# On the representation of normative sentences in FOL

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January 23, 2012

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## Abstract

Rules, regulations and policy statements quite frequently contain nested sequences of normative modalities as in, for example:

- The database manager is obliged to permit the deputy-manager to authorise access for senior departmental staff.
- Parking on highways ought to be forbidden. Marcus (1966)

Accordingly, a knowledge-representation language for such sentences must be able to accommodate nesting of this kind. However, if—as some have proposed—normative modalities such as *obligatory, permitted,* and *authorised* are to be interpreted as first-order predicates of named actions, then nesting appears to present a problem, since the scope formula of *obligatory* in "obligatory that it is permitted that $a$" (where $a$ names an action) is not a name but a sentence.

The 'disquotation' theory presented in Kimbrough ("A Note on Interpretations for Federated Languages and the Use of Disquotation", and elsewhere) may provide a candidate solution to this FOL problem. In this paper we rehearse parts of that theory and evaluate its efficacy for dealing with the indicated normative nesting problem.

# 1 Introduction

Rules, regulations and policy statements quite frequently contain nested sequences of normative modalities as in, for example:

- The database manager is obliged to permit the deputy-manager to authorise access for senior departmental staff.

Accordingly, a knowledge-representation language for such sentences must be able to accommodate nesting of this kind. Applications require it and it is the use of logic in applications that we mainly have in mind. However, if—as some have proposed—normative modalities such as *obligatory, permitted,* and *authorised* are to be interpreted as first-order predicates of named actions, then nesting appears to present a problem, since the scope formula of *obligatory* in "obligatory that it is permitted that $a$" (where $a$ names an action) is not a name but a sentence.

To illustrate, for certain purposes it may work to represent $\mathcal{O}P$ ("$P$ is obligatory") in first-order logic (FOL) by (1) using a predicate to approximate $\mathcal{O}$ (say *Obligatory*), (2) identifying $P$ with a named action, say $p$, and (3) applying the predicate to the named action. This would yield, in the example, say $\mathit{Obligatory}(p)$. Once we wish to nest operators, however, this approach would seem to fail at the outset. $\mathit{Obligatory}(\mathit{Forbid}(p))$ is simply ill-formed in FOL and there is no immediately apparent alternative available in the spirit of the original proposal.

The 'disquotation' theory presented in "A Note on Interpretations for Federated Languages and the Use of Disquotation" (Kimbrough, 2001, 2005), and elsewhere, may provide a candidate solution to this FOL problem. In this paper we rehearse parts of that theory and evaluate its efficacy for dealing with the indicated normative nesting problem. That theory encompasses both quotation—a means of naming sentences—and disquotation, a means of recovering the named sentences and using them for logical inference. Our present main concern—representation of nested modalities—will lead us to focus on the quotation aspect. We defer for future work discussion of how disquotation may be used in the present context.

We shall propose a general approach for the representation in FOL of modalities, particularly normative modalities. This approach employs two principal moves: embedding of sentence operators in FOL as functions, and quotation. The bulk of the paper is devoted to explaining and illustrating these moves. We begin, in the next section, with the representation of sentence logic by embedding it within FOL. Modalities soon follow. We post here for the sake of the reader's convenience a table, Table 1, whose meaning will become apparent in the sequel.[1]

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[1] We are aware that other work has been done on the representation of modalities in FOL. For instance, one of the reviewers of this paper has drawn our attention to (van Benthem, 2010), sections 7.1 and 7.2. However, we postpone to later work a comparison of the relative merits of these approaches.

| Interpretation | Sentence logic operator | Embedding function in FOL |
|---|---|---|
| not | $\neg$ | $n$ |
| or | $\vee$ | $d$ |
| and | $\wedge$ | $c$ |
| material implication | $\rightarrow$ | $m$ |
| biconditional | $\leftrightarrow$ | $b$ |
| necessity | $\Box$ | $l$ |
| obligation | $\mathcal{O}$ | $o$ |
| action, stit | $\mathcal{E}$ | $e$ |

Table 1: Embedding correspondances used

## 2 Sentence Logic (SL) Formulations

It will be useful to begin by applying our basic moves to sentence logic. The aim is to *embed* a source language—here, sentence logic—into FOL in such a way that the two representations are equivalent (cf., (Bhargava and Kimbrough, 1995)). First, then, a formulation of sentence logic (SL), which is here to be our *source language.* It will be translated into FOL, which will serve as the *target language.*

$\mathcal{P} = \{p_1, p_2, \ldots\}$ is the set of atomic sentences.

1. If $\bigcirc \in \mathcal{P}$ then $\bigcirc$ is a well-formed formula (wff) in SL.
2. If $\bigcirc$ is a wff in SL, then $\neg\bigcirc$ is a wff in SL.
3. If $\bigcirc$ and $\triangle$ are wffs in SL, then $(\bigcirc \vee \triangle)$ is a wff in SL.
4. Nothing else is a wff in SL.

For our proof mechanism we are using a truth tree method, as in Jeffrey (1991). The basic rules of inference are:

1. Path closure

$$\begin{array}{c} \bigcirc \\ \neg\bigcirc \\ \hline \times \end{array}$$

2. Double negation

$$\begin{array}{c} \neg\neg\bigcirc \\ \hline \bigcirc \end{array}$$

3. Disjunction

$$\begin{array}{c} (\bigcirc \lor \triangle) \\ \hline \bigcirc \qquad\qquad \triangle \end{array}$$

4. Denied disjunction

$$\begin{array}{c} \neg(\bigcirc \lor \triangle) \\ \hline \neg\bigcirc \\ \neg\triangle \end{array}$$

We require, in addition, that rules of inference not be invoked if their resulting path additions are redundant. For example, if $\neg(\bigcirc \lor \triangle)$ occurs in a path, and in every open path underneath both $\neg\bigcirc$ and $\neg\triangle$ occur, then denied disjunction is not available on $\neg(\bigcirc \lor \triangle)$. This is simply to prevent failure to halt.

To effect the embedding we apply two transformation rules, in sequence, to expressions in the source language, here SL. We denote the translation of the SL expression $\bigcirc$ into FOL, as:

$$\mathbf{ITr}(\mathbf{Tr}(\bigcirc)) \tag{1}$$

where

$$\mathbf{ITr}(\mathbf{Tr}(\bigcirc)) = S(\mathbf{Tr}(\bigcirc)). \tag{2}$$

The **ITr** function (mnemonic: "initial translation") embeds its argument in a FOL predicate, arbitrarily chosen to be $S$ (mnemonic: "sentence"). The **Tr** function (mnemonic: "translation") converts source language propositional formulas to FOL names and well-formed function expressions (wfts: well-formed terms). It works as follows.

1. If $\bigcirc \in \mathcal{P}$ then $\mathbf{Tr}(\bigcirc) = \ulcorner\bigcirc\urcorner$.
2. If $\bigcirc = \neg\triangle$, then $\mathbf{Tr}(\bigcirc) = n(\mathbf{Tr}(\triangle))$.
3. If $\bigcirc = (\triangle \lor \triangledown)$, then $\mathbf{Tr}(\bigcirc) = d(\mathbf{Tr}(\triangle), \mathbf{Tr}(\triangledown))$.

Add the following rules of inference to FOL:

1. **Tr** Path closure

$$\begin{array}{c} S(\bigcirc) \\ S(n(\bigcirc)) \\ \hline \times \end{array}$$

2. **Tr** Double negation

$$\begin{array}{c} S(n(n(\bigcirc)) \\ \hline S(\bigcirc) \end{array}$$

3. **Tr** Disjunction

$$\begin{array}{c} S(d(\bigcirc, \triangle)) \\ \hline S(\bigcirc) \qquad\qquad S(\triangle) \end{array}$$

4. **Tr** Denied disjunction

$$\begin{array}{c} S(n(d((\bigcirc, \triangle))) \\ \hline S(n(\bigcirc)) \\ S(n(\triangle)) \end{array}$$

Note that the FOL function $n$ corresponds to the SL sentence operator $\neg$ (negation). Similarly, $d$ corresponds to $\vee$ (disjunction). Refer to Table 1 for all of the operators and correspondences we cover in this paper.

While this is all that is necessary, it will be convenient in what follows to have rules for conjunction, material implication and for biconditional. How this should be done should be obvious. Here are the results.

5. Conjunction

$$\begin{array}{c} (\bigcirc \wedge \triangle) \\ \hline \bigcirc \\ \triangle \end{array}$$

6. Denied conjunction

$$\begin{array}{c} \neg(\bigcirc \wedge \triangle) \\ \hline \neg\bigcirc \qquad\qquad \neg\triangle \end{array}$$

7. Conditional

$$\begin{array}{c} (\bigcirc \rightarrow \triangle) \\ \hline \neg\bigcirc \qquad\qquad \triangle \end{array}$$

8. Denied conditional

$$\begin{array}{c} \neg(\bigcirc \rightarrow \triangle) \\ \hline \bigcirc \\ \neg\triangle \end{array}$$

9. Biconditional

$$\begin{array}{c} (\bigcirc \leftrightarrow \triangle) \\ \hline \end{array}$$

$$\begin{array}{ccc} \bigcirc & \qquad\qquad & \neg\bigcirc \\ \triangle & & \neg\triangle \end{array}$$

10. Denied biconditional

$$\begin{array}{c} \neg(\bigcirc \leftrightarrow \triangle) \\ \hline \begin{array}{ccc} \bigcirc & \qquad\qquad & \neg\bigcirc \\ \neg\triangle & & \triangle \end{array} \end{array}$$

They go into FOL as follows:

5. **Tr** Conjunction

$$\begin{array}{c} S(c(\bigcirc, \triangle)) \\ \hline S(\bigcirc) \\ S(\triangle) \end{array}$$

($c$ is the FOL function we use to correspond to the conjunction operator.)

6. **Tr** Denied conjunction

$$\begin{array}{c} S(n(c(\bigcirc, \triangle))) \\ \hline \begin{array}{ccc} S(n(\bigcirc)) & \qquad\qquad & S(n(\triangle)) \end{array} \end{array}$$

7. **Tr** Conditional

$$\begin{array}{c} S(m(\bigcirc, \triangle)) \\ \hline \begin{array}{ccc} S(n(\bigcirc)) & \qquad\qquad & S(\triangle) \end{array} \end{array}$$

($m$ is the FOL function we use to correspond to the material conditional operator.)

8. **Tr** Denied conditional

$$\begin{array}{c} S(n(m(\bigcirc, \triangle))) \\ \hline S(\bigcirc) \\ S(n(\triangle)) \end{array}$$

9. **Tr** Biconditional

$$\begin{array}{c} S(b(\bigcirc, \triangle)) \\ \hline \begin{array}{ccc} S(\bigcirc) & \qquad\qquad & S(n(\bigcirc)) \\ S(\triangle) & & S(n(\triangle)) \end{array} \end{array}$$

($b$ is the FOL function we use to correspond to the biconditional operator.)

10. **Tr** Denied biconditional

$$\frac{S(n(b(\bigcirc,\triangle)))}{\begin{array}{cc} S(\bigcirc) & S(n(\bigcirc)) \\ S(n(\triangle)) & S(\triangle) \end{array}}$$

Points arising:

1. We have with **Tr** converted atomic sentences in sentence logic to 'quotations' of themselves in FOL. $p_1$ goes to $\lceil p_1 \rceil$ and so on. The quoted expressions are logically names in FOL. They are names of the atomic sentences in the embedded language. See (Kimbrough, 2001, 2005) for details.

2. The embedding translation is invertible.
   (a) $\mathbf{Tr}(\bigcirc) = \mathbf{ITr}^{-1}(S(\mathbf{Tr}(\bigcirc))$.
   (b) $\bigcirc = \mathbf{Tr}^{-1}(\mathbf{Tr}(\bigcirc))$.
   (c) $\mathbf{Tr}^{-1}(\lceil \bigcirc \rceil) = \bigcirc$ where $\bigcirc$ is a sentence letter of the embedded language.
   (d) $\mathbf{Tr}^{-1}(n(\bigcirc)) = \neg\mathbf{Tr}^{-1}(\bigcirc)$.
   (e) $\mathbf{Tr}^{-1}(d(\bigcirc,\triangle)) = (\mathbf{Tr}^{-1}(\bigcirc) \vee \mathbf{Tr}^{-1}(\triangle))$.

   And so on.

3. In consequence, the translation emulates the embedded language exactly. So long as the only formulas or expressions introduced into the FOL representation are through the translation from sentence logic, the two systems will be isomorphic.

4. The embedding strategy—using quotation to create names of atomic sentences and using FOL functions to represent SL sentence operators—is general in the sense that there is no evident reason why the same approach cannot be used for other sentence operators. To that issue next.

## 3 System T

Modal system **T** as given in (Hughes and Cresswell, 1968, chapter three) is an extension of sentence logic, with the addition of two axioms and one transformation rule. The two axioms are

1. $\Box P \rightarrow P$ [The axiom of necessity, aka: T axiom]

2. $\Box(P \rightarrow Q) \rightarrow (\Box P \rightarrow \Box Q)$ [The K axiom]

The transformation rule, called the Rule of Necessitation (or N) is: If $\bigcirc$ is a thesis, then $\Box\bigcirc$ is a thesis.

The proof system for sentence logic used by Hughes and Cresswell (1968) is distinct from ours (i.e., the truth tree method described by Jeffrey (1991)) and so we first need to set out system **T** in terms of our preferred proof system. That is easily done. The two axioms become additions to the tree rules of inference we already have.

5. The axiom of necessity

   On any open path it is permitted to write once and only once
   $$\Box\bigcirc \rightarrow \bigcirc$$
   uniformly substituting a wff for $\bigcirc$.

6. The K axiom

   On any open path it is permitted to write once and only once
   $$\Box(\bigcirc \rightarrow \triangle) \rightarrow (\Box\bigcirc \rightarrow \Box\triangle)$$
   uniformly substituting wffs for $\bigcirc$ and $\triangle$.

The Rule of Necessitation is also easily handled. In the truth tree method, proofs work by listing the premises of the argument followed by the denial of the conclusion, and then manipulating the tree with the rules of inference until all branches are closed because they contain a contradiction, in which case the argument is valid, or until no more rules can be applied, in which case the argument is invalid. A thesis is simply a wff whose denial is contradictory. The Rule of Necessitation tells us that if $\bigcirc$ is a thesis, then so is $\Box\bigcirc$. So, to prove $\Box\bigcirc$ we can simply prove that $\bigcirc$ is a thesis. To capture this, we simply add the Rule of Necessitation to our sentence logic proof rules.

Here is an example proof of $(P \rightarrow \Diamond P)$ in system **T** given in (Hughes and Cresswell, 1968, page 33). (For simplicity, we eschew $\Diamond$ in favor of $\neg\Box\neg$, so we want to prove $(P \rightarrow \neg\Box\neg P)$ .)

1. $\Box\neg P \rightarrow \neg P$

   (By the Axiom of Necessity and substitution of $\neg P$ for $\bigcirc$ , i.e., $\bigcirc/\neg P$.)

2. $\neg\neg P \rightarrow \neg\Box\neg P$

   (By (1) and sentence logic contraposition.)

3. $P \rightarrow \neg\neg P$

   (By the propositional calculus.)

4. $P \rightarrow \neg\Box\neg P$

   (By the propositional calculus, using lines (2) and (3), above.)

Now the proof using our truth tree method. Here we avoid using →, and simply prove $(\neg P \vee \neg\Box\neg P)$.

1. $\neg(\neg P \vee \neg\Box\neg P)$

   (Denial of what is to be proved, for proof by contradiction.)

2. $\neg\neg P$

   (Line 1, denied disjunction.)

3. $\neg\neg\Box\neg P$

   (Line 1, denied disjunction.)

4. $\Box\neg P$

   (Line 3, double negation.)

5. $\Box\neg P \rightarrow \neg P$

   (Axiom of necessity, $\bigcirc/\neg P$.)

6. $\neg\Box\neg P$ (5, conditional)

7. $\times$ (4, 6, path closure)

8. $\neg P$ (5, conditional)

9. $\times$ (2, 8, path closure)

This example illustrates proofs in our system **T** truth tree proof mechanism. This mechanism is equivalent to that of Hughes and Cresswell (1968, chapter three).

To embed system **T** in FOL we extend our previous embedding of sentence logic by adding one new rule for **Tr**:

4. If $\bigcirc = \Box\triangle$, then $\mathbf{Tr}(\bigcirc) = l(\mathbf{Tr}(\triangle))$.

   (Note: $l$ is an FOL function, serving to represent the necessity operator of system **T** in its propositional formulations.)

We also need to add the two inference rules, as translated.

11. **Tr** axiom of necessity

    On any open path it is permitted to write once and only once

    $$S(m(l(\bigcirc), \bigcirc))$$

    uniformly substituting a wft for $\bigcirc$. Note: $\mathbf{ITr}(\mathbf{Tr}(\Box\bigcirc \rightarrow \bigcirc)) = S(m(l(\mathbf{Tr}(\bigcirc)), \mathbf{Tr}(\bigcirc)))$.

12. **Tr** K axiom

    On any open path it is permitted to write once and only once

    $$S(m(l(m(\bigcirc, \triangle)), m(l(\bigcirc), l(\triangle))))$$

    uniformly substituting wfts for $\bigcirc$ and $\triangle$. Note:

    $\mathbf{ITr}(\mathbf{Tr}(\Box(\bigcirc \to \triangle) \to (\Box\bigcirc \to \Box\triangle))) =$

    $S(m(l(m(\mathbf{Tr}(\bigcirc), \mathbf{Tr}(\triangle))), m(l(\mathbf{Tr}(\bigcirc)), l(\mathbf{Tr}(\triangle)))))$

Finally, $S(\bigcirc)$ is a thesis iff $S(n(\bigcirc))$ yields a contradiction (that is, initiating a tree with only $S(n(\bigcirc))$ results in all paths being closed upon full application of the proof method). Further, if $S(\bigcirc)$ is a thesis, then $S(l(\bigcirc))$ is a thesis.

To illustrate, we revisit our previous proofs of $(\neg P \vee \neg\Box\neg P)$ .

1. $S(n(d(n(\ulcorner P \urcorner), n(l(n(\ulcorner P \urcorner))))))$

   Note: $\mathbf{ITr}(\mathbf{Tr}(\neg(\neg P \vee \neg\Box\neg P)))$.

   (Denial of what is to be proved, for proof by contradiction.)

2. $S(n(n(\ulcorner P \urcorner)))$

   Note: $\mathbf{ITr}(\mathbf{Tr}(\neg\neg P))$.

   (Line 1, denied disjunction.)

3. $S(n(n(l(n(\ulcorner P \urcorner)))))$

   Note: $\mathbf{ITr}(\mathbf{Tr}(\neg\neg\Box\neg P))$.

   (Line 1, denied disjunction.)

4. $S(l(n(\ulcorner P \urcorner)))$

   Note: $\mathbf{ITr}(\mathbf{Tr}(\Box\neg P))$.

   (Line 3, double negation.)

5. $S(m(l(n(\ulcorner P \urcorner)), n(\ulcorner P \urcorner)))$

   (Axiom of necessity, $\bigcirc / n(\ulcorner P \urcorner)$.)

| | |
|---|---|
| 6. $S(n(l(n(\ulcorner P \urcorner))))$ (5, conditional) | 8. $S(n(\ulcorner P \urcorner))$ (5, conditional) |
| 7. × (4, 6, path closure) | 9. × (2, 8, path closure) |

This example illustrates proofs in our FOL system embedding **T**, using our truth tree proof mechanism. Our claim is that this mechanism is equivalent to that of Hughes and Cresswell (1968, chapter three). Again, invertibility is present and serves to clinch the claim (provided, as before, that no further wffs or expressions are introduced on the FOL side).

System **S4** may be defined by adding the basic **S4** axiom to **T** (Hughes and Cresswell, 1968, page 46):

- $\Box\bigcirc \rightarrow \Box\Box\bigcirc$

This in turn gets translated into our truth tree proof method as

- Basic **S4** axiom

  On any open path it is permitted to write once and only once

  $$S(m(l(\bigcirc), l(l(\bigcirc))))$$

  uniformly substituting a wft for $\bigcirc$. Note:

  $\mathbf{ITr}(\mathbf{Tr}(\Box\bigcirc \rightarrow \Box\Box\bigcirc)) = S(m(l(\mathbf{Tr}(\bigcirc)), l(l(\mathbf{Tr}(\bigcirc)))))$.

System **S5** may be defined by adding the basic **S5** axiom to **T** Hughes and Cresswell (1968, page 49):

- $\neg\Box\neg\bigcirc \rightarrow \Box\neg\Box\neg\bigcirc$

This in turn gets translated into our truth tree proof method as

- Basic **S5** axiom

  On any open path it is permitted to write once and only once

  $$S(m(n(l(n(\bigcirc))), l(n(l(n(\bigcirc))))))$$

  uniformly substituting a wft for $\bigcirc$. Note:

  $\mathbf{ITr}(\mathbf{Tr}(\neg\Box\neg\bigcirc \rightarrow \Box\neg\Box\neg\bigcirc)) = S(m(n(l(n(\mathbf{Tr}(\bigcirc)))), l(n(l(n(\mathbf{Tr}(\bigcirc)))))))$.

## 4 SDL: Standard Deontic Logic

Standard Deontic Logic (SDL) may be handled straightforwardly with the methods used for systems **T**, **S4**, and **S5**. Since there are multiple (equivalent) formulations of SDL, we need to select one to discuss. We select Hilpinen's (Hilpinen, 1970) (see also `http://home.utah.edu/~nahaj/logic/structures/systems/standard-deontic.html`). This system is constructed by adding three axioms to sentence logic.

1. $\mathcal{O}P \rightarrow \neg\mathcal{O}\neg P$
2. $\mathcal{O}(P \rightarrow Q) \rightarrow (\mathcal{O}P \rightarrow \mathcal{O}Q)$
3. $\mathcal{O}(P \vee \neg P)$

We will handle the third axiom on a par with the Rule of Necessitation in the modal systems. The Rule of Deontic Necessitation tells us that if $\bigcirc$ is a thesis, then so is $\mathcal{O}\bigcirc$. So, to prove $\mathcal{O}\bigcirc$ we can simply prove that $\bigcirc$ is a thesis. To capture this, we simply add the Rule of Deontic Necessitation to our sentence logic proof rules.

To get the FOL embedding, we proceed as before. We extend our previous embedding of sentence logic by adding one new rule for **Tr**:

- If $\bigcirc = \mathcal{O}\triangle$, then $\mathbf{Tr}(\bigcirc) = o(\mathbf{Tr}(\triangle))$.

  (Note: $o$ is a FOL function, serving to represent the obligation operator $\mathcal{O}$ in its propositional formulations.)

Axioms 1 and 2 become inference rules in the usual way.

1. SDL 1

   On any open path it is permitted to write once and only once

   $$S(m(o(\bigcirc), n(o(n(\bigcirc)))))$$

   uniformly substituting a wft for $\bigcirc$. Note:

   $\mathbf{ITr}(\mathbf{Tr}(\mathcal{O}\bigcirc \rightarrow \neg\mathcal{O}\neg\bigcirc)) = S(m(o(\mathbf{Tr}(\bigcirc)), n(o(n(\mathbf{Tr}(\bigcirc)))))).$

2. SDL 2

   On any open path it is permitted to write once and only once

   $$S(m(o(m(\bigcirc, \triangle)), m(o(\bigcirc), o(\triangle))))$$

   uniformly substituting wfts for $\bigcirc$ and $\triangle$. Note:

   $\mathbf{ITr}(\mathbf{Tr}(\mathcal{O}(\bigcirc \rightarrow \triangle) \rightarrow (\mathcal{O}\bigcirc \rightarrow \mathcal{O}\triangle))) =$

   $S(m(o(m(\mathbf{Tr}(\bigcirc), \mathbf{Tr}(\triangle))), m(o(\mathbf{Tr}(\bigcirc), o(\mathbf{Tr}(\triangle)))))).$

Finally, $S(\bigcirc)$ is a thesis iff $S(n(\bigcirc))$ yields a contradiction (that is, initiating a tree with only $S(n(\bigcirc))$ results in all paths being closed upon full application of the proof method). Further, if $S(\bigcirc)$ is a thesis, then $S(o(\bigcirc))$ is a thesis.

# 5 Intensionality

It is well known that modal logics, including SDL, are intensional. Materially equivalent wffs may not in general be substituted *salva veritate* (Mackie, 1985). For example, in any of the modal systems we have considered, $\Box P, P \leftrightarrow Q \not\vdash \Box Q$, and $\Diamond P, P \leftrightarrow Q \not\vdash \Diamond Q$. It is also well known that first-order logic is extensional. Do we in consequence have a problem with our embedding? We do not.

Here is the tree proof for $\Diamond P, P \leftrightarrow Q \not\vdash \Diamond Q$:

1. $\neg\Box\neg P$

   $(=\Diamond P)$

2. $(P \leftrightarrow Q)$

3. $\neg\neg\Box\neg Q$

   $(= \neg\Diamond Q)$

   (denial of conclusion)

4. $\Box\neg Q$

   (3, double negation)

From here the path splits into two.

5. $P$ (2, biconditional)

6. $Q$ (2, biconditional)

7. $\Box\neg Q \rightarrow \neg Q$

   (axiom of necessity, $\bigcirc/\neg Q$)

8. $\neg\Box\neg Q$

   (7, material implication)

9. $\times$ (4, 8, path closure)

10. $\neg Q$

    (7, material implication)

11. $\times$ (6, 10, path closure)

12. $\neg P$ (2, biconditional)

13. $\neg Q$ (2, biconditional)

14. $\Box\neg Q \rightarrow \neg Q$

    (axiom of necessity, $\bigcirc/\neg Q$)

15. $\neg\Box\neg Q$

    (14, material implication)

16. $\times$ (4, 15, path closure)

17. $\neg Q$

    (14, material implication)

18. (finished, and open)

Now the same proof in the embedding.

1. $(S(n(l(n(\lceil P\rceil)))))$
2. $S(b(\lceil P\rceil, \lceil Q\rceil))$
3. $S(n(n(l(n(\lceil Q\rceil)))))$
   (denial of conclusion)
4. $S(l(n(\lceil Q\rceil)))$
   (3, double negation)

Again, the path now splits in two.

5. $S(\lceil P\rceil)$ (2, biconditional)
6. $S(\lceil Q\rceil)$ (2, biconditional)
7. $S(m(l(n(\lceil Q\rceil)), n(\lceil Q\rceil)))$
   (axiom of necessity, $\bigcirc/n(\lceil Q\rceil)$)
   - 8. $S(n(l(n(\lceil Q\rceil))))$
     (7, material implication)
   - 9. × (4, 8, path closure)
   - 10. $S(n(\lceil Q\rceil))$
     (7, material implication)
   - 11. × (6, 10, path closure)

12. $S(n(\lceil P\rceil))$ (2, biconditional)
13. $S(n(\lceil Q\rceil))$ (2, biconditional)
14. $S(m(l(n(\lceil Q\rceil)), n(\lceil Q\rceil)))$
    (axiom of necessity, $\bigcirc/n(\lceil Q\rceil)$)
    - 15. $S(n(l(n(\lceil Q\rceil))))$
      (14, material implication)
    - 16. × (4, 15, path closure)
    - 17. $S(n(\lceil Q\rceil))$
      (14, material implication)
    - 18. (finished, and open)

The proofs track one another exactly, of course.

## 6 Multiple Operators

It should by now be plain that the FOL representation, the embedding with quotation, can handle arbitrary nesting of operators.

Already mentioned is $\mathcal{O}(\mathcal{F}A)$ for "Parking on highways ought to be forbidden" (Marcus, 1966) (using $\mathcal{O}$ for ought and $\mathcal{F}$ for forbiddance). Our representation—using **ITr** as developed—for $\mathcal{O}(\mathcal{F}A)$ (using $\mathcal{O}\neg$ instead of $\mathcal{F}$ for forbiddance) is

$$S(o(o(n(\ulcorner A \urcorner)))) \tag{3}$$

As a second example, we consider $\mathcal{P}(\mathcal{O}B)$ for "It is permitted that filing of tax forms is obligatory." This goes to

$$S(n(o(n(o(\ulcorner B \urcorner))))) \tag{4}$$

Our third example has three operators. "The police ought to be permitted to forbid Nazi rallies," is approximately rendered in the standard way (with no mention of the police) as $\mathcal{O}(\mathcal{P}(\mathcal{F}N))$. Our direct representation would be:

$$S(o(n(o(n(o(n(\ulcorner N \urcorner))))))) \tag{5}$$

Also, mixtures of nested operators can be expressed. "It is necessary that murder is forbidden."

$$S(l(o(n(\ulcorner M \urcorner)))) \tag{6}$$

We shall see other examples below, but first an excursion into event semantics, which shall prove useful to us.

## 7 The Machinery and Motivation (for ES$\Theta$)

Consider, from (Parsons, 1990, page 101), the following statements:

1. Brutus stabbed Cæsar violently in the back.
2. Brutus stabbed Cæsar violently.
3. Brutus stabbed Cæsar in the back.
4. Brutus stabbed Cæsar.
5. Brutus stabbed violently.
6. Brutus stabbed in the back.
7. Cæsar was stabbed violently.
8. Cæsar was stabbed in the back.

9. Brutus stabbed.
10. Cæsar was stabbed.

One way to symbolize these statements—the way normally taught in logic texts—would be to use 10 different predicates, one for each of

$S_1$ "… stabbed … violently in the back"

$S_2$ "… stabbed … violently,"

…

$S_{10}$ "… was stabbed"

But this seems, at the least, quite strange. Notice especially that this symbolization entirely misses out on a great deal of logical structure. For example,[2] (1) → (2), but it is not true that $S_1(b,c) \rightarrow S_2(b,c)$. Also, (1) → (2) ∧ (3) but not (2) ∧ (3) → (1), yet the $S_i$ representation is irrelevant to this logical structure. Nor are any of very many other logical relations among (1) … (10) captured. In short, something is wrong with the standard representation if we cannot infer a stabbing from a stabbing violently in the back.

Although a number of writers have addressed this problem (cf., Bennett (1988); Davidson (1980); Moore (1995); Reichenbach (1947)), Parsons (1990) has perhaps the most sustained and thorough treatment of it, and we shall largely follow his account, so far as it goes. There are excellent treatments of, and developments of, event semantics in other, often more recent, work (cf., Larson and Segal (1995); Higginbotham et al. (2000); Ludlow (1999)). Focusing on Parsons's account, however, is convenient and for present purposes will not lead us astray.

> The thesis … is that semantics of simple sentences of English require logical forms that are somewhat more complex than is normally assumed in investigations of natural language semantics. In particular, the semantics of a simple sentence such as 'Brutus stabbed Cæsar' requires a form of at least the following complexity:
>
> For some event e, e is a stabbing, and the agent of e is Brutus, and the object of e is Cæsar, and e culminated at some time in the past.
>
> This form, which is typical, is dominated by an existential quantification over events. Since no such quantification is explicitly indicated in the sentence 'Brutus stabbed Cæsar', I call it an "underlying" quantification. A main theme of the theory I investigate is that such underlying quantification over events (and states) is ubiquitous in natural language. (Parsons, 1990, page 1)

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[2] The numbers in parentheses refer to items in the list of 10 statements above, beginning with "Brutus stabbed Cæsar violently in the back."

The theory under investigation, referred to in this passage, is called the *underlying event theory of verb phrase (VP) modification.* We shall usually abbreviate this to the *underlying event theory* or *event semantics*[3] Since thematic roles are so important, the theory is also often referred to as *event semantics with thematic roles* or ES$\Theta$ theory (Larson and Segal, 1995), an abbreviation we find congenial. We shall now sketch the elements of the theory. Once that is done, we shall put it to use in intensional contexts, contexts with modal operators. This is a category of use not explored by Parsons.

The underlying event theory is a theory of the semantics of VP modification. Parsons (1990) and Larson and Segal (1995) (who develop a general semantics, called ES$\Theta$ theory, for event semantics with thematic roles) contain the most developed treatments of the theory. Important contributions include Davidson (1984); Kowalski and Sergot (1986); Pustejovsky (1995); Reichenbach (1947); Schein (1993); Voorst (1988). The theory aims to provide an account of the semantics of VPs, especially VP modifiers (e.g., adverbs, prepositional phrases), in extensional (transparent, or non-opaque) contexts.

The underlying event theory (event semantics) posits three kinds of underlying entities: events, states, and processes. Together, these are called *eventualities,* so that the theory is perhaps better named the *underlying eventuality theory.* Criteria for identifying and distinguishing eventualities are not part of the theory. What these criteria are is of course an important question, but for present purposes, it suffices to give rough indicators in the form of examples. Events are things that happen. There are two kinds. ‘Clinton won the election’ reports an *achievement event.* Notice that while it makes sense to ask when it happened, it does not make sense to ask how long it took. Achievement events are instantaneous. *Accomplishment events,* the other kind, may or may not consume an extent of time, but it always makes sense to ask of them how long they took. ‘Clinton made a speech’ reports an accomplishment event. Events typically have a more or less definite point of finishing, called their *culmination* in the theory. States on the other hand are “beings” rather than “happenings.” ‘Clinton is President’ reports a state, or state of affairs. It doesn’t make sense to ask how long a state took, although we can ask how long it lasted. As in systems analysis, we might think of events as transitions between states. Roughly, state = description of a system (at a given time), and event = change of state. A process, or activity, is a series of events.[4]

The basic claim for the underlying event theory is that, for a certain range of linguistic phenomena (VP modifiers in extensional contexts), the theory provides representations that get the logic right, or at least more right than competing theories. This is also the claim

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[3] Because they would not substantially affect the points we want to make here, we are leaving out very many details. For the record, Parsons advances, with much discussion, the underlying event theory with thematic roles and with the underlying conjunct hypothesis in place. There is much to be said about this, but we largely agree with Parsons and for the moment are simply availing ourselves of his results.

[4] For the sake of keeping things as simple as possible, we will not distinguish among eventualities in what follows. At different times, the $e$’s are best understood as events, or states, or processes. I’m ignoring these distinctions as being peripheral to the main points of the paper.

being advanced—sketched—for sentences with embedded propositional content: these representational ideas when applied properly will get the logic (the inferential relations) right. A word now by way of example for the underlying event theory.

In the underlying event theory, 'Brutus stabbed Cæsar' goes into first-order logic as:

$$\begin{aligned} &\exists I \exists e \exists t(before(t, now) \wedge t \in I \wedge stab(e) \wedge \\ &Subject(e, Brutus) \wedge Obj(e, Caesar) \wedge Cul(e, t)) \end{aligned} \tag{7}$$

(Our representation assumes a typed variable regime. $I$ is a temporal (or spatio-temporal) interval, $e$ is an eventuality, and $t$ is a time.) Similarly, 'Brutus stabbed Cæsar in the back with a knife violently' goes into first-order logic as:

$$\begin{aligned} &\exists I \exists e \exists t(before(t, now) \wedge t \in I \wedge stab(e) \wedge \\ &Subject(e, Brutus) \wedge Obj(e, Caesar) \wedge \\ &in(e, \textit{the-back}) \wedge with(e, knife) \wedge \\ &violent(e) \wedge Cul(e, t)) \end{aligned} \tag{8}$$

(The analysis is not complete, since *the-back* remains not fully articulated. Doing that is more or less a straightforward matter, but it is one that digresses from the issues at hand.)

Notice that expression (8) logically implies expression (7). Further, notice that this approach to representation works for every sentence in the list about the stabbing of Cæsar that began this section. (We leave it as an exercise for the reader to work out the details. It's quite a simple problem.)

The essential strategy is to break down the VPs into components of meaning that are assembled with logical conjunction. The basic intuition here is, e.g., that to do something violently is to do that thing and to do it violently. This is why, in the representation, doing something violently entails doing that thing: VP modifiers attach as conjuncts. To make this work, there must be some common, quantified variable (or shared name) that links the several predicates in a representation. Here, that variable ($e$) names the event (the stabbing) which has the properties indicated by the predicates in the representation. For present purposes, the underlying event theory may be seen as a carefully considered articulation of this idea.

The underlying event theory—particularly as developed by Parsons (1990), who is largely successful in keeping representations within the confines of first-order logic—offers great promise as a (partial) theory for natural language representation. We have explored in several places how the theory can be put to good use in computer-to-computer communication in electronic commerce (cf. Kimbrough (1997, 99); Kimbrough and Tan (2000)).

Although we shall make extensive use of the theory in what follows, its finer details would only be distracting in the present context. Two further points nevertheless need to be made. First, as developed by Parsons and in most of this literature, ESΘ theory does not encompass normative or other modalities. (Larson and Segal (1995) is an exception, but they do not offer a representation in FOL.) Second, we cannot say that our logical embedding of the various modalities yields *directly* the sorts of elegant logical inferences on display in this section and developed so well by Parsons. Instead, we believe that the embedding affords expression of information, in the form of axioms and meaning postulates, that supports inference as required.

# 8 Extending the Embedding with Event Semantics

## 8.1 Initial Examples

Consider "The police forbid that $P$." We have this as simply $S(o(n(\ulcorner P \urcorner)))$, leaving out the police, as does $\mathcal{O}\neg P$. By abstracting our embedding, we arrive at a natural representation that affords an ES$\Theta$ perspective. Consider

$$\exists x(S(o(x, n(\ulcorner P \urcorner))) \wedge \mathit{Agent}(x, \mathit{the\text{-}police})) \tag{9}$$

Here, $x$ is a variable ranging over eventualities (events, states, processes). We are employing exactly the apparatus of ES$\Theta$ theory.

Now our previous example again, "The police ought to be permitted to forbid Nazi rallies," which, recall, is approximately rendered in the standard way (with no mention of the police) as $\mathcal{O}(\mathcal{P}(\mathcal{F}N))$. Our direct representation was:

$$S(o(n(o(n(o(n(\ulcorner N \urcorner))))))) \tag{10}$$

Using the abstraction move shown in expression (9), we get

$$\exists x \exists y \exists z(S(o(x, n(o(y, n(o(z, n(\ulcorner N \urcorner))))))) \wedge \mathit{Agent}(z, \mathit{the\text{-}police})) \tag{11}$$

for (the stylistic variant) "It ought to be the case that it is permitted that the police forbid Nazi rallies." For the somewhat different "It ought to be the case that the police permit that Nazi rallies are forbidden" we have

$$\exists x \exists y \exists z(S(o(x, n(o(y, n(o(z, n(\ulcorner N \urcorner))))))) \wedge \mathit{Agent}(y, \mathit{the\text{-}police})) \tag{12}$$

Inevitably we have

$$\exists x \exists y \exists z(S(o(x, n(o(y, n(o(z, n(\ulcorner N \urcorner))))))) \wedge \mathit{Agent}(x, \mathit{the\text{-}police})) \tag{13}$$

which represents "The police make it obligatory (oblige) that it is permitted that Nazi rallies are forbidden."

## 8.2 Conflicting Obligations in SDL

Standard Deontic Logic views conflicting obligations as contradictory. Recalling §4, this is implemented in the Hilpinen's system by the first inference rule:

1. SDL 1

$$\mathcal{O}\bigcirc \rightarrow \neg\mathcal{O}\neg\bigcirc$$

From

1. $\mathcal{O}\bigcirc$
2. $\mathcal{O}\neg\bigcirc$

we can add SDL 1 to get

3. $\mathcal{O}\bigcirc \rightarrow \neg\mathcal{O}\neg\bigcirc$

We then apply the material conditional rule to split the path,

| | |
|---|---|
| 4. $\neg\mathcal{O}\bigcirc$ | 6. $\neg\mathcal{O}\neg\bigcirc$ |
| 5. × (1, 4, path closure) | 7. × (2, 6, path closure) |

producing explicit contradictions. How does this work if we expand our notation as in the previous subsection? It would seem we want $\mathbf{ITr}(\mathbf{Tr}(\mathcal{O}\bigcirc)) = \exists x(S(o(x, \mathbf{Tr}(\bigcirc))))$ for premises of the argument. (Roughly, the translation "It is obliged that $\bigcirc$" goes to "There is a state named by $\mathbf{Tr}(\bigcirc)$ and it is obligatory.") Given this, SDL 1 is naturally represented in FOL as:

1. **ITr**(**Tr-a** SDL 1), abstracted

$$\forall x(S(m(o(x, \bigcirc)), n(o(x, n(\bigcirc)))))$$

2. **ITr**(**Tr-a** SDL 2), abstracted

$$\forall x(S(m(o(x, m(\bigcirc, \triangle)), m(o(x, \bigcirc), o(x, \triangle)))))$$

Now we'll repeat the proof in the new, abstracted representation.

1. $\exists x(S(o(x, \bigcirc)))$ (Premise)
2. $\exists x(S(o(x, n(\bigcirc))))$ (Premise)
3. $\forall x(S(m(o(x, \bigcirc)), n(o(x, n(\bigcirc)))))$ (SDL 1)

But these premises are not inconsistent! Applying existential instantiation to premises 1 and 2 yields

4. $S(o(n_1, \bigcirc))$
5. $S(o(n_2, n(\bigcirc)))$

where $n_1$ and $n_2$ are new names. There is no inconsistency unless $n_1 = n_2$, but to have that we have to have more information.

What has gone wrong? Nothing, or so we submit. The example tells us two things. First, the proposed existentially quantified representation for $\mathbf{ITr}(\mathbf{Tr}(\mathcal{O}\bigcirc))$ (and for SDL 1 and SDL 2 and by extension, the other axioms) will *not* mimic the original logic. One way to make it do so would be to add an axiom:[5]

- Universality of Obligation

$$\forall x \forall y((S(o(x, \bigcirc)) \land S(o(y, \bigcirc))) \to x = y)$$

This, however, defeats the purpose of having a unique way of referring to, and distinguishing, obligations (and necessities, etc.). A better alternative would be to add

- Uniformity of Obligation.

$$\forall x \forall y(S(o(x, \bigcirc)) \to \neg S(o(y, n(\bigcirc))))$$

Adding Uniformity of Obligation to the path allows us to find an inconsistency.

The second thing the example tells us is that it is possible to support conflicting obligations without throwing away the baby (with the bath water) as well and without acceding to Universality or Uniformity of Obligation. Let us suppose a richer information set. Obligations have types and we forbid conflicting obligations of the same type. In short, we have the Typed Obligation Consistency axiom.

- Typed Obligation Consistency

  $\forall x \forall y \forall u \forall y((S(o(x, \bigcirc)) \land S(o(y, n(\bigcirc))) \land \mathit{Type}(x, u) \land \mathit{Type}(y, v)) \to u \neq v)$

To illustrate consider these two premises in which the stated obligations are of the same type.

1. $\exists x(S(o(x, \bigcirc)) \land \mathit{Type}(x, t_1))$ (Premise)
2. $\exists x(S(o(x, n(\bigcirc))) \land \mathit{Type}(x, t_1))$ (Premise)

Assuming Typed Obligation Consistency, deriving a contradiction is easy. (As is standard in FOL, we count $\bigcirc \neq \bigcirc$ as contradictory and path-closing, where $\bigcirc$ is a well-formed term.) With this apparatus in place, we can model situations in which obligations conflict, without having to fall into inconsistency. This is, of course, not to say that this could not be done in an appropriate modal logic, especially if it contained a relativised action operator.

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[5] For the remainder of this section, §8.2, we revert without prejudice to the more standard FOL notation for logical connectives.

### 8.3 Nesting Abstracted Operators

Recall that we were motivated on ES$\Theta$ grounds to abstract our FOL representations of sentence operators to include an index argument in the embedding FOL function for sentence logic operators. For example

$$\mathbf{ITr}(\mathbf{Tr}(\mathcal{O}P) \rightsquigarrow S(o(\lceil P \rceil))) \tag{14}$$

becomes

$$\mathbf{ITr}(\mathbf{Tr\text{-}a}(\mathcal{O}P) \rightsquigarrow \exists x(S(o(x, \lceil P \rceil)))) \tag{15}$$

under the proposed abstraction. We then pointed out that thematic roles could be attached and exploited to encode various meanings. For example, "The police forbid Nazi rallies" might adequately be expressed as

$$\exists x(S(o(x, n(\lceil N \rceil))) \wedge \mathit{Agent}(x, \mathit{the\text{-}police})) \tag{16}$$

This issue arises, with perhaps distinguishable motivations, in embedding modal operators that are relativized to, for example, times, persons, or institutions. Viewing these as thematic roles in the embedding representation does the general approach we have presented continue to work and if so, how?

We will work with a simple example. A relativized version of a classical, but not normal, modal system (in the sense of (Chellas, 1980)) is used by Jones and Sergot (1996) in their discussion of the action component of institutionalised power. The intended interpretation is that the relativized operator $\mathcal{E}_x$ means "$x$ sees to it that ...." System for this action logic add to modal system **E** an axiom schema and one rule of inference. The latter does not concern us at present. The axiom schema is the **T** schema with relativization:

$$\mathcal{E}_\circ \triangle \rightarrow \triangle \tag{17}$$

We use $\circ$ as a metavariable for an individual name (the logic is not quantified; expressions with freely-occurring variables are not well defined) and $\triangle$ etc. for particular wffs.

In specifying our translation function we now need to be a bit more fussy.

- If $\bigcirc$ is a complete sentence to be translated, then $\mathbf{ITr}(\bigcirc) = S(\mathbf{Tr\text{-}a}(\bigcirc))$.
- If $\bigcirc = \Phi[\mathbf{Tr\text{-}a}(\mathcal{E}_\circ \triangle)]$, then $\mathbf{Tr\text{-}a}(\bigcirc) =$
  $\exists x(\Phi[e(x, \mathbf{Tr\text{-}a}(\triangle))] \wedge \mathit{Agent}(x, \circ))$, where $x$ is suitably chosen to be unique.

Points arising:

1. The function $e$ corresponds to the modal operator $\mathcal{E}$ and might be read "the bringing about of ...."
2. The expression $\Phi[\triangle]$ signifies a formula produced by the translation process in which the expression $\triangle$ occurs (once).

3. The condition that "$x$ is suitably chosen to be unique" requires that $x$ be a new individual variable, not occurring in $\bigcirc$. ($x$ also has to be different than $\circ$. Since $x$ is a variable and $\circ$ is a name, we assume the formation rules obviate the problem.)

4. To illustrate how this works, consider: $\bigcirc = \mathcal{E}_{n_1}\mathcal{E}_{n_2}\mathcal{E}_{n_3}P$. ("Guy 1 sees to it that guy 2 sees to it that guy 3 sees to it that $P$.") Here is the embedding translation, step by step.

   (a) $\mathbf{ITr}(\mathcal{E}_{n_1}\mathcal{E}_{n_2}\mathcal{E}_{n_3}P) = S(\mathbf{Tr\text{-}a}(\mathcal{E}_{n_1}\mathcal{E}_{n_2}\mathcal{E}_{n_3}P))$

   (b) $S(\mathbf{Tr\text{-}a}(\mathcal{E}_{n_1}\mathcal{E}_{n_2}\mathcal{E}_{n_3}P)) = \exists x(S(e(x, \mathbf{Tr\text{-}a}(\mathcal{E}_{n_2}\mathcal{E}_{n_3}P))) \wedge \mathit{Agent}(x, n_1))$

   (c) $\exists x(S(e(x, \mathbf{Tr\text{-}a}(\mathcal{E}_{n_2}\mathcal{E}_{n_3}P))) \wedge \mathit{Agent}(x, n_1)) =$
   $\exists y(\exists x(S(e(x, e(y, \mathbf{Tr\text{-}a}(\mathcal{E}_{n_3}P)))) \wedge \mathit{Agent}(x, n_1)) \wedge \mathit{Agent}(y, n_2))$

   (d) $\exists y(\exists x(S(e(x, e(y, \mathbf{Tr\text{-}a}(\mathcal{E}_{n_3}P)))) \wedge \mathit{Agent}(x, n_1)) \wedge \mathit{Agent}(y, n_2)) =$
   $\exists z(\exists y(\exists x(S(e(x, e(y, e(z, \mathbf{Tr\text{-}a}(P))))) \wedge \mathit{Agent}(x, n_1)) \wedge \mathit{Agent}(y, n_2)) \wedge \mathit{Agent}(z, n_3))$

   (e) $\exists z(\exists y(\exists x(S(e(x, e(y, e(z, \mathbf{Tr\text{-}a}(P))))) \wedge \mathit{Agent}(x, n_1)) \wedge \mathit{Agent}(y, n_2)) \wedge \mathit{Agent}(z, n_3))$
   $=$
   $\exists z(\exists y(\exists x(S(e(x, e(y, e(z, \lceil P \rceil)))) \wedge \mathit{Agent}(x, n_1)) \wedge \mathit{Agent}(y, n_2)) \wedge \mathit{Agent}(z, n_3))$

5. The general pattern and approach should be amply clear. Other thematic roles are easily added, say for time, benefactive, patient, and so on. Other sentence operators may easily be mixed in, say for obligation.

## 9 Discussion and Future Directions

The scheme we have described answers to the worry about representing nested sequences of normative modalities (or indeed any sequence of sentence operators) in FOL. It can be done, using quotation and embedding of sentence operators as functions in the arguments of FOL predicates (our $S$).

Nothing we have presented here constitutes new logic. We have merely contributed a way of representing logics with normative (and other) modalities in FOL. There are, we believe, a number of practical advantages for doing this. With the embedding, FOL theorem provers can be used for modal logics. The abstracting of the approach affords graceful hookup with ES$\Theta$ theory. There is every reason to think that this can only be helpful for the programme of developing logical representations for contracts, sales transactions, electronic data interchange messages, and so on. Future developments will, however, be required to validate the claim of usefulness.

There is also theoretical work to be done. We note three areas in particular. First, operators for the counts-as conditional and (thereby) institutionalised power, for speech acts, and indeed for a broad array of communicative documents (e.g., purchase orders, invoices, contracts) need to be investigated in detail, both with regard to their representation and their logic. Second, computational complexity issues need to be investigated, especially for representations that are candidates for actual application. The hope, of course, is that much can be done with Horn clauses, or some other suitable, fast computing fragment of FOL. We are optimistic on this score. Third, the extension of this approach to quantified modal logics remains to be investigated.

The observations and analyses made in this paper indicate that there are (at least) two ways of representing normative sentences in formal logic: as expressions in a suitable modal-logical language, and as expressions in the language of first-order logic (FOL).[6] And our particular aim has been to show that FOL representations are available even when nesting of normative modalities occurs in the natural-language and modal-logical formulations of rules, regulations and policies.

But we would also like to suggest that these two modes of formal-logical representation serve different purposes. If the concern is primarily to clarify and disambiguate the normative structure and content of, e.g., natural-language policy-statements, then modal logic provides a far more perspicuous and easily handled analytical tool than FOL, not least because the logical structures afforded by modal logic are usually much closer to the original natural-language structures than those exhibited by FOL representations. In our view, the principal advantage of applying modal logic to the representation of normative sentences lies in its ability to provide a clear characterization of formal conceptual structure in an easily accessible, highly intuitive manner. (Since the constituent modalities and their inter-relations are given a well-defined semantics, relations of consistency and implication in formalized sets of norms may then also be systematically investigated.)

But if, on the other hand, the concern is essentially implementation-facing, and the principal aim is to construct a computationally tractable representation of policy-statements, then there are obvious advantages in using FOL, as a basis for defining a computational framework. This distinction between two levels of formal representation, the one focused on *conceptual characterization,* the other on *computational tractability,* was alluded to in the closing sections of Jones and Kimbrough (2005).

The topic was taken further in Jones et al. (2011b), and has now been developed in some detail as part of a proposal for a methodology for engineering socio-technical systems, in Jones et al. (2011a).

## 10 Afterword

This paper originates from a discussion that took place in about 1990 between Robert Kowalski, Marek Sergot and Andrew Jones, concerning the question of whether such normative concepts as obligation, permission and prohibition could appropriately be represented

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[6] We observe, or remind the reader, that our FOL representations are semantically conventional. Quoted atomic sentences from the embedded language are simply notational devices for individual names. The standard model theoretic semantics apply unproblematically.

as FOL predicates. At the time, Jones suggested that nested sequences of normative terms might constitute a problem for an FOL approach, and gave ‘The police ought to be permitted to forbid Nazi rallies’ as an example. We felt that the festschrift for Marek would provide a suitable opportunity to return to this issue, by offering what we believe to be a novel way of dealing with it. May the discussion continue!

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