Chapter 2: Static Game Theory.

In this chapter we will look at how static games can be represented, and examine some ways that have been suggested for how they might be solved. A solution to a game is a prediction of what each player in that game will do. In static games the players make their moves in isolation without knowing what other players have done. This does not necessarily mean that all decisions are made at the same time, but rather only as if the decisions were made at the same time. An example of a static game is a one-off sealed bid auction. In this type of auction each player submits only one bid, without knowing what any of the other players have bid. The highest bid is then accepted as the purchase price. In contrast to static games, dynamic games have a sequence to the order of play and players observe some, if not all, of one another’s moves as the game progresses. An example of a dynamic game is a so called English auction. Here players openly bid up the price of an object. The final and highest bid is accepted as the purchase price.

2.1. Normal Form and Extensive Form Games.

In non-cooperative game theory there are two alternative ways in which a game can be represented. The first type is called a normal form game or strategic form game. The second type is called an extensive form game. Both are widely used in economics and we examine each in turn.

a. Normal Form Games.

A normal form game is any game where we can identify the following three things:

(i) The players.

The players in a game are the individuals who make the relevant decisions. For there to be interdependence we need to have at least two players in the game. In most of the applications we look at there will only be two players. In some games “Nature” is considered a further player, whose function is to determine the outcome of certain random events, such as the weather or the “type” of players in the game.

(ii) The strategies available to each player.

A strategy is a complete description of how a player could play a game. This does not necessarily just list the player’s alternative actions. Instead it describes how the player’s actions are dependent on what he observes other players in the game to have done. For example, if I am thinking about selling my car then my actions are limited to selling it or keeping it. My chosen strategy, however, tells me how these possible actions are dependent on what other people do. If someone offers me £5000 or more for my car I will certainly sell it. If they offer me less than £5000 I will keep the car. In dynamic
games such as this a player’s strategy set will be much larger than his or her possible actions. In static games, however, the two are the same. This is because in static games decisions are taken in isolation and so players cannot make their actions dependent on what other players do. In the example of me trying to sell my car, this would correspond to the very strange game where I have to accept or reject someone’s offer without knowing what it is! In this case my strategies are the same as my actions: to sell or not to sell. (In this discussion we have ignored the possibility of players adopting mixed strategies. These are discussed later in this chapter.)

(iii) The payoffs.

A payoff is what a player will receive at the end of the game contingent upon the actions of all the players in the game. A normal form game shows the payoffs for every player, except Nature, for every possible combination of available strategies. These are then represented in the form of a matrix or matrices. The payoffs are defined so that the players in the game always prefer higher to smaller payoffs. For example, the payoffs may correspond to monetary rewards, such as profits, or the utility each player obtains at the end of the game. A player is said to be rational when he seeks to maximize his payoff. If a player does not have this objective they are said to be irrational, because they are not acting in their own self interest.

To make the ideas discussed more specific we will look at one well known static game called “The Prisoners’ Dilemma”. In this game the police have arrested two suspects of a crime. However they lack sufficient evidence to convict either of them unless at least one of them confesses. The police hold the two suspects in separate cells and explain the consequences of their possible actions. If neither confess then both will be convicted of a minor offense and sentenced to one month in prison. If both confess they will be sent to prison for six months. Finally, if only one of them confesses, then that prisoner will be released immediately while the other one will be sentenced to nine months in prison - six months for the crime and a further three months for obstructing the course of justice.

The above description of the game satisfies the three requirements of a normal form game. We have two players, each of whom has two strategies (which in this static game are the same as the prisoners’ actions, to confess or not confess), and payoffs for each possible combination of strategies. The normal form for this game is shown in Figure 2.1. The payoffs are shown as the negative number of months in prison for each outcome and for each prisoner. This assumes that each suspect, if rational, seeks to minimize the amount of time spent in prison. By convention the first payoff listed in each cell refers to the row player, prisoner 1, and the second payoff refers to the column player, prisoner 2.
**b. Extensive Form Games.**

In extensive form games greater attention is placed on the **timing** of the decisions to be made, as well as on the amount of **information** available to each player when a decision has to be made. This type of game is represented not with a matrix but with a decision, or game, tree. The extensive form for the prisoners' dilemma is shown in Figure 2.2.

![Figure 2.2. The Prisoners' Dilemma Game in Extensive Form.](image)

Starting at the left of the diagram the open circle represents the first decision to be made in the game. It is labeled 1 to show that it is prisoner 1 that makes this decision. The branches coming out of this initial node represent the actions available to the player at that point in the game.
Prisoner 1 can either confess to the crime or not confess. At the end of these branches there is a node representing prisoner 2’s decision. Again this prisoner can either confess to the crime or not confess, as given by the branches coming from his decision nodes. However prisoner 2 makes this decision without knowing what prisoner 1 has done. This is shown by joining prisoner 2’s decision nodes with a dotted line. This dotted line shows that the connected nodes are in the same information set. This means that prisoner 2 is unable to distinguish which of the two nodes he is at, at the time this decision is made. This is because he does not know if prisoner 1 has confessed or not confessed to the crime. Finally at the end of the game we have the payoffs for each player. These are again dependent on what each prisoner has done in the game, and they are listed in the order of the players in the game, i.e. prisoner 1’s payoff is first, and prisoner 2’s payoff is second.

Generalizing from Figure 2.2 we can state that extensive form games have the following four elements in common:

**Nodes.** This is a position in the game where one of the players must make a decision. The first position, called the initial node, is an open dot, all the rest are filled in. Each node is labeled so as to identify who is making the decision.

**Branches.** These represent the alternative choices that the person faces, and so correspond to available actions.

**Vectors.** These represent the payoffs for each player, with the payoffs listed in the order of players. When we reach a payoff vector the game ends. When these payoff vectors are common knowledge the game is said to be one of complete information. (Information is common knowledge if it is known by all players, and each player knows it is known by all players, and each player knows that it is known that all players know it, and so on ad infinitum.) If, however, players are unsure of the payoffs other players can receive then it is an incomplete information game.

**Information Sets.** When two or more nodes are joined together by a dashed line this means that the player whose decision it is does not know which node he is at. When this occurs the game is characterized as one of imperfect information. When each decision node is its own information set the game is said to be one of perfect information, as all players know the outcome of previous decisions.

A fundamental assumption of game theory is that the structure of the game is common knowledge. This places three specific requirements on information sets. The **first** is that players always remember whether they have moved previously in the game. This does not, however, means that they always remember what decision they previously made, only that a decision...
was made. The **second** requirement is that nodes in the same information set have the same player moving. The **final** condition is that nodes in the same information set have the same possible actions coming from them. If this were not true a player could differentiate between the nodes by examining their available actions. Again generalizing from Figure 2.2 we can state one further requirement that is always satisfied for extensive form games:

Each node has at least one branch pointing out of it (some action is available to the player) and at most one branch pointing into it. (The initial node has no branch pointing to it.)

This means that at whatever node we begin at there is only one possible path back to the initial node and we never cycle back to the node we started from. For this reason extensive form games always look like trees. From the initial node we always branch out and a branch never grows back into itself.

We have now seen that there are two different ways of representing the same game, either as a normal form game or as an extensive form game. The normal form gives the minimum amount of information necessary to describe a game. It lists the players, the strategies available to each player, and the resulting payoffs to each player. The extensive form gives additional details about the game concerning the timing of the decisions to be made and the amount of information available to each player when each decision has to be made. Clearly the two forms are closely related and we can state the following two results:

For every extensive form game there is one and only one corresponding normal form game.

For every normal form game there are, in general, several corresponding extensive form games.

The reason for this lack of one-to-one correspondence between a normal form game and an extensive form game is that, as described above, the extensive form game includes additional information. This implies that different extensive forms can be drawn from the same normal form game, depending on what is assumed about these additional details of the game.
**Exercise 2.1**

Depict the following situation as both a normal form game and an extensive form game:

Two rival firms are thinking of launching a similar product at the same time. If both firms launch the product then they will each make a profit of £40,000. If only one firm launches its product then it can act as a monopolist and will make a profit of £100,000. If either firm decides not to launch the product that firm makes a loss of £50,000, due to costs already incurred in developing the product.

**Exercise 2.2.**

Interpret the following diagrams and discuss whether they represent valid extensive form games.

(i).

(ii).

(iii).

(iv).
2.2. Solution Techniques for Solving Static Games.

As stated at the beginning of this chapter a solution to a game is a prediction of what each player in that game will do. This may be a very precise prediction, where the solution gives one optimal strategy for each player. When this occurs the solution is said to be unique. However, it is often the case that the solution to a particular game is less precise, even to the extent that none of the available strategies are ruled out. As may be expected many different solution techniques have been proposed for different types of games. For static games two broad solution techniques have been applied. The first set of solution techniques rely on the concept of dominance. Here the solution to a game is determined by attempting to rule out strategies that a rational person would never play. Arguments based on dominance seek to answer the question “What strategies would a rational player never play ?” The second set of solution techniques are based on the concept of equilibrium. In non-cooperative games an equilibrium occurs when none of the players, acting individually, have an incentive to deviate from the predicted solution. With these solution techniques a game is solved by answering the question “What properties does a solution need to have for it to be an equilibrium ?”

In the following section we examine various dominance techniques that can be applied to static games, and two equilibrium concepts. In subsequent chapters further equilibrium concepts that are commonly used in game theory will be presented and discussed.

a. Strict Dominance.

A strategy is said to be strictly dominated if another strategy always gives improved payoffs whatever the other players in the game do. This solution technique makes the seemingly reasonable assumption that a rational player will never play a strictly dominated strategy. If a player knowingly plays a strictly dominated strategy they cannot be maximizing their expected payoff, given their beliefs about what other players will do. In this sense a player who plays a strictly dominated strategy is said to be irrational. Applying the principle of strict dominance rules out this type of irrational behaviour. To illustrate this technique we use it to solve the prisoners’ dilemma game. In applying the principle of strict dominance we examine each player in turn and exclude all those strategies that are strictly dominated. This process may rule out all but one strategy for each player. This is true for the prisoners’ dilemma game, and so this technique produces a unique solution for this game.

Consider first the dilemma facing prisoner 1. Should he confess or should he remain quiet hoping the other prisoner does the same. The principle of strict dominance argues that prisoner 1 should confess. The
reason for this is that whatever prisoner 2 decides to do prisoner 1 is always better off confessing. This means not confessing is strictly dominated and so it seems reasonable to suppose it will not be played. The same logic applies equally to prisoner 2 and so strict dominance predicts that he will also confess. The solution to this game based on strict dominance is that both prisoners confess even though both would be better off if neither confessed. As at least one of the players in this game can, with a different outcome, be made better off without the other player being made worse off this solution is said to be Pareto inefficient. (In fact if neither player confesses both would be better off.) This is a very common feature of many games used in economics, and it will be illustrated in many contexts throughout this book.

It should be noted here that the cause of Pareto inefficiency is not that the players cannot communicate, but rather that they cannot commit themselves to the Pareto efficient outcome. Even if both prisoners agreed before being arrested that neither of them will confess, once in custody it is in their individual self interest to do the opposite. This illustrates the difference between non-cooperative and cooperative game theory. In cooperative game theory the two prisoner’s could enter into a binding and enforceable agreement not to confess and so be made better off. This is not possible in non-cooperative game theory.

<table>
<thead>
<tr>
<th>Exercise 2.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve the previous product launch game described in Exercise 2.1. using the principle of strict dominance.</td>
</tr>
</tbody>
</table>

b. Weak Dominance.

A strategy is said to be weakly dominated if another strategy makes the person better off in some situations and leaves them indifferent in all others. Again it seems reasonable to assume that a rational player will not play a weakly dominated strategy, as they could do at least as well, and possibly even better, by playing the dominant strategy. Consider the normal form game shown in Figure 2.3. In this game there are two players each with two possible strategies. Player 1 can move either “up” or “down”, and player 2 can move either “left” or “right”. The payoffs are given in the matrix, where the first figure is the payoff for player 1 and the second figure is the payoff for player 2. For this game none of the available strategies are ruled out using the principle of strict dominance. This is because no strategy makes that player worse off in all circumstances. For example, if player 1 plays “up” then player 2 is indifferent between “left” and “right”. Similarly if player 2 plays “left” player 1 is indifferent between “up” and “down”. Although we cannot appeal to the principle of strict dominance to rule out any of the available strategies, we can apply the principle of weak dominance.
According to the principle of weak dominance player 1 will never play “down” and so this can be ruled out. Similarly player 2 will never play “right”, and so this can also be ruled out. This leaves only one remaining strategy for each player. The predicted outcome is that player 1 will move “up” and player 2 will move “left”. Again this is a Pareto inefficient solution. This is because the outcome “down/left” makes player 2 better off and player 1 no worse off. The reason player 1 does not switch to playing “down”, even though this leads to a Pareto improvement, is that it entails greater risk for this player. If player 2 were to play “right” then player 1 is definitely worse off moving “down” instead of “up”. This element of avoiding unnecessary risk is reflected in the principle of weak dominance.

**c. Iterated Strict Dominance.**

Iterated strict dominance assumes that strict dominance can be applied successively to different players in a game. For example, if one player rules out a particular strategy, because it is strictly dominated by another, then it is assumed other players recognize this and that they also believe the other player will not play this dominated strategy. This in turn may lead them to exclude dominated strategies, and so on. In this way it may be possible to exclude all but one strategy for each player, and so make a unique prediction for the game being analysed. Consider the game shown in Figure 2.4.
Player 1

<table>
<thead>
<tr>
<th></th>
<th>LEFT</th>
<th>MIDDLE</th>
<th>RIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td>1,0</td>
<td>1,2</td>
<td>0,1</td>
</tr>
<tr>
<td>DOWN</td>
<td>0,3</td>
<td>0,1</td>
<td>2,0</td>
</tr>
</tbody>
</table>

Player 2

**Figure 2.4. An Application of Iterated Strict Dominance.**

In this game player 1 has two possible strategies, “up” and “down”, and player 2 has three possible strategies, “left”, “middle” and “right”. Initially neither “up” nor “down” are strictly dominated by the other for player 1. However for player 2 “right” is strictly dominated by “middle”. Appealing to strict dominance we can reason that player 2 will never play “right”. If player 1 also knows that player 2 is rational and will not play “right”, then “up” now strictly dominates “down” for player 1. Iterated strict dominance now predicts that “down” will not be played. Finally if player 2 knows that player 1 will never move “down” then iterated strict dominance predicts that player 2 will play “middle”. The unique solution to this game based on successive or iterated strict dominance is therefore “up/middle”.

d. Iterated Weak Dominance.

The final dominance technique is iterated weak dominance. This is the same as iterated strict dominance except here it is weak dominance that is applied successively to different players in the game. Again it is possible that this technique can produce a unique solution to a particular game.

One problem with iterated weak dominance, which is not shared by iterated strict dominance, is that the predicted solution can depend on the order in which players’ strategies are eliminated. This is true for the game shown in Figure 2.5. If we start by applying weak dominance to player 1 then we predict that the players will choose the unique solution “up/middle”. If we first apply weak dominance to player 2 then all we can conclude is that player 2 will not play “right”. Clearly the order in which we apply weak dominance significantly affects the predicted outcome of the game. Unfortunately for most games this choice is totally arbitrary.
It should be noted that in applying iterated dominance arguments we are assuming a stronger version of rationality than we did with mere dominance. With dominance we assumed that rational players will not play dominated strategies. With iterated dominance we assume that rational players will not play dominated strategies, and also that players assume that other players are rational and will not do this. For iterated dominance to predict accurately people must not only be rational but assume that others are rational as well, and this requirement needs to be strengthened with each iteration. (For example, I need to assume that you believe that I believe that you believe that I am rational, and so on. If this sequence of reasoning *ad infinitum* we have the frequently used assumption of **common knowledge of rationality**.) As the number of iterations becomes large these additional assumptions become increasingly more dubious. An example of a game where the principle of iterated strict dominance is taken to extreme lengths is Rosenthal’s (1981) centipede game. This dynamic game is discussed at the end of chapter 3.

If a game yields a unique solution by applying either strict, weak or iterated dominance then that game is said to be dominance solvable. The main problem with all these solution techniques is that often they give very imprecise predictions about a game. Consider the game shown in Figure 2.6. In this game arguments based on dominance lead to the very imprecise prediction that anything can happen! If a more specific solution to this type of game is needed then a stronger solution technique must be applied. This leads us on to solution techniques based not on dominance but on the concept of equilibrium.

---

**Figure 2.5. An Application of Iterated Weak Dominance**

<table>
<thead>
<tr>
<th>Player 1</th>
<th>LEFT</th>
<th>MIDDLE</th>
<th>RIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UP</strong></td>
<td>10 , 0</td>
<td>5 , 1</td>
<td>4 , -2</td>
</tr>
<tr>
<td><strong>DOWN</strong></td>
<td>10 , 1</td>
<td>5 , 0</td>
<td>1 , -1</td>
</tr>
</tbody>
</table>
Player 1

<table>
<thead>
<tr>
<th></th>
<th>LEFT</th>
<th>MIDDLE</th>
<th>RIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td>0, 4</td>
<td>4, 0</td>
<td>5, 3</td>
</tr>
<tr>
<td>CENTER</td>
<td>4, 0</td>
<td>0, 4</td>
<td>5, 3</td>
</tr>
<tr>
<td>DOWN</td>
<td>3, 5</td>
<td>3, 5</td>
<td>6, 6</td>
</tr>
</tbody>
</table>

**Figure 2.6. An Illustration of the Problem with Dominance Techniques.**

**e. Nash Equilibrium.**

As stated in the introduction to this section arguments based on dominance ask the question “What strategies would a rational player never play?” In contrast the concept of Nash equilibrium is motivated by the question “What properties must an equilibrium have?” The answer to this question from John Nash (1951), based on much earlier work by Cournot (1838), was that in equilibrium each player’s chosen strategy is optimal given that every other player chooses the equilibrium strategy. If this were not the case then at least one player would wish to choose a different strategy and so we could not be in an equilibrium. Again this concept seeks to apply the economist’s assumption that individuals are rational in the sense that they seek to maximize their own self interest.

Finding the Nash equilibrium for any game involves two stages. **First,** we identify each player’s optimal strategy in response to what the other players might do. This involves working through each player in turn and determining their optimal strategies. This is done for every combination of strategies by the other players. **Second,** a Nash equilibrium is identified when all players are playing their optimal strategies simultaneously.

Strictly speaking the above methodology only identifies pure strategy Nash equilibria. It does not identify mixed strategy Nash equilibria. A pure strategy equilibrium is where each player plays one specific strategy. A mixed strategy equilibrium is where at least one player in the game randomizes over some or all of their pure strategies. This means that players place a probability distribution over their alternative strategies. For example, players might decide to play each of two available pure strategies with a probability of 0.5, and never play any other strategy. A pure strategy is therefore a restricted mixed strategy with a probability of one given to the chosen strategy, and zero to all the others. The concept of mixed strategy Nash
equilibrium is discussed later in this section.

To illustrate the two stage methodology for finding a (pure strategy) Nash equilibrium we apply it to the prisoners’ dilemma game. This is shown in Figure 2.7.

<table>
<thead>
<tr>
<th></th>
<th>PRISONER 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CONFESS</td>
</tr>
<tr>
<td>PRISONER 1</td>
<td></td>
</tr>
<tr>
<td>CONFESS</td>
<td>-6 , -6</td>
</tr>
<tr>
<td>DON’T CONFESS</td>
<td>-9 , 0</td>
</tr>
</tbody>
</table>

Figure 2.7. The Nash Equilibrium of the Prisoners’ Dilemma Game.

Stage One. We first need to identify the optimal strategies for each prisoner, dependent upon what the other prisoner might do. If prisoner 1 expects prisoner 2 to confess then prisoner 1’s best strategy is also to confess (-6 is better than -9). This is shown in Figure 2.7 by underlining this payoff element for prisoner 1 in the cell corresponding to both prisoners confessing. If prisoner 1 expects prisoner 2 not to confess, then prisoner 1’s best strategy is still to confess (this time 0 is better than -1). Again we show this by underlining this payoff element for prisoner 1. The same analysis is undertaken for prisoner 2 and his best strategy payoffs are underlined.

Stage Two. Next we determine whether a Nash equilibrium exists by examining the occurrence of the previously identified optimal strategies. If all the payoffs in a cell are underlined then that cell corresponds to a Nash equilibrium. This is true by definition, since in a Nash equilibrium all players are playing their optimal strategy given that other players also play their optimal strategies. In the prisoners’ dilemma game only one cell has all its elements underlined. This corresponds to both prisoners confessing, and so this is the unique Nash equilibrium for this game.

This prediction for the prisoners’ dilemma game is the same as that derived using strict dominance. In fact it is always true that a unique strict dominance solution is the unique Nash equilibrium. The reverse of this statement is, however, not always true. A unique Nash equilibrium is not
always a unique strict dominant solution. In this sense the Nash equilibrium is a stronger solution concept than strict dominance. For this reason the Nash equilibrium concept may predict a unique solution to a game where strict dominance does not. This is illustrated in the game used previously to demonstrate that a game may not be dominance solvable. This game shown in Figure 2.6 is reproduced below in Figure 2.8. As stated before arguments based on dominance applied to this game predict that anything can happen. Using the two stage methodology of finding a (pure strategy) Nash equilibrium however yields the unique prediction that player 1 will choose “down” and player 2 will choose “right”. The concept of Nash equilibrium may therefore be particularly useful when dominance arguments do not provide a unique solution.

\[
\begin{array}{|c|c|c|}
\hline
\text{Player 2} & \text{LEFT} & \text{MIDDLE} & \text{RIGHT} \\
\hline
\text{TOP} & 0, 4 & 4, 0 & 5, 3 \\
\hline
\text{CENTRE} & 4, 0 & 0, 4 & 5, 3 \\
\hline
\text{BOTTOM} & 3, 5 & 3, 5 & 6, 6 \\
\hline
\end{array}
\]

Figure 2.8. A Further Application of Nash Equilibrium.

One important result from game theory is that for any finite game (i.e. games with a finite number of players and strategies) there always exists at least one Nash equilibrium. Before thinking that this result means that we can always make a definite prediction about what people will do in any game the following two qualifications need to be stated.

**First**, the above result is only true if we include mixed strategies, as well as pure strategies. This means that we cannot always state for certain what all players in a game will do, but instead we may only be able to give the probabilities for various outcomes occurring. This possibility is discussed below.

**Second**, the above result does not rule out the possibility of multiple Nash equilibria. Indeed many games do exhibit multiple Nash equilibria. With multiple equilibria the problem is how to select one equilibrium from many. In answer to this question numerous refinements of Nash equilibrium have been proposed to try and restrict the set of possible equilibria. Some of these refinements are discussed in later chapters.
Exercise 2.4.

State whether the following games have unique pure strategy solutions, and if so what they are and how they can be found.

(i).

<table>
<thead>
<tr>
<th>Player 1</th>
<th>LEFT</th>
<th>MIDDLE</th>
<th>RIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td>4,3</td>
<td>2,7</td>
<td>0,4</td>
</tr>
<tr>
<td>DOWN</td>
<td>5,5</td>
<td>5,-1</td>
<td>-4,-2</td>
</tr>
</tbody>
</table>

(ii).

<table>
<thead>
<tr>
<th>Player 1</th>
<th>LEFT</th>
<th>MIDDLE</th>
<th>RIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td>4,10</td>
<td>3,0</td>
<td>1,3</td>
</tr>
<tr>
<td>DOWN</td>
<td>0,0</td>
<td>2,10</td>
<td>1,3</td>
</tr>
</tbody>
</table>

(iii).

<table>
<thead>
<tr>
<th>Player 1</th>
<th>LEFT</th>
<th>MIDDLE</th>
<th>RIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td>10,10</td>
<td>4,3</td>
<td>7,2</td>
</tr>
<tr>
<td>DOWN</td>
<td>5,6</td>
<td>8,10</td>
<td>6,12</td>
</tr>
</tbody>
</table>


To illustrate that there may be multiple Nash equilibria to a particular game, and also the idea of mixed strategies, we look at another classic game called the “Battle of the Sexes”. In this game a husband and wife are trying to decide where to go for an evening out. Whilst apart they must choose to either go to a boxing match, or to the ballet. Both players would rather go anywhere together, but given this the man prefers the boxing and the woman the ballet. (This game was proposed in the 1950’s, which partly explains its stereotypical views.) These preferences are represented in the normal form game shown in Figure 2.9.
Applying the two stage method of identifying a pure strategy Nash equilibrium we can see that the above game has two such equilibria. These are that either both will go to the boxing or both will go to the ballet. This means that each person will go wherever they think the other person will go. This is not very helpful, as it tells neither player what the other person is likely to do. As there is no unique pure strategy Nash equilibrium neither player can confidently predict what the other person will do. Playing a mixed strategy is a response to this uncertainty. A mixed strategy is when a player randomizes over some or all of their available pure strategies. This means that the player places a probability distribution over their alternative strategies. A mixed strategy equilibrium is where at least one player plays a mixed strategy and no-one has the incentive to deviate unilaterally from that position.

The key feature of a mixed strategy Nash equilibrium is that every pure strategy played as part of the mixed strategy has the same expected value. If this were not true a player would play the strategy that yields the highest expected value to the exclusion of all others. This means the initial situation could not have been an equilibrium. Here we show how to identify the mixed strategy Nash equilibrium for the battle of the sexes game.

Let \( pr(\text{boxing})_H \) be the probability that the husband goes to the boxing match, and \( pr(\text{boxing})_W \) the probability that the wife goes to the boxing match. Similarly let \( pr(\text{ballet})_H \) be the probability that the man goes to the ballet, and \( pr(\text{ballet})_W \) the probability that the woman goes to the ballet. As these are the only two alternatives it must be true that \( pr(\text{boxing}) + pr(\text{ballet}) = 1 \) for both the husband and wife. Given these probabilities we can calculate the expected value of each persons possible action.

From the normal form game the expected payoff value for the wife if she chooses to go to the boxing match is given as
Similarly the expected payoff value if she goes to the ballet is

\[ EV(ballet)_w = \text{pr}(boxing)_h(0) + \text{pr}(ballet)_h(2) \]

\[ = 2 \text{pr}(ballet)_h \]

In equilibrium the expected value of these two strategies must be the same and so we get

\[ EV(boxing)_w = EV(ballet)_w \]

\[ \therefore \text{pr}(boxing)_h = 2 \text{pr}(ballet)_h \]

\[ \therefore 1 - \text{pr}(ballet)_h = 2 \text{pr}(ballet)_h \]

\[ \therefore 1 = 3 \text{pr}(ballet)_h \]

\[ \therefore \text{pr}(ballet)_h = \frac{1}{3} \text{ and } \text{pr}(boxing)_h = \frac{2}{3} \]

This means that in the mixed strategy equilibrium the husband will go to the ballet with a 1/3 probability and the boxing with a 2/3 probability. We can perform the same calculations for the husband’s expected payoff and derive the similar result that in equilibrium his wife will go to the ballet with a probability of 2/3 and the boxing with a probability of 1/3. With these individual probabilities we can calculate that they will both go to the boxing with a probability of 2/9, both go to the ballet with a probability of 2/9, and go to separate events with a probability of 5/9.

This combination of mixed strategies constitutes a third Nash equilibrium for this game. Intuitively this seems the most reasonable Nash equilibrium of the three, as it explicitly takes into account the inherent uncertainty in the game. It should be noted that playing a mixed strategy does not mean that players flip a coin or roll a dice to make their decisions. Rather playing a mixed strategy is a rational response to uncertainty about what other players will do.

One curious aspect of a mixed strategy equilibrium is that because each of the chosen pure strategies in the mixed strategy have the same expected payoff value, each player is indifferent as to which strategy he or she actually plays. A mixed strategy equilibrium is, therefore, said to be a weak equilibrium because none of the players are made worse off if they abandon their mixed strategy, and play any one of the pure strategy components of their mixed strategy. This feature of a mixed strategy Nash equilibrium has caused its application within economics to be controversial. In particular this solution
technique has been criticised as imposing unacceptable constraints on players' beliefs. Some of these criticisms are discussed in chapter 12.

**Exercise 2.5.**

Draw the normal form game for the following game and identify both the pure and mixed strategy equilibria. In the mixed strategy Nash equilibrium determine each firm’s expected profit level if it enters the market.

There are two firms that are considering entering a new market, and must make their decision without knowing what the other firm has done. Unfortunately the market is only big enough to support one of the two firms. If both firms enter the market then they will each make a loss of £10 million. If only one firm enters the market, that firm will earn a profit of £50 million, and the other firm will just break even.

### 2.3. Conclusions

Static games are where players make decisions in isolation. Each decision is made without knowing what the other players have done. These games can be represented as either normal or extensive form games. Normal form games give the minimum amount of information necessary to describe a game. They list the players in the game, the strategies available to each player, and the payoffs dependent on the outcome of the game. Extensive form games give additional details on the timing of decisions and the amount of information players have when making these decisions. Static games are predominantly represented as normal form games. This is because in such games the amount of information available to players does not vary within the game, and the timing of decisions has no effect on players' choices. In the next chapter we examine dynamic games where the timing of decisions and information constraints critically determine the outcome of the game.

In attempting to predict the outcome of static games various solution techniques have been suggested. These are either based on the concept of dominance or equilibrium. These solution techniques try and predict what rational players will do in specified games. Sometimes they yield a definite prediction of what each player will do. Often, however, the solution is less precise. These solution techniques can also be applied to dynamic games, but as we will see in the next chapter additional assumptions are typically needed so that reasonable predictions are generated.
2.4 Solutions to Exercises.

Exercise 2.1.

The normal and extensive forms for this static game are shown in Figures 2.10 and 2.11 respectively:

Figure 2.10.

<table>
<thead>
<tr>
<th>FIRM A</th>
<th>LAUNCH</th>
<th>DON'T LAUNCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAUNCH</td>
<td>£40,000</td>
<td>£100,000</td>
</tr>
<tr>
<td>DON'T LAUNCH</td>
<td>- £50,000</td>
<td>- £50,000</td>
</tr>
</tbody>
</table>

Figure 2.11.

Exercise 2.2.

(i). This is a one player static game against nature with imperfect information. Nature determines the outcome of the toss of an unbiased coin. Without knowing whether the outcome is heads (H) or tails (T), player A calls chooses either heads or tails. If the call is correct the player wins a payoff of
1. If the call is wrong the player receives nothing. This diagram is a valid extensive form game. In such games we assume that players simply maximise their expected payoff. In particular there are no strategic considerations in one player games. For this reason this book only analyses games with two or more (rational) players.

(ii). This is a dynamic game with imperfect recall. Player A initially decides between A1 and A2. This is observed by player B who then decides between B1 and B2. If B1 is chosen the game ends. If B2 is chosen player A moves again, playing either A1 or A2. Significantly these two final decision nodes are in the same information set, which means that player A does know which one they are at. However, the only difference in the paths to these nodes is player A’s initial move. This means that player A must have forgotten what their first move was! This is a valid extensive form game, and indeed some economic models have assumed that agents have imperfect recall. In this book, however, we limit ourselves to games where all players have perfect recall. This means that players do not forget any information that has been previously revealed to them.

(iii). This is not a valid extensive form game as it entails a logical contradiction. In the diagram player B’s decision nodes are in the same information set, which means that they cannot be distinguish. However at the decision node following A1 there are three possible actions, while at the node following A2 there are only 2 options. Player B must know the actions available to them and so based on this information they will be able to distinguish between their decision nodes. This contradicts the fact they are shown as being in the same information set. To avoid such logical contradictions it is required that the set of possible actions from nodes in the same information set must be identical.

(iv). This is not a valid extensive form game, as it violates one of the previous assumptions. This is the requirement that each node has at most one branch pointing to it. This is not true for player C’s decision node. The reason this assumption is made is to guarantee a unique path from any decision node back to the initial node. (This is important for the application of backward induction discussed in the next chapter.) This diagram does not satisfy this feature, as there are two possible paths back to the initial node from player C’s decision node.

Exercise 2.3.

Strict dominance predicts that both firms will launch their respective products because this gives each firm a higher pay-off whatever the other firm does.

Exercise 2.4.

(i). The unique pure strategy equilibrium is “down/left”. This is both a Nash equilibrium and an iterated strict dominant solution. The process of elimination for the dominant solution is “right”, “up”, “middle”.

(ii). The unique pure strategy Nash equilibrium this time is “up/left”, and is
both a Nash equilibrium and an iterated weak dominant solution. The process of elimination in the latter case is “down”, “middle”, “right”.

(iii). This game is not dominance solvable, but “up/left” is a Nash equilibrium.

**Exercise 2.5.**

The normal form for this static entry game is given in Figure 2.12.

**Figure 2.12.**

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>ENTERS</th>
<th>STAYS OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENTERS</td>
<td>(- £10m, - £10m)</td>
<td>(£50m, 0)</td>
</tr>
<tr>
<td>STAYS OUT</td>
<td>(0, £50m)</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

Using the two stage method for finding a pure strategy Nash equilibrium we can see that there are two such equilibria. Both involve one firm entering the market, and the other firm staying out.

We can determine the mixed strategy Nash equilibrium in the following way. Let \(pr(\text{enter})_1\) and \(pr(\text{enter})_2\) be the probabilities of firm 1 and firm 2 entering the market respectively. And let \(pr(\text{stay out})_1\) and \(pr(\text{stay out})_2\) be the probabilities of the two firms staying out of the market.

Expected profits for firm 1 if it enters the market are therefore

\[
EV(\text{enter})_1 = -10 \cdot pr(\text{enter})_2 + 50 \cdot pr(\text{stay out})_2
\]

and its expected profit if it stays out of the market is 0. In equilibrium these expected values must equal each other and so we get
\[-10 \cdot \text{pr(enter)}_2 + 50 \cdot \text{pr(stay out)}_2 = 0\]

\[\therefore 50 \cdot \text{pr(stay out)}_2 = 10 \cdot \text{pr(enter)}_2\]

\[\therefore 5 \cdot \text{pr(stay out)}_2 = \text{pr(enter)}_2\]

\[\therefore 5 \cdot \text{pr(stay out)}_2 = 1 - \text{pr(stay out)}_2\]

\[\therefore 6 \cdot \text{pr(stay out)}_2 = 1\]

\[\therefore \text{pr(stay out)}_2 = \frac{1}{6} \text{ and pr(enter)}_2 = \frac{5}{6}\]

We could do the same calculations to find the same probabilities of firm 1 entering and staying out of the market.

Substituting these probabilities back into the equation for EV(enter)_1 we can find the expected value for firm 1 of entering the market.

\[\text{EV(enter)}_1 = -10 \cdot \text{pr(enter)}_2 + 50 \cdot \text{pr(stay out)}_2\]

\[\therefore \text{EV(enter)}_1 = -10 \cdot \frac{5}{6} + 50 \cdot \frac{1}{6}\]

\[\therefore \text{EV(enter)}_1 = 0\]

The same result holds for firm 2. In the mixed strategy Nash equilibrium the expected value for both firms if they enter the market is zero. This could have been found by noting that this equals the expected value of not entering which equals zero.

**Further Reading.**


