Alternatives to Ideal Rationality

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Outline

1. Introduction
2. Classical Setup
3. Epistemic Challenge
4. Models
   - Bertrand
   - Cournot
5. PROBE AND ADJUST
   - Cournot Competition
   - Bertrand Competition
6. Discussion
7. End Matter
Classical Game Theory & Neoclassical Economics

- Both make very strong assumptions about the rationality of the agents they model.
- Agents are assumed to maximize their expected utilities.
- Note: “maximize” here \(\neq\) “make a strong effort to achieve a maximum.” Rather, it means *actually* achieving a maximum.
- We may give these assumptions a name: Rational Choice Theory (RCT).
- From their inception these assumptions have been subject to challenge and criticism. Yet they endure.
Goals of the Paper/Talk

1. To rebut a commonly-given defense of rationality point.
2. In doing so,
   1. To engage in clarifying reconstruction of ideal rationality, and then
   2. To offer a positive view on “post-classical game theory,” [Kimbrough, 2012]
   3. To present examples embodying this view.
   4. To say something towards a theory of non-ideal rationality.
Commonly Heard Point

... made in defense of Rational Choice Theory.

... orthodox economists had a pretty good comeback to these kinds of objections, namely “Well, what other way is there?” [McKibben, 2007, page 32]
The Defense of Rationality point, often made in discussion, is roughly:

Yes of course rationality—ideal rationality—as defined in game theory and economics is utterly unrealistic. Humans, let alone other animals, simply do not have the epistemic and computational powers to operate in accordance with the theory. The thing is, ideal rationality allows us to make unique predictions. There is only one way to be (ideally) rational in any given situation. Ideal rationality is

1. Clearly defined,
2. Mathematically grounded,
3. Principled, and
4. Leads to unique predictions.

There are infinitely many ways to behave nonrationally, and so any alternative to ideal rationality has to be ad hoc and so unprincipled. And with many alternatives, these alternatives cannot give us a unique prediction.
Concluding with

- The claim that individual deviations from rationality are unimportant in the general scheme of things because (a) they appear as noise and will be averaged out in real situations, and (b) failures of rationality will quickly be punished by arbitragers, yielding systemic rationality after all.

- And then, with or without the above point, the conclusion is drawn (or insinuated) that modeling with the assumption of (ideal) rationality should continue and be the preferred mode for Serious People doing modeling in the social sciences, and that there is no genuine credible alternative to Rational Choice Theory.
This is a terrible argument.
(Looking for keys beneath the lamppost.)

I am not primarily concerned to rebut it.
(Although I will make some passing comments in that direction.)

I am mainly concerned with addressing the important question touched upon by the Defense of Rationality point:

*If not Rational Choice Theory in our models, then what?*
Note: All is not well with the classical theory

Including, prediction failure.

Ironically, [classical] game theory is often hoisted on its own pétard: many of its most fundamental predictions—predictions that would have been too vague to test with any confidence in the pre-game-theoretic era—are decisively and repeatedly disconfirmed, in laboratory settings, with substantial agreement among experimenters, regardless of their theoretical priors. [Gintis, 2000, page xxiv]
Some have even gone so far as to say this

It is better to drop the term “rational” altogether, . . . . Dispensing with the rationality postulate does not imply that people are irrational (whatever that means). The point is that the concept of “rationality” does not help us understand the world. [Gintis, 2000, pages xxv–xxvi]
Classical rationality setup. We need three items:

1. $\Omega^*$. A set of outcomes, possible states of the world, consumption bundles to be had, profits, . . . . (There are subtleties here, but I will pass on them. I use * to indicate true or actual.)

2. $\succeq$. A preference relation with the right properties on $\Omega^*$ (complete, transitive, conforming to the substitution axiom and the Archimedean axiom). (There are various formulations here. See, e.g., [Kreps, 1990, Sen, 1993]. These are the axioms of (ideal) rationality.)

3. $\mathcal{A}^*$. A set of possible actions, where for each $a \in \mathcal{A}^*$, there is a probability distribution (lottery) $x$ on $\Omega^*$ such that $a \rightsquigarrow x$ (choosing $a$ leads to $x$ being realized). (Note: Actually $\mathcal{A}^{+*}$.)
Given this setup . . .

- It has been proved that there exists a utility function, $u(\cdot)$, such that for all lotteries $x$ and $y$ on $\Omega^*$ ($\forall x, y \in \Omega^+$),

$$E(u(x)) \geq E(u(y)) \text{ if and only if } x \succeq y \quad (1)$$

and $u(\cdot)$ is unique up to a positive linear transformation. ($E(u(x))$ is the expected value of $u(x)$.) (von Neumann-Morgenstern utility function)

- Rationality assumption: (1) the setup conditions obtain, and (2) the agent actually chooses an $a \in A^+$ that maximizes its expected utility.
The equilibrium prediction in games and economic situations is immediate. If the strategies chosen by the players in a game are not in equilibrium, then at least one player has violated the rationality assumption and hence is “irrational.”

When $x$, what is led to by the action of player $i$, $a_i$, depends on actions taken by other players, $a_{-i}$, i.e., if $a_i(a_{-i}) \rightsquigarrow x$, classical game theory offers no account of how what happens happens. Game theory simply asserts that somehow an equilibrium will be reached. This is a well-recognized problem and goes by the names of “too many equilibria” and “the equilibrium selection problem.”
Consider the Stag Hunt game as an example

- Just in pure strategies, the one-shot game has two equilibria.

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<tr>
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<th>C</th>
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<tr>
<td>C</td>
<td>3</td>
<td>2</td>
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<tr>
<td>D</td>
<td>0</td>
<td>1</td>
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There is a third, so-called mixed equilibrium, in which agents play a probabilistic mixture of C and D. Setting R=3, T=2, S=0, and P = 1, that equilibrium is at

\[
P(C) = \frac{(P-S)}{((R+P)-(T+S))} = \frac{1}{4-2} = \frac{1}{2}.
\]

(Note: if R increases the agent plays C with lower probability.)

How are the players to achieve coordination?
So this is a problem. Accounting for coordination and
telling a plausible procedural story in the face of multiple
equilibria. (Recall the defense: there’s only one way to be rational!)

Item $\succeq$, the rationality axioms, has often been attacked.

For example, as Sen notes [Sen, 1993], if $\Omega^* = \{\omega_1, \omega_2\}$
and the agent picks $a_1 \succsim \omega_1$ and if $\Omega^* = \{\omega_1, \omega_2, \omega_3\}$
and the agent picks $a_2 \succsim \omega_2$, then the agent is irrational
because it has violated the rationality axioms.

So, (1) the rationality axioms fail on uniqueness and (2)
they fail to recognize intuitively unproblematic preferences,
e.g., preference for the second largest piece of remaining
cake. (Broadly: *positional* preferences.)
Note in passing

Many economists and political theorists buy in on the rationality axioms and assert further that to be rational is to be egoistic and entirely self-regarding.

This might be true, but if it is, it is not true in virtue of the rationality axioms (and $\succ$), which permit any complying preference ordering.

One may be entirely rational according to the theory in preferring others’ welfare always before one’s own. Think: Hume’s finger.
From an advocate of the classical view

Indeed, the Bayesian framework is too roomy in that it admits belief functions that would be entertained only by a fool, and value assignments that would be entertained only by a monster. But I take it that the formulation and critique of particular probability and value assignments must largely be conducted in situ, with the aid of facts about the agent, his language, his community and his special situation; and that although such activity should use the Bayesian framework, it belongs to other disciplines—say, to inductive logic, and to ethics. [Jeffrey, 1983, page 211]
Without knowledge, rationality loses its bite

- Rationality, however, is insufficient as an account of behavior, even of idealized behavior.
- Agents presented with \( \langle \Omega^*, \succeq, \mathcal{A}^+ \rangle \) must also somehow have knowledge of the situation [Aumann and Brandenburger, 1995].
- This is normally handled by assuming that the agents have “full knowledge” of the circumstances, that is that they know what is in \( \Omega^* \), what is in \( \mathcal{A}^+ \), and they know—up to the limits of probability distributions and the structure of the game at hand—how choices from \( \mathcal{A}^+ \) lead to probabilistic mixtures of outcomes in \( \Omega^* \).
Let us drop the assumption of full knowledge.

Instead, an agent is presented with and knows

\[ \langle \Omega, \succeq, A \rangle \text{ where } A \subseteq A^+\ast \text{ and } \Omega \subseteq \Omega^* \] (where \( A \) and \( \Omega \) are subsets that result from the limited knowledge of the agent)

This takes us to an epistemic framework, Figure 1.
Figure 1: An epistemic framework

\[
\begin{array}{c|c}
\mathcal{A}^{++} & \Omega^{++} \\
\hline
1. A \times \Omega & 2. A \times \Omega^c \\
3. A^c \times \Omega & 4. A^c \times \Omega^c \\
\end{array}
\]

Figure: Epistemic framework (subset version).

\[\mathcal{A}^{++} \times \Omega^{++} = 1 \cup 2 \cup 3 \cup 4. 1 = A \subseteq \mathcal{A}^{++} \times \Omega \subseteq \Omega^{++}.\]
\[2 = A \subseteq \mathcal{A}^{++} \times \Omega^c \subseteq \Omega^*. \ 3 = A^c \subseteq \mathcal{A}^{++} \times \Omega \subseteq \Omega^{++}.\]
\[4 = A^c \subseteq \mathcal{A}^{++} \times \Omega^c \subseteq \Omega^*.\]
Four cases

1. Agent chooses (decides on) \( a \in A, a \mapsto x \in \Omega \), and \( u(x) \geq u(y) \) for all \( y \in \Omega^+ = (\Omega \cup \Omega^c) \). The agent is lucky and has actually maximized its expected utility.

2. Agent chooses (decides on) \( a \in A, a \mapsto x \in \Omega \), but \( \exists y \in (A \times \Omega)^c \) (i.e., 2, 4) such that \( u(x) < u(y) \). The agent is unlucky and has failed to actually maximize its utility, even though \( u(x) \geq u(z) \ \forall z \in (A \times \Omega) \).
Four cases

3 Agent is frustrated because it is aware of the $x \in \Omega$ that (actually) maximizes its utility but is unaware of a choice, $a \in A^c$ that would lead to $x$. The agent fails to maximize utility because it is unaware of an action available to do so. ([Cherniak, 1986] considers such cases, although from a different framework than ours, e.g., the agent likes state of affairs $P$ best, but fails to choose $Q$ because of ignorance of the fact that $Q \rightarrow P$.)

4 The agent is clueless. Given the setup, its most preferred $x$ is unknown to the agent because $x \in \Omega^c$. Further, while there exists an $a \in A^{++}$ such that $a \sim x$, the agent is also unaware of this. The agent cannot even make a lucky guess.
What should we say about the rationality of an agent in these cases?

Remember: throughout our discussion the agent is ideally rational in the sense that it maximizes its expected utility on the assumption of the choice situation to which it has epistemic access.

1. **Strong ideal rationality.** The agent has full knowledge of \( \langle \Omega^*, \succeq, A^{++*} \rangle \) and acts, maximizing expected utility with respect to the presumably true frame, \( \langle \Omega^*, \succeq, A^{++*} \rangle \).

2. **Weak ideal rationality.** The agent has only partial knowledge, but does have full knowledge of \( \langle \Omega, \succeq, A \rangle \) and acts, maximizing expected utility according to the \( \langle \Omega, \succeq, A \rangle \) frame.
The upshot

The expression *full knowledge* is equivocal.

- Is it full knowledge of $\langle \Omega^*, \succeq, A^{++} \rangle$ or of $\langle \Omega, \succeq, A \rangle$ where $\langle \Omega, \succeq, A \rangle \neq \langle \Omega^*, \succeq, A^{++} \rangle$? What is at play, strong or weak ideal rationality?

- Claim: Many (most? all?) models in the literature presume full knowledge but at best deliver results for weak ideal rationality. This has led to lots of puzzled musings in the literature, which I will skip for present purposes. But note: “Rational Fools” [Sen, 1977].

- Now, on to examples and alternatives.
Each period all firms offer a price and the market takes all demand from the low-price firm.

Economics theory: collusion is impossible. Even with just two firms in the market they will compete away their profits.

If firm 1 really believes that firm 2 will charge a price \( \hat{p} \) that is greater than the marginal cost, it will always pay firm 1 to cut its price to \( \hat{p} - \varepsilon \). But firm 2 can reason the same way! Thus any price higher than marginal cost cannot be an equilibrium; the only equilibrium is the competitive equilibrium. [Varian, 2003, page 488]

Note the business literature on this: Don’t do it!
Cournot Competition

- Classic model of oligopoly.
- Quantity competition: Firms offer quantities of a good and the market sets the price.
- The classic theory is undermotivated mathematically. Assumes “best response” behavior. This leads to the Cournot equilibrium, which lies between the monopoly quantity and the competitive quantity.
Roughly:

- A market for a particular product supplied by \( n \) firms.
- During each time step each of the supplying firms offers quantity \( Q_i \) \((i = 1, 2, \ldots, n)\) to the market, so that the total supply in a given period is

\[
Q = \sum_{i=1}^{n} Q_i \quad (2)
\]

- The unit price resulting is determined by the demand function—

\[
P = \max\{a - \text{slope} \times Q, 0\} \quad (3)
\]

- Each firm \( i \) receives revenues of \( P \times Q_i\).
Cournot reference model (con’t.)

- Firms may independently and without communication with each other adjust the quantities they offer to the market, their \( Q_i \)s.
- In setting their \( Q_i \)s each firm takes into account its unit cost of production, \( k_i \), and the behavior of the other firms.
- Each firm follows the *best response* strategy.
- If all of the firms do this they will reach the Cournot equilibrium in which the individual firm Cournot quantities are

\[
Q^C_i(n, k_i) = \frac{(a - k_i)}{(n + 1) \cdot \text{slope}}
\]  

(4)
PROBE AND ADJUST

- A kind of reinforcement learning for a continuous quantity.
- Episode (= round of play). Epoch (= a number of episodes).
- Probe uniform $\pm \delta$ in each episode. Adjust $\pm \varepsilon$ at the end of each epoch.

Diagram:
- Anchor value
- $+\delta$
- $-\delta$
- $-\varepsilon$
- $-\varepsilon$
Is there another way of modeling this?

- The Cournot conclusion follows mathematically—provided you make the *best response* assumption.

- But why should you? Behaviorally implausible.

- What if the agents follow PROBE AND ADJUST in learning to set their quantities?

- See [Kimbrough and Murphy, 2009], “Learning to Collude Tacitly on Production Levels by Oligopolistic Agents” or Chapter 10 of [Kimbrough, 2012].
Agents collectively reach the *Cournot quantity*. That is, they individually and collectively put to the market the total quantity that is predicted by the Cournot model. Without the implausible Cournot assumptions! And under a plausible behavioral procedure. Also observed: number effects consistent with behavioral experiments.
Part of PROBE AND ADJUST is the *update policy* used by the agent.

What should the agent track for deciding on adjustments at the end of epochs?

The above results were obtained for agents using the Own Returns policy.

What happens if instead the agents track the total returns to the industry, Market Returns?

They collectively arrive at the monopoly quantity of the low-cost producer!

But

This is exploitable.
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Is there another way?

- **MR-COR**
  Market Returns, Constrained by Own Returns

- When all agents use it, they collectively arrive at the monopoly quantity of the low-cost producer.

- If an agent defects (to Own Returns), they return to the Cournot equilibrium.

- In the defecting scenario, the MR-COR agents do just slightly worse than the Own Returns agent(s).

- But all agents are better off if they all choose MR-COR. (Stag Hunt-like)
Contrary to the Cournot model, quantity putting in an ongoing market is an indefinitely iterated game, subject to the Folk Theorem.

**PROBE AND ADJUST** with MR-COR has found a natural, credible ‘equilibrium’ for the game.

Bertrand Competition

**Competition on price**

- Each period all firms offer a price and the market takes all demand from the low-price firm.

- Economics theory: collusion is impossible. Even with just two firms in the market they will compete away their profits.

  *If firm 1 really believes that firm 2 will charge a price \( \hat{p} \) that is greater than the marginal cost, it will always pay firm 1 to cut its price to \( \hat{p} - \varepsilon \). But firm 2 can reason the same way! Thus any price higher than marginal cost cannot be an equilibrium; the only equilibrium is the competitive equilibrium. [Varian, 2003, page 488]*

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With Own Returns, agents indeed compete away their profits. A race to the bottom.

With MR-COR, tacit collusion to reach the monopoly price, is possible.

It depends on
- The number of firms in the market
- The epoch lengths of the firms
- How tolerant the firms are to sub-par returns

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Summing up

Defense of rationality point: One way to be rational, indefinitely many ways to be not rational; no good alternative to ideal rationality for modeling and theory.

After noting problems with the defense of rationality point (e.g., failure of uniqueness, actually; problems with the axioms), the main argument:

1. Ideal rationality is useless and basically irrelevant unless the agents have sufficient (full?) knowledge to exploit it.

2. Considering carefully the basic rationality setup, we distinguish two subjects of necessary knowledge, even granting the basic rationality set and the rationality axioms: $\Omega^*$ and $A^{++}$. 

The main argument, con’t.

3. Thus “full knowledge” is equivocal. Of $\Omega^*$? Of $\mathcal{A}^{++}$? Of both?

4. Existing models (Bertrand, Cournot, and I would claim many others) assume full knowledge of $\Omega^*$ but systematically exclude elements of $\mathcal{A}^{++}$. See in particular the standard view and the role of MR-COR in Bertrand competition. (NB: Cournot and reaction consistency.)

5. Claim: Realistic limitations on agent knowledge is a serious problem for classical rationality, worse than failure of the axioms due to positional preferences. This holds even granting the rationality axioms. (So the defense is beside the point.)
What then are the alternatives?

The possibilities are rich and are being explored (but more to do):

- Minimal rationality models. PROBE AND ADJUST, reinforcement learning models (welcome back behavioral psychology!), etc. These can be used to demonstrate possibilities, e.g., of tacit collusion (above).
- Note: By now large literature on artificial markets with agent-based modeling. Epstein and Axtell, Growing Artificial Societies.
- Machine learning of effective strategies in complex situations.
- Behaviorally realistic cognitive models.

Plus, two themes: robustness (multiple models welcome), and simulation (often necessary).


*Agents, Games, and Evolution: Strategies at Work and Play.*
CRC Press, Boca Raton, FL.

Learning to collude tacitly on production levels by oligopolistic agents.
*Computational Economics, 33*(1):47–78.
http://dx.doi.org/10.1007/s10614-008-9150-6

*A Course in Microeconomic Theory.*
Princeton University Press, Princeton, NJ.


Neoclassical Economics (the “economic approach”)

- Resembles classical game theory in its program, but adds assumptions (perhaps).

  The combined assumptions of maximizing behavior, market equilibrium, and stable preferences, used relentlessly and unflinchingly, form the heart of the economic approach as I see it. [Becker, 1976, page 5]

NB. “Maximizing” means “actually achieving a maximization,” not “attempting, and possibly failing, to achieve a maximization.”
Neoclassical Economics

I am saying that the economic approach provides a valuable unified framework for understanding all human behavior, although I recognize, of course, that much behavior is not yet understood, and that non-economic variables and the techniques and findings from other fields contribute significantly to the understanding of human behavior.

The heart of my argument is that human behavior is not compartmentalized, sometimes based on maximizing, sometimes not, sometimes motivated by stable preferences, sometimes by volatile ones, sometimes resulting in an optimal allocation of information, sometimes not. Rather, all human behavior can be viewed as involving participants who maximize their utility from a stable set of preferences and accumulate an optimal amount of information and other inputs in a variety of markets.

[Becker, 1976, page 14]
Note that the last paragraph quoted above from Becker is a clear case of *false dilemma*, a well known fallacy.

*A reasoner who unfairly presents too few choices and then implies that a choice must be made among this short menu of choices commits the *false dilemma* fallacy...*

*Internet Encyclopedia of Philosophy,*
http://www.iep.utm.edu/fallacy/
What does this message mean?

Operated on this morning. Diagnosis not yet complete but results seem satisfactory and already exceed expectations.
The preceding paragraph:

Yet the devastation of Berlin was small compared to what had become possible that same afternoon of Monday, July 16. What Truman did not yet know, what none of them knew, was that at a remote part of the Alamogordo Air Base in the desert of New Mexico, at 5:29 in the morning (1:29 in the Afternoon in Berlin) there had been a blinding flash, “a light not of this world,” from the first nuclear explosion in history. Stimson received word at his Babelsberg quarters that evening at 7:30, a top-secret telegram from George Harrison in Washington, which Stimson took directly to Truman.