Reasoning about the Objects of Attitudes and Operators

Towards a Disquotation Theory for Representation of Propositional Content

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ABSTRACT

S believes that P, S promises that P, S says that P, and so on are examples of sentences with embedded propositional content (that P in these examples). Such sentences are ubiquitous in everyday reasoning, in legal reasoning, and in conducting business. This paper sketches an approach to formalizing such sentences for purposes of automated reasoning. The method advocated, called the disquotation theory of propositional content, applies to modeling formally propositional attitude sentences, as well as modal and deontic sentences. Exploiting the resources of event semantics, the method generates intensional contexts of the highest degree, then allows relaxations via added axioms.

Categories and Subject Descriptors

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1. INTRODUCTION

Sentences in ordinary language may be simple—as in The cat is on the mat and Es regnet—or they may be complex in the sense that they embed other sentences. In Bush believes that he got more votes than Gore the embedded sentence is

he got more votes than Gore. We may think of this sentence as the object of the verb believes. Sentences may also be compound in the sense that they combine, using broadly logical operators, more than one sentence. An example: Bush won the electoral vote, but Gore won the popular vote. Here, Bush won the electoral vote and Gore won the popular vote are two distinct sentences combined via the broadly logical term but.

My concern in this paper is the matter of formalizing complex sentences, those embedding other sentences. The embedded sentences are said to represent the propositional content of the governing (embedding) sentence. Complex sentences, embedding other sentences, occur ubiquitously in everyday discourse and reasoning, in the law, and in the conduct of business. All talk of belief, intention, desire, knowing, promising, requesting, needing, saying, and so on falls within this ambit. The class is a large one, and there is no definitive means of characterizing it. We must work with and generalize upon examples. In English embedded content is often signalled by ‘that’ followed by a complete sentence. Examples include:

- Saying that (indirect discourse):
  - Galileo said that the earth moves
  - Bush said that the U.S. economy was in deep trouble and only massive tax reductions, especially for the wealthy, could save the day.

- Propositional attitude descriptions (aka: clausal complements [17]), such as:
  - Jane believes that Tom loves Mary
  - Jane desires that Tom does not love Mary
  - Jane intends that Tom will marry Susan

and their stylistic equivalents. For example, Tom intends to marry Mary is arguably a variant of the more stilted Tom intends that he (Tom) will marry Mary.

- Speech act descriptions, such as:
  - Tom promised that he will marry Mary.

Many other constructions are possible, e.g., ‘to’ as in Mary promises to come to the party, which I read as a variant of Mary promises that she (Mary) will come to the party.


Sam promised Sue a diamond.²

Bush asserted that his administration would operate on a bipartisan basis.

The Supreme Court declared that the ballots will not be counted.

- Modal descriptions, such as:
  - It is impossible that Gore can appeal a Supreme Court decision.
  - Necessarily, it is raining or not

- Deontic descriptions, such as:
  - Jake is obliged to Tom that Jake read(s) the letter.
  - Parking is not permitted here.

- Perceiving-that descriptions, such as:
  - Jane saw that Tom kissed Mary
    (as distinguished from Jane saw Tom kiss Mary.)

- Others, e.g.,
  - Gore needs it to be the case that a miracle occurs.

Several comments. First, there has been no general representation theory covering all these (and related) cases. The closest we have to this are the various semantic theories of propositional attitudes (see, e.g., [17, chapter 11] and the discussion below) and various modal logics.

Second, formalization for purposes focusing on reasoning and inference (as distinguished from semantic purposes) has typically relied on inventing and investigating various sentence operators. Notable success in this regard has been achieved for modal logics, for which necessity and possibility sentence operators (□, ◦) are used to embed sentences. Deontic logics are also well-developed. They typically use and embellish such sentence operators as $O$ (“It is obligatory that . . .”) and $P$ (“It is permitted that . . .”). But while formal operator-based systems have been well developed for necessity and obligation, this is not the case for many other cases of sentence embedding, e.g., belief, intention, desire, hope, promising, requesting, seeing that, and so on.

Third, embedded (or propositional content) sentences typically appear in opaque (or intensional) contexts. This—and the fact that they are so common—is what makes them so puzzling and hence so interesting. In general, if $E$ embeds $\Phi$ and $\Phi \rightarrow \Psi$, then from $E(\Phi)$ it will not follow that $E(\Psi)$. For example, any true sentence is logically equivalent to a tautology. Let $I$: President Clinton was impeached. Since $I$ is in fact true, it has the same truth value as any tautology, e.g., $(P \lor \neg P)$. But tautologies are necessarily true, so $\Box (P \lor \neg P)$ and $I \rightarrow (P \lor \neg P)$ but not $\Box (I)$. Although Clinton was impeached, it could have been otherwise. So, both $I$ is necessary that . . . and □ create intensional, or opaque, contexts.

Failure of substitution to preserve truth may occur for referring expressions as well as sentence expressions. Standardly, from Bill knows that the morning star is very bright and The morning star is one and the same as the evening star it does not follow that Bill knows that the evening star is very bright. This could happen if Bill didn’t know that two were identical. More generally and compactly, then, we can say that in many cases $E(\Phi(\alpha))$ and $\alpha = \beta$ do not imply $E(\Phi(\beta))$.³

In the presence of these phenomena, three questions present themselves most forcefully:

1. The (formal) modeling question
   How can, how should, we formalize, e.g. for purposes of machine-to-machine communication and for automated inferencing, such embedded-sentence talk?

2. The semantic question
   What does, what could, such talk possibly mean? What would a rigorous and formal theory of such meanings be?

3. The etiology question
   How do such meanings arise and in what do they consist? How can and do purely physical systems come to have such meanings?

An example may help communicate these distinctions. Consider the sentence $S$ knows that $P$. The semantic question (in the broad sense in which I take it) asks for an analysis of what such sentences means. Theories abound. There is the justified-true-belief theory, which in paraphrase holds that $S$ knows that $P$ is true if and only if it is also true that $S$ believes that $P$, $P$ is true, and $S$ is justified in believing that $P$. There is the information-caused-belief theory, which in paraphrase holds that $S$ knows that $P$ is true if and only if it is also true that $S$ believes that $P$ and $S$’s belief was caused by $S$’s having the information that $P$. The etiology question asks how, for a given semantic theory, an agent can come know that $P$. What are the causal mechanisms and the physical structures? The modeling question is related, but quite distinct. Here, the focus is on formal representation for reasoning with and about ascriptions of knowledge. If $S$ knows that $P$, and we take this to imply that $S$ believes that $P$, then we should be able to deduce this formally. If $S$ believes that $P$, and being $P$ is to be superstitious, then $S$ is superstitious, and we should be able to deduce this formally.

These questions are valid and important. They are distinct questions, whose answers should be closely related. The etiology question is a broadly scientific one, asking as it does for a causal theory of explanation. The modeling and semantic questions may be usefully compared to the distinction in logic between proof theory and model theory. In proof theory we are concerned with developing a formal calculus for a particular language. Working with intuitive interpretations of the expressions in the calculus we aim for adequate expressive richness (Can everything we wish to say informally be expressed formally, be modeled, in our calculus?) and deductive correctness (Can what should follow informally be automatically deduced in the calculus? Is what should not follow informally not deducible in the calculus?). Modal logics, and indeed many other logics, were originally developed in just this manner, from a purely proof-theoretic or formal modeling perspective.

²I take this kind of construction as shorthand for, here, something akin to Sam promised Sue that she (Sue) will get a diamond.

³The relevant literature is extensive. See [19] for a particularly clear statement of the problems.
In model theory we are concerned with developing a rigorous, plausible, and revealing account of what the expressions in the calculus mean. Now we are asking, e.g., what the truth conditions are for the expressions, whether the deductions allowed by the calculus preserve truth, and whether the calculus misses proper deductions. For example, the standard semantics for modal logic explicates modal inferences by appeal to possible worlds and to truth in a world. Something is possible if there is some possible world in which it is true. Necessities are true in every possible world. And so on.

Much of the relevant literature (on propositional attitudes, etc.) is directly concerned with the semantic question. My focus here is on the formal modeling question, which I take up in earnest in the next section.

2. THE MACHINERY

Consider, from [22, page 101], the following statements:

1. Brutus stabbed Cæsar violently in the back.
2. Brutus stabbed Cæsar violently.
3. Brutus stabbed Cæsar in the back.
4. Brutus stabbed Cæsar.
5. Brutus stabbed violently.
7. Cæsar was stabbed violently.
8. Cæsar was stabbed in the back.
10. Cæsar was stabbed.

One way to symbolize these statements—the way normally taught in logic texts—would be to use 10 different predicates, one for each of

\[
S_1 \text{ "... stabbed ... violently in the back"}
\]

\[
S_2 \text{ "... stabbed ... violently,"
\]

\[
S_{10} \text{ "... was stabbed"}
\]

But this seems, at least, quite strange. Notice especially that this symbolization entirely misses out on a great deal of logical structure. For example, \((1) \rightarrow (2)\), but it is not true that \(S_1(b,c) \rightarrow S_2(b,c)\). Also, \((1) \rightarrow (2) \land (3)\) but not \((2) \land (3) \rightarrow (1)\), yet the \(S_i\) representation is irrelevant to this logical structure. Nor are any of very many other logical relations among \((1) \ldots (10)\) captured. In short, something is wrong with the standard representation if we cannot infer a stabbing from a stabbing violently in the back.

Although a number of writers have addressed this problem (cf., [2, 5, 21, 24]), Parsons [22] has perhaps the most sustained and thorough treatment of it, and I shall largely follow his account, so far as it goes. There are excellent treatments of, and developments of, event semantics in other, often more recent, work (cf. [5, 22, 17, 7, 18]). Focusing on Parsons’s account, however, is convenient and for present purposes will not lead us astray.

The thesis ... is that semantics of simple sentences of English require logical forms that are somewhat more complex than is normally assumed in investigations of natural language semantics. In particular, the semantics of a simple sentence such as ‘Brutus stabbed Cæsar’ requires a form of at least the following complexity:

For some event \(e\), \(e\) is a stabbing, and the agent of \(e\) is Brutus, and the object of \(e\) is Cæsar, and \(e\) culminated at some time in the past.

This form, which is typical, is dominated by an existential quantification over events. Since no such quantification is explicitly indicated in the sense ‘Brutus stabbed Cæsar’, I call it an “underlying” quantification. A main theme of the theory I investigate is that such underlying quantification over events (and states) is ubiquitous in natural language. [22, page 1]
of affairs. It doesn’t make sense to ask how long a state took, although we can ask how long it lasted. As in systems analysis, we might think of events as transitions between states. Roughly, state = description of a system (at a given time), and event = change of state. A process, or activity, is a series of events. The basic claim for the underlying event theory is that, for a certain range of linguistic phenomena (VP modifiers in extensional contexts), the theory provides representations that get the logic right, or at least more right than competing theories. This is also the claim being advanced—sketched—for sentences with embedded propositional content: these representational ideas when applied properly will get the logic (the inferential relations) right. A word now by way of example for the underlying event theory as originally proposed by Parsons.

In the underlying event theory, ‘Brutus stabbed Caesar’ goes into first-order logic as:

\[ \exists e, \exists t \exists (\text{before}(t, \text{now}) \land t \in I \land \text{stab}(e) \land \text{Subject}(e, \text{Brutus}) \land \text{Obj}(e, \text{Caesar}) \land \text{Cul}(e, t)) \]

(Our representation assumes a typed variable regime. I is a temporal (or spatio-temporal) interval, \( e \) is an eventuality, and \( t \) is a time.) Similarly, ‘Brutus stabbed Caesar in the back with a knife violently’ goes into first-order logic as:

\[ \exists e, \exists t \exists (\text{before}(t, \text{now}) \land t \in I \land \text{stab}(e) \land \text{Subject}(e, \text{Brutus}) \land \text{Obj}(e, \text{Caesar}) \land \text{in}(e, \text{the-back}) \land \text{with}(e, \text{knife}) \land \text{violent}(e) \land \text{Cul}(e, t)) \]

(The analysis is not complete, since the-back remains not fully articulated. Doing that is more or less a straightforward matter, but it is one that digresses from the issues at hand.)

Notice that expression 2 logically implies expression 1. Further, notice that this approach to representation works for every sentence in the list about the stabbing of Caesar that began this section. (I leave it as an exercise for the reader to work out the details. It’s quite a simple problem.)

The essential strategy is to break down the VPs into components of meaning that are assembled with logical conjunction. The basic intuition here is, e.g., that to do something violently is to do that thing and to do it violently. This is why, in the representation, doing something violently entails doing that thing: VP modifiers attach as conjuncts. To make this work, there must be some common, quantified variable (or shared name) that links the several predicates in a representation. Here, that variable (\( e \)) names the event (the stabbing) which has the properties indicated by the predicates in the representation. For present purposes, the underlying event theory may be seen as a carefully considered articulation of this idea.

The underlying event theory—particularly as developed by Parsons [22], who is largely successful in keeping representations within the confines of first-order logic—offers great promise as a (partial) theory for natural language representation. I have explored in several places how the theory can be put to good use in computer-to-computer communication in electronic commerce (cf. [10, 11, 15]).

Although we shall make extensive use of the theory in what follows, its finer details would only be distracting. I have made every effort at simplification. In particular, I have disposed of talk of thematic roles (Agent, Benefactive, Theme, etc. and have stuck to related and more familiar, but less discriminating, concepts: direct object (Obj), indirect object (IndObj), and subject (Subject).

3. THE CORE IDEA

Propositional content has (at least) two important aspects. First, it is about something, that is to say it is true-or-false or rather it is a description, accurate or not, of something. Second, it is itself something about which we attribute certain properties, e.g., that Mary believes it or hopes it or asserts it or promises it. Summarizing (perhaps sloganizing), we might put the point by saying that the sentences of interest here essentially have the structure: content + comment (on the content). The core idea I wish to develop involves directly recognizing and representing these two aspects (content, comment) of sentences with propositional content. Treating examples will best tell the story.

Consider the simple propositional content (and speech act) sentence:

**Expression 1.** Mary asserts that Sam arrived yesterday.

My idea is to represent this (and similar) sentence(s) with two kinds of expression: (a) a fundamental expression and (b) one or more axiom schemas, used to articulate meaning for the fundamental expressions. First, we can represent *Sam arrived yesterday* in what is more or less standard event semantics:

**Expression 2.** \( \phi' \exists e (\text{arrive}(e') \land \text{Subject}(e', \text{Sam}) \land \text{Cul}(e', \text{yesterday})) \)

Let \( \phi \) represent Expression 2.

The fundamental expression for the sentence (in Expression 1) becomes, in shorthand:

**Expression 3.** \( \exists e (\text{assert}(e) \land \text{Subject}(e, \text{Mary}) \land \text{Obj}(e, [\phi])) \)

or fully written out:

**Expression 4.** \( \exists e (\text{assert}(e) \land \text{Subject}(e, \text{Mary}) \land \text{Obj}(e, [\phi'] (\text{arrive}(e') \land \text{Subject}(e', \text{Sam}) \land \text{Cul}(e', \text{yesterday})))) \)

Thus, the main idea in the fundamental expressions is to treat a quoted sentence (the propositional content) as an object or individual about which a comment is made. In particular, the quoted sentence is the direct object of an event (or eventuality). Moreover, a special form of quotation is used: [\( \cdot \cdot \)]. By quoting an expression in this way—as in 3 and 4—we treat it as an individual and so capture (I argue) the second aspect noted about it. (I ask the reader to hold off for the moment questions about what is really going on here. I do note that this representation move does not conflate the distinction between Mary said *hello* and Mary said “Hello.” For now, it is only the former to which the theory applies.)

We need additional expressions to capture the content—the first—aspect of Sam’s arriving yesterday. Well, if Mary’s assertion is true—or *Veridical*—then what she asserted (the content) is true. Formally we have the following rule:
4. THE CORE IDEA APPLIED

In this section we’ll revisit the examples, above. Our context makes inappropriate a careful and detailed study of each of these cases. Rather, the purpose here is to make a plausibility argument. Read with charity.

4.1 Saying that

Consider

The cognoscenti will notice a similarity between the present theory and quotational theories of propositional attitudes. I shall take this matter up in the sequel. Briefly, part of why I call it the disquotation theory is to distinguish it from the quotational theories and to suggest similarity with Tarski-style T sentences.

Expression 5. *Galileo said that the earth moves.*

Neglecting such fine points as matters of time and tense, we represent *Galileo said that the earth moves* (in the style of event semantics) as follows.

Expression 6. \( \exists e (\text{saying}(e) \land \text{Subject}(e, \text{Galileo}) \land \text{Obj}(e, [\exists e (\text{move}(e) \land \text{Subject}(e, \text{Earth}))])) \)

More generally, we have:

Fundamental Schema 1 (Saying that). \( \exists e (\text{saying}(e) \land \text{Subject}(e, s) \land \text{Obj}(e, [\Phi]) \land \Gamma) \)

In Fundamental Schema 1, \( \Gamma \) holds the place of any additional qualifiers on \( e \).

It is well to note that Expression 5—*Galileo said that the earth moves*—is a stylistic variant of *Galileo said the earth moves*. Both are quite distinct from

Expression 7. *Galileo said “The earth moves.”*

The former are true, the latter—Expression 7—is false. Galileo didn’t speak English. And in English, so far as I know, there is no verb for specifically distinguishing a saying-that (e.g., Expression 5) from a saying-quote (e.g., Expression 7). So we’ll make one up:

Fundamental Schema 2 (Saying quote). \( \exists e (\text{saying-quote}(e) \land \text{Subject}(e, s) \land \text{Obj}(e, [\Phi]) \land \Gamma) \)

Notice that a saying-quote does not in general imply the corresponding saying. That is, in general

Expression 8. \( \exists e (\text{saying-quote}(e) \land \text{Subject}(e, s) \land \text{Obj}(e, [\Phi])) \neq \exists e (\text{saying}(e) \land \text{Subject}(e, s) \land \text{Obj}(e, [\Phi])) \)

does not hold, for the same reason there is a distinction between sentence meaning and speaker’s meaning. One may say one thing and mean (say that) another. Happens all the time. Representational correctness requires we recognize it. My principal concern in this paper is with saying that, not with saying-quote. (See [6] for a useful, and I think largely correct, discussion of saying that.)

Characteristically, the object of a saying that may, or may not be, *Veridical*. Thus:

Axiom Schema 2 (Saying that Success Rule).

\( \forall e ((\text{saying}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\text{Veridical}(e) \rightarrow \phi)) \)

If and only if what you said is true, your saying that is veridical.

4.2 Speech Act Descriptions

For the sake of the discussion we may follow Searle and Vanderveken [26] in identifying five fundamental illocutionary forces: assertives, commissives (promises), declaratives, requestives, and emotives.\(^7\) Everything that can be said, they claim, is, or involves, one or more of these five forces. What distinguishes the fundamental forces are their (fundamental) points. The point of an assertive is to say that something is true; the point of a requestive is to indicate a desire to get someone to do something, and so on.

\(^6\)This is a simplification for the sake of getting on with the discussion. There are several competing theories of speech acts. The disquotation theory can for present purposes be considered neutral with respect to speech act theories.
Following ideas developed in my earlier work on representation of speech acts (cf. [9, 12, 10, 14, 13, 11, 15]) we identify a characteristic success predicate for each of the various fundamental illocutionary forces. An assertive succeeds if what is asserted is true, in which case the assertion is Veridical. A request succeeds when what is requested occurs, in which case the request is Honored. A commissive, or promise, succeeds when what is promised in fact comes about, in which case the promise is Kept. A declarative succeeds if the declaration is made with sufficient authority (and under the right circumstances), in which case the declaration is Authoritative. Success criteria for emotives—”Yeah!”, “Boo!” and so on—are in general problematic, especially since they need not have any propositional content at all. Nevertheless we can define fundamental expressions for them. What is problematic is finding general axiom schema.

Consider now in detail the other fundamental speech acts. Besides asserting, we have fundamental schemas and axiom schemas for promising, declaring, and requesting. (See [15] and references therein for elaboration.) The fundamental schemas for the fundamental illocutionary forces are as follows.

**Fundamental Schema 3** (Assert). \( \exists e (\text{assert}(e) \land \text{Obj}(e, [\Phi]) \land \Gamma) \)

In Fundamental Schema 3, \( \Gamma \) holds the place of any additional qualifiers on \( e \). Thus, Expression 4 is an instance of Fundamental Schema 3. The other fundamental illocutionary forces have corresponding fundamental schemas.

**Fundamental Schema 4** (Promise). \( \exists e (\text{promise}(e) \land \text{Obj}(e, [\Phi]) \land \Gamma) \)

**Fundamental Schema 5** (Request). \( \exists e (\text{request}(e) \land \text{Obj}(e, [\Phi]) \land \Gamma) \)

**Fundamental Schema 6** (Declare). \( \exists e (\text{declare}(e) \land \text{Obj}(e, [\Phi]) \land \Gamma) \)

**Fundamental Schema 7** (Emote). \( \exists e (\text{emote}(e) \land \text{Obj}(e, [\Phi]) \land \Gamma) \)

The various fundamental illocutionary force success rules are straightforward. A promise is kept if and only if what was promised is/becomes true.

**Axiom Schema 3** (Promise Success Rule). \( \forall e ((\text{promise}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\text{Kept}(e) \leftrightarrow \phi)) \)

A request is honored if and only if what was requested is done/becomes true.

**Axiom Schema 4** (Request Success Rule). \( \forall e ((\text{request}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\text{Honored}(e) \leftrightarrow \phi)) \)

Sometimes saying so makes it so. If the umpire says “You’re out” then you’re out, provided that the umpire’s saying this is authoritative (properly done under the right circumstances). And a declaration is/becomes true if (but not only if, since someone else could properly call you out) what was declared was declared with the requisite authority.

**Axiom Schema 5** (Declaration Success Rule). \( \forall e ((\text{declare}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\text{Authoritative}(e) \leftrightarrow \phi)) \)

And, as hinted, we’ll have no success rule for emotives.

It is tempting, however, to combine all of these into a more general schema.

**Axiom Schema 6** (General Success Rule). \( \forall e ((\text{promise}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\text{Successful}(e) \leftrightarrow \phi)) \)

letting \( \Psi \) stand for any member of a proper list of illocutionary forces.

Convenient as they are (as I shall argue) these axiom schemas do come with a certain amount of baggage: they fail to rule out very many—indeed any—invalid or otherwise flawed utterances. Some examples. First, suppose at time \( t \) a promise is made—or rather an utterance to that effect is given—that something will be done at time \( t’ \), where \( t’ \) is before \( t \). Well, usually you can’t promise to do things in the past. To say “I promise that I did it” is to make a solemn avowal perhaps, but it is not to make a promise. The representational devices to hand—the fundamental schemas and the axiom schemas—do not support exclusion of flawed utterances of this sort. If we are to model our practices and conventions pertaining to promising, we need to do better.

Second, the axiom schemas other than that for assertions seem too strong in that the biconditional should be replaced by a plain conditional. Suppose I request to you that the beer arrives by 6, you do everything in your power to prevent the beer from arriving at all, and yet the beer does arrive on time. Axiom Schema 4 would have it that my request has in fact been honored, contra received intuitions. Similarly, suppose in response to my request you (disingenuously) promise to get the beer here on time. Despite all your good efforts it does arrive as promised. Has the promise been kept? Not likely. Finally, suppose that I am a policeman and acting on my believed authority I declare you under arrest for a certain charge. Unknown to me, another policeman has arrested you for the charge and my authority has been destroyed because I have also been charged with a crime and been stripped of my position. Since in fact you are under arrest for the specified charge, Axiom Schema 5 implies—contra assumption—that I have the authority to place you under arrest.

I draw two lessons from these examples. First, validity conditions for the Fundamental Schemas need to be defined and instances of the schemas should be taken as true only if valid. In this way, we simply rule out, e.g., promises about the past.

Second, the schemas themselves tell us something about the validity conditions for the modeled utterances. Requests to an agent (addressee) must be for the agent to do something, e.g., cause or try as best it can. For example, one (s the speaker) cannot validly request of (addressee) a (only) a delivery of something, \( g \).

**Expression 9.** \( \exists e ((\text{request}(e) \land \text{Subject}(e, s) \land \text{IndObj}(e, a) \land \text{Obj}(e, [\text{deliver}(e_1) \land \text{Obj}(e_1, g) \land \text{IndObj}(e_1, s)])) \)

Instead, one must specify that \( a \) in some appropriate sense do the delivering.\(^9\)

\(^9\)There are some subtle issues here, which can be noted without our having to be much troubled by them for present purposes. Suppose a hires the carrier FedEx to deliver the
Finally, for present purposes, we need a means of linking a specific event to the, here, request that occasions it. Two different request events may request delivery by a of , particularly when is stuff of a common type, rather than an identified object. The car you buy has a unique VIN (vehicle identification number), but your order for the car does not mention it. Instead, you specify a make and model number, etc. Similarly, you order a dozen eggs of a particular grade and quality. We need a means of mapping a particular delivery to a particular request, and or its analogs will not do for us. We need to add a predicate, , for the sake of event , and is to count as contributing to the success of .

This presents us a problem: How can we quantify into a quoted sentence? The solution is straightforward. Think of as “Necessarily, if is then the case that “ or as “It necessarily is the case that “. I’ll use a shorthand notation for this: . So we have , provided that , provided that , provided that , and provided that .

Expression 11. \[ \exists e \text{(request(e)) \land \text{Subject(e, s) \land IndObj(e, a) \land Obj(e, [\exists e_1 (\text{deliver(e_1)} \land \text{Sub(e_1, a) \land Obj(e_1, g) \land IndObj(e_1, s) \land Sake(e_1, s)])])} \]

Similarly, we can stipulate that a valid promise requires that the Subject of the promising event specifically be also the subject of the event promised and that that event is to occur on or after the time of the promise itself. Finally, can be used as a required part of the content of a declaration. The declaring event is authoritative if and only if the declared event obtains for the sake of, or in virtue of, the declaring event. Let’s see how this works. declares that is under arrest:

Expression 12. \[ \exists e \text{(declare(e)) \land \text{Subject(e, s) \land IndObj(e, a) \land Obj(e, [\exists e_1 (\text{arrest(e_1)} \land \text{Sub(e_1, s) \land Obj(e_1, a) \land Sake(e_1, s)])})} \]

The problems identified above, e.g., a has been arrested but not by s, are obviated with this representation.

4.3 Propositional Attitudes

The general move applies as well to propositional attitude descriptions, that is descriptions of psychological states. I’ll discuss the basics: believing, intending, and desiring.

Expression 10. \[ \exists e \text{(request(e) \land \text{Subject(e, s) \land IndObj(e, a) \land Obj(e, [\exists e_1 (\text{deliver(e_1)} \land \text{Sub(e_1, a) \land Obj(e_1, g) \land IndObj(e_1, s) \land Sake(e_1, s)])])} \]

4.4 Perceiving that

Perceiving-that sentences work similarly.

Expression 13. \[ \exists e_1 \text{(deliver(e_1)} \land \text{Sub(e_1, a) \land Obj(e_1, g) \land IndObj(e_1, s) \land Sake(e_1, s)} \]

Axiom Schema 7 (Belief Rule).

Axiom Schema 8 (Desire Rule).

Axiom Schema 9 (Intention Rule).

Axiom Schema 10 (Seeing that Rule).

4.5 Deontic Expressions

Deontic sentences—about obligations and permissions—are an interesting category. A first step in the general direction of the disquotation theory is to combine the Anderson reduction with event semantics. The Anderson reduction first [1, 20]. Instead of for “It ought to be the case that ” we have \[ \Box (\neg \phi \rightarrow V) \] where is the bad (violation) condition. That is, “ought to be true” is unpacked as “Necessarily, if is not true, then the bad happens” (and that’s not good!).

Focusing for present purposes on simple sentences, having a single verb and governing event, the Anderson reduction may be reframed so as to exploit the underlying events. Suppose that a delivery is obligated:

Expression 14. \[ \exists e_1 \text{(deliver(e_1)} \land \text{Sub(e_1, a) \land Obj(e_1, g) \land IndObj(e_1, s) \land Sake(e_1, s)} \]

(Axiom Schema for Ought follows the usual form)

Note that beliefs, assertions, and sayings—that have contents that may or may not be veridical. The representation theory on offer here does not by itself distinguish beliefs from assertions (or knowing that or saying that). Nor should we expect this of a representation theory. It is enough that the fundamental schemas capture the distinctions. It is the job of semantics to account for them. How does belief differ from knowledge? Tell me and the disquotation theory will be useful in formalizing their characteristic differences. The disquotation theory doesn’t say how belief and knowledge (etc.) differ; that’s not its job.

What if is already in use in the expression? This is a technical issue and can be handled by carefully stating rules for introducing and replacing names.
Fundamental Schema 8 (Ought). \(\exists e (\text{ought}(e) \land \text{Obj}(e, [\phi])) \land \Gamma)\)

and our example (Expression 13) instantiates in the predictable fashion:

Expression 14. \(\exists e (\text{ought}(e) \land \text{Obj}(e, [\exists e_1 (\text{deliver}(e_1) \land \text{Sub}(e_1, a) \land \text{Obj}(e_1, g) \land \text{IndObj}(e_1, s) \land \text{Sake}(e_1, \square)])])\)

Corresponding closely to the spirit of the Anderson reduction gives us the weak ought rule:

Axiom Schema 11 (Weak Ought Rule).
\[\forall e (\text{ought}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\neg \phi \rightarrow V(e))\]

Note that we have \((\neg \phi \rightarrow V(e))\) rather than \[\square(\neg \phi \rightarrow V(e))\]
as in the Anderson reduction. This is as it should be. The fundamental schema ensures sufficient intensionality, so that if \(\phi\) ought to be the case and \(\phi \rightarrow \psi\), it does not follow merely from the fundamental schema that \(\phi\) ought to be the case. (It does follow from the rule of inference in \(D^*\), see below.) Moreover, it does follow from the axiom schema that if \(\neg \phi\) then the same violation condition obtains when \(\neg \phi\). Of course, they go together as it happens.

Our use of event semantics permits distinguishing \(V\) more finely as \(V(e)\); instead of the the violation condition, \(e\) names a violation condition. This allows us to employ the strong ought rule:

Axiom Schema 12 (Strong Ought Rule).
\[\forall e (\text{ought}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\neg \phi \rightarrow V(e))\]

Permission works similarly.

Expression 15 (Permission). \(\exists e (\text{permit}(e) \land \text{Obj}(e, [\exists e_1 (\text{deliver}(e_1) \land \text{Sub}(e_1, a) \land \text{Obj}(e_1, g) \land \text{IndObj}(e_1, s) \land \text{Sake}(e_1, \square)])])\)

Fundamental Schema 9 (Permission). \(\exists e (\text{permit}(e) \land \text{Obj}(e, [\phi])) \land \Gamma)\)

Permissions don’t lead to violations. You can’t violate a permission.

Axiom Schema 13 (Permission Rule).
\[\forall e (\text{permit}(e) \land \text{Obj}(e, [\phi])) \rightarrow \neg V(e)\]

Finally, prohibition, for which we have weak and strong rules, as with obligation.

Axiom Schema 14 (Weak Prohibition Rule).
\[\forall e (\text{prohibit}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\phi \rightarrow V(e))\]

Axiom Schema 15 (Strong Prohibition Rule).
\[\forall e (\text{prohibit}(e) \land \text{Obj}(e, [\phi])) \rightarrow (\phi \rightarrow V(e))\]

Some comments.

4.5.0.1 Deontic Axiom Systems.
There are many distinct systems of deontic logic, distinguished by different axiomatizations. Just as, e.g., \(O^P\) can be used to represent It is obligatory that \(P\) but is not of itself a logic, so our expressions similarly fall short. How do we get from them to a fully fledged deontic logic? One example should be enough. Briefly, consider system \(D^*\), aka standard deontic logic [3, page 190]. \(D^*\) is characterized by a rule of inference
\[(\phi_1 \land \ldots \land \phi_n) \rightarrow \phi \vdash (O\phi_1 \land \ldots \land O\phi_n) \rightarrow O\phi \geq 0\]
plus an axiom scheme
\[\neg(O\phi \land O\neg \phi)\]
The rule of inference directly can be translated directly using the disquotation representation method. One simply expands the \(O\phi\) expressions appropriately. The revision required of the \(D^*\) axiom schema is mainly due to our Anderson-like reduction and the fact that we are using event semantics. The revised axiom schema is:
\[\forall e (\neg V(e) \land \neg V(e))\]
which is a tautology and hence not needed at all.\(^{12}\) ought implies can and may. These are straightforward extensions, exercises I leave to the reader.

4.5.0.2 Systems of Obligation and Permission.
What etiquette forbids the law may allow. And in general, what is permitted or even obligated under one system of norms may or may not be permitted or obligated by another. The disquotation theory can handle such phenomena simply and directly. Interpret IsUnderNSystem(e, n) as Event e is governed by normative system n.\(^{13}\) Then add it as a qualifier to the fundamental schema expressions, where needed. Etiquette forbids loud burping at table then could be represented as:

Expression 16. \(\exists e (\text{prohibit}(e) \land \text{IsUnderNSystem}(e, \text{etiquette}) \land \text{Obj}(e, [\exists e_1 (\text{burp}(e_1) \land \text{loud}(e_1) \land \text{at}(e_1, \text{table}))])\)

4.5.0.3 Directed Obligation.
The move here is the same as in systems of obligation and permission: add predicates to qualify the underlying event. If a has an obligation to b under system of norms n that \(\phi\), we then have:

Fundamental Schema 10 (Directed Obligation).
\[\exists e (\text{ought}(e) \land \text{Subject}(e, a) \land \text{IndObj}(e, b) \land \text{IsUnderNSystem}(e, n) \land \text{Obj}(e, [\phi]) \land \Gamma)\]

(Tan and his group have done useful work on directed obligation. See [15] and papers cited therein.)

4.6 Necessity and Possibility.
With the apparatus to hand, we can extend the Anderson reduction from deontic logic to alethic modal logic.

Fundamental Schema 11 (Necessity).
\[\forall e (\text{necessity}(e) \land \text{Obj}(e, [\phi]) \land \Gamma)\]

Axiom Schema 16 (Strong Necessity Rule).
\[\forall e (\text{necessity}(e) \land \text{Obj}(e, [\phi]) \rightarrow (\phi \rightarrow T(e)))\]

\(^{12}\)This strikes me as very pretty aspect of the Anderson-style reduction. Developing the thought, however, is matter for a different paper.

\(^{13}\)Perhaps the disquotation approach can be made to do the formal work behind Jones and Sergot’s insightful analysis of institutional power [8]. This must be stuff for future papers.
∀(T(e) generalizes T. Just as T is an axiom, so is ∀e(T(e)).
(And we might add: ∀e(T(e) → T)).

As in the case of deontic representations, various axioms can be translated more or less directly into the argot of the disquotation theory. The T axiom in modal logic is □φ → φ. This becomes:

**Axiom Schema 17** (Necessity T Rule).
∀e((necessity(e) ∧ Obj(e, [φ])) → φ)

(Note that Strong Necessity (Axiom Schema 16) implies the T Rule. Should Strong Necessity be weakened?)

The other modal axioms can be treated similarly. Also, the previous comments about systems of norms for deontic logics applies here. With appropriate qualification ( ∧ Γ), we can identify various kinds of necessity (e.g., logical, nomic, legal).

Impossibility corresponds to prohibition in deontic logics.

**Fundamental Schema 12** (Impossibility). ∃e(possibility(e) ∧ Obj(e, [φ]))

**Axiom Schema 18** (Strong Impossibility Rule).
∀e((possibility(e) ∧ Obj(e, [φ])) → (φ → T(e)))

What about possibility? Well something is possible if it is not impossible.

**Axiom Schema 19** (Possibility Rule).
∀e(possibility(e) ↔ impossibility(e))

4.7 Needs

Finally, needs or rather needing that. The familiar patterns apply.

**Fundamental Schema 13** (Need that).
∃e(need-that(e) ∧ Obj(e, [φ])) ∧ Γ

**Axiom Schema 20** (Need that).
∀e(((need-that(e) ∧ Obj(e, [φ]))) → (Satisfied(e) ↔ φ))

5. DISCUSSION

To many the disquotation theory might seem thin gruel. Admittedly, I am offering no complete account of what it is to, e.g., make a promise, believe something, or say something. Rather, I am offering a story on how to say that a promise is made and I am claiming that if you have a more elaborate and complete theory, then the modeling, or representational, means at hand is sufficient to express your more detailed account of promising (believing, saying, etc.). See [15] for further discussion.

The main point is this: the disquotation mode of representation gives you all the intensionality you could ever need. You can take it back with additional axiom schemas and you can capture desired relationships easily and directly via rules on the eventuality. And we get all of this without ontologically leaving the actual world. A powerful idea, I submit. Some examples to illustrate the point.

Take an arbitrary case of a fundamental schema (Fundamental Schema 13):

**Expression 17.** ∃e(need-that(e) ∧ Obj(e, [φ]))

Expression 17 is logically equivalent to

**Expression 18.** ∃e(need-that(e) ∧ Obj(e, [ψ]))

if and only if [φ] = [ψ]. Assuming, as I have throughout, that in general [φ] = [ψ] if and only if the expression of φ is identical to the expression of ψ, then only literally identical substitutions will be (guaranteed to) preserve truth. For example if φ is the expression P ∧ Q and ψ is the expression Q ∧ P, they are not literally identical and the substitution does not work. Notice that φ and ψ might have the same meaning (if that’s possible), but so long as their expressions literally differ they cannot be intersubstituted with a guarantee of truth preservation. This is indeed as much intensionality as one can ask for.

Perhaps we have too much intensionality. Indeed we do, but reducing intensionality is much easier and more straightforward than increasing it. We simply add axioms that allow us to make the substitutions we wish to allow. For example, in standard deontic logic, if Oφ and (φ ↔ ψ) then Oψ. This, or its analog, will obtain in our disquotational representation, provided we accept the rule of inference of D∗ (equation 3). Similarly, one might argue that needs-that is extensional: substitution can be freely made without failing to preserve any truths. Formalizing this is straightforward.

**Axiom Schema 21** (Need that Extensionality).
∀e((need-that(e) ∧ Obj(e, [φ]) ∧ (φ ↔ ψ)) → Obj(e, [ψ]))

This generalizes: if substitution preserves truth when meanings or references are identical, we can have an axiom schema for that; similarly for logical equivalence, co-extensionality (above), nomic equivalence, and anything else you can identify.

Finally, the representation approach allows a very fine-grained treatment of propositional content and various relations to it. Suppose an agent believes P whenever it believes P ∧ Q, then write an axiom schema for this. Or perhaps your agent can do modus tollens only about flowers, then say so. Our axioms may be supported by logical or conceptual principles, as emphasized here, or by empirical considerations. In any event, event semantics coupled with the disquotation theory for representing propositional content gives us an attractive toolbox for modeling sentences with embedded propositional content.

6. CONCLUSION

So what do we have? The disquotation theory (only adumbrated, sketched here) gives us a principled, general representation scheme for embedded propositional content. This allows us to reason formally with and about such expressions, without at the same time having to settle on a full and complete semantic or etiological theory. The claim—disprovable but not disproved—is that the disquotation approach allows us to say formally whatever we need to say, and to get the inferences right. It remains to explore and exploit the approach, so there is much yet to do. In the meantime the theory is usable incrementally. It will not, I believe, support invalid inferences. When it fails to support needed inferences, axiom schemas can be added. Not a bad position to be in from a practical perspective.

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14One might think of ⌈·⌉ as an infinitely-discriminating hash function, taking well-formed formulas in its argument.
8. REFERENCES


