s}}$ *and* $p_c = \sqrt{(c-s)\cdot(v-s)} + s$*, and all customers will try to buy. If* $\beta > \beta^*_{mySn}$*, in the RE equilibrium, the firm's price and quantity choices are characterized by* $\bar{F}_{\beta D}(Q^*_{mySn}) = \frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}$ *and* $p^*_{mySn} = \frac{v+s+\sqrt{(v-s)^2+4k(c-s)}}{2}$*, and only snobs will try to buy.*

Proof of Proposition B1.2: Follows from the results of Lemma B1.1. If $\beta \leq \beta^*_{mySn}$ then it is more profitable to apply the *Regular Production* strategy since $\Pi^*_{N,mySn} \leq \Pi^*_{N,c}$ and if $\beta > \beta^*_{mySn}$ then it is more profitable to apply the *Limited Production* strategy since $\Pi^*_{N,mySn} > \Pi^*_{N,c}$. □

Before we prove more results, we prove Proposition B1.3.

Proposition B1.3. *There exists another unique level of percentage of snobs,* $\beta_{Q,mySn}$*, where* $Q^*_{mySn} < Q^*_c$ *when* $\beta < \beta_{Q,mySn}$ *and* $Q^*_{mySn} > Q^*_c$ *when* $\beta > \beta_{Q,mySn}$*. This unique threshold level is*

$$\beta_{Q,mySn} = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) / \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)$$

Proof of Proposition B1.3: Recall the proof of Proposition 4. We use the same techniques to prove the results of Proposition B1.3. We define $Q^*_{s,my}(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. Also, we define $\triangle Q(\beta) = Q^*_{s,my}(\beta) - Q^*_c$. We show that the same conditions hold as in the proof of Proposition 4 under the same assumptions:

- $\triangle\mathbf{Q}(\mathbf{0}) < \mathbf{0}$: $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q^*_c > 0$. Then, $\triangle Q(0) < 0$.

[B1.1] $\beta^*_{mySn} = \sqrt{(c-s)(v-s)} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} uf_D(u)\,du / \frac{(v-s)+\sqrt{(v-s)^2+4k(c-s)}}{2} \int_0^{\bar{F}_D^{-1}(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k})} uf_D(u)\,du$

- $\triangle\mathbf{Q}(\mathbf{1}) > \mathbf{0}$: $\triangle Q(1) = \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) > 0$ by (B1.5) within the proof of Lemma B1.1. Then, $\triangle Q(1) > 0$.
- $\frac{\partial\triangle\mathbf{Q}(\beta)}{\partial\beta} > \mathbf{0}$: $\frac{\partial\triangle Q(\beta)}{\partial\beta} = \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right) > 0$. The inequality follows directly from the assumption that the support of D is non-negative and $Q^*_{s,my} > 0$. Then, $\frac{\partial\triangle Q(\beta)}{\partial\beta} > 0$.

Then, there exists a unique root $\beta_{Q,mySn}$ such that $\triangle Q(\beta_{Q,mySn}) = 0$:

$$\Rightarrow \beta_{Q,mySn} = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) / \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)$$

Hence, we have shown that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. □

Appendix B2: Snobs have a lower value for commonly available products

The RE Equilibrium remains as described in 4.3.

Proposition B2.1.

(Limited Production) *In the RE equilibrium under limited production, all customers (snobs & commoners) can buy, and the firm's price and quantity choices are characterized by* $P(D > Q_s^*) = \sqrt{\frac{c-s}{k+v_s-s}}$ *and* $p_s^* = \sqrt{(c-s)\cdot(k+v_s-s)} + s$.

Proof of Proposition B2.1: The firm sets the reservation price of snobs so the product is available to everyone. The RE equilibrium conditions reduce to

$$p = \bar{F}_D(Q_s^*)\cdot(k+v_s-s)+s \tag{B2.1}$$

Solving the firm's critical fractile together with (B2.1) provides the equilibrium quantity, $\bar{F}_D(Q_s^*) = \sqrt{\frac{c-s}{k+v_s-s}}$. Putting this implicit term back in (B2.1) provides the equilibrium price:

$$p_s^* = \bar{F}_D(Q_s^*)(k+v_s-s)+s = \sqrt{\frac{c-s}{k+v_s-s}}(k+v_s-s)+s = \sqrt{(c-s)(k+v_s-s)}+s$$

□

Proof of Proposition 6: The firm sets the reservation price of commoners so the snobs are excluded from consideration ($(1-\beta)D$ instead of β) (Recall that this is the case where the snobs have a *lower* valuation than the commoners due to abundant availability). The RE equilibrium conditions reduce to

$$p = \bar{F}_{(1-\beta)D}(Q_c^*)\cdot(v-s)+s = P((1-\beta)D > Q_c^*)\cdot(v-s)+s \tag{B2.2}$$

The firm will obtain the critical fractile quantity choice and solving the critical fractile quantity together with (B2.2) provides the equilibrium quantity, $\bar{F}_{(1-\beta)D}(Q_c^*) = \sqrt{\frac{c-s}{v-s}}$.Putting the implicit term back in (B2.2) provides the equilibrium price:

$$p_c^* = \bar{F}_{(1-\beta)D}(Q_c^*)(v-s)+s = \sqrt{\frac{c-s}{v-s}}(v-s)+s = \sqrt{(c-s)(v-s)}+s$$

□

Before we prove more results, we proceed with Lemma B2.1.

Lemma B2.1. *There exists a unique threshold of snobs, β^*, where $\Pi^*_{N,s} \leq \Pi^*_{N,c}$ when $\beta \leq \beta^*$, and $\Pi^*_{N,s} > \Pi^*_{N,c}$ when $\beta > \beta^*$.*[B2.1]

Proof of Lemma B2.1: Recall the proof of Lemma A2. We use the same approach to prove the results of Lemma B2.1. In the new setting, the optimal profit of the firm becomes,

$$\Pi^*_{N,s} = \sqrt{(c-s)(k+v_s-s)} \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v_s-s}}\right)} u f_D(u) du.$$

In addition to that, the optimal profit of the firm serving to the commoners only becomes,

$$\Pi^*_{N,c} = \sqrt{(c-s)(v-s)}(1-\beta) \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u f_D(u) du$$

We define $\Pi^*_{N,c}(\beta)$ to represent the firm's optimal profit function when regular production strategy is applied given that the percentage of snobs is β. Also, we define $\triangle\Pi(\beta) = \Pi^*_{N,s} - \Pi^*_{N,c}(\beta)$. We show that the same conditions hold as in the proof of Lemma A2 under the same assumptions.

- $$\begin{aligned} \triangle\mathbf{\Pi}(\mathbf{0}) < \mathbf{0}: \ \triangle\Pi(0) &= \sqrt{(c-s)(k+v_s-s)} \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v_s-s}}\right)} u f_D(u) du \\ &\quad - \sqrt{(c-s)(v-s)} \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u f_D(u) du \end{aligned}$$

  $\sqrt{(c-s)(v-s)} \geq \sqrt{(c-s)(k+v_s-s)}$ since $0 < k \leq v - v_s$. Also, $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) \geq \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v_s-s}}\right)$, since $\sqrt{\frac{c-s}{v-s}} \leq \sqrt{\frac{c-s}{k+v-s}}$. Then, the last equality above must be negative.

- $\triangle\mathbf{\Pi}(\mathbf{1}) > \mathbf{0}: \triangle\Pi(1) = \sqrt{(c-s)(k+v_s-s)} \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v_s-s}}\right)} u f_D(u) du$. $\sqrt{(c-s)(k+v_s-s)}$ is positive since $v > v_s > c > s$. The support of D is non-negative and $Q^*_s > 0$. Then, the last term of the equality must be non-negative.

- $\frac{\partial\triangle\mathbf{\Pi}(\beta)}{\partial\beta} > \mathbf{0}: \ \frac{\partial\triangle\Pi(\beta)}{\partial\beta} = \sqrt{(c-s)\cdot(v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u f_D(u) du$. The first term of the equality is positive since $v > c > s$. The support of D is non-negative and $Q^*_c(\beta)/(1-\beta) > 0$. Then, the equality must be positive.

Then, there exists a unique root β^* such that $\triangle\Pi(\beta^*) = 0$:

$$\Rightarrow \beta^* = 1 - \left(\sqrt{k+v-s} \cdot \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v_s-s}})} u f_D(u)\, du / \sqrt{v-s} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u)\, du\right)$$

[B2.1] The unique threshold level is $\beta^* = 1 - \left(\sqrt{k+v-s} \cdot \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v_s-s}})} u f_D(u)\, du / \sqrt{v-s} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u)\, du\right)$

Hence, we have shown that $\triangle\Pi(\beta)$ changes sign only at β^* as β increases. $\square$

Proposition B2.2. *If $\beta \leq \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_{(1-\beta)D}(Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ and $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$, and only commoners can buy. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_D(Q_s^*) = \sqrt{\frac{c-s}{k+v_s-s}}$ and $p_s^* = \sqrt{(c-s)\cdot(k+v_s-s)} + s$, and all customers can buy.*

Proof of Proposition B2.2: Follows from the proof of Lemma B2.1. If $\beta \leq \beta^*$ then it is more profitable to apply the "Regular Production" strategy since $\Pi^*_{N,s} < \Pi^*_{N,c}$ and if $\beta > \beta^*$ then it is more profitable to apply the "Limited Production" strategy since $\Pi^*_{N,s} > \Pi^*_{N,c}$. $\square$

Proposition B2.3. *There exists a unique level of percentage of snobs, β_Q, where $Q_s^* < Q_c^*$ when $\beta < \beta_Q$ and $Q_s^* > Q_c^*$ when $\beta > \beta_Q$. This threshold level is given by*

$$\beta_Q = 1 - \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v_s-s}})}{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}$$

Proof of Proposition B2.3: Recall the proof of Proposition 4. We use the same technique to prove the results of Proposition B2.3. We define $Q_c^*(\beta)$ as the equilibrium quantity choice under the regular production strategy when snobs allocate β percentage of the market. Also, we define $\triangle Q(\beta) = Q_s^* - Q_c^*(\beta)$. We show that the same conditions hold as in the proof of Proposition 4 under the same assumptions:

- $\triangle\mathbf{Q}(\mathbf{0}) < \mathbf{0}$: $\triangle Q(0) = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v_s-s}}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. We have shown that $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v_s-s}}\right) \leq \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ within the proof of Lemma B2.1. Then, $\triangle Q(0) < 0$.

- $\triangle\mathbf{Q}(\mathbf{1}) > \mathbf{0}$: $\triangle Q(1) = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v_s-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v_s-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q_s^* > 0$. Then, $\triangle Q(1) > 0$.

- $\frac{\partial \triangle\mathbf{Q}(\beta)}{\partial\beta} > \mathbf{0}$: $\frac{\partial \triangle Q(\beta)}{\partial\beta} = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) > 0$. The inequality follows directly from the assumption that the support of D is non-negative and $Q_c^*(\beta)/(1-\beta) > 0$. Then, $\frac{\partial \triangle Q(\beta)}{\partial\beta} > 0$.

Then, there exists a unique root β_Q such that $\triangle Q(\beta_Q) = 0$:

$$\Rightarrow \beta_Q = 1 - \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v_s-s}})}{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}$$

Hence, we have shown that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. $\square$

Appendix B3: Non-Linear utility from stockouts

Case A:

Proposition B3.1. i) ***(Limited Production)*** *In the RE equilibrium under limited production, only snobs can buy, and the firm's price and quantity choices are characterized by $k.\bar{F}_{\beta D}(Q_s^*)^{n+1} + \bar{F}_{\beta D}(Q_s^*)^2.(v-s) = c-s$ and $p_s^* = k.\bar{F}_{\beta D}(Q_s^*)^n + \bar{F}_{\beta D}(Q_s^*).(v-s) + s$.*

*ii) **(Regular Production)** In the RE equilibrium, all customers (snobs & commoners) can buy, and the firm's price and quantity decisions are characterized by* $\bar{F}_D(Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ *and* $p_c^* = \sqrt{(c-s).(v-s)} + s$.

Proof of Proposition B3.1(1): The firm sets the reservation price of snobs so the commoners are excluded from consideration (βD instead of β). The RE equilibrium conditions reduce to

$$p = k.\bar{F}_{\beta D}(Q_s)^n + \bar{F}_{\beta D}(Q_s).(v-s) + s \tag{B3.1}$$

Solving the critical fractile quantity together with (B3.1) provides the implicit equation for the equilibrium quantity:

$$\Rightarrow k.\bar{F}_{\beta D}(Q_s^*)^{n+1} + \bar{F}_{\beta D}(Q_s^*)^2.(v-s) = c-s \tag{B3.2}$$

Putting the implicit term, $\bar{F}_{\beta D}(Q_s^*)$, back in (B3.1) provides the equilibrium price:

$$p_s^* = k.\bar{F}_{\beta D}(Q_s^*)^n + \bar{F}_{\beta D}(Q_s^*).(v-s) + s$$

□

Proof of Proposition B3.1(2): Recall that the utility function of the commoners does not change within this extension. This implies that the reservation price of the commoners stay same. Hence, the firm's price and quantity decisions follow from the proof of Proposition 2(2).

Before we proceed with more results, we prove Lemma B3.1.

Lemma B3.1. *There exists a unique threshold of snobs,* β^**, where* $\Pi_{N,s}^* < \Pi_{N,c}^*$ *when* $\beta \leq \beta^*$*, and* $\Pi_{N,s}^* > \Pi_{N,c}^*$ *when* $\beta > \beta^*$ *under both formulations.*

Proof of Lemma B3.1: We use the same approach of proof of Lemma A2 to prove the results of Lemma B3.1. In this setting the optimal profit of the firm serving to the whole market does not change and the optimal profit of the firm serving to snobs only becomes:

$$\begin{aligned}
\Pi_{N,s}^* &= \left(k.\bar{F}_D(Q_s^*/\beta)^n + \bar{F}_D(Q_s^*/\beta).(v-s)\right) \int_0^{\frac{Q_s^*}{\beta}} \beta u f_D(u) du \qquad (by\ (B3.2)) \\
&= \frac{c-s}{\bar{F}_D(Q_s^*/\beta)} \beta \int_0^{\frac{Q_s^*}{\beta}} u f_D(u) \quad (by\ (B3.2))
\end{aligned}$$

We define $\Pi_{N,s}^*(\beta)$ to represent the firm's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. Also, we define $\triangle\Pi(\beta) = \Pi_{N,s}^*(\beta) - \Pi_{N,c}^*$. We show that the same conditions hold as in the proof of Lemma A2 under the same assumptions:

- $\triangle\Pi(0) < 0$**:** $\triangle\Pi(0) = -\sqrt{(c-s).(v-s)} \int_0^{Q_c^*} u f_D(u) du$. $\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q_c^* > 0$. Then, the equality must be non-positive.

- $\triangle\Pi(1) > 0$**:**

$$\triangle\Pi(1) = \left(k.\bar{F}_D(Q_s^*)^n + \bar{F}_D(Q_s^*).(v-s)\right) \int_0^{Q_s^*} u f_D(u) du - \sqrt{(c-s).(v-s)} \int_0^{Q_c^*} u f_D(u) du$$

  It is easy to show that $k.\bar{F}_D(Q_s^*)^n + \bar{F}_D(Q_s^*).(v-s) > \sqrt{(c-s)(v-s)}$ and $Q_s^* > Q_c^*$ by using (B3.2). Hence, $\Pi(1) > 0$.

- $\frac{\partial \triangle\Pi(\beta)}{\partial\beta} > 0$**:** First, we use Equation (B3.2) to show that $\frac{\partial Q_s^*}{\partial\beta} - \frac{Q_s^*}{\beta} = 0$. This result simplifies the derivative of $\triangle\Pi(\beta)$ with respect to β and this simplification reveals the sign of $\frac{\partial \triangle\Pi(\beta)}{\partial\beta}$. The derivative of Equation (B3.2) with respect to β is:

$$\frac{\partial \bar{F}_D(Q_s^*/\beta)}{\partial\beta}.\left(k.(n+1).\bar{F}_D(Q_s^*/\beta)^n. + 2.(v-s).\bar{F}_D(Q_s^*/\beta)\right) = 0$$

$v-s>0$ since $v>c>s$, and $k>0$. Then, the term in the parenthesis must be positive. This implies that $\frac{\partial \bar{F}_D(Q_s^*/\beta)}{\partial\beta} = 0$. The support of D is positive and $\beta \in (0,1)$. Hence, $\frac{\partial Q_s^*}{\partial\beta} = \frac{Q_s^*}{\beta}$. The derivative of $\triangle\Pi(\beta)$ with respect to β is:

$$\frac{\partial \triangle\Pi(\beta)}{\partial\beta} = \frac{c-s}{\bar{F}_D(Q_s^*/\beta)^2}.\int_0^{\frac{Q_s^*}{\beta}} u f_D(u)du \qquad \left(since\ \frac{\partial Q_s^*}{\partial\beta} = \frac{Q_s^*}{\beta}\right)$$

$c-s>0$ since $v>c>s$ and $Q_s^*>0$ except for $\beta=0$. Then, the last equality above must be positive.

Hence, we have shown that $\Pi(\beta)$ changes sign only at β^* as β increases. □

Proposition B3.2. *If $\beta \le \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_D(Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ and $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$, and all customers can buy. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $k.\bar{F}_{\beta D}(Q_s^*)^{n+1} + \bar{F}_{\beta D}(Q_s^*)^2.(v-s) = c-s$ and $p_s^* = k.\bar{F}_{\beta D}(Q_s^*)^n + \bar{F}_{\beta D}(Q_s^*).(v-s)+s$, and only snobs can buy.*

Proof of Proposition B3.2: Follows from the results of Lemma B3.1. If $\beta < \beta^*$ then it is more profitable to apply the "Regular Production" strategy since $\Pi^*_{N,s} < \Pi^*_{N,c}$ and if $\beta > \beta^*$ then it is more profitable to apply the "Limited Production" strategy since $\Pi^*_{N,s} > \Pi^*_{N,c}$. □

<u>**Case B:**</u>

Proposition B3.3. *i) **(Limited Production)** In the RE equilibrium under limited production, only snobs can buy, and the firm's price and quantity choices are characterized by $\bar{F}_{\beta D}(Q_s^*) = \frac{(c-s).(1-\bar{F}_{\beta D}(Q_s^*))}{v-(1-\bar{F}_{\beta D}(Q_s^*))^{n+1}.v-\bar{F}_{\beta D}(Q_s^*).(1-\bar{F}_{\beta D}(Q_s^*))^n.s}$ and $p_s^* = \frac{v}{(1-\bar{F}_{\beta D}(Q_s^*))^n} - (1-\bar{F}_{\beta D}(Q_s^*)).v - \bar{F}_{\beta D}(Q_s^*).s + s$.*

*ii) **(Regular Production)** In the RE equilibrium, all customers (snobs & commoners) can buy, and the firm's price and quantity decisions are characterized by $\bar{F}_D(Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ and $p_c^* = \sqrt{(c-s).(v-s)} + s$.*

Proof of Proposition B3.3(1): The firm sets the reservation price of snobs so the commoners are excluded from consideration (βD instead of β). The RE equilibrium conditions reduce to:

$$p = \frac{v}{(1-\bar{F}_{\beta D}(Q_s))^n} - (1-\bar{F}_{\beta D}(Q_s)).v - \bar{F}_{\beta D}(Q_s).s + s \tag{B3.3}$$

The firm will obtain the critical fractile quantity choice. Solving the critical fractile together with (B3.3) provides the implicit equation for the equilibrium quantity:

$$\Rightarrow v.\bar{F}_D(Q_s^*/\beta).(1-(1-\bar{F}_D(Q_s^*/\beta))^n) + \bar{F}_D(Q_s^*/\beta)^2.(1-\bar{F}_D(Q_s^*/\beta))^n.(v-s) = (c-s).(1-\bar{F}_D(Q_s^*/\beta))^n \tag{B3.4}$$

Putting the implicit term, $\bar{F}_{\beta D}(Q_s^*)$, back in (B3.3) provides the equilibrium price:

$$p_s^* = \frac{v}{(1-\bar{F}_{\beta D}(Q_s^*))^n} - (1-\bar{F}_{\beta D}(Q_s^*)).v - \bar{F}_{\beta D}(Q_s^*).s + s$$

□

Proof of Proposition B3.3(2): Recall that the utility function and the reservation price of commoners stay unchanged. Hence, the firm's price and quantity decisions follow from the proof of Proposition 2(2). □

Before we proceed with more results, we prove Lemma B3.2.

Lemma B3.2. *There exists a unique threshold of snobs, β^*, where $\Pi^*_{N,s} < \Pi^*_{N,c}$ when $\beta \leq \beta^*$, and $\Pi^*_{N,s} > \Pi^*_{N,c}$ when $\beta > \beta^*$ under both formulations.*

Proof of Lemma B3.2: Recall the proof of Lemma A2. We use the same technique to prove the results of Lemma B3.2.

In this setting, the optimal profit of the firm serving to the whole market does not change and the optimal profit of the firm serving to snobs only becomes:

$$\Pi^*_{N,s} = \left(\frac{v.(1-(1-\bar{F}_D(Q^*_s/\beta))^n)}{(1-\bar{F}_D(Q^*_s/\beta))^n} + \bar{F}_D(Q^*_s/\beta).(v-s)\right).\int_0^{\frac{Q^*_s}{\beta}} \beta.u.f_D(u)du \qquad (by\ (B3.4))$$

$$= \frac{c-s}{\bar{F}_D(Q^*_s/\beta)}\beta\int_0^{\frac{Q^*_s}{\beta}} u\cdot f_D(u) \qquad (by\ (B3.4))$$

We define $\Pi^*_{N,s}(\beta)$ to represent the firm's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. Also, we define $\triangle\Pi(\beta) = \Pi^*_{N,s}(\beta) - \Pi^*_{N,c}$. We show that the same conditions hold as in the proof of Lemma A2 under the same assumptions:

$\triangle\Pi(0) < 0$**:** $\triangle\Pi(0) = -\sqrt{(c-s).(v-s)}\int_0^{Q^*_c} uf_D(u)du$. $\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q^*_c > 0$. Then, the equality must be non-positive.

$\triangle\Pi(1) > 0$**:**

$$\triangle\Pi(1) = \left(\frac{v.(1-(1-\bar{F}_D(Q^*_s))^n)}{(1-\bar{F}_D(Q^*_s))^n} + \bar{F}_D(Q^*_s).(v-s)\right)\int_0^{Q^*_s} uf_D(u)du - \sqrt{(c-s).(v-s)}\int_0^{Q^*_c} uf_D(u)du$$

It can be shown that $\frac{v.(1-(1-\bar{F}_D(Q^*_s))^n)}{(1-\bar{F}_D(Q^*_s))^n} + \bar{F}_D(Q^*_s).(v-s) > \sqrt{(c-s)(v-s)}$ and $Q^*_s > Q^*_c$ by using (B3.4). Hence, $\Pi(1) > 0$.

$\frac{\partial\triangle\Pi(\beta)}{\partial\beta} > 0$**:** First, we use Equation (B3.4) to show that $\frac{\partial Q^*_s}{\partial\beta} - \frac{Q^*_s}{\beta} = 0$. This result simplifies the derivative of $\triangle\Pi(\beta)$ with respect to β and this simplification reveals the sign of $\frac{\partial\triangle\Pi(\beta)}{\partial\beta}$.

The derivative of Equation (B3.4) with respect to β is:

$$\Rightarrow\ v\frac{\partial\bar{F}_D(Q^*_s/\beta)}{\partial\beta}.\left((1-(1-\bar{F}_D(Q^*_s))^n) + n.\bar{F}_D(Q^*_s)(1-\bar{F}_D(Q^*_s))^{n-1}\right) + 2(v-s)\frac{\partial\bar{F}_D(Q^*_s/\beta)}{\partial\beta}\bar{F}_D(Q^*_s/\beta)(1-\bar{F}_D(Q^*_s))^n$$
$$= n(1-\bar{F}_D(Q^*_s))^{n-1}\frac{\partial\bar{F}_D(Q^*_s/\beta)}{\partial\beta}\left((v-s)\bar{F}_D(Q^*_s/\beta)^2 - (c-s)\right) \qquad (B3.5)$$

It is easy to show that $\bar{F}_D(Q^*_c)^2 > \bar{F}_D(Q^*_s)^2 > \bar{F}_D(Q^*_s/\beta)^2$ by using (B3.4). This leads to the following result:

$$\Rightarrow \bar{F}_D(Q^*_c)^2 = \frac{c-s}{v-s} > \bar{F}_D(Q^*_s/\beta)^2 \Rightarrow (v-s).\bar{F}_D(Q^*_s/\beta)^2 - (c-s) < 0$$

Then, the terms in the parenthesis on the righthand side of Equation B3.5 is negative. On the other hand, the rest of the terms, except for $\frac{\partial\bar{F}_D(Q^*_s/\beta)}{\partial\beta}$, is positive on both sides of the equation. Therefore, the equality in Equation B3.5 holds if and only if $\frac{\partial\bar{F}_D(Q^*_s/\beta)}{\partial\beta} = 0$. The support of D is positive and $\beta \in (0,1)$. Thus, $\frac{\partial Q_s^*}{\partial \beta} = \frac{Q_s^*}{\beta}$. The derivative of $\triangle\Pi(\beta)$ with respect to β is:

$$\frac{\partial \triangle\Pi(\beta)}{\partial \beta} = \frac{c-s}{\bar{F}_D(Q_s^*/\beta)^2} . \int_0^{\frac{Q_s^*}{\beta}} u f_D(u) du \qquad \left(since \ \frac{\partial Q_s^*}{\partial \beta} = \frac{Q_s^*}{\beta} \right)$$

$c - s > 0$ since $v > c > s$ and $Q_s^* > 0$ except for $\beta = 0$. Then, the last equality above must be positive. Hence, we have shown that $\Pi(\beta)$ changes sign only at β^* as β increases. □

Proposition B3.4. *1. If $\beta \leq \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_D(Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ and $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$, and all customers can buy. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_{\beta D}(Q_s^*) = \frac{(c-s).(1-\bar{F}_{\beta D}(Q_s^*))^n}{v-(1-\bar{F}_{\beta D}(Q_s^*))^{n+1}.v-\bar{F}_{\beta D}(Q_s^*).(1-\bar{F}_{\beta D}(Q_s^*))^n.s}$ and $p_s^* = \frac{v}{(1-\bar{F}_{\beta D}(Q_s^*))^n} - (1-\bar{F}_{\beta D}(Q_s^*)).v - \bar{F}_{\beta D}(Q_s^*).s + s$, and only snobs can buy.*

Proof of Proposition B3.4: Follows from the results of Lemma B3.2. If $\beta \leq \beta^*$ then it is more profitable to apply the "Regular Production" strategy since $\Pi_{N,s}^* \leq \Pi_{N,c}^*$ and if $\beta > \beta^*$ then it is more profitable to apply the "Limited Production" strategy since $\Pi_{N,s}^* > \Pi_{N,c}^*$. □

Proposition B3.5. *There exists a unique level of percentage of snobs, β_Q, where $Q_s^* \leq Q_c^*$ when $\beta \leq \beta_Q$ and $Q_s^* > Q_c^*$ when $\beta > \beta_Q$ under both formulations.*

Proof of Proposition B3.5: Recall the proof of Proposition 4. We use the same technique to prove the results of Proposition 5. We define $Q_s^*(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. Also, we define $\triangle Q(\beta) = Q_s^*(\beta) - Q_c^*$. We show that the same conditions hold as in the proof of Proposition 4 under the same assumptions:

Case A:

- $\triangle \mathbf{Q(0)} < \mathbf{0}$: $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q_c^* > 0$. Then, $\triangle Q(0) < 0$.
- $\triangle \mathbf{Q(1)} > \mathbf{0}$: $\triangle Q(1) = Q_s^*(1) - Q_c^*$. It is easy to show that $Q_s^*(1) > Q_c^*$ by using (B3.2). Then, $\triangle Q(1) > 0$.
- $\frac{\partial \triangle \mathbf{Q}(\beta)}{\partial \beta} > \mathbf{0}$: $\frac{\partial \triangle Q(\beta)}{\partial \beta} = \frac{\partial Q_s^*}{\partial \beta}$. We show that $\frac{\partial Q_s^*}{\partial \beta} = \frac{Q_s^*}{\beta}$ within the proof of Lemma B3.1. $Q_s^* > 0$ except for $\beta = 0$ and $\beta > 0$. Then, $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$.

Hence, we have shown that $\triangle Q(\beta)$ changes sign only at β_Q as β increases.

Case B:

- $\triangle \mathbf{Q(0)} < \mathbf{0}$: $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q_c^* > 0$. Then, $\triangle Q(0) < 0$.
- $\triangle \mathbf{Q(1)} > \mathbf{0}$: $\triangle Q(1) = Q_s^*(1) - Q_c^*$. It is easy to show that $Q_s^*(1) > Q_c^*$ by using (B3.4). Then, $\triangle Q(1) > 0$.
- $\frac{\partial \triangle \mathbf{Q}(\beta)}{\partial \beta} > \mathbf{0}$: $\frac{\partial \triangle Q(\beta)}{\partial \beta} = \frac{\partial Q_s^*}{\partial \beta}$. We show that $\frac{\partial Q_s^*}{\partial \beta} = \frac{Q_s^*}{\beta}$ within the proof of Lemma B3.2. $Q_s^* > 0$ except for $\beta = 0$ and $\beta > 0$. Then, $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$.

Hence, we have shown that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. □

Appendix B4: Beliefs based on Fill-Rate

Proposition B4.1. *1. **(Limited Production)** In the RE equilibrium under limited production, only snobs can buy, and the firm's price and quantity choices are characterized by* $\bar{F}_{\beta D}(Q_s^*) = \frac{c-s}{v+k.\frac{\int_{Q_s^*}^{\infty}(u-Q_s^*)f_{\beta D}(u)du}{E_{\beta D}[D]}-s}$ *and* $p_s^* = \frac{c-s}{\bar{F}_{\beta D}(Q_s^*)} + s$*.*

*2. **(Regular Production)** In the RE equilibrium, all customers (snobs & commoners) can buy, and the firm's price and quantity decisions are characterized by* $P(D > Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ *and* $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$*.*

Proof of Proposition B4.1(1): The RE equilibrium conditions reduce to

$$p = v + k\frac{\int_{Q_s}^{\infty}(u-Q_s)f_{\beta D}(u)du}{E_{\beta D}[D]} \tag{B4.1}$$

Solving the critical fractile choice together with (B4.1) provides the implicit equation for the equilibrium quantity:

$$\bar{F}_D(\frac{Q_s^*}{\beta}) = \frac{c-s}{p-s} = \frac{c-s}{v+k\frac{\int_{Q_s^*}^{\infty}(u-Q_s^*)f_{\beta D}(u)du}{E_{\beta D}[D]}-s} \tag{B4.2}$$

Solving this equation with uniformly distributed demand on (0,N) leads to $Q_s^* = \beta N\left(1-\left(\frac{c-s}{k+v-s}\right)^{1/3}\right)$. We obtain the equilibrium price using (B4.2), $p_s^* = \frac{c-s}{\bar{F}(\frac{Q_s^*}{\beta})} + s$. □

Proof of Proposition B4.1(2): Again note that the utility function and the reservation price of the commoners remain unchanged. Hence, the firm's price and quantity decisions follow from the proof of Proposition 2(2). □

Lemma B4.1. *There exists a unique threshold level of snobs,* β^**, where* $\Pi_{N,s}^* \leq \Pi_{N,c}^*$ *when* $\beta \leq \beta^*$*, and* $\Pi_{N,s}^* > \Pi_{N,c}^*$ *when* $\beta > \beta^*$*.*

Proof of Lemma B4.1: Approach to proving this Lemma B4.1 follows the the proof of Lemma A2. In this setting, the optimal profit of the firm does not change and the optimal profit of the firm serving to snobs only becomes:

$$\Pi_{N,s}^* = ((c-s)^{2/3}(k+v-s)^{1/3})\int_0^{Q_s^*/\beta}\beta.uf_D(u)du$$

We define $\Pi_{N,s}^*(\beta)$ to represent the firm's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. Also, we define $\triangle\Pi(\beta) = \Pi_{N,s}^*(\beta) - \Pi_{N,c}^*$. We show that the same conditions hold as in the proof of Lemma A2 under the same assumptions:

- $\triangle\Pi(0) < 0$**:** $\triangle\Pi(0) = -\sqrt{(c-s)(v-s)}\int_0^{Q_c^*}uf_D(u)du$. $\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q_c^* > 0$. Then, the equality must be non-positive.
- $\triangle\Pi(1) > 0 : \triangle\Pi(1) = ((c-s)^{2/3}(k+v-s)^{1/3})\int_0^{Q_s^*}uf_D(u)du - \sqrt{(c-s)(v-s)}\int_0^{Q_c^*}uf_D(u)du$. It is easy to show that $(c-s)^{2/3}(k+v-s)^{1/3} > \sqrt{(c-s)(v-s)}$ and $Q_s^* > Q_c^*$ by using (B4.2). Hence, $\Pi(1) > 0$.

- $\frac{\triangle\Pi(\beta)}{\beta} > 0$: All conditions are implicit. Nevertheless, the approach is same as before. For instance, if the demand is distributed $U(0,N)$, we can show $\frac{\triangle\Pi(\beta)}{\beta} = (c-s)^{2/3}.(k+v-s)^{1/3}\int_0^{N\left(1-\left(\frac{c-s}{k+v-s}\right)^{\frac{1}{3}}\right)} uf_D(u)du > 0$. We know that $s < c < v$ and $Q_s^*/\beta > 0$. Then, $\frac{\triangle\Pi(\beta)}{\beta} > 0$.

Hence, we have shown that $\Pi(\beta)$ changes sign only at β^* as β increases. □

Proposition B4.2. *If $\beta \le \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_D(Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ and $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$, and all customers can buy. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_{\beta D}(Q_s^*) = \frac{c-s}{v+k.\frac{\int_{Q_s^*}^{\infty}(u-Q_s^*)f_{\beta D}(u)du}{E_{\beta D}[D]}-s}$ and $p_s^* = \frac{c-s}{\bar{F}_{\beta D}(Q_s^*)} + s$, and only snobs can buy.*

Proof of Proposition B4.2: Follows from the results of Lemma B4.1. If $\beta \le \beta^*$ then it is more profitable to apply the "Regular Production" strategy since $\Pi_{N,s}^* \le \Pi_{N,c}^*$ and if $\beta > \beta^*$ then it is more profitable to apply the "Limited Production" strategy since $\Pi_{N,s}^* > \Pi_{N,c}^*$. □

Proposition B4.3. *There exists a unique level of percentage of snobs, β_Q, where $Q_s^* \le Q_c^*$ when $\beta \le \beta_Q$ and $Q_s^* > Q_c^*$ when $\beta > \beta_Q$.*

Proof of Proposition B4.3: We use the same approach as proof of Proposition 4 to prove the results of Proposition B4.3. We define $Q_s^*(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. Also, we define $\triangle Q(\beta) = Q_s^*(\beta) - Q_c^*$. We show that the same conditions hold as in the proof of Proposition 4 under the same assumptions:

- $\triangle Q(0) < 0$**:** $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q_c^* > 0$. Then, $\triangle Q(0) < 0$.

- $\triangle Q(1) > 0$**:** $\triangle Q(1) = Q_s^* - Q_c^*$. It is easy to show that and $Q_s^* > Q_c^*$ by using (B4.2). Then, $\triangle Q(1) > 0$.

- $\frac{\partial\triangle Q(\beta)}{\partial\beta} > 0$**:** Recall Q_s^* within the proof of Proposition B4.1(1): $\frac{\partial\triangle Q(\beta)}{\partial\beta} = N.\left(1-\left(\frac{c-s}{k+v-s}\right)^{1/3}\right)$. We know that $s < c < v$. Then, $\frac{\triangle\Pi(\beta)}{\beta} > 0$.

Hence, we have shown that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. □

Appendix B5: Beliefs based on Expected Sales

Proposition B5.1. 1. ***(Limited Production)*** *In the RE equilibrium under limited production, only snobs can buy, and the firm's price and quantity choices are characterized by $\bar{F}_{\beta D}(Q_s^*) = \frac{c-s}{v+\frac{k}{Q_s^*-\beta\int_0^{Q_s^*/\beta}F_D(u)du}-s}$ and $p_s^* = \frac{c-s}{\bar{F}_D\left(\frac{Q_s^*}{\beta}\right)} + s$.*

2. ***(Regular Production)*** *In the RE equilibrium, all customers (snobs & commoners) can buy, and the firm's price and quantity decisions are characterized by $P(D > Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ and $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$.*

Proof of Proposition B5.1(1): The firm sets the price at the reservation price of snobs so the commoners are excluded from consideration (βD instead of β). The RE equilibrium conditions reduce to:

$$p = v + \frac{k}{Q_s - \beta \int_0^{Q_s/\beta} F_D(u)du} \tag{B5.1}$$

The firm will obtain the critical fractile quantity choice. Solving the critical fractile together with(B5.1) provides the implicit equation for the equilibrium quantity:

$$\Rightarrow \bar{F}_D(\frac{Q_s^*}{\beta}).(v-s) + k.\frac{\bar{F}_D(\frac{Q_s^*}{\beta})}{Q_s^* - \beta \int_0^{Q_s^*/\beta} F_D(u)du} = c - s \tag{B5.2}$$

We obtain the equilibrium price using (B5.2): $p_s^* = \frac{c-s}{\bar{F}(\frac{Q_s^*}{\beta})} + s$. □

Proof of Proposition B5.1(2): Recall that the reservation price of the commoners stay same. Hence, the firm's price and quantity decisions follow from the proof of Proposition 2(2). □

Lemma B5.1. *There exists a unique threshold level of snobs, β^*, where $\Pi_{N,s}^* \leq \Pi_{N,c}^*$ when $\beta \leq \beta^*$, and $\Pi_{N,s}^* > \Pi_{N,c}^*$ when $\beta > \beta^*$.*

Proof of Lemma B5.1: Proof for Lemma B5.1 is similar to the proof of Lemma A2.

In this setting, the optimal profit of the firm under regular production does not change and the optimal profit of the firm serving to snobs only becomes:

$$\Pi_{N,s}^* = (p_s^* - s).\int_0^{Q_s^*/\beta} \beta.uf_D(u)du$$

We define $\Pi_{N,s}^*(\beta)$ to represent the firm's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. Also, we define $\triangle\Pi(\beta) = \Pi_{N,s}^*(\beta) - \Pi_{N,c}^*$. We show that the same conditions hold as in the proof of Lemma A2 under the same assumptions:

- $\triangle\Pi(0) < 0$**:** $\triangle\Pi(0) = -\sqrt{(c-s)(v-s)} \int_0^{Q_c^*} uf_D(u)du$. $\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q_c^* > 0$. Then, the equality must be non-positive.
- $\triangle\Pi(1) > 0$: $\triangle\Pi(1) = (p_s^* - s). \int_0^{Q_s^*} uf_D(u)du - \sqrt{(c-s).(v-s)}. \int_0^{Q_c^*} uf_D(u)du$. It is easy to show that $p_s^* > \sqrt{(c-s).(v-s)}$ and $Q_s^* > Q_c^*$ by using (B5.2). Hence, $\triangle\Pi(1) > 0$.
- $\frac{\triangle\Pi(\beta)}{\beta} > 0$: General distributions are intractable. We can derive results for the specific uniform distribution, or computationally for any distribution. The profit obtained from Limited Edition Strategy, $\Pi_{N,s}^*(\beta)$ increases with β. Thus, $\frac{\triangle\Pi(\beta)}{\beta} > 0$.

Hence, we have shown that $\Pi(\beta)$ changes sign only at β^* as β increases. □

Proposition B5.2. *If $\beta \leq \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_D(Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ and $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$, and all customers can buy. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_{\beta D}(Q_s^*) = \frac{c-s}{v + \frac{k}{Q_s^* - \beta\int_0^{Q_s^*/\beta} F_D(u)du} - s}$ and $p_s^* = \frac{c-s}{\bar{F}_D\left(\frac{Q_s^*}{\beta}\right)} + s$, and only snobs can buy.*

Proof of Proposition B5.2: Follows from the results of Lemma B5.1. If $\beta \leq \beta^*$ then it is more profitable to apply the "Regular Production" strategy since $\Pi_{N,s}^* \leq \Pi_{N,c}^*$ and if $\beta > \beta^*$ then it is more profitable to apply the "Limited Production" strategy since $\Pi_{N,s}^* > \Pi_{N,c}^*$. □

Proposition B5.3. *There exists a unique level of percentage of snobs, β_Q, where $Q_s^* < Q_c^*$ when $\beta < \beta_Q$ and $Q_s^* > Q_c^*$ when $\beta > \beta_Q$.*

Proof of Proposition B5.3: Recall the proof of Proposition 4. We use the same technique to prove the results of Proposition B5.5. We define $Q_s^*(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. Also, we define $\triangle Q(\beta) = Q_s^*(\beta) - Q_c^*$. We show that the same conditions hold as in the proof of Proposition 4 under the same assumptions:

- $\triangle Q(0) < 0$**:** $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q_c^* > 0$. Then, $\triangle Q(0) < 0$.
- $\triangle Q(1) > 0$**:** $\triangle Q(1) = Q_s^*(1) - Q_c^*$. We can show that $Q_s^*(1) > Q_c^*$ by using (B5.2). Then, $\triangle Q(1) > 0$.
- $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$**:** The result can be shown for specific distributions with finite support such as uniform or using numerical analysis. The equilibrium quantity under Limited Edition Strategy, $Q_s^*(\beta)$ increases with β. Thus, $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$.

Hence, we have shown that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. □

Appendix B6: Beliefs based on Firm's Production Quantity

Proposition B6.1. 1. ***(Limited Production)*** *In the RE equilibrium under limited production, only snobs can buy, and the firm's price and quantity choices are characterized by $\bar{F}_{\beta D}(Q_s^*) = \frac{c-s}{v+\frac{k}{Q_s^*}-s}$ and $p_s^* = v + \frac{k}{Q_s^*}$.*

2. ***(Regular Production)*** *In the RE equilibrium, all customers (snobs & commoners) can buy, and the firm's price and quantity decisions are characterized by $P(D > Q_c^*) = \sqrt{\frac{c-s}{v-s}}$ and $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$.*

Proof of Proposition B6.1(1): The firm sets the price at the reservation price of snobs so the commoners are excluded from consideration (βD instead of β). The RE equilibrium conditions reduce to:

$$p = v + \frac{k}{Q_s} \tag{B6.1}$$

Solving the critical fractile together with (B6.1) provides the implicit equation for the equilibrium quantity:

$$\Rightarrow \bar{F}(\frac{Q_s^*}{\beta}).(v-s) + k.\frac{\bar{F}(\frac{Q_s^*}{\beta})}{Q_s^*} = c - s \tag{B6.2}$$

Solving this equation with uniformly distributed demand on (0,1) leads to $Q_s^* = \frac{v-c-\frac{k}{\beta}+\sqrt{(v-c-\frac{k}{\beta})^2+4.\frac{(v-s)}{\beta}.k}}{2.\frac{(v-s)}{\beta}}$.
We use (B6.2) to obtain the equilibrium price: $p_s^* = \frac{c-s}{\bar{F}(\frac{Q_s^*}{\beta})} + s$. □

Proof of Proposition B6.1(2): Recall that the utility function and the reservation price of the commoners remain unchanged. Hence, the firm's price and quantity decisions follow from the proof of Proposition 2(2).

Lemma B6.1. *There exists a unique threshold level of snobs, β^*, where $\Pi_{N,s}^* < \Pi_{N,c}^*$ when $\beta \leq \beta^*$, and $\Pi_{N,s}^* > \Pi_{N,c}^*$ when $\beta > \beta^*$.*

Proof of Lemma B6.1: We use the the proof of Lemma A2 to prove the results of Lemma B6.1. The optimal profit of the firm under regular production does not change, and the optimal profit of the firm serving to snobs only becomes,

$$\Pi^*_{N,s} = (p^*_s - s).\int_0^{Q^*_s/\beta} \beta.uf_D(u)du$$

We define $\Pi^*_{N,s}(\beta)$ to represent the firm's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. Also, we define $\triangle\Pi(\beta) = \Pi^*_{N,s}(\beta) - \Pi^*_{N,c}$. We show that the same conditions hold as in the proof of Lemma A2 under the same assumptions:

- $\triangle\Pi(0) < 0$**:** $\triangle\Pi(0) = -\sqrt{(c-s)(v-s)}\int_0^{Q^*_c} uf_D(u)du$. $\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q^*_c > 0$. Then, the equality must be non-positive.
- $\triangle\Pi(1) > 0$:

$$\triangle\Pi(1) = (p^*_s - s).\int_0^{Q^*_s} uf_D(u)du - \sqrt{(c-s).(v-s)}.\int_0^{Q^*_c} uf_D(u)du$$

  It is easy to show that $p^*_s > \sqrt{(c-s)(v-s)}$ and $Q^*_s > Q^*_c$ by using (B6.2). Hence, $\triangle\Pi(1) > 0$.
- $\frac{\partial\triangle\Pi(\beta)}{\partial\beta} > 0$: Similar to above proofs.

Hence, we have shown that $\Pi(\beta)$ changes sign only at β^* as β increases. □

Proposition B6.2. *If $\beta \le \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_D(Q^*_c) = \sqrt{\frac{c-s}{v-s}}$ and $p^*_c = \sqrt{(c-s)\cdot(v-s)} + s$, and all customers can buy. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_{\beta D}(Q^*_s) = \frac{c-s}{v+\frac{k}{Q^*_s}-s}$ and $p^*_s = v + \frac{k}{Q^*_s}$, and only snobs can buy.*

Proof of Proposition B6.2: Follows from the results of Lemma B6.1. If $\beta \le \beta^*$ then it is more profitable to apply the "Regular Production" strategy since $\Pi^*_{N,s} \le \Pi^*_{N,c}$ and if $\beta > \beta^*$ then it is more profitable to apply the "Limited Production" strategy since $\Pi^*_{N,s} > \Pi^*_{N,c}$. □

Proposition B6.3. *There exists a unique level of percentage of snobs, β_Q, where $Q^*_s \le Q^*_c$ when $\beta \le \beta_Q$ and $Q^*_s > Q^*_c$ when $\beta > \beta_Q$.*

Proof of Proposition B6.3: To prove Proposition B6.3, we follow the proof of Proposition 4. We define $Q^*_s(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. Also, we define $\triangle Q(\beta) = Q^*_s(\beta) - Q^*_c$. We show that the same conditions hold as in the proof of Proposition 4 under the same assumptions:

- $\triangle Q(0) < 0$**:** $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q^*_c > 0$. Then, $\triangle Q(0) < 0$.
- $\triangle Q(1) > 0$**:** $\triangle Q(1) = Q^*_s - Q^*_c$. It is easy to show that and $Q^*_s > Q^*_c$ by using (B6.2). Then, $\triangle Q(1) > 0$.

- $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$**:** Recall equation (B6.2). Taking the derivative of the equation with respect to β leads to the following result:

$$f_D\left(\frac{Q_s^*(\beta)}{\beta}\right).Q_s^*(\beta).\frac{1}{\beta}.\left[\frac{\partial Q_s^*(\beta)}{\partial \beta} - \frac{Q_s^*(\beta)}{\beta}\right] = \frac{\frac{-k\bar{F}_D\left(\frac{Q_s^*(\beta)}{\beta}\right)}{Q_s^*(\beta)}.\frac{\partial Q_s^*}{\partial \beta}}{\frac{k}{Q_s^*(\beta)} + v - s}$$

  The equality holds if and only if $0 < \frac{\partial Q_s^*(\beta)}{\partial \beta} < \frac{Q_s^*}{\beta}$.

Hence, we have shown that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. □

Appendix B7: Price Dependent Stockout Beliefs

Proposition B7.1. *1. **(Limited Production)** In the RE equilibrium under limited production, for those values of the price set larger than the commoners' reservation price but smaller than or equal to the snobs' reservation price, i.e., $r_c < p \leq r_s$, only snobs can buy, and the firm's quantity decisions are characterized by $\bar{F}_D\left(\frac{Q^*(p)}{\beta}\right) = \frac{c-s}{p-s}$.*

*2. **(Regular Production)** In the RE equilibrium, for those values of the price set smaller than or equal to the commoners' reservation price, i.e., $p \leq r_c$ all customers (snobs & commoners) can buy, and the firm's price and quantity decisions are characterized by $\bar{F}_D(Q^*(p)) = \frac{c-s}{p-s}$.*

Proof of Proposition B7.1(1): The firm sets a price higher than the anticipated reservation price of the commoners and smaller than or equal to the anticipated reservation price of the snobs, i.e., $\varepsilon_{r_c} < p \leq \varepsilon_{r_s}$, so the commoners are excluded from consideration (βD instead of β). The RE equilibrium conditions reduce to:

$$p = \bar{F}_D\left(\frac{Q(p)}{\beta}\right).(k + v - s) + s \tag{B7.1}$$

Then, the firm will obtain the critical fractile quantity based on this price. This provides the equilibrium quantity: $\bar{F}_D\left(\frac{Q^*(p)}{\beta}\right) = \sqrt{\frac{c-s}{k+v-s}}$. Putting $\bar{F}_D\left(\frac{Q^*(p)}{\beta}\right)$ back in (B7.1) provides the equilibrium price, $p_s^* = \sqrt{(c-s)(k+v-s)} + s$. □

Proof of Proposition B7.1(2): For those values of the price smaller than or equal to the anticipated reservation price of the commoners, i.e., $p \leq \varepsilon_{r_c}$, the RE equilibrium conditions reduce to:

$$p = \bar{F}_D(Q(p)).(v - s) + s \tag{B7.2}$$

Then, the firm will obtain the critical fractile quantity based on this price. This provides the equilibrium quantity: $\bar{F}_D(Q^*(p)) = \sqrt{\frac{c-s}{v-s}}$. Putting $\bar{F}(Q^*(p))$ back in (B7.2) provides the equilibrium price: $p_c^* = \sqrt{(c-s)(v-s)} + s$. □

Proof of Proposition 7: Propositions B7.1(1) and B7.1(2) show that the firm sets the same equilibrium price and produces the same equilibrium quantity as in Proposition 2 for the corresponding selling strategies. This concludes the proof. □

Appendix C: Proofs for Section 5

Appendix C1: Endogenous Salvage Pricing

Proof of Proposition 8(1): The firm sets the price of the good at the reservation price of snobs so the commoners are excluded from the consideration (βD instead of β). The RE equilibrium conditions reduce to

$$p = \bar{F}_D(\frac{Q}{\beta}).\left(k + v - s_0 + s_1 \int_0^{\frac{Q}{\beta}} \frac{(Q-\beta u)}{Q} dF_D(u)\right) + s_0 - s_1 \int_0^{\frac{Q}{\beta}} \frac{(Q-\beta u)}{Q} dF_D(u) \qquad \text{(B7.3)}$$

The firm will produce the quantity that sets the FOC with respect to Q to 0:

$$\frac{s_1}{Q_s^{*2}} \int_0^{\frac{Q_s^*}{\beta}} u^2 dF_D(u) + \bar{F}_D(\frac{Q_s^*}{\beta}).\left(\bar{F}_D(\frac{Q_s^*}{\beta}).(v+k-s_0) + s_1 - s_1.F_D(\frac{Q_s^*}{\beta}). \int_0^{\frac{Q_s^*}{\beta}} \frac{(Q_s^* - \beta u)}{Q_s^*} dF_D(u)\right) = c-(s_0-s_1) \qquad \text{(B7.4)}$$

Putting Q_s^* back in (B7.3) provides the equilibrium price:

$$p_s^* = \bar{F}_D(\frac{Q_s^*}{\beta}).\left(k + v - s_0 + s_1 \int_0^{\frac{Q_s^*}{\beta}} \frac{(Q_s^*-\beta u)}{Q_s^*} dF_D(u)\right) + s_0 - s_1 \int_0^{\frac{Q_s^*}{\beta}} \frac{(Q_s^*-\beta u)}{Q_s^*} dF_D(u)$$

□

Proof of Proposition 8(2): The RE equilibrium conditions reduce to

$$p = \bar{F}_D(Q).\left(v - s_0 + s_1 \int_0^{Q} \frac{(Q-u)}{Q} dF_D(u)\right) + s_0 - s_1 \int_0^{Q} \frac{(Q-u)}{Q} dF_D(u) \qquad \text{(B7.5)}$$

The firm will produce the quantity that sets the FOC with respect to Q to 0:

$$\frac{s_1}{Q_c^{*2}} \int_0^{Q_c^*} u^2 dF_D(u) + \bar{F}_D(Q_c^*).\left(\bar{F}_D(Q_c^*).(v-s_0) + s_1 - s_1.F_D(Q_c^*). \int_0^{Q_c^*} \frac{(Q_c^* - u)}{Q_c^*} dF_D(u)\right) = c-(s_0-s_1) \qquad \text{(B7.6)}$$

Putting the implicit term, Q_c^*, back in (B7.5) provides the equilibrium price:

$$p_c^* = \bar{F}_D(Q_c^*).\left(k + v - s_0 + s_1 \int_0^{Q_c^*} \frac{(Q_c^*-u)}{Q_c^*} dF_D(u)\right) + s_0 - s_1 \int_0^{Q_c^*} \frac{(Q_c^*-u)}{Q_c^*} dF_D(u)$$

□

Before we proceed with more result, we prove Lemma 1.

Lemma C.1. *There exists a unique threshold level of snobs, β^*, where $\Pi_{N,s}^* < \Pi_{N,c}^*$ when $\beta \leq \beta^*$, and $\Pi_{N,s}^* > \Pi_{N,c}^*$ when $\beta > \beta^*$.*

Proof of Lemma C.1: Recall the proof of Lemma A2. We use the same technique to prove the results of Lemma C.1.

In this setting, the optimal profit of the firm serving to the whole market becomes:

$$\Pi_{N,c}^* = p_c^*.Q_c^*.\bar{F}_D(Q_c^*) - c.Q_c^* + s_0.Q_c^*.F_D(Q_c^*) - s_0. \int_0^{Q_c^*} u dF_D(u) - \frac{s_1}{Q_c^*} \int_0^{Q_c^*} (Q_c^* - u)^2 dF_D(u)$$

Also, the optimal profit of the firm serving to the snobs only becomes:

$$\Pi^*_{N,s} = p^*_s.Q^*_s.\bar{F}_D(\frac{Q^*_s}{\beta}) - c.Q^*_s + s_0.Q^*_s.F_D(\frac{Q^*_s}{\beta}) - s_0.\beta.\int_0^{\frac{Q^*_s}{\beta}} udF_D(u) - \frac{s_1}{Q^*_s}\int_0^{\frac{Q^*_s}{\beta}} (Q^*_s - \beta u)^2 dF_D(u)$$

We define $\Pi^*_{N,s}(\beta)$ to represent the firm's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. Also, we define $\triangle\Pi(\beta) = \Pi^*_{N,s}(\beta) - \Pi^*_{N,c}$. We assume that we are considering the set of parameters which gives non-negative profits for both strategies. We show that the same conditions hold as in the proof of Lemma A2 under the same assumptions:

- **$\triangle\Pi(0) < 0$:** $\triangle\Pi(0) = -\Pi^*_{N,c}$. We know that $\Pi^*_{N,c} > 0$. Then, $\triangle\Pi(0) < 0$.
- **$\triangle\Pi(1) > 0$:** For $\beta = 1$ the profit obtained from *Limited Edition Strategy*, $\Pi^*_{N,s}(\beta)$ is always higher than the profit obtained from *Regular Production Strategy*, $\Pi^*_{N,c}$. Thus, $\triangle\Pi(1) > 0$.
- $\frac{\triangle\Pi(\beta)}{\beta} > 0$: Result shown as before for uniform distribution. Computational tests confirm the result for general distributions.

Hence, we have shown that $\Pi(\beta)$ changes sign only at β^* as β increases. □

Proposition C.1. *If $\beta \leq \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by Equations (3) and (4), and all customers can buy. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by Equations (1) and (2), and only snobs can buy.*

Proof of Proposition C.1: Follows from the results of Lemma C.1. □

Proposition C.2. *There exists a unique level of percentage of snobs, β_Q, where $Q^*_s < Q^*_c$ when $\beta < \beta_Q$ and $Q^*_s > Q^*_c$ when $\beta > \beta_Q$.*

Proof of Proposition C.2: We show that the difference between Q^*_s and Q^*_c changes sign only at a particular threshold level, β_Q, as β increases. We define $Q^*_s(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. We assume that $Q^*_s(\beta) > 0$ except for $\beta = 0$ and $Q^*_c > 0$ without loss of generality. Also, we define $\triangle Q(\beta) = Q^*_s(\beta) - Q^*_c$. Then, showing that $\triangle Q(0) < 0$, $\triangle Q(1) > 0$, and $\frac{\partial\triangle Q(\beta)}{\partial\beta} > 0$ is sufficient to say there exists unique β_Q such that $\triangle Q(\beta_Q)$:

- **$\triangle Q(0) < 0$:** $\triangle Q(0) = -Q^*_c$. We know that $Q^*_c > 0$. Then, $\triangle Q(0) < 0$.
- **$\triangle Q(1) > 0$:** Again, for $\beta = 1$ the equilibrium quantity under Limited Edition Strategy, Q^*_s is always higher than Q^*_c. Thus, $\triangle Q(1) > 0$.
- **$\frac{\partial\triangle Q(\beta)}{\partial\beta} > 0$:** The equilibrium quantity under Limited Edition Strategy, $Q^*_s(\beta)$ increases with β. Thus, $\frac{\partial\triangle Q(\beta)}{\partial\beta} > 0$.

Hence, we have shown that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. □

Appendix C2: Clearance Pricing Model

Proof of Proposition 9(1): RE equilibrium conditions reduce to

$$s_2^*(q, D) = \arg\max_{s_2}\{s_2.p_2(s_2)|s_2 \leq (q-\beta D)^+\}; \quad p_2^*(q, D) = p_2(s_2^*(q, D))$$

$$p_1 = \bar{F}_D(\frac{q}{\beta}).\,(v + k - E_D[p_2^*(q, D)]) + E_D[p_2^*(q, D)]$$

We use the backward induction method to find the RE equilibrium of this game. First, we start with Period 2. We find the equilibrium number of remaining units to sell, $s_2^*(q, \xi)$, hence the equilibrium price set, $p_2^*(q, \xi)$, for every possible realization of demand from Period 1 (ξ) and number of units produced in Period 1 (q). Let $R_2(s_2) = s_2(p_a - p_b s_2)$ be the unconstrained revenue function of the firm from Period 2. Note that $\frac{\partial R_2^2(s_2)}{\partial s_2^2} = -2p_b < 0$, hence $R_2(s_2)$ is concave in s_2. Then, $s_2^* = \frac{p_a}{2p_b}$ that sets $\frac{\partial R_2(s_2)}{\partial s_2}$ to 0, maximizes the Period 2 revenue. However, number of units to sell in Period 2 is constrained by the number of remaining units from Period 1, i.e., $(q - \beta D)^+$. If s_2^* turns out to be higher than $(q - \beta D)^+$, the firm earns the most if he sells the whole remaining inventory from Period 1, i.e., $(q-\beta D)^+$ because of the concave structure of $R_2(s_2)$. We can write the corresponding optimal selling quantity, which also provides the optimal clearance price in Period 2 through the clearance price function as

$$s_{2,s}^*(q, D) = \begin{cases} \frac{p_a}{2p_b} & \text{if } \frac{p_a}{2p_b} \leq (q - \beta D)^+, \\ (q - \beta D)^+ & \text{if } \frac{p_a}{2p_b} > (q - \beta D)^+. \end{cases}$$

$$p_{2,s}^*(q, D) = \begin{cases} \frac{p_a}{2} & \text{if } \frac{p_a}{2p_b} \leq (q - \beta D)^+, \\ p_a - p_b(q - \beta D)^+ & \text{if } \frac{p_a}{2p_b} > (q - \beta D)^+. \end{cases}$$

We move to Period 1 after finding the Period 2 equilibrium price and selling quantity decisions. The firm's profit from the whole game can be written as

$$\begin{aligned} \Pi(q, p_1) &= E_D[p_1 \min\{q, \beta D\} - c.q + p_2^*(q, D).s_2^*(q, D)] \\ &= (p_1 - c)q - p_1 \int_0^{\frac{q}{\beta}} (q - \beta u) dF_D(u) + \frac{p_a^2}{4p_b} F_D\left(\frac{q - \frac{p_a}{2p_b}}{\beta}\right) \\ &\quad + \int_{\frac{q-\frac{p_a}{2p_b}}{\beta}}^{\frac{q}{\beta}} \left(p_a - p_b(q - \beta u)\right)(q - \beta u) dF_D(u) \end{aligned}$$

Note that $\frac{\partial \Pi(q,p_1)}{\partial p_1} = q - \int_0^{\frac{q}{\beta}} (q - \beta u) dF_D(u) > q.(1 - F_D\left(\frac{q}{\beta}\right)) > 0$. Then, the firm sets the reservation price to the maximum possible value that is the reservation price of the customers. The firm's Period 1 price choice can be written as:

$$p_{1,s}^* = \int_0^{\frac{Q_s^*}{\beta}} \left(\bar{F}_D\left(\frac{Q_s^*}{\beta}\right).(v + k - p_2^*(Q_s^*, u)) + p_2^*(Q_s^*, u)\right) dF_D(u) \tag{B7.7}$$

The firm will obtain the equilibrium production quantity choice as:

$$\frac{\partial \Pi(p_1, q)}{\partial q} = (p_1 - c) - p_1.F_D\left(\frac{q}{\beta}\right) + \int_{\frac{q-\frac{p_a}{2p_b}}{\beta}}^{\frac{q}{\beta}} (p_a - 2p_b(q - \beta u)) dF_D(u)$$

$$\Rightarrow \frac{\partial \Pi(p_1,q)}{\partial q} = (p_1^* - c) - p_1^*.F_D\left(\frac{Q_s^*}{\beta}\right) + \int_{\frac{Q_s^* - \frac{p_a}{2p_b}}{\beta}}^{\frac{Q_s^*}{\beta}} (p_a - 2p_b(Q_s^* - \beta u))dF_D(u) = 0 \qquad \text{(B7.8)}$$

Solving equations (B7.7) and (B7.8) provides the equilibrium quantity characterized by the following equation:

$$\Rightarrow \quad \bar{F}_D\left(\frac{Q_s^*}{\beta}\right).F_D\left(\frac{Q_s^*}{\beta}\right).\left((v+k-p_a).\bar{F}_D\left(\frac{Q_s^*}{\beta}\right)+p_a\right) - \bar{F}_D\left(\frac{Q_s^*}{\beta}\right).F_D\left(\frac{Q_s^*}{\beta}\right).F_D\left(\frac{Q_s^* - \frac{p_a}{2p_b}}{\beta}\right)\frac{p_a}{2}$$
$$-(\bar{F}_D\left(\frac{Q_s^*}{\beta}\right).F_D\left(\frac{Q_s^*}{\beta}\right)+2).p_b\int_{\frac{Q_s^* - \frac{p_a}{2p_b}}{\beta}}^{\frac{Q_s^*}{\beta}} (Q_s^* - \beta u)dF_D(u) - c + p_a.\left(F_D\left(\frac{Q_s^*}{\beta}\right) - F_D\left(\frac{Q_s^* - \frac{p_a}{2p_b}}{\beta}\right)\right) = 0$$

□

Proof of Proposition 9(2): RE equilibrium conditions reduce to

$$s_2^*(q,D) = argmax_{s_2}\{s_2.p_2(s_2)|s_2 \le (q-D)^+\}; \qquad p_2^*(q,D) = p_2(s_2^*(q,D))$$

$$p_1 = \bar{F}_D(q).\,(v+k-E_D[p_2^*(q,D)]) + E_D[p_2^*(q,D)]$$

We use the backward induction method to find the RE equilibrium of this game. First, we start with Period 2. We find the equilibrium number of remaining units to sell, $s_2^*(q,D)$, hence the equilibrium price set, $p_2^*(q,D)$, for every possible realization of demand from Period 1 (D) and number of units produced in Period 1 (q). Let $R_2(s_2) = s_2(p_a - p_b s_2)$ be the unconstrained revenue function of the firm from Period 2. Note that $\frac{\partial R_2^2(s_2)}{\partial s_2^2} = -2p_b < 0$, hence $R_2(s_2)$ is concave in s_2. Then, $s_2^* = \frac{p_a}{2p_b}$ that sets $\frac{\partial R_2(s_2)}{\partial s_2}$ to 0, maximizes the Period 2 revenue. However, number of units to sell in Period 2 is constrained by the number of remaining units from Period 1, i.e., $(q-D)^+$. If s_2^* turns out to be higher than $(q-D)^+$, the firm earns the most if he sells the whole remaining inventory from Period 1, i.e., $(q-D)^+$ because of the concave structure of $R_2(s_2)$. We can write the corresponding optimal selling quantity, which also provides the optimal clearance price in Period 2 through the clearance price function as

$$s_{2,c}^*(q,D) = \begin{cases} \frac{p_a}{2p_b} & \text{if } \frac{p_a}{2p_b} \le (q-D)^+, \\ (q-D)^+ & \text{if } \frac{p_a}{2p_b} > (q-D)^+. \end{cases}$$

$$p_{2,c}^*(q,D) = \begin{cases} \frac{p_a}{2} & \text{if } \frac{p_a}{2p_b} \le (q-a)^+, \\ p_a - p_b(q-\xi)^+ & \text{if } \frac{p_a}{2p_b} > (q-\xi)^+. \end{cases}$$

We move to Period 1 after finding the Period 2 equilibrium price and selling quantity decisions. The firm's profit from the whole game can be written as

$$\begin{aligned} \Pi(q,p_1) &= E_D[p_1.min\{q,D\} - c.q + p_2^*(q,D).s_2^*(q,D)] \\ &= (p_1-c)q - p_1\int_0^q (q-u)dF_D(u) + \frac{p_a^2}{4p_b}F_D\left(q - \frac{p_a}{2p_b}\right) + \int_{q-\frac{p_a}{2p_b}}^{q} (p_a - p_b(q-u))\,(q-u)dF_D(u) \end{aligned}$$

Note that $\frac{\partial \Pi(q,p_1)}{\partial p_1} = q - \int_0^q (q-u)dF_D(u) > q.(1-F_D(q)) > 0$. Then, the firm sets the reservation price to the maximum possible value that is the reservation price of the customers. The firm's Period 1 price choice can be written as:

$$p_{1,c}^* = \int_0^{Q_c^*} (\bar{F}_D(Q_c^*).(v - p_2^*(Q_c^*, u)) + p_2^*(Q_c^*, u))dF_D(u) \tag{B7.9}$$

The firm will obtain the equilibrium production quantity choice as:

$$\frac{\partial \Pi(p_1, q)}{\partial q} = (p_1 - c) - p_1.F_D(q) + \int_{q-\frac{p_a}{2p_b}}^{q} (p_a - 2p_b(q - u))dF_D(u)$$

$$\Rightarrow \frac{\partial \Pi(p_1, q)}{\partial q} = (p_{1,c}^* - c) - p_{1,c}^*.F_D(Q_c^*) + \int_{Q_c^*-\frac{p_a}{2p_b}}^{Q_c^*} (p_a - 2p_b(Q_c^* - u))dF_D(u) = 0 \tag{B7.10}$$

Solving equations (B7.9) and (B7.10) provides the equilibrium quantity characterized by the following equation:

$$\Rightarrow \quad \bar{F}_D(Q_c^*).F_D(Q_c^*).((v - p_a).\bar{F}_D(Q_c^*) + p_a) - \bar{F}_D(Q_c^*).F_D(Q_c^*).F_D\left(Q_c^* - \frac{p_a}{2p_b}\right)\frac{p_a}{2}$$
$$- (\bar{F}_D(Q_c^*).F_D(Q_c^*) + 2).p_b \int_{Q_c^*-\frac{p_a}{2p_b}}^{Q_c^*} (Q_c^* - u)dF_D(u) - c + p_a.\left(F_D(Q_c^*) - F_D\left(Q_c^* - \frac{p_a}{2p_b}\right)\right) = 0$$

□

Before we proceed with more results, we prove Lemma C2.

Lemma C.2. *There exists a unique threshold of snobs, β^*, where $\Pi_s^* < \Pi_c^*$ when $\beta \leq \beta^*$, and $\Pi_s^* > \Pi_c^*$ when $\beta > \beta^*$.*

Proof of Lemma C.2: We use distributions with finite support, such as uniform, to prove the result (verified for general IGFR distributions). The equilibrium profit under Limited Edition Strategy, Π_s^*, increases with β. Since, the profit difference changes from negative to positive, which implies that there exists a unique threshold of snobs, β^*, where $\Pi_s^* < \Pi_c^*$ when $\beta \leq \beta^*$, and $\Pi_s^* > \Pi_c^*$ when $\beta > \beta^*$. □

Proposition C.3. *If $\beta \leq \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by equations in Proposition 9(2), and all customers can buy in Period 1. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by equations in Proposition 9(1), and only snobs can buy in Period 1.*

Proof of Proposition C.3: Follows from the results of Lemma C2. □

Appendix C3: Signaling through Sourcing Investments

Proof of Proposition 10: We show that the threshold level for limited production decreases with the marginal cost of the supply c. Recall that $\beta^* = \sqrt{\frac{v-s}{k+v-s}} \frac{\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} uf_D(u)du}{\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} uf_D(u)du}$ is defined $\forall c \in [s, v]$ from Lemma A2. Showing that the first derivative of β^* with respect to c is non-positive will be sufficient for the argument:

$$\begin{aligned}
\frac{\partial\beta^*}{\partial c} &= \sqrt{\frac{v-s}{k+v-s}}\frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}}))\frac{\frac{-1}{2\sqrt{(c-s)(v-s)}}}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}}))}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}uf_D(u)du}{(\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}uf_D(u)du)^2} \\
&- \sqrt{\frac{v-s}{k+v-s}}\frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}))\frac{\frac{-1}{2\sqrt{(c-s)(k+v-s)}}}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}))}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}uf_D(u)du}{(\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}uf_D(u)du)^2} \\
&= \sqrt{\frac{v-s}{k+v-s}}\frac{1}{(\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}uf_D(u)du)^2}\frac{1}{2\sqrt{c-s}} \\
&\cdot \left(\frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}{\sqrt{k+v-s}}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}uf_D(u)du - \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}{\sqrt{v-s}}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}uf_D(u)du\right)
\end{aligned}$$

Terms outside the parentheses are positive since $s < c < v$ and D has a non-negative support. Thus, the last equality is non-positive if and only if terms in the parentheses give non-positive value. The analysis of the terms in parentheses will provide the sufficient and necessary condition:

$$\begin{aligned}
\frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}{\sqrt{k+v-s}}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}uf_D(u)du &\leq \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}{\sqrt{v-s}}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}uf_D(u)du \\
\sqrt{\frac{v-s}{k+v-s}}\frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}uf_D(u)du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}uf_D(u)du} &\leq \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} \Rightarrow \beta^* \leq \beta_Q
\end{aligned}$$

Recall that we provide one sufficient condition in Proposition A3 for the last inequality to hold. Then, this is sufficient to say that $\frac{\partial\beta^*}{\partial c} \leq 0$. We have shown that the threshold level for limited production decreases with the marginal cost of the supply c. Therefore, the threshold level for the more expensive source, $\beta^*_{c_H}$, is less than the threshold level for the cheaper source, $\beta^*_{c_L}$. Simply, $\beta^*_{c_H} < \beta^*_{c_L}$ since $c_L < c_H$. □

Proof of Proposition 11: We derive the conditions which dictate the choice of the source by the profit maximizing firm for the high intensity region. We define $v' = v + k$ without loss of generality. (Proof for the low intensity region can be obtained by setting $k = 0$ and $\beta = 1$). To generalize our results for all demand distributions, we derive the structure of the profit function for changing cost of the supplier. We show that there exists a global maximum at c^*, at least one inflection point in (c^*, v'), and a global minimum at v'. Recall $\Pi^*_{N,s}(c)$ from the proof of Lemma A2 that stands for the optimal profit of the firm experiencing high intensity of snobs under limited production strategy:

$$\Pi^*_{N,s}(c) = \sqrt{(c-s)(v'-s)}\beta\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})}uf_D(u)du$$

$\Pi^*_{N,s}(c)$ is a continuous function on the closed interval $[s, v']$, and differentiable on the open interval (s, v'), where $s < v'$. Note that $\Pi^*_{N,s}(s) = 0$ and $\Pi^*_{N,s}(v') = 0$. Then, there exists at least one c^* in (s, v') such that $\frac{\partial\Pi^*_{N,s}(c^*)}{\partial c} = 0$ by the mean value theorem. Now that we show there must be at least one extreme point within (s, v'), the next step is to show that there can only be one extreme point which is a global maximum in (s, v'). We check the first derivative of $\Pi^*_{N,s}(c)$ with respect to c:

$$\begin{aligned}
\frac{\partial \Pi_{N,s}(c)}{\partial c} &= \frac{\beta}{2}\sqrt{\frac{v'-s}{c-s}}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} u f_D(u)du - \frac{\beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v'-s}}\right)}{2} \\
&= \frac{\beta}{2}\sqrt{\frac{v'-s}{c-s}}\left(-\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})\bar{F}_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})) + \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} \bar{F}_D(u)du\right) - \frac{\beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v'-s}}\right)}{2} \\
&\qquad (Int.\ by\ parts) \\
&= \frac{\beta}{2}\sqrt{\frac{v'-s}{c-s}}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} \bar{F}_D(u)du - \beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v'-s}}\right)
\end{aligned}$$

Any extreme point c^* in $[s, v']$ must satisfy $\frac{\partial \Pi_{N,s}(c^*)}{\partial c} = 0$:

$$\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}})} \bar{F}_D(u)du = 2\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right)\sqrt{\frac{c^*-s}{v'-s}} \tag{B7.11}$$

We check the sign of the second derivative of $\Pi^*_{N,s}(c)$ with respect to c the extreme points:

$$\frac{\partial^2 \Pi_{N,s}(c)}{\partial c^2} = \frac{\beta}{4}\frac{1}{\sqrt{(c-s)(v'-s)}}\left(-\frac{v'-s}{c-s}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} \bar{F}_D(u)du + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}}))}\right)$$

$$\begin{aligned}
\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} &= \frac{1}{4}\frac{\beta}{\sqrt{(c^*-s)(v'-s)}}\left(-\frac{v'-s}{c^*-s}2\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right)\sqrt{\frac{c^*-s}{v'-s}} + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}}))}\right) \quad (by\ (B7.11)) \\
&= \frac{1}{4}\frac{\beta}{\sqrt{(c^*-s)(v'-s)}}\left(-\frac{2\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right)}{\sqrt{\frac{c^*-s}{v'-s}}} + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}}))}\right)
\end{aligned}$$

Terms outside the parentheses are positive since $s < c < v'$. Thus, the sign of the last equality is dictated by the terms in the parentheses. We define $\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right) = \xi^*$. Note that $\frac{\partial \xi^*}{\partial c^*} \leq 0$. The analysis of the terms in the parentheses reveals the following result:

$$-\frac{2\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right)}{\sqrt{\frac{c^*-s}{v'-s}}} + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}}))} = -\frac{2\xi^*}{\bar{F}_D(\xi^*)} + \frac{1}{f_D(\xi^*)} = \frac{1}{f_D(\xi^*)}\left(-\frac{2f_D(\xi^*)\xi^*}{\bar{F}_D(\xi^*)} + 1\right)$$

Hence, the structure of the function at the potential extreme point is dictated by the following conditions:

- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} < 0$ if and only if $\frac{f_D(\xi^*)\xi^*}{\bar{F}_D(\xi^*)} > \frac{1}{2}$
- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} > 0$ if and only if $\frac{f_D(\xi^*)\xi^*}{\bar{F}_D(\xi^*)} < \frac{1}{2}$
- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} = 0$ if and only if $\frac{f_D(\xi^*)\xi^*}{\bar{F}_D(\xi^*)} = \frac{1}{2}$

Recall that the demand in our model has an increasing generalized failure rate (IGFR) property. Then, when we move from a potential extreme point to a higher potential extreme point, $g(\xi^*)$ must decrease since ξ^* decreases with c^*. This property eliminates the possibility of more than one combination of local maximum and local minimum in (s, v').

We know that $\Pi^*_{N,s}(s) = 0$, $\Pi^*_{N,s}(v') = 0$ and $\Pi^*_{N,s}(c) > 0$ in (s, v'). Note that $\frac{\partial \Pi_{N,s}(s)}{\partial c}$ and $\frac{\partial^2 \Pi_{N,s}(s)}{\partial c^2}$ are undefined so the function might be tangent at s since we know that the function is continuous and differentiable in (s, v'). Since $\lim_{c \to s^+} \frac{\partial^2 \Pi_{N,s}(c)}{\partial c^2} < 0$, the function can only be increasing concave after s. Having increasing concave structure $\forall c \in (s, s + \varepsilon)$ implies that the first extreme point in (s, v') can either be an inflection point or a local maximum.

It is easy to show that the first extreme point can not be an inflection point by contradiction. Having inflection point first as an extreme point would imply that there exists no local maximum since $g(\xi)$ can not attain values larger than $1/2$ anymore. This contradicts with the fact that function returns back to 0 at v'. Thus, the first extreme point must be a local maximum which we state as c^*. Since there is no possibility of more than one combination of local maximum and local minimum in (s, v'), next possible set of extreme points after c^* is a set of inflection points plus a local minimum point. In fact, it can be immediately shown that there exists at least one inflection point in (c^*, v') by the mean value theorem. Hence, the unique local minimum is v' since $\frac{\partial \Pi_{N,s}(v')}{\partial c} = 0$ and $\frac{\partial^2 \Pi_{N,s}(v')}{\partial c^2} > 0$.

We have shown that $\Pi^*_{N,s}$ reaches a global maximum at c^*, has at least one inflection point in (c^*, v'), and reaches a global minimum at v'. Therefore,

1. If $(c_H >) c_L \geq c^*$ then $\Pi^*_{N,s}(c_L) \geq \Pi^*_{N,s}(c_H)$. [Region D]
2. If $(c_L <) c_H \leq c^*$ then $\Pi^*_{N,s}(c_L) \leq \Pi^*_{N,s}(c_H)$. [Region A]
3. If $c_L < c^* < c_H < c_{equal}$ then $\Pi^*_{N,s}(c_L) \leq \Pi^*_{N,s}(c_H)$. [Region B]
4. If $c_L < c^* < c_{equal} < c_H$ then $\Pi^*_{N,s}(c_L) \geq \Pi^*_{N,s}(c_H)$. [Region C]

where $\Pi^*_{N,s}(c_L) = \Pi^*_{N,s}(c_H)$ when $c_H = c_{equal}$. □